Liquid Crystal Colloids

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Abstract

Colloids are abundant in nature, science, and technology, with examples ranging from milk to quantum dots and the colloidal atom paradigm. Similarly, liquid crystal ordering is important in contexts ranging from biological membranes to laboratory models of cosmic strings and liquid crystal displays in consumer devices. Some of the most exciting recent developments in both of these soft matter fields emerge at their interface, in the fast-growing research arena of liquid crystal colloids. Mesoscale self-assembly in such systems may lead to artificial materials and to structures with emergent physical behavior arising from patterning of molecular order and nano- or microparticles into precisely controlled configurations. Liquid crystal colloids show exceptional promise for new discovery that may impinge on composite material fabrication, low-dimensional topology, photonics, and so on. Starting from physical underpinnings, I review the state of the art in this fast-growing field, with a focus on its scientific and technological potential.
1. INTRODUCTION

Colloids are soft matter systems in which microscopic particles of one substance are suspended throughout another substance (1). These gas, solid, or liquid particles, each with typical dimensions ranging from nanometers to micrometers, are kept suspended in the host surrounding medium by thermal fluctuations (1). Widely familiar naturally occurring colloids include fog, paint, and milk, whereas colloidal dispersions of synthetic particles, such as quantum dots and plasmonic nanoparticles, promise to revolutionize modern technologies and consumer products ranging from new generations of solar cells to information displays (1–3). Colloids can form liquid, crystalline, and glassy condensed matter states (1), which often serve as model systems for understanding the physics of their molecular and atomic counterparts. Colloids also offer the possibility of harnessing self-assembly to realize unusual composite materials and to understand the complex behavior of biological systems (1–3).

Liquid crystals (LCs), another important class of soft matter systems, are formed by weakly interacting anisotropic building blocks, such as molecules or micelles with anisometric rodlike or disclike shapes (1). These molecules self-organize in states with orientational order that are liquids (they flow) but exhibit elastic-type stiffness to certain orientational deformations such that they also display anisotropic properties and elastic behavior typically associated with solids (1). Weak external stimuli can dramatically alter interactions between the building blocks of LCs, so that, for example, LCs can melt into isotropic fluids as a result of modest temperature changes or can respond to low-voltage electric fields (1). This facile response to weak electric fields and other external stimuli enables many practical applications of LCs, ranging from information displays to biomedical detectors.

LC colloids are formed by dispersions of colloidal particles in the LC host medium rather than in conventional isotropic liquids (2). The discovery of novel colloidal interactions in LCs (2), which are mediated by the elasticity of the LC medium hosting the colloidal nanometer- or micrometer-sized particles, prompted the emergence of a new subfield of soft matter at the interface of LC and colloidal sciences. The fast growth of this new research arena is fueled by the richness of fundamental physics phenomena and a strong potential for technological applications. LC colloids further bridge colloidal and LC sciences within fields as diverse as topology, biology, and photonics (2–7). They may enable technological breakthroughs in the development of flexible information displays; the efficient conversion of solar energy to electricity; and the tunable photonic crystals and metamaterials and other novel optically controlled materials capable, in turn, of controlling light. For example, LC colloids may allow for realizing artificial composite materials with pre-engineered optical and mechanical properties defined through exquisite control of the structural organization of metal and semiconductor nanoparticles at mesoscopic scales, as I discuss in Section 8. The excitement that LC colloids bring from the fundamental science standpoint stems from their hierarchy of important length scales and timescales, which leads to the creation of entirely new concepts and generalizations. Two decades after its origin (2, 8–15), the field of LC colloids has become a well-established, fast-growing area of soft matter research. The main goal of this review is to summarize the recent key developments in this field and to provide an overview of the dazzling variety of current activities and future developments emerging on the horizon, with a focus on colloidal dispersions with LC building blocks much smaller than colloidal particles (Figure 1a).

2. LIQUID CRYSTAL–COLLOIDAL INTERFACES, BOUNDARY CONDITIONS, AND ORIENTATIONAL ELASTICITY

One key difference between conventional isotropic host–based and LC colloids is that the molecular interactions at the interfaces of colloidal particles and the surrounding LC are highly anisotropic.
Liquid crystal (LC) colloids and control of surface boundary conditions (BCs). (a) Schematic showing colloidal particles with dimensions roughly in the range 10 nm–10 μm dispersed in LC composed of rodlike molecules approximately 1 nm in length. The director \( \mathbf{n} \) represents the average direction of ordering of anisotropic building blocks such as molecules in LC phases. (b) Surface functionalization or natural material behavior of particles enables the control of surface BCs for \( \mathbf{n} \) to be perpendicular (left), tangential (middle), or tilted and conically degenerate (right). (c–e) Interplay of elastic and surface anchoring forces defines configurations of the director field \( \mathbf{n}(\mathbf{r}) \), ranging from (c) director structure with weak deviations of \( \mathbf{n}(\mathbf{r}) \) away from \( \mathbf{n}_0 \) (the far-field uniform director) and violation of BCs at the particle surface for \( D \ll \xi_e \) to the structures with (d) ringlike line and (e) pointlike topological defects that are observed at \( D > \xi_c \). \( D \) is the diameter of the colloidal particle; \( \xi_e = K/W \) is the extrapolation length, where \( K = (K_{11} + K_{22} + K_{33})/3 \) is the average elastic constant \( [K_{11}, K_{22}, \text{and } K_{33}] \) are Frank elastic constants that pertain to splay, twist, and bend distortions of \( \mathbf{n}(\mathbf{r}) \), respectively. The colors highlight deviations of \( \mathbf{n}(\mathbf{r}) \) away from \( \mathbf{n}_0 \), depicting regions with \( \mathbf{n}(\mathbf{r}) \) along \( \mathbf{n}_0 \) (blue) and regions with \( \mathbf{n}(\mathbf{r}) \) tilted away from \( \mathbf{n}_0 \) to progressively larger angles (from smallest to largest: cyan, green, yellow, and red).

(Figure 1). These interactions are characterized by the easy-axis orientation of the LC director \( \mathbf{n} \) (the average direction of ordering of anisotropic building blocks such as molecules in LC phases) at the particle’s surface with respect to the local surface normal, which depends on the particle’s chemical composition and surface morphology and is controlled by chemical functionalization and other means. Similar to the case of flat surfaces (1, 15–17), boundary conditions (BCs) on colloidal particles can define the energy-minimizing local orientation of \( \mathbf{n} \) to be tangential, perpendicular, or tilted with respect to the surfaces (Figure 1b). In a uniformly aligned nematic phase LC, the director field \( \mathbf{n}(\mathbf{r}) \) is subjected to both the BCs on particle surfaces and the far-field uniform alignment of the director, \( \mathbf{n}_0 \). BCs are characterized by the surface anchoring energy and a corresponding coefficient \( W \). Typically, \( W < 1 \text{ mJ/m}^2 \), which is much smaller than the isotropic LC director (\( \mathbf{n} \)):
- average direction of ordering of anisotropic building blocks of LC phases with nonpolar
- symmetry
part of the corresponding surface energy. For example, in the case of perpendicular BCs with the easy axis \( \mathbf{v} \) (a unit vector) locally orthogonal to the particle surface, the surface anchoring free energy can be expressed as an integral over the particle surface (18):

\[
F_{\text{anchoring}} = -\frac{W}{2} \int (\mathbf{v} \cdot \mathbf{n})^2 \, dS.
\]

The bulk free energy due to orientational elasticity of a nematic LC reads as (1, 18)

\[
F_{\text{elastic}} = \int \left\{ \frac{K_{11}}{2} (\mathbf{n} \cdot \mathbf{n})^2 + \frac{K_{22}}{2} (\mathbf{n} \cdot (\nabla \times \mathbf{n}))^2 + \frac{K_{33}}{2} (\mathbf{n} \times (\nabla \times \mathbf{n}))^2 \right\} dV,
\]

where \( K_{11}, K_{22}, \) and \( K_{33} \) are Frank elastic constants that pertain to splay, twist, and bend distortions of \( \mathbf{n}(\mathbf{r}) \), respectively, and the integration is done over the sample’s volume. The LC–particle interactions are determined by a competition of bulk elastic and surface anchoring energies, which is characterized by the so-called extrapolation length \( \xi_e = K/W \), where \( K = (K_{11} + K_{22} + K_{33})/3 \) is the average elastic constant (typically within 1–20 pN, say \( K \approx 6.5 \) pN for pentylycyanobiphenyl) and \( W \) is typically within \( 10^{-3}–10^{-4} \) J/m². When the colloidal particle’s diameter \( D \ll \xi_e \) (\( \xi_e \) is within 50–10,000 nm), it induces no defects and distortions of \( \mathbf{n}(\mathbf{r}) \) can be neglected (Figure 1c).

The director in this case stays aligned along \( \mathbf{n}_0 \) while violating BCs on the colloidal surface. In contrast, elastic distortions and bulk or surface topological LC defects (singularities) occur when \( D \gg \xi_e \), in which case the BCs are met and matched to \( \mathbf{n}_0 \) by distorting \( \mathbf{n}(\mathbf{r}) \) around the particle. The emergence of line and point defects in the latter case (1, 12, 18) is illustrated by the example of perpendicular BCs in Figure 1d,e. Overall, the structural configurations of \( \mathbf{n}(\mathbf{r}) \) are determined by minimization of the total free energy, the sum of Equations 1 and 2. Because the bulk elastic free energy roughly scales as \( \propto KD \) and the surface energy as \( \propto WD^2 \), the total free energy minimization leads to violations of BCs for small \( D \) and \( W \) and to elastic distortions and defects in the opposite case. Similar considerations also apply to the cases of tangential and tilted BCs. The nature of particle-induced defects depends on the particle’s topology and geometry, as discussed in Section 7.

### 3. ELASTIC MULTipoles

Multipole expansion, which physicists and engineers learn as part of an undergraduate-level introduction to electromagnetism (19), is widely used in many branches of physics and cosmology. In describing the far-field distortions in electric, magnetic, gravitational, and other fields, say due to localized charge or current distributions, truncation of the multipole expansion to its leading-order nonzero term is useful for theoretical calculations and for explaining physical behavior (19). The leading-order multipoles are typically monopoles, dipoles, and quadrupoles, with higher-order multipoles rarely playing dominant roles, though they can be important for certain types of nematic colloids (see below) (2, 8–13, 20–24). Much like in the case of electrostatic charge distributions, the far-field distortions of \( \mathbf{n}(\mathbf{r}) \) due to a colloidal particle can be represented as elastic multipoles (2, 13, 20–24). Far from a colloidal particle, the deviations of the director \( n_{\mu} \) \((\mu = x, y)\) from the far-field uniform director \( n_0 = (0, 0, 1) \) are small (24, 25). Assuming one-elastic-constant approximation and representing the field as \( \mathbf{n}(\mathbf{r}) \approx (n_x, n_y, 1) \), the LC elastic free energy is expressed as

\[
F_{\text{elastic}} = \frac{K}{2} \sum_{\mu=x,y} \int d\mathbf{r} \nabla n_{\mu} \cdot \nabla n_{\mu}.
\]
The corresponding Euler–Lagrange equations arising from the minimization of $F_{\text{elastic}}$ are of the Laplace type, $\Delta n_\mu = 0$, with solutions expanded into multipoles:

$$n_\mu (r) = \sum_{l=1}^{N} a_l (-1)^l \partial_\mu \frac{\tilde{\alpha}^{l-1}}{r},$$

where $a_l = b_l r_0^{l+1}$ is the elastic multipole moment of the $l$th order ($2^l$-pole), $r_0$ is the particle’s radius, and expansion coefficients $b_l$ can be found from experiments (24, 25). This electrostatics analogy provides a framework for understanding, predicting, and engineering both the structure of the director and self-assembly of nematic colloids.

At no external fields, the lowest-order elastic multipoles are dipoles (2, 13), such as the ones formed by a colloidal microsphere with perpendicular BCs and a bulk hedgehog point defect.
**Disclination:** a discontinuity in the orientational order in the form of a line defect along which \( \mathbf{n} \) cannot be defined.

**Boojum:** a localized pointlike singularity at the surface of LC.

(Figures 1e and 2a–c). This particle-induced dipolar structure of \( \mathbf{n}(\mathbf{r}) \) (Figure 2a–c) is invariant with respect to rotations around \( \mathbf{n}_0 \) but lacks mirror symmetry with respect to a plane passing through the particle’s equator orthogonally to \( \mathbf{n}_0 \). Microspheres with perpendicular BCs can also induce a disclination loop defect around the particle’s equator, often called Saturn ring, which yields an elastic quadrupole (Figure 2d–f). In contrast to the elastic dipoles, this quadrupolar \( \mathbf{n}(\mathbf{r}) \) structure (Figure 2e) is mirror symmetric with respect to the plane passing through the particle’s equator orthogonally to \( \mathbf{n}_0 \). Colloidal spheres with strong tangential anchoring at their surface also form \( \mathbf{n}(\mathbf{r}) \) distortions of quadrupolar type (Figure 2g–i). They induce two boojums at the poles along \( \mathbf{n}_0 \). For weak surface anchoring BCs or particle diameters comparable to or smaller than \( \xi_n \), both the tangential and perpendicular BCs lead to quadrupolar configurations, similar to the elastic quadrupole with the Saturn ring or boojums, but the defects become virtual (25, 26) (within a particle’s volume), as the director is allowed to deviate away from the easy-axis orientations. Particles with tilted, conically degenerate BCs (Figure 2j–l) locally distort \( \mathbf{n}(\mathbf{r}) \), so that the polarizing optical micrographs (Figure 2j) feature eight bright lobes around the particle, separated by eight dark regions within which \( \mathbf{n}(\mathbf{r}) \) at the particle’s perimeter is parallel to \( \mathbf{n}_0 \). These features of polarizing micrographs are different from the cases of dipoles and quadrupoles, which exhibit alternation of two bright and two dark regions (Figure 2a) and four bright and four dark regions (Figure 2d,g), respectively. The tilt of \( \mathbf{n}(\mathbf{r}) \) away from \( \mathbf{n}_0 \) switches between clockwise and counterclockwise directions eight times as one circumnavigates the hexadecapole (Figure 2l), which is different from what occurs in dipoles, where it switches two times, and quadrupoles, where it switches four times. Both the surface boojums at the particle’s poles along \( \mathbf{n}_0 \) and the Saturn ring at the particle’s equator are induced by hexadecapoles (24). To minimize the free energy cost of bulk elastic distortions, interaction of conically degenerate BCs on the microsphere with the uniform \( \mathbf{n}_0 \) lifts the conical degeneracy of surface anchoring and yields an axially symmetric \( \mathbf{n}(\mathbf{r}) \) that can be thought of as a superposition of the quadrupolar structures with boojums and the Saturn ring (Figure 2l) (24).

To illustrate the analogies between elastic and electrostatic multipoles, a projection \( n_x \) of \( \mathbf{n}(\mathbf{r}) \) onto the \( x \)-axis orthogonal to \( \mathbf{n}_0 \) can be visualized using colors that highlight positive, near-zero, and negative \( n_x \) (Figure 2cf,i,k). Away from the particle surface and from singularities such as boojums, hedgehogs, and disclination loops, \( \mathbf{n}(\mathbf{r}) \) is continuous (24). The maps of \( n_x \), plotted on spherical surfaces encompassing particles and defects, clearly illustrate the multipolar nature of elastic distortions, similar to what is observed in the electrostatic analogs (19). The color presentations of elastic distortions induced by colloidal dipoles (Figure 2e), two different quadrupoles (Figure 2fi), and hexadecapoles (Figure 2k) resemble the corresponding dipolar, quadrupolar, and hexadecapolar electrostatic charge distributions described by \( \sigma f^m(\theta, \phi) = A \cos(m \phi) P_l^m(\cos \theta) \) with \( l, m = (1, 1), (2, \pm 1) \) and \( (4, 1) \), respectively, where \(-l \leq m \leq l \), \( A \) is a normalization constant, \( \theta \) is a polar angle and \( \phi \) is an azimuthal angle, and \( P_l^m(\cos \theta) \) is the associated Legendre polynomial. Similar to electrostatic charge distributions, the odd moments vanish when \( \mathbf{n}(\mathbf{r}) \) is symmetric about the particle center, as in the cases of elastic quadrupoles and hexadecapoles presented in Figure 2e,b,l (which have coefficients \( b_1, b_3 \), and \( b_5 \) and so on equal to zero, as required by symmetry). However, both odd and even moments are present for particles with asymmetric \( \mathbf{n}(\mathbf{r}) \), such as elastic dipoles, for which all \( b_l \) can be nonzero (13, 21–24). The design of elastic colloidal multipoles not only takes advantage of symmetry considerations but also builds on the versatile means of controlling \( \mathbf{n}(\mathbf{r}) \) using shape and BCs, so that the coefficients \( b_l \) corresponding to lower-order multipoles can be tuned to be close to zero, yielding higher-order multipoles. For example, the hexadecapole shown in Figure 2j–l was designed by tuning \( b_8 \) to be close to zero while precluding the dipole and octupole moments on the basis of symmetry, as discussed above (24). In a similar way, an elastic octupole could potentially be obtained by tuning \( b_1 \) and \( b_2 \) to zero.
while maximizing $b_3$, though this has not been achieved so far. Although it is widely assumed that elastic monopoles cannot be realized without external torques applied to colloidal particles (18), certain particles can effectively behave as elastic monopoles when they are attached to surfaces and are exerting torques on the LC director through surface anchoring (23). Both elastic monopoles and octupoles require further study.

4. COLLOIDAL INTERACTIONS MEDIATED BY ORIENTATIONAL ELASTICITY OF LIQUID CRYSTALS

The analogy between electrostatic and elastic multipoles may help in devising approaches for self-assembly of mesostructured colloidal composite materials. The potential energy of elastic interaction between nematic colloidal multipoles, which arises from the changes in free energy (which the medium tends to minimize) that result from superposition of elastic distortions induced by interacting particles, is also analogous to the energy of interaction between electrostatic multipoles (2, 13, 21, 24). For example, the potential energy of interaction between two nematic colloidal hexadecapoles scales as $P_6(\cos \theta)/R^7$ (24), that between two quadrupoles as $P_4(\cos \theta)/R^5$ (8–13, 20–22), that between two octupoles as $P_6(\cos \theta)/R^7$ (24, 25), and that between two dipoles as $P_2(\cos \theta)/R^3$ (2, 13), where $P_n(\cos \theta)$ are Legendre polynomials and $R$ is the center-to-center separation distance. Mixed interactions between elastic multipoles of different order can also be predicted by exploiting the electrostatic analogy and scale, for example, as $P_6(\cos \theta)/R^7$ between quadrupoles and hexadecapoles. Experimentally, colloidal interactions can be probed using a combination of laser tweezers and video microscopy, as shown for the case of hexadecapoles in Figure 3. Interaction between two nematic colloidal hexadecapoles (24) is predicted to have eight angular sectors of attraction and eight sectors of repulsion as the center-to-center separation vector $R$ is rotated (Figure 3a,b) by $2\pi$ with respect to $n_0$, which is indeed observed by releasing particles from laser traps and directly visualizing the directionality of interaction forces with video microscopy (Figure 3c). Tracking the motion of particles as they interact reveals that the experimental time dependencies of $R$ and distance dependencies of interaction potential (Figure 3d) at different orientations of $R$ are all consistent with the model (24). Similar characterization of pair interactions was also done for elastic dipoles and quadrupoles, leaving no doubts that multipolar expansion can provide the necessary foundation for understanding the physics of nematic colloids (2, 12, 27–33). However, only some of the elastic multipoles are widely studied (the most common ones are shown in Figure 2); the others often require particles with nonspherical shapes or patterned surface BCs. For example, elastic dipoles with moments orthogonal to $n_0$ could be induced by triangular and pentagonal platelets but so far have not been realized with microspheres (28).

The description of structure and interactions between nematic colloidal particles using Legendre polynomials $P_l^m$ or spherical harmonics with indices $(l, m)$, where $-l \leq m \leq l$, also resembles the description of atomic orbitals of elements in the periodic table (Figure 3e), where the analogous indices $(l, m)$ are quantum numbers. Therefore, the multipolar analysis of nematic colloids not only brings about many interesting analogies and helps us to assess the diversity of structures that can be realized but also is of interest from the standpoint of the colloidal atom paradigm (3–5), as discussed next in Section 5.

5. NEMATIC COLLOIDAL ATOM PARADIGM AND COLLOIDAL CRYSTALS

Since the works of Einstein and Perrin showed how particles in colloidal dispersions obey the same statistical thermodynamics as atoms, the colloid–atom analogy has provided insights into the physics of atomic and molecular systems by enabling investigators to probe the dynamics
of colloidal crystals and glasses (5). This colloidal atom paradigm has inspired the development of colloidal self-assembly to reproduce or exceed the diversity of atomic systems (3, 33–35). LC colloids have several advantages in addressing these grand scientific and engineering challenges. The LC host naturally allows for the realization of anisotropic interparticle interactions (2), including spontaneous alignment of shape-anisotropic particles on the basis of surface anchoring
interactions (28, 33) and exploitation of the rich multipolar behavior of elastic interaction forces (2), which would be difficult or impossible to achieve in conventional isotropic fluids (3). Furthermore, arrays of topological defect lines within the ground-state blue and cholesteric phases, or in other LC phases when stabilized by confinement, can be used to entangle or entrap colloidal particles to form various sparse structures, quasicrystals, and other assemblies (34, 36–41). In conventional colloidal dispersions, high-symmetry crystals with 3D cubic and 2D hexagonal lattices have been studied extensively (3–7, 42), confirming and extending Einstein’s colloidal atom analogy by giving direct experimental confirmations of theoretical models originally developed for atomic and molecular crystals (3, 5). However, experimental realization of colloidal architectures with low crystallographic diversity remained challenging, even though colloids are amenable to designed control of their self-assembly through DNA functionalization (35). A series of self-assembled 2D and tetragonal 3D crystal lattices and other colloidal architectures have been achieved in LC colloidal crystals through the use of guided assembly with laser tweezers (42–48), though these colloidal superstructures could be built only particle by particle and only up to several primitive cells in dimension because of the reliance on laser tweezers rather than self-assembly alone. However, it was recently shown that low-symmetry triclinic colloidal crystals can self-assemble in LC dispersions as a result of competition between the highly anisotropic elastic interactions discussed above and (more conventional) electrostatic repulsive forces (33, 43). Realization of such triclinic colloidal crystals suggests that the high crystallographic diversity previously established for atomic and molecular crystals is now accessible in LC colloids as well.

The triclinic colloidal crystal serves as an illustration of how competition of long-range electrostatic and anisotropic quadrupolar elastic interactions in nematic colloids can lead to self-assembly of colloidal particles with long-range orientational and positional ordering (Figure 4) (33). The long-range electrostatic repulsions arise from weakly screened Coulomb-type forces in a low-ionic-strength nonpolar LC with a large (typically ≥500 nm) Debye screening length. Because the elasticity-mediated interactions are also long range, colloidal crystal assemblies with micrometer-range lattices emerging from the competition of these electrostatic and elastic forces can be an order of magnitude larger than the size of constituent colloidal particles (Figure 4a–d) (33). The crystallographic axes of the triclinic lattices and the colloidal nanorods within them tend to follow the director, which can be controlled on large scales via approaches used to manufacture LC displays (18). Triclinic crystallization of particles at packing factors ≲1% shows potential for fabricating mesostructured composites through self-assembly on device scales, tuned by weak external stimuli, such as low voltages (43). From a fundamental science standpoint, it is interesting that the thermal fluctuations of particles within the lattice and the melting and crystallization transitions (Figure 4e,f) are consistent with what is expected for their atomic counterparts (33). Mean square displacement (MSD) exhibited by colloidal particles within these dispersions saturates with time when in a crystal but continues growing when in a colloidal fluid state (inset of Figure 4f). Characterization of MSD $\langle \Delta r^2(t) \rangle$ relative to the crystal lattice, quantified by the dependence of the Lindemann criterion parameter $\delta L = \left[ \left\langle \Delta r^2(t\to\infty) / (4R^2) \right\rangle \right]^{1/2}$ on the nanorod number density $\rho_N$, provides a means of demonstrating how low-symmetry positional ordering (Figure 4f) emerges from the interplay of weakly screened electrostatic and elasticity-mediated interactions in a nematic host. The triclinic pinacoidal lattices of orientationally ordered nanorods (Figure 4) are the lowest-symmetry 3D colloidal crystals achieved so far (Figure 4a–c) and may prompt the realization of LC colloidal architectures with other symmetries, ranging from cubic to triclinic pedial lattices.

Though so far demonstrated only for elastic quadrupoles and electrostatic monopoles, the use of competition of elastic, electrostatic, and other (e.g., magnetic) interactions to guide symmetry and density of colloidal crystals can be extended to other nematic colloidal multipoles, such as
Colloidal crystals. (a) 3D micrograph showing a primitive unit cell of a triclinic colloidal crystal, which was reconstructed on the basis of confocal imaging. It shows spatial arrangements of colloidal particles as they explore the potential energy landscape near their minimum-energy triclinic lattice sites. (b–d) Schematics (not to scale) of a primitive cell of a triclinic colloidal crystal, (b) showing the director field $n(r)$ distortions (blue lines) induced by nanorods in the background of the uniform far-field director $n_0$, (c) defining parameters of a triclinic lattice, and (d) showing it unfolded. Crystallographic planes, axes, and angles are marked on the schematics. (e) The potential energy $\Delta u$ landscape corresponding to four lattice sites in the (100) crystallographic plane of a triclinic crystal. The inset shows a local distance $\Delta$ dependence of the relative $\Delta u$ experienced by colloidal particles. (f) Lindemann parameter $\delta_L$ versus $\rho_N$ of nanorods, characterizing the crystallization–melting transition. Inset shows the mean square displacement (MSD) of particles versus time at $\rho_N = 1.4$ $\mu$m$^{-3}$ (▲) and $4.5$ $\mu$m$^{-3}$ (▼). For more details, see Reference 33.

Dipoles and hexadecapoles (Figure 2), which for the most part were previously exploited only from the standpoint of interactions guided by laser tweezers (24, 44–48). Considering the intrinsic anisotropy and diversity of elastic multipoles, one can expect a large variety of low-symmetry colloidal architectures emerging from such interactions. Furthermore, because atomic orbitals and the outermost occupied electron shells of chemical elements (Figure 3e) are described by spherical harmonics $Y_{lm}(\theta, \phi)$, they are analogous (in terms of symmetry) to the nematic colloidal atoms. Because the symmetry of the director wiggle wave function of nematic hexadecapolar colloids (Figure 2k,l) would correspond to chemical elements with filled $g$ shells (Figure 3e),
Magnetic hysteresis: a characteristic of ferromagnets associated with the lack of retraceability of the magnetization curve when the magnetic field is relaxed.

which have not been discovered, nematic colloidal atoms already exceed the diversity of their atomic counterparts (24).

6. MOLECULAR-COLLOIDAL ANISOTROPIC FLUIDS

The realization of thermodynamic phases that combine low-symmetry order and fluidity is one of the grand challenges in soft matter research. Both molecular and conventional colloidal LCs are classic examples of how various degrees of orientational and partial positional order can be combined with fluidity in nematic, smectic, and columnar phases (18), though they tend to be nonpolar and they exhibit high symmetry. Starting from the early work of Brochard & de Gennes (49), researchers predicted and demonstrated self-alignment of shape-anisotropic colloidal particles such as rods and platelets in LCs (28, 33, 36, 50–56). This LC-mediated spontaneous orientation of anisotropic particles relative to \( n_0 \) emerges from elastic (49, 55) and/or surface anchoring–based (56) interactions. It allows the system-minimizing surface anchoring energy at the LC–colloidal interface given by Equation 1 or elastic energy due to the bulk distortions near the particle-induced BCs at its surface given by Equation 2, though surface anchoring and elastic energies are often comparable and both contribute to defining particle orientations (57–68). In most cases, dilute dispersions of anisotropic colloidal particles in LCs do not alter the symmetry of the LC dispersion as compared to that of the LC, as the orientations of particles typically simply mimic the ordering of the LC host (69) and can be treated in terms of the behavior of individual inclusions (70–88). LC–colloidal interactions, along with the direct magnetic interparticle interactions, can also lead to the polar alignment of magnetic colloidal inclusions (49–54), although the orientations of the magnetic dipoles of colloidal particles are commonly slaved to \( n_0 \), orienting either parallel or perpendicular to it without breaking uniaxial symmetry. Experimental realization of low-symmetry ordered fluids, such as orthorhombic biaxial LCs (89), is challenging for both molecular and colloidal systems, especially in the case of nematic phases without positional ordering. However, similar to the case of low-symmetry colloidal crystals (33), LC colloidal dispersions of various nanoparticles also provide a series of advantages and promise realization of hybrid LC–colloidal fluids with low-symmetry orientational order (16). For example, a recently introduced molecular-colloidal complex fluid formed by a dispersion of magnetic nanoplates in a thermotropic nematic LC exhibits coexisting polar and biaxial ordering of organic molecular and magnetic colloidal building blocks, with the lowest \( C_s \) symmetry orientational order in a fluid medium, which so far has not been realizable in purely molecular or colloidal systems (16).

In the molecular-colloidal LC with the \( C_s \) symmetry, guided by interactions at different length scales (16), rodlike organic molecules of the host fluid spontaneously orient along the director, whereas magnetic colloidal nanoparticles order with their dipole moments \( m \) (Figure 5a–d) parallel to each other, but pointing at an angle to the director, yielding macroscopic magnetization at no external fields (Figure 5e,f). This unusual physical behavior is enabled by the control of conically degenerate surface anchoring at the magnetic nanoparticle surfaces (16, 17), which assures tilted orientations of magnetic moments \( m \) and magnetization \( M \) with respect to \( n_0 \). Magnetic hysteresis loops (Figure 5e,f) and facile switching of such ordered ferromagnetic fluids emerge from the competing actions of elastic and magnetic torques and a rich magnetic domain structure (16). This and other molecular-colloidal complex LC fluids promise rich behavior arising from the properties of solid nanoparticles and their long-range ordering prompted by interactions with a host medium at the mesoscale. Future research may lead to hybrid molecular-colloidal soft matter phases with different point group symmetries describing the order of molecular and colloidal building blocks, such as an orthorhombic biaxial (89) nematic molecular–colloidal fluid in which anisotropic nanoparticles (such as nanorods) could orientationally order in a direction orthogonal...
Molecular-colloidal liquid crystal (LC) fluids. (a) Schematic of a ferromagnetic nanoplate, coated with silica and then further functionalized with polyethylene glycol (PEG). (b) Scanning transmission electron microscopy (TEM) image of such magnetic nanoplates, with a zoomed-in TEM image of a nanoplatelet’s edge revealing the approximately 5-nm-thick silica shell (inset). (c) Schematic of an individual nanoplate in LC, with the magnetic moment $\mathbf{m}$ tilted away from the far-field director $\mathbf{n}_0$ to an angle $\theta_{\text{me}}$. (d) A schematic of the biaxial ferromagnetic molecular-colloidal hybrid LC with magnetization $\mathbf{M}$ tilted away from $\mathbf{n}_0$. (e) Experimental (open triangle symbols) and theoretical (solid curve, with the relative domain size coded according to the color scheme shown on the right side of the panel, where the average ratio of lateral size of domains to cell thickness varies from 0.5 to 2) magnetic hysteresis loop measured for magnetic field along $\mathbf{n}_0$. Schematics in the top-left inset show relative orientations of $\mathbf{M}$ and the director $\mathbf{n}$ within domains that correspond to the part of the hysteresis loop indicated by a gray arrow. (f) Hysteresis loop probed for the same sample as in panel e, but for magnetic field $\mathbf{H} \perp \mathbf{n}_0$; relative orientations of $\mathbf{M}$ and $\mathbf{n}$ corresponding to different parts of the hysteresis loop are indicated by gray arrows. For more details, see Reference 16.

to the ordering direction of rodlike LC molecules. Such hybrid molecular-colloidal systems, in which colloidal self-organization is enriched by elastic and surface anchoring interactions, are expected to yield soft condensed matter phases that could not be realized previously.

7. TOPOLOGY, GEOMETRY, AND CHIRALITY: SURFACES WITH AND WITHOUT BOUNDARIES

The interaction of colloidal particles with the LC director field can be understood and generalized on the basis of the interplay between the topologies of fields and surfaces widely studied by mathematicians (90). All surfaces are characterized (up to homeomorphism) by their genus, orientability, and number of boundary components, as stated by the classification theorem (90).
Colloidal particles with all dimensions larger than $\xi_e = K/W$ produce well-defined BCs and can be considered as having orientable surfaces without boundaries, so that their interaction with the $n(r)$ depends on the particle surface’s genus $g$ (or, equivalently, its Euler characteristic $\chi = 2 - 2g$). Colloidal particles tend to have surfaces with spherical or other topologically simple shapes with $g = 0$ and $\chi = 2$, such as prisms (continuously deformable to spheres). However, one can fabricate colloidal particles with different numbers of handles and $g \neq 0$ (91–95). Fabrication approaches include conventional photolithography (28), direct laser writing (91, 92), two-photon polymerization (93, 94), and laser-induced reduction of colloidal graphene oxide nanoflakes (95).

In LCs, these particles induce 3D $n(r)$ and topological defects dictated by colloidal topology. 3D nonlinear optical imaging and numerical modeling (91–94) reveal that topological charge induced by the particles is conserved and that the net hedgehog charge $m$ of particle-induced bulk defects obeys predictions of the Gauss–Bonnet and Poincaré–Hopf index theorems. Although particles can induce different $n(r)$ structures, depending on confinement, external fields, and material properties, the sum of hedgehog charges due to induced point defects and disclination loops, $\sum m_i = -m_e = \pm \chi/2$, always compensates for the colloidal particles’ hedgehog charge $m_e$ due to $n(r)$ at their surfaces and is uniquely predetermined by the particles’ topologies (Figure 6). The use of a plus or a minus sign in this relation depends on the choice of the vectorizing $n(r)$-field direction, but the relative signs of the induced defects are fixed. This relation allowed the establishment of a procedure for the assignment and summation of topological charges in 3D director fields (91).

Particles with tangential BCs additionally induce boojums at the LC–colloidal interface. The total boojum winding number in the projection of $n(r)$ to the particle surface, $n_e(r)$, is always $\Sigma_s \iota_s = \chi$, also satisfying topological theorems (92). These simple relations describe defects induced both by the $g = 0$ spherical surfaces (Figures 1 and 2) and by surfaces such as handlebodies and torus knots with $g \neq 0$ (Figure 6), though additional self-compensating defects can appear to reduce the energetic cost of elastic distortions associated with matching BCs on colloidal surfaces to $n_e$. For example, $\Sigma_s \iota_s = 0$ for self-compensating boojums induced by a trefoil knot colloidal particle (Figure 6g–i), as expected for the torus knot’s surface with $\chi = 0$. However, beyond the predictions of topological theorems, nematic colloids can also have mutually tangled physical knots of particles and line defects in $n(r)$, as illustrated in Figure 6j–l (93). For all nematic colloidal particles, the interplay of topologies of surfaces, fields, and defects guides $n(r)$ to comply with the particle shape, generating knotted, linked, and other 3D configurations (91–94) and thereby providing experimental insights into aspects of mathematical knot theory (90, 93, 94).

Nematic colloidal particles in the form of thin foil with two lateral dimensions $\gg \xi_e$ but one dimension (thickness) $\ll \xi_e$ behave as analogs of orientable surfaces with boundaries, to which the constraints on induced defects discussed above cannot apply and, generally, no defects need to be induced (91, 96). Yet, by morphing the shapes of such foil-based particles, one can pattern the defects to achieve various elastic multipoles and drive LC-mediated self-assembly similar to that exhibited by nematic colloidal particles with surfaces without boundaries (96). Their behavior is different from that of both microparticles (larger than $\xi_e$) with strong BCs (Figures 1d,e, 2, and 3) and nanoparticles (smaller than $\xi_e$) with effectively weak surface anchoring (Figures 4 and 5), being capable of inducing no defects or various self-compensating defect structures depending on their geometric shape and orientation relative to $n_0$ (96). Nonorientable colloidal particles with boundaries have also been recently theoretically considered (97), though their experimental realization still presents a challenge.

The shape of colloidal particles plays a key role in defining the locations and types of induced defects even when the topology of particle surfaces stays simple (Figure 7). For example, by shaping $g = 0$ colloidal particles with perpendicular BCs as truncated pyramids (Figure 7a–d), one preselects the topology-complying defect with $m = \pm 1$ to take the form of a half-integer

Genus ($g$):
a maximum integer number of cuttings along nonintersecting closed simple curves that do not render the resultant manifold disconnected, equal to the number of handles on it

Euler characteristic ($\chi$): a topological invariant $\chi = 2 - 2g$ that describes a topological space’s shape or structure regardless of the way it is bent

Hedgehog charge: degree of $n$ along $S$, of space $V$ bounded by $S = \partial V$, calculated by integrating the Jacobian of $n(r)$, $m = (1/4\pi) \int_S dS \nabla \cdot n$

Boojum winding number: the number of times that the director $n_e$ at the LC–particle interface rotates by $2\pi$ as one circumnavigates the surface boojum’s defect core once

Handlebody: a submanifold of Euclidean space comprising a ball with handles attached to it along its boundary

Trefoil knot: the simplest nontrivial knot obtained by joining together the two loose ends of a common overhand knot, yielding a knotted loop
disclination loop localized near the pyramid’s base, and one uniquely defines the orientation of the ensuing elastic dipole (Figure 7d). Polygonal platelets (Figure 7e–f) with odd and even numbers of edge faces behave as dipoles and quadrupoles, respectively, illustrating that the elastic multipole nature can be controlled by tuning the particle’s geometric shape (28). Particles shaped as letters of the Latin alphabet (Figure 7k,l), on the other hand, exhibit well-defined orientations with respect to \( n_0 \) and induce surface boojums with \( \Sigma_j = \chi \) correlated with their shapes. Other colloidal shapes studied recently include pyramids, spirals, shape-morphing elastomeric rods, gourd-shaped dimers, particles with nanoscale surface roughness, and so on (98–113), in all cases demonstrating that the geometric shape predefines the locations of topological defects and colloidal interactions.

Figure 6

Topology-controlled nematic colloidal behavior. (a,b) Transmission-mode optical micrographs of a genus \( g = 5 \) colloidal handlebody with tangential boundary conditions (BCs) in a nematic liquid crystal (LC) obtained (a) between crossed polarizer P and analyzer A and (b) without polarizers. Boojums are visible as discontinuities in bright brushes in panel a and as dark light-scattering localized regions in panel b. (c,d) Polarizing optical micrographs of a \( g = 5 \) colloidal handlebody with perpendicular BCs in a nematic LC obtained between crossed P and A (e) without and (f) with an additional 530-nm phase-retardation plate with the slow axis \( Z \) marked by a blue arrow. (e,f) A \( g = 3 \) colloidal handlebody with perpendicular BCs in a nematic LC (g) imaged using three-photon excitation fluorescence polarizing microscopy and (h) shown schematically. The overlaid fluorescence patterns obtained for the two mutually orthogonal polarizations (green and magenta double arrowheads) of excitation light are shown using green and magenta coloration. Black lines represent \( n(r) \), and the red or magenta points and rings depict topological defects with opposite hedgehog charges. (g–i) A trefoil knot particle with tangential BCs in an aligned LC imaged (g,h) between crossed P and A (g) without and (b) with an additional 530-nm retardation plate whose slow axis \( Z \) is marked by a blue arrow. Boojums are indicated by yellow arrows in panel g. (i) 3D representation of the director field \( n(r) \) deviating away from the far-field director \( n_0 \) due to the incorporated trefoil knot particle. Colors depict the azimuthal orientation of \( n(r) \) when projected onto a plane orthogonal to \( n_0 \) according to the scheme shown in the inset. The structure is visualized on a tube following the knot particle’s surface. Points where different colors meet are boojums. (j–l) A colloidal trefoil knot with perpendicular BCs. (j,k) Three-photon excitation fluorescence polarizing microscopy images of \( n(r) \) for excitation light polarizations (magenta double-headed arrows) at different orientations with respect to \( n_0 \). Yellow arrows mark the defect lines. (l) Computer-simulated \( n(r) \) around a trefoil knot with perpendicular BCs and the torus plane oriented orthogonally to \( n_0 \). Green and magenta lines show regions with reduced scalar order parameter corresponding to the cores of two knotted defect lines (panels j and k). Inset shows a schematic of mutual linking between the particle knot (blue) and defect knots (green and magenta). For more details, see References 92–94.
Often closely related to particle shape and topology, chirality of nematic colloids can also play an important role in defining their physical behavior. For example, the knot-shaped particles shown in Figure 6g–l are chiral in nature and induce 3D director distortions that mimic this chirality (93, 94), similar to the surface-attached chiral particles studied in Reference 23. However, even mirror-symmetric particles can induce 3D chiral distortions of \( \mathbf{n}(r) \) due to nontrivial conformation states and transformations of the topological defects (102). The chiral nature of LC hosts also drives formation of chiral distortions in \( \mathbf{n}(r) \) and chiral colloidal superstructures. For example, it can lead to formation of twist-escaped disclination loops around particles (114), to formation of ...
sparse assemblies enabled by cholesteric quasi-layers (40), and to LC topological solitons bound to particles (115, 116) and also to other types of chirality-enabled colloidal behavior (117–126).

8. PROPERTY ENGINEERING IN NEMATIC COLLOIDAL COMPOSITES

Self-organization of nanosized functional units is an exceptionally promising way of designing artificial composite materials with new macroscopic physical behavior (127). Many approaches focus on the material behavior arising from ordering, colocalization, and alignment of nanoparticles, which lead to controlled plasmon–exciton and other interactions enabled by the nanoscale confinement (127). In the field of LC colloids, recent studies explore such interactions at the levels of individual quantum dots, photon upconversion and plasmonic nanoparticles (128–138), and novel composite materials (43, 55, 56, 61, 63, 64, 66, 67, 69, 73, 80, 139–144), demonstrating a means of controlling nanoparticle and composite optical characteristics such as luminescence lifetimes. For example, plasmon–exciton interactions can alter blinking statistics of quantum dot nanoparticles and can shorten luminescence lifetimes by an order of magnitude (128). The host medium’s intrinsic order translates into the self-organization of immersed inclusions depending on the shapes and/or sizes of particles and on how anisotropic colloidal structures interact with the molecular alignment fields, as discussed above. Shape-dependent colloidal interactions and ordering of quantum dots, dye-doped nanostructures, and plasmonic metal nanoparticles reveal underpinning physical mechanisms that can ultimately determine composite properties (55, 61). Facile control of nanoparticle and molecular organization and the ensuing composite properties can be achieved by applying electric, magnetic, and optical fields (80). In addition to the equilibrium structures and various crystal or LC colloidal phases, future designs of composite material behavior may also involve various types of LC colloidal dynamics (145–150), though additional studies are still needed to make this possible.

9. CONCLUSIONS AND OUTLOOK

Mimicking and ultimately exceeding the diversity of organization of atomic elements are among the main goals of colloidal science, which could allow for self-assembly of a broad range of mesostructured composite materials. Self-organization of colloidal nanometer- and micrometer-sized particles dispersed in LCs is a promising approach to achieve this goal. Due to unusual but controlled molecular interactions at colloidal surfaces, the solid particles can locally perturb the uniform molecular alignment of the nematic host, forming elastic multipolar distortions of ordering that drive highly anisotropic interactions and lead to low-symmetry organization in the form of colloidal crystals with tunable lattice parameters and colloidal fluids with orientational ordering. The interdisciplinary research on LC colloids addresses some of the most exciting problems at the interfaces of materials engineering, soft matter physics, nanoscience, and photonics. The emerging scientific frontiers at the nexus of these fields show exceptional promise for significant new discoveries and applications. They will advance our fundamental knowledge of self-assembly in complex fluids and will impinge on fields as diverse as information displays, metamaterial and photonic crystal fabrication, and energy conversion.

DISCLOSURE STATEMENT

The author is not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.
ACKNOWLEDGMENTS

I gratefully acknowledge research contributions and discussions over many years with my former and current students and postdocs working on LC colloids, including P. Ackerman, M. Campbell, J. Evans, D. Gardner, J. Giller, A. Hess, L. Jiang, C. Lapointe, T. Lee, Q. Liu, A. Martinez, H. Mundoor, M. Pandey, S. Park, O. Puls, B. Senyuk, R. Trivedi, O. Trushkevych, C. Twombly, M. Varney, Y. Yuan, and Q. Zhang. I also acknowledge support from National Science Foundation grant DMR-1410735.

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Errata

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