

A liquid-crystal analogue of the skyrmion Hall effect

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Topology provides a unifying framework across disciplines, ranging from particle physics and cosmology to condensed-matter systems. In condensed matter, one of its most striking manifestations is the skyrmion Hall effect, where skyrmions—emergent spin configurations characterized by a conserved topological charge—acquire a transverse velocity under driving. Recent advances have revealed that chiral liquid crystals host a wide variety of topological solitons, including torons and skyrmion tubes. Here we show that such solitons exhibit a Hall-like response under shear flow, undergoing a transverse drift away from the shear direction. Our theoretical analysis predicts a weak net transverse mass flow whose magnitude scales as the cube of the shear rate. However, tracer-particle experiments and numerical simulations show that the solitons do not behave as passive tracers of this flow: their transverse motion is much stronger, and in simulations, it occurs opposite to the net transverse mass current. A liquid-crystal analogue of the Thiele equation provides a simple physical framework that highlights the central role of flow vorticity in the effect. These results establish a liquid-crystal analogue of the skyrmion Hall effect and open pathways for microfluidic technologies that exploit soft topological solitons as functional building blocks.

Liquid crystals (LCs) combine fluidity and anisotropy within a single phase and, therefore, exhibit an exceptionally diverse range of properties. Examples include the facile response of the LC order parameter to external stimuli, which is exploited in advanced photonic applications and in reconfigurable, on-demand composite nanomaterials^{1–6}. From a fundamental perspective, LCs also provide unique testbeds for the study of topology, offering unifying concepts that link condensed-matter physics with high-energy physics and cosmology.

For example, cholesteric LCs support a wide variety of thermodynamically stable topological solitons, including skyrmions^{7–9},

torons^{10,11}, hopfions^{12,13}, heliknotons¹⁴, möbiusons¹⁵ and even topological solitonic macromolecules¹⁵. The first four structures have also been observed in chiral magnets^{16–21}, where the characteristic size of magnetic solitons is ~10–100 nm, to be compared with ~0.5–200 μm (ref. 10) for their soft-matter counterparts.

One of the most striking manifestations of topology in chiral magnets is the emergence of effective electromagnetic fields when non-coplanar skyrmion spin textures couple to conduction electrons^{17,22,23}. This underlies the skyrmion Hall effect^{24,25}, in which driven skyrmions acquire a transverse velocity component. In this

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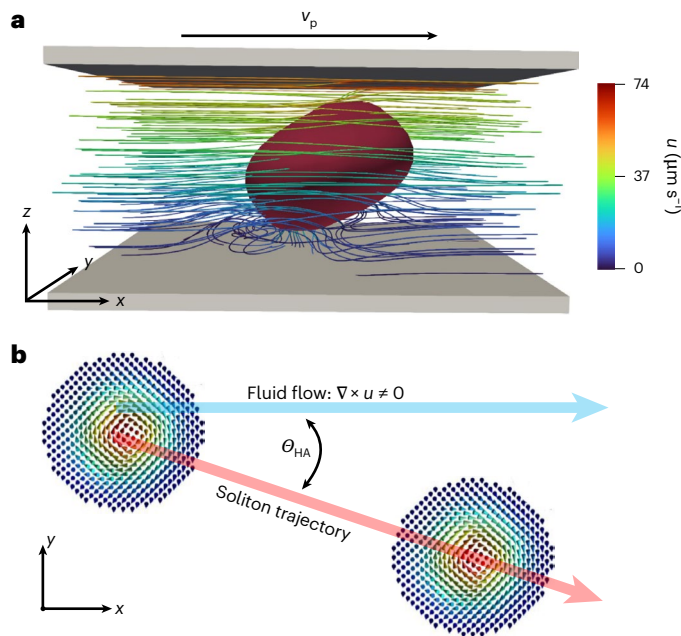


Fig. 1 | System geometry and an LC toron in shear flow. **a**, Schematic of the system geometry and representative configuration of an LC toron in a Couette flow, characterized by $Er \approx 4.6$ (see the main text for details). The LC is confined between two parallel plates. The bottom plate is fixed, whereas that on the top moves at a constant speed in the direction indicated by the black arrow. The confining parallel plates impose rigid homeotropic boundary conditions on the LC director. The curves represent fluid streamlines that are coloured to depict the magnitude of flow velocity. The red surface in the middle represents the toron shape, corresponding to the isosurface on which the z component of the director field $n_z = 1$ ($n_z = -1$ far from the toron). **b**, Schematics of the x - y cross-section of the system in **a** showing the essence of the LC Hall effect. The soliton in the shear flow acquires a velocity component perpendicular to the direction of the primary driving flow. The effect is characterized by the soliton Hall angle $\theta_{HA} \equiv \arctan(v_2/v_1)$, where v_1 and v_2 are the soliton velocity components parallel and perpendicular to the driving flow, respectively.

Article, we ask whether LC topological solitons can exhibit an analogous Hall-like transport under externally imposed material flows. It is known that solitons in LCs can be driven by oscillating electric fields^{15,26–28}, but much less is known about their response to shear flows. Here we address this question using experiments, numerical simulations, perturbation analysis and a coarse-grained description. The system geometry, a representative shear flow configuration and the basic Hall-like deflection mechanism are illustrated in Fig. 1.

The experiments demonstrate a Hall-like transverse drift of LC solitons in shear flows—a behaviour akin to the skyrmion Hall effect in magnets—and rule out passive advection by the weak transverse mass flow as the origin of the effect. Moreover, the numerical simulations show that the solitons move in the opposite direction to the emergent transverse mass currents. A Thiele-like collective-coordinate model, constructed for skyrmion tubes, reveals the key role of flow vorticity in the reported phenomenon and establishes a link with the skyrmion Hall effect in ferromagnets.

LC toron Hall effect: experiments

The experimental LC cell consists of two parallel glass plates separated by freely rolling silica spheres, which define the cell thickness L_z and allow one plate to slide relative to the other (Supplementary Fig. 1). We use local laser-induced melting of the LC order to generate torons that emerge spontaneously on switching off the laser tweezers (Supplementary Video 1). The deflection of torons in the y direction, that is, perpendicular to the shear plane (x - z) is illustrated by polarizing optical microscopy (POM) images in Fig. 2a–i, and Supplementary Video

2 shows the toron's displacement in the early time regime, just after shearing has been imposed. Unexpectedly, we observe the deflection of torons in both $+y$ and $-y$ directions (Fig. 2a–i (yellow and blue arrows); Extended Data Fig. 1 shows the parametric trajectories ($x(t)$, $y(t)$) of the tracked torons at different values of Ericksen number Er (Methods). Under the present imaging conditions, the generated objects display a similar gross optical appearance under crossed polarizers. However, the optical imaging used here was optimized for trajectory tracking over a relatively large field of view under shear and, therefore, does not resolve the internal director structure of each toron with sufficient detail to unambiguously determine whether oppositely drifting torons are structurally identical or distinct. The numbers of torons deflecting in opposite directions are approximately equal, as evidenced by the behaviour of the toron Hall angle $\theta_{HA} \equiv \arctan(v_2/v_1)$, where v_2 and v_1 are the two in-plane velocity components of the toron (Fig. 2j). By reversing the direction of shear, the toron velocities also reverse (Fig. 2d–f). The full dynamics shown in Fig. 2a–f is presented in Supplementary Videos 3 and 4. As shown below, numerical simulations identify the orientation of a preimage vector connecting the preimages of the north and south poles in the middle plane $z = L_z/2$ as a robust geometrical descriptor of the sign of θ_{HA} . However, the present experiments do not probe the internal director structure of each toron in sufficient detail, for example, through three-dimensional imaging as in ref. 26, to determine what sets this vector for each individual trajectory.

Increasing the shear rate has pronounced effects on both director configuration away from the torons (as evidenced by the change in light transmission intensity (colour at the rear sides); Fig. 2g–i) and toron structure. The toron roughly halves its lateral extension after 1 s (Fig. 2g, h) and subsequently retains this shape (Supplementary Video 5). Additionally, the director distribution loses the initial axial symmetry, and resembles that induced by a sufficiently strong voltage in LCs with negative dielectric anisotropy²⁶. Relative to the shear flow, the rear sides of the toron appear dark, as the director is oriented perpendicular to the image plane, whereas the brighter frontal sides correspond to regions in which the director has an in-plane component.

We observe some torons to elongate predominantly in the y direction, that is, perpendicular to the shearing direction (Fig. 2g–i, red arrows). A more careful analysis reveals that this effect is due to the pinning of one of the toron's hedgehogs to the closest cell plate. Indeed, as shown in Supplementary Video 5, the bottom parts of these elongating torons are immobilized because of the aforementioned pinning. Despite these structural changes, most of the torons retain their original quasi-circular shape, which is in sharp contrast to our earlier study²⁹ in which we reported the strong elongation of torons in pressure-driven Poiseuille flow for $v_z = 0$.

Is the observed transverse motion of torons a genuine Hall-like effect arising from their intrinsic topological structure, or merely a trivial consequence of advection by a transverse mass flow? To address this question, we dispersed small tracer particles in the unwound cholesteric phase, where the director field is uniformly aligned perpendicular to the confining plates, and tracked their dynamics under shear at various shear rates. As shown in Supplementary Video 6, the colloidal particles move along the shear direction x within the experimental resolution, without any detectable transverse drift, in clear contrast to the behaviour of torons. We repeated these experiments for multiple values of pitch over cell thickness ratio $P/L_z > 1.4$ (to suppress toron formation) and Er , consistently obtaining the same result. These observations indicate that any transverse mass flow under the present experimental conditions is negligible on the scale of the tracer-particle dynamics and cannot account for the observed transverse motion of torons. This supports the interpretation of the toron dynamics as a genuine Hall-like response associated with the soliton structure, rather than passive advection by the flow.

Next, we theoretically address the (non-)existence of transverse flows in uniform unwound cholesteric LCs subject to shearing

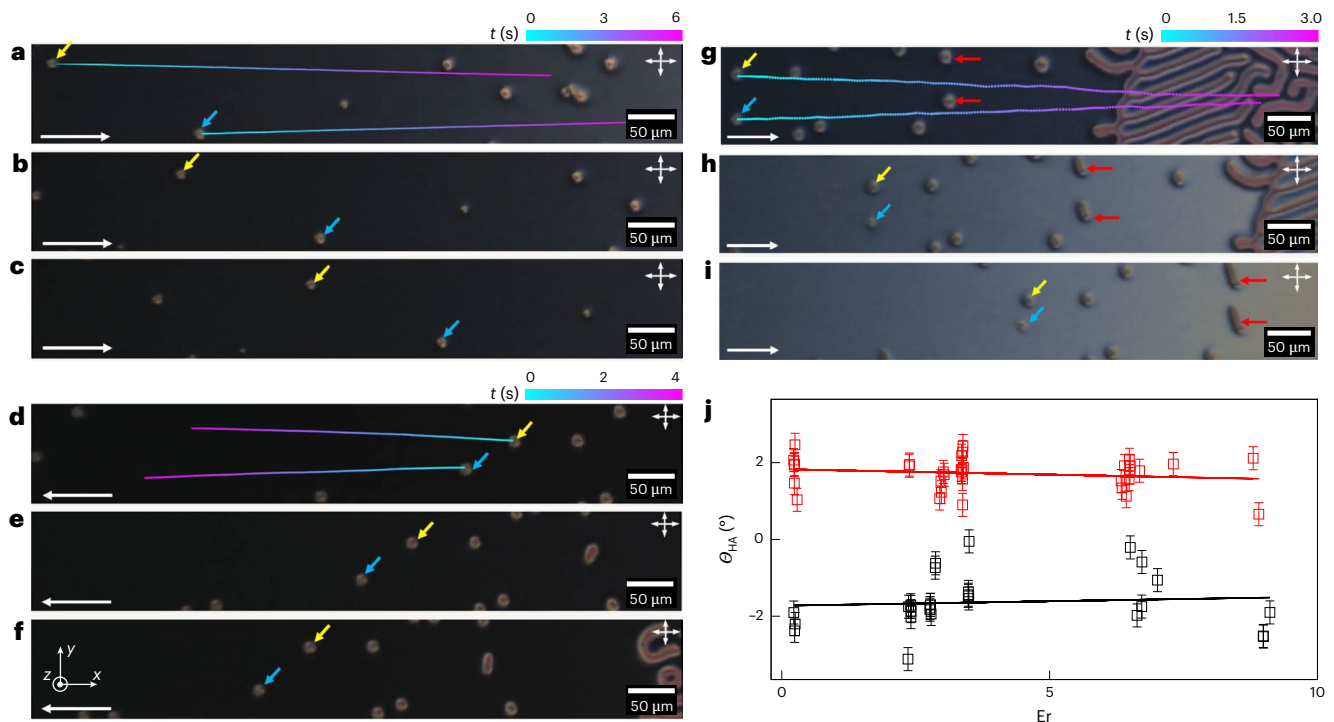


Fig. 2 | Toron trajectories in shear flows. a–i, Micrographs taken at crossed polarizers (schematically shown by two crossed white arrows at the top right corners of the panels) demonstrating Hall-like cross-stream motion of the torons in a Couette flow. The top glass plate of the LC cell is translated horizontally relative to the bottom plate. The top plate moves as follows: from left to right at a speed of $v_p \approx 68 \mu\text{m s}^{-1}$, which corresponds to $Er \approx 3.0$ (a–c); from right to left at $v_p \approx 55 \mu\text{m s}^{-1}$, $Er \approx 2.4$ (d–f); from left to right at $v_p \approx 160 \mu\text{m s}^{-1}$, $Er \approx 7.0$ (g–i). The displacement direction of the top plate is indicated by the horizontal white arrows at the bottom left corners of the panels. The micrographs are taken at times $t = 0$ (a and d); $t = 2$ s (b and e); $t = 4$ s (c and f); $t = 0$ (g); $t = 1$ s (h); and $t = 2$ s

(i). The toron trajectories in a, d and g, reconstructed from videos, are colour coded by time according to the scale. The torons exhibit a non-zero transverse velocity perpendicular to the shear plane (x – z). The inclined yellow and blue arrows point at the torons, which follow the trajectories illustrated in a, d and g. The two selected torons drift in opposite directions along the transverse y axis. The red arrows in g–i point to torons that elongate in the direction perpendicular to the shear plane. Scale bar, $50 \mu\text{m}$. j, Toron Hall angle $\theta_{HA} \equiv \arctan(v_2/v_1)$ as a function of Er . Each data point corresponds to a single toron trajectory. Error bars indicate the measurement resolution ($\pm 0.3^\circ$) rather than statistical uncertainties. The solid line is a linear fit to the experimental data.

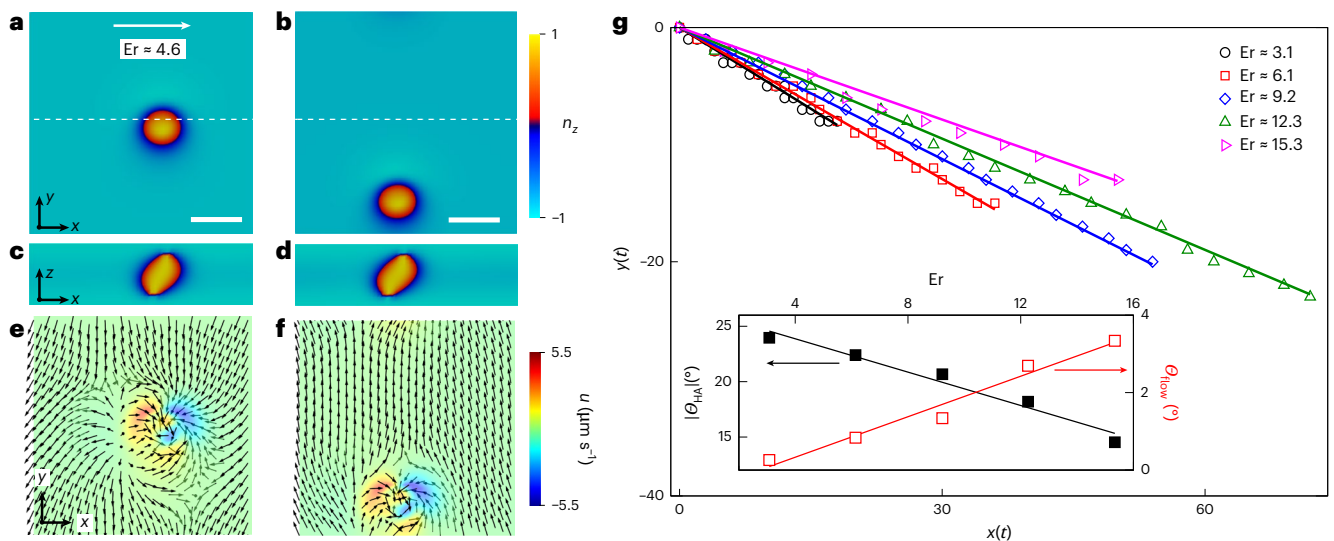


Fig. 3 | Torons in shear flows: simulations. a, b, Snapshots of toron configurations shown by the colour-coded z component n_z of the director field in the middle plane $z = L_z/2$. The results of the LBM simulations obtained at $Er = 4.6$ and at times of 0.2 s (a) and 2.3 s (b). The toron trajectory exhibits $\theta_{HA} \approx -20.7^\circ$. The white arrow indicates the direction of the moving plate. Scale bars (white), $10 \mu\text{m}$. c, d, n_z in the plane $y = y_{TCM}(t)$, where $y_{TCM}(t)$ is the instantaneous y coordinate of the toron's centre of mass. e, f, Relative velocity field (local velocity minus the velocity far from the toron) in the plane $z = L_z/2$. The arrows indicate the direction of velocity field. g, Parametric plots $x(t), y(t)$ of the toron trajectories,

where t is time, for several Er values, obtained from the LBM simulations. Solid lines are linear fits to the data points (symbols). The inset shows the Hall angle θ_{HA} as a function of the Er (solid squares), and the angle $\theta_{flow} = \arctan(\langle u_y \rangle / \langle u_x \rangle)$ (empty squares) quantifying the magnitude of the net mass flow in the y direction, and $\langle u_y \rangle$ and $\langle u_x \rangle$ are space-averaged components of the steady-state flow field. The red and black lines in the inset represent linear fits to the data points. x and y in g are in the units of the lattice grid used in the simulations (the conversion to real units is specified in Table 1). The numerical calculations are carried out for $L_x = L_y = 35 \mu\text{m}$ and $L_z = P = 8.75 \mu\text{m}$.

Table 1 | Parameters used in the simulation and physical units

Symbol	Simulation units	Physical units
ρ	1	$1,088 \text{ kg m}^{-3}$
Δx	1	$0.625 \mu\text{m}$
Δt	1	$2 \times 10^{-9} \text{ s}$
K_{11}	1.67×10^{-7}	$6.4 \times 10^{-12} \text{ N}$
K_{22}	7.88×10^{-8}	$3.0 \times 10^{-12} \text{ N}$
K_{33}	2.62×10^{-7}	$9.98 \times 10^{-12} \text{ N}$
α_1	0.0373	0.0036 Pa s
α_2	-0.4496	-0.044 Pa s
α_3	-0.0203	-0.0020 Pa s
α_4	0.9318	0.091 Pa s
α_5	0.3084	0.030 Pa s
α_6	-0.1617	-0.016 Pa s

deformations. The results of the perturbation analysis of the Ericksen–Leslie model for $Er \ll 1$ (Methods and Extended Data Figs. 2–5) demonstrate that the leading-order contribution to the transverse component of the flow field is $u_y \approx Er^3$, and it is driven by a source term $\propto \partial_z (n_x^{(1)} n_y^{(1)})$ in the governing Laplace in equation (45), where ∂_z denotes the partial derivative with respect to z , and $n_x^{(1)}$ and $n_y^{(1)}$ are the first-order contributions to the x and y components of the director, respectively. Although $n_x^{(1)}$ is also expected for achiral nematics, $n_y^{(1)}$ emerges only due to the chiral term $\propto q_0$ in the Frank–Oseen free energy density (equation (10)). This term enables the linear momentum transfer from the x to y direction.

LC toron Hall effect: simulations

In this section, we explore the driven dynamics of torons by adopting the lattice Boltzmann method (LBM) to numerically solve the Ericksen–Leslie model (Methods). First, we consider a toron-free case and obtain the steady-state transverse flow field profiles $u_y(z)$ (Extended Data Fig. 6a), which qualitatively agree with the results of the perturbation analysis (Extended Data Fig. 5a). Surprisingly, the net transverse current $\langle u_y(z, t) \rangle = L_z^{-1} \int_0^{L_z} u_y(z, t) dz$ exhibits a non-trivial dynamics at early times with a local minimum/maximum for left/right handedness, and a reversal of the current direction at intermediate times (~ 0.2 s; Extended Data Fig. 6b). In the presence of a toron, $u_y(z)$ is similar but slightly enhanced.

In contrast to pressure-driven flows²⁹, which elongate torons and skyrmions in the flow direction, the torons in a Couette flow maintain their quasi-circular shape, as revealed by the steady-state cross-sectional director profiles (Fig. 3a–d) and flow field configurations (Fig. 3e,f). The relative flow velocity, in the plane $z = L_z/2$, exhibits an interesting pumping structure: the toron pulls in fluid from its front and rear and expels it along the vertical direction (normal to the viewing plane). The near-toron flows are also characterized by a complex pattern of vorticity (Extended Data Fig. 7).

By increasing Er , but keeping it below the toron disintegration threshold $Er^* \approx 16$ (Extended Data Fig. 8i,j), the toron maintains its circular shape in the plane $z = L_z/2$ (Extended Data Fig. 8a–c), but tilts in the direction of the shear (Extended Data Fig. 8d–f). These findings are supported by experiments (Extended Data Fig. 8g,h). The POM image shown in Extended Data Fig. 8h is taken at $Er \approx 7.0$, and the corresponding midplane director tilts by 45° away from the z axis, as shown by the brightening of the image on increasing Er (Extended Data Fig. 8g,h). We observe a similar director tilt in the simulations.

The parametric trajectories $(x(t), y(t))$ of the torons (Fig. 3g) yield Hall angles in the range of $-25^\circ \lesssim \theta_{\text{HA}} \lesssim -15^\circ$, substantially larger in magnitude than in the experiments. In agreement with the experiments, the toron steady-state transverse speed is much larger than the

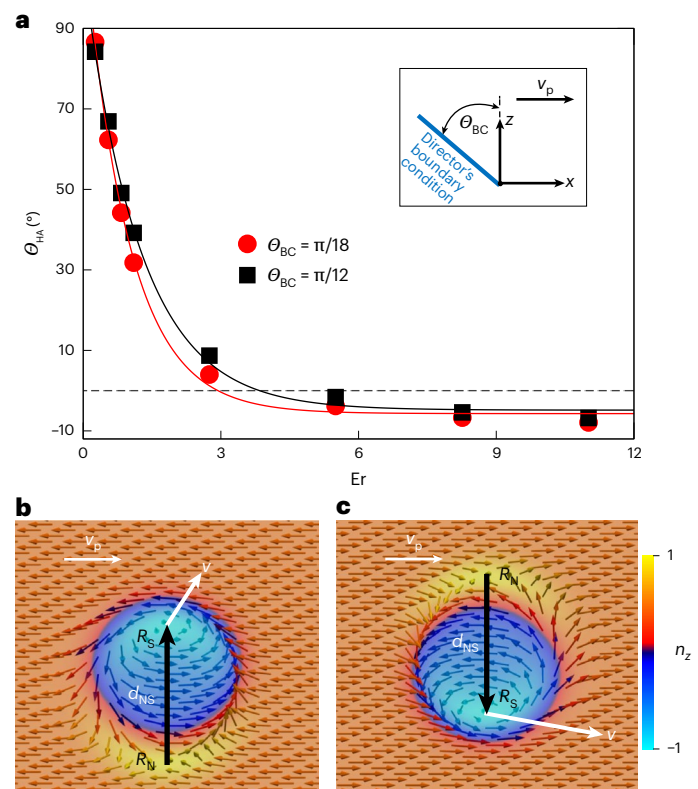


Fig. 4 | Effect of boundary conditions on the toron's Hall angle. **a**, Toron Hall angle θ_{HA} as a function of Er for two values of the director tilt angle θ_{BC} at the cell-confining surfaces. The director field is being optimized in simulations, keeping the shear flow field $\mathbf{u}(\mathbf{r}) = \dot{\gamma} z \hat{\mathbf{e}}_x$ unchanged. **b, c**, The toron middle-plane ($z = L_z/2$) configurations shown by the vectorized director field and the colour-coded z component n_z of the director field. v_p denotes the direction of shear flow, and $\mathbf{v}$ is the toron velocity. **b**, When the cell surfaces impose boundary conditions that oppose flow-induced alignment, the director field away from the toron tilts in the direction opposite to v_p , resulting in $\theta_{\text{HA}} > 0$. **c**, For homeotropic boundary conditions, the director aligns with the flow, which leads to $\theta_{\text{HA}} < 0$. The black arrows represent the preimage vector $\mathbf{d}_{\text{NS}} = \mathbf{R}_S - \mathbf{R}_N$, where $\mathbf{R}_S$ and $\mathbf{R}_N$ are the south-pole (blue region, with $n_z = -1$) and north-pole (yellow regions, with $n_z = 1$) preimages, respectively. The direction of $\mathbf{d}_{\text{NS}}$ determines the sign of θ_{HA} . The numerical calculations are carried out for $L_x = L_y = 40 \mu\text{m}$, $L_z = 11.25 \mu\text{m}$ and $P = 12.5 \mu\text{m}$.

net transverse flow $\langle u_y \rangle$, where $\langle \dots \rangle$ denotes space averaging. Figure 3g (inset) shows the angle $\theta_{\text{flow}} = \arctan(\langle u_y \rangle / \langle u_x \rangle)$ as a function of Er for $0.1^\circ \lesssim \theta_{\text{flow}} \lesssim 3.4^\circ$. Remarkably, the toron transverse velocity v_z and net transverse flow $\langle u_y \rangle$ have opposite signs. This shows that the toron dynamics cannot be explained by passive advection by the transverse mass flow, but instead are controlled by the soliton structure and its coupling to the flow field.

The quantitative disagreement between the values $|\theta_{\text{HA}}| \approx 2^\circ$ (baseline experiments) and $|\theta_{\text{HA}}| \approx 20^\circ$ (simulations) reflects the system-specific magnitude of the effect. Indeed, Supplementary Video 7 shows a different experimental realization in which substantially larger toron Hall angles, up to -10° , can be achieved. In this case, we consider $L_z/P = 2$, where torons are stabilized by a high-frequency sinusoidal voltage ($f = 2,000 \text{ Hz}$)³⁰. Because the voltage is applied at high frequency, it does not produce additional large-scale director distortions, whereas the larger cell-gap-to-toron separation reduces the effective toron–surface friction.

In the baseline tracer-particle experiments (Supplementary Video 6), the colloidal particles move along the shear direction within the experimental resolution and no detectable transverse drift is observed. In the a.c.-stabilized regime (Supplementary Video 7), torons exhibit a broader spread of Hall-like responses than in the baseline case: the middle and

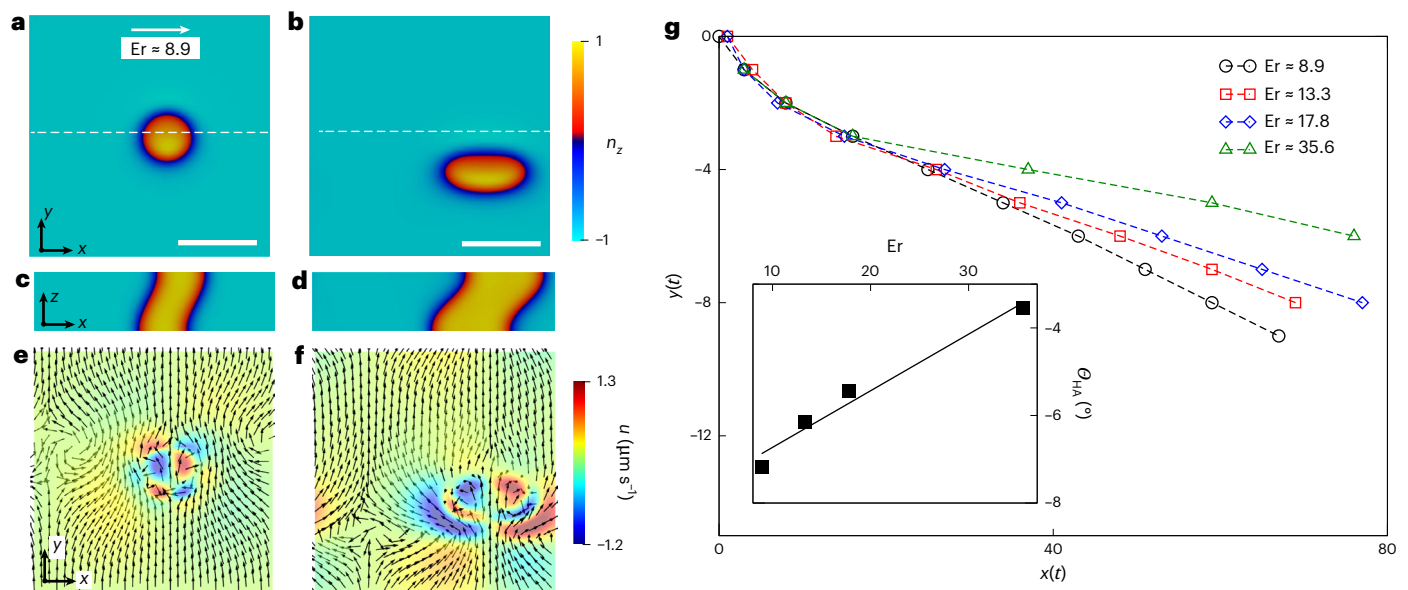


Fig. 5 | Skymion tubes in shear flows: numerics. **a–f**, Snapshots of skymion tubes in a shear flow obtained from the LBM simulations at $Er \approx 8.9$ and at two times: 0.1 s (**a**, **c** and **e**) and 1.16 s (**b**, **d** and **f**). The soliton trajectory exhibits $\Theta_{HA} \approx -7.2^\circ$. **a, b**, The z component n_z of the director field in the midplane $z = L_z/2$. The white arrow indicates the direction of the moving plate. Scale bars (white), $10 \mu\text{m}$ (L_z). **c, d**, n_z in the plane $y = y_{TCM}(t)$. **e, f**, Relative velocity field (local velocity minus the velocity far from the toron in the plane $z = L_z/2$). The arrows indicate

the direction of velocity field. **g**, Parametric plots $(x(t), y(t))$ of the skymion trajectories, where t is time, for several Er values. The inset shows the Hall angles Θ_{HA} as a function of Er obtained from the above skymion trajectories, where the solid line is a linear fit to the data points. x and y in **g** are in the units of the lattice grid used in the simulations (the conversion to real units is specified in Table 1). The numerical calculations are carried out for $L_x = L_y = 35 \mu\text{m}$, $L_z = 10 \mu\text{m}$ and $P = 8.75 \mu\text{m}$.

bottom portions of the video show torons at $Er \approx 0.5$ with Hall angles of $\Theta_{HA} \approx 10^\circ$ and $\Theta_{HA} \approx 2^\circ$, respectively, deflected in opposite transverse directions. Under stronger shear, Supplementary Video 7 (top) shows that some torons become strongly elongated by the flow and may acquire a finger-like morphology and still exhibit a transverse Hall-like drift. Because our purpose here is to illustrate the Hall-like transport accessible in this regime, rather than to classify these elongated states in detail, we do not pursue their analysis further. The broader spread of the observed deflection angles in the a.c.-stabilized regime suggests a possible route towards the electrically controlled steering of topological soliton motion, although a systematic investigation of this voltage-dependent behaviour is beyond the scope of the present work.

Numerical simulations provide empirical evidence that the sign of Θ_{HA} for torons is related to the direction of the preimage vector $\mathbf{d}_{NS} = \mathbf{R}_S - \mathbf{R}_N$ in the $z = L_z/2$ plane (Fig. 4b,c), which connects the north-pole $\mathbf{R}_N$ and south-pole $\mathbf{R}_S$ preimages (or south- and north-pole ones for the opposite director vectorization). Specifically, the direction of the toron transverse drift is given by $\mathbf{d}_{NS}/d_{NS}$, which may be controlled by, for example, a combination of the shear rate and the surface-anchoring direction at the LC cell plates (Fig. 4a and Supplementary Videos 8–12). Moreover, by introducing position-dependent surface anchoring, we may achieve position-dependent Θ_{HA} , both in terms of its sign and magnitude (Supplementary Videos 13 and 14). These results show that weak spatial variations in the effective anchoring conditions are sufficient in the simulations to reverse the sign of the Hall angle and, therefore, provide one plausible explanation for the bidirectional toron drift observed experimentally (Fig. 2j). However, because the present optical measurements do not resolve the internal director structure of each toron in detail, we cannot exclude the fact that subtle toron-to-toron structural differences or transient-flow-induced distortions may also contribute to the observed sign reversal. The unusual reversal trajectory (Extended Data Fig. 1b) suggests that such additional perturbations may indeed affect the experimental dynamics. A similar effect is expected from a bulk field that aligns the director field opposite to the flow.

Skymion tubes and bags

Up to this point, we have considered the influence of shear flow only on torons, which naturally raises the question of whether similar behaviour occurs for other types of LC topological soliton. Figure 5 and Supplementary Videos 15 and 16 provide a positive answer, demonstrating the LC Hall effect for skymion tubes. These are energetically favoured for weak surface anchoring³⁰, and in simulations, they are obtained by assuming vanishing surface anchoring and a very weak fictitious bulk field to align the director along the z axis. The skymions, in contrast to torons, are stretched in the direction of shear—resembling the behaviour of torons and skymions under pressure-driven flows^{29,31}.

We have also examined skymion bags³² containing varying numbers of enclosed skymions, and Supplementary Videos 17 and 18 demonstrate the motion of skymion bags enclosing three and seven antiskyrmions (with the skymion number opposite to that of the enclosing skymion), respectively. We have found a weak dependence of Θ_{HA} on the net skymion number, finding $\Theta_{HA} \approx 11.5^\circ$ and $\Theta_{HA} \approx 9.0^\circ$ for the three and seven skymion bags, respectively, compared with $\Theta_{HA} \approx 12.1^\circ$ for a single skymion tube.

Mechanism

To gain an insight into the physical mechanism underlying the LC topological soliton Hall effect, we use the Thiele equation approach³³, which has been widely used in magnetic systems³⁴. In a nutshell, the Thiele equations describe the driven motion of magnetic skymions in terms of the relevant collective coordinates, and in its simplest realization, it neglects the deformations of the skymion texture, focusing only on the motion of the skymion geometric centre.

We start from equation (11) (Methods), which governs the dynamics of the director field $\mathbf{n}(\mathbf{r}, t)$ in the presence of a time-independent flow field $\mathbf{u}(\mathbf{r})$. Equation (11) plays here the role of the Landau–Lifshitz–Gilbert equation for the dynamics of the magnetization field $\mathbf{m}$. In the context of the skymion Hall effect, the z -averaged second term on the left-hand side of equation (11) corresponds to the adiabatic torque $\propto (\mathbf{j} \cdot \nabla)\mathbf{m}$ when the skymion Hall effect is driven by the spin-transfer

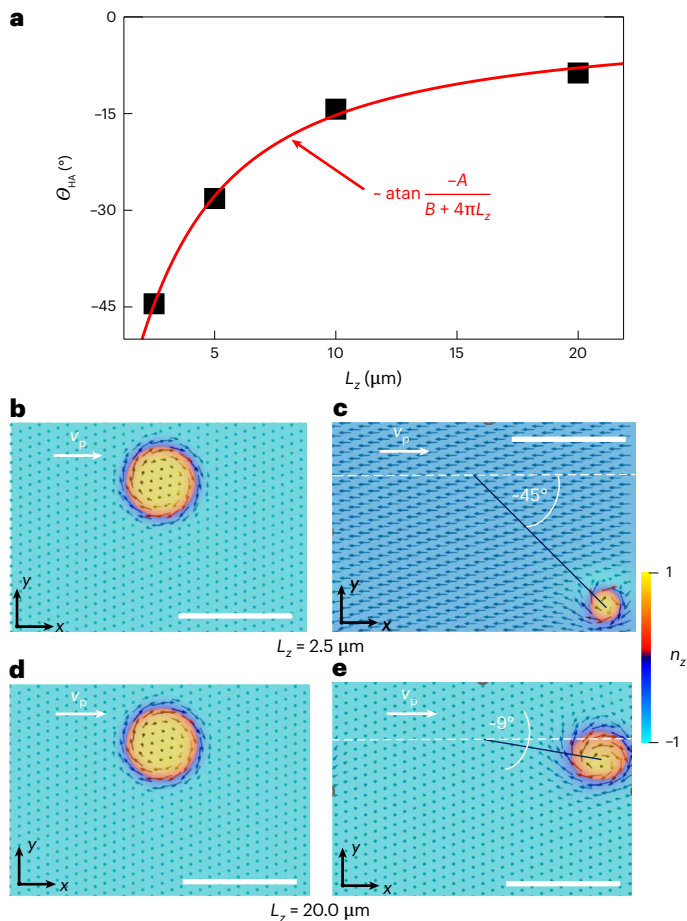


Fig. 6 | Skyrmion tubes in shear flows: theory versus numerics. **a**, Θ_{HA} as a function of L_z obtained from numerical simulations, where the director field changes, keeping the shear flow field unchanged (solid squares); $v_p = 6.25 \mu\text{m s}^{-1}$, $P = 20 \mu\text{m}$, $L_x = L_y = 60 \mu\text{m}$. The solid red curve represents the fitting curve; A and B are fitting parameters. **b, c**, Snapshots of the skyrmion tube corresponding to $t = 0$ (**b**) and $t = 15.3 \text{ s}$ (**c**); $L_z = 2.5 \mu\text{m}$. The skyrmion trajectories are shown by the dark blue lines in **c** and **e**. The white arrows in **b–e** indicate the flow direction. Scale bars (white), one cholesteric pitch.

torque³⁵, as a spin-polarized electron current density $\mathbf{j}$ flows in a thin ferromagnetic film hosting skyrmion textures. In our case, this term alone contributes to the skyrmion drift along the flow direction $\hat{\mathbf{e}}_x$ (refs. 29,31,36). By contrast, the last term on the right-hand side of equation (11) drives the motion of the skyrmion in the $\hat{\mathbf{e}}_y$ direction. This term is the analogue of the field-like torque ($\propto \mathbf{m} \times \mathbf{s}$) in the magnetic skyrmion motion under a spin-orbit torque³⁴, experimentally realized in heavy metal/ultrathin ferromagnet heterostructures^{24,25}, where $\mathbf{s}$ is the spin current density injected perpendicular to the ferromagnetic film from the underlying heavy metal layer with a lateral charge current density $\mathbf{j}_{\text{hm}}$.

In the Methods, we present the Thiele-like equation for a skyrmion tube configuration, which drifts without changing its intrinsic structure with a constant velocity $\mathbf{v} = (v_1, v_2, 0)^T$. The Thiele equation then renders the following expression for Hall angle:

$$\tan \theta_{\text{HA}} = \tan \frac{v_2}{v_1} = -\frac{T_{11}}{T_{21} + 4\pi L_z N_{\text{sk}}}, \quad (1)$$

where N_{sk} is the skyrmion number and T_{ij} is the so-called torque tensor (both are specified in the Methods). To check the validity of the theoretical result for Θ_{HA} , we numerically integrate equation (11) for a

skyrmion tube in a time-independent moderate shear flow, to minimize flow-induced skyrmion distortions. Figure 6 favourably compares the numerical results for Θ_{HA} (solid squares) as a function of L_z with the theoretical prediction in equation (1), where T_{11} and T_{21} are treated as fitting parameters. The inverse dependence of Θ_{HA} on N_{sk} is also qualitatively supported by our numerical results on skyrmion bags (see the ‘LC toron Hall effect: simulations’ section and Supplementary Videos 17 and 18).

In contrast to the full numerical simulation results reported in Fig. 5, which show a dependence of Θ_{HA} on Er , equation (1) predicts that the Hall angle is independent of the flow velocity. This is a direct consequence of the assumption that the skyrmion shape does not change during motion. A similar effect was also reported for magnetic skyrmions in ref. 24.

Conclusions

We found particularly intriguing dynamics of LC solitons, such as torons, skyrmion tubes and skyrmion bags, when subjected to shear-driven flows. Solitons undergo Hall-like drifts in a direction perpendicular to the shear plane. Rather unexpectedly, some torons with different optical appearances under our imaging conditions were deflected in opposite transverse directions; in some cases, their trajectories exhibited non-monotonic transverse drift. Our numerical analysis shows that weak spatial variations in the effective anchoring conditions are sufficient to reverse the sign of the Hall angle and, therefore, provide one plausible explanation for this bidirectional drift. However, the present study does not allow us to uniquely determine whether the observed sign reversal originates from anchoring heterogeneity, subtle differences in internal toron structure, transient-flow-induced director tilting and distortions or a combination of these factors. A perturbation analysis of the dynamic Ericksen–Leslie equations for small shear rates showed that the chiral term in the elastic torque is the key feature responsible for the transfer of linear momentum into the transverse direction, with the associated transverse mass flow. The magnitude of the transverse flow was found to scale as the third power of the shear rate at low shear rates. However, both experiments and simulations convincingly show that the soliton transverse motion is unrelated to the emergent transverse mass currents.

The LBM applied to the Ericksen–Leslie model resolved the structure of torons in shear flows and provided results for the toron Hall angle over a range of shear rates. We found that Θ_{HA} decreases markedly with the shear rate, which is in qualitative agreement with the behaviour found in experiments, where Θ_{HA} exhibits a weak decreasing trend. In addition, the simulations showed that the steady-state soliton trajectories are deflected in the direction opposite to that of the transverse flow field. These findings qualitatively agree with the experiments on tracer colloidal particles, which, in contrast to solitons, do not exhibit transverse motion due to the weakness of the transverse mass flow. We also identified an a.c.-stabilized $L_z/P = 2$ regime in which toron Hall angles reach -10° . In the baseline geometry, by contrast, passive particles show no detectable transverse drift, in qualitative agreement with the weak transverse mass flow predicted by the LBM simulations. The a.c.-stabilized regime further suggests that the Hall-like transport of topological solitons may be tunable by external electric fields. These observations confirm that the solitons do not behave as passive tracers advected by the LC flow, but exhibit a genuine topological Hall-like response associated with their intrinsic structure. Moreover, the experimental observations and simulations support the universal nature of the reported LC soliton Hall effect, which does not depend on a particular experimental realization and is valid for different types of LC topological soliton, such as torons, skyrmion tubes and skyrmion bags.

We proposed a simple physical explanation of the reported effect, in terms of an effective collective-coordinate model akin to the Thiele equation for the skyrmion Hall effect. For axially symmetric skyrmion

tubes moving without changing their shapes under time-independent shear flows, we derived an LC analogue of the Thiele equation, which predicts a surprising inverse dependence of the Hall angle on the product of the skyrmion number and the LC cell thickness. We showed that the flow vorticity plays the role of the spin current density in the magnetic skyrmion motion under a spin–orbit torque realized in heavy metal/ultrathin ferromagnet heterostructures^{24,25}.

This study supports the use of LC topological solitons as elementary building blocks for programmable active and driven soft-matter systems. A wide range of LC solitons, such as hopfions, heliknotons, möbiusons, or topological solitonic macromolecules and crystals, enriched by the possibility of mixing these quasiparticles with real colloidal particles, provides an extraordinarily rich experimental platform for both fundamental condensed-matter studies and applied physics research.

Online content

Any methods, additional references, Nature Portfolio reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at <https://doi.org/10.1038/s41567-026-03379-0>.

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Methods

Experiments

Chiral LCs are prepared by mixing 4-cyano-4'-pentylbiphenyl (EM Chemicals) with a left-handed chiral additive, cholesterol pelargonate (Sigma-Aldrich), whose helical twisting power $h_{\text{htp}} = 6.25 \mu\text{m}^{-1}$. The cholesteric pitch of the ensuing chiral LCs is $P = 1/C_{\text{dopant}} h_{\text{htp}}$, where C_{dopant} is the weight fraction of the added chiral dopant. For $L_z/P = 1$, the concentration of the chiral dopant is 1.6 wt%.

The sample cells are assembled from indium-tin-oxide-coated glass slides treated with polyimide SE5661 (Nissan Chemicals) to obtain strong perpendicular (homeotropic) boundary conditions. The polyimide is applied to the surfaces by spin coating at 2,700 rpm for 30 s followed by baking (5 min at 90 °C and then 1 h at 180 °C). The parallel glass plates (Supplementary Fig. 2a) were separated by freely rolling silica spheres/spacers (diameter, $9.9 \pm 0.09 \mu\text{m}$), which define the cell thickness L_z (Supplementary Fig. 1). The spacer particles were mixed with ultraviolet adhesive (NOA 61, Norland Products). The adhesive covering the microspheres has moderate viscosity and was not cured, which allowed one plate to slide relative to the other.

Spectrometer measurements (Supplementary Fig. 2b–f) showed that the cell gap slightly varied from the centre to the sides, probably due to partial curing of the adhesive near the cell edges. As the LC pitch was set to $\sim 10 \mu\text{m}$ (as determined via the Grandjean–Cano wedge cell method; Supplementary Fig. 3) for the toron Hall effect experiments, the slight variation in the L_z/P ratio could affect only the weak transverse mass flow, as shown by the simulation results and perturbation theory. The topological Hall effect, by contrast, is rooted in the intrinsic properties of the solitons. On the basis of numerical simulations and experiments, we conclude that no appreciable changes in the gross morphology and stability of the torons are expected when L_z/P varies by $\pm 9\%$.

To impose shear on the confined LC, we fixed one glass plate and attached the other one to the microscope stage (slide holder; Supplementary Fig. 1) that can be controlled electronically and moved in one direction at speed v_p . We quantify the flow intensity by $\text{Er} \equiv \frac{\mu v_p L_z}{K}$, where μ is the viscosity and K is the average elastic constant, and used the following values for the model parameters: $L_z = 10 \mu\text{m}$, $\mu = 28 \text{ mPa s}$ and $K = 6.46 \text{ pN}$.

We utilized a ytterbium-doped fibre laser (YLR-10-1064, IPG Photonics; operating at 1,064 nm) to generate torons, by melting the LC locally with the laser power around 30 mW. When we switch off the laser tweezers, the torons are generated spontaneously as the LC quenches back (Supplementary Video 1). The POM images are obtained using a multimodal imaging setup built around an IX-81 Olympus inverted microscope and charge-coupled device cameras (Grasshopper, Point Grey Research). Olympus objectives ($\times 20$ and $\times 50$) with numerical apertures of 0.4 and 0.5, respectively, are used. The speed and trajectories of the torons are analysed using ImageJ (freeware from NIH).

For the tracer-particle experiments in the unwound cholesteric phase, we used samples with $L_z/P = 0.7, 0.6$ and 0.3 , corresponding to chiral-dopant concentrations of 1.1 wt%, 0.96 wt% and 0.48 wt%, respectively. Two types of tracer particle were used: melamine resin spheres (Aldrich) with a diameter of $4 \mu\text{m}$, and alumina nanoplates (Novarials) with a lateral size of $\sim 1 \mu\text{m}$ and a thickness of $\sim 0.1 \mu\text{m}$. Both particle types impose tangential anchoring on the director field $\mathbf{n}(\mathbf{r})$ at their surfaces. The melamine resin spheres were obtained by evaporating water from the supplied aqueous dispersion, whereas the alumina nanoplates were used as received in the dry powdered form. The particles were then added to the LC and sonicated to reduce aggregation.

LBM simulation of the Ericksen–Leslie model

The Ericksen–Leslie model^{37–39} describes the coupled dynamics of the LC director $\mathbf{n}$ and material flow $\mathbf{u}$ fields. It consists of two sets of equation expressing the balance of local force and torque. For the material flow field, the local force balance is the Navier–Stokes

equation augmented with the continuity equation for the conservation of mass:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = \nabla \cdot (\Pi \mathbf{I} + \boldsymbol{\sigma}^v + \boldsymbol{\sigma}^e) \quad (2)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (3)$$

where ρ is the fluid density, Π is a scalar Lagrange multiplier that arises from the assumed incompressibility in equation (3) and has the meaning of hydrostatic pressure, and $\mathbf{I}$ is the identity tensor. $\boldsymbol{\sigma}^v$ is the viscous stress tensor with the following components:

$$\sigma_{\alpha\beta}^v = \alpha_1 n_\alpha n_\beta n_\mu n_\mu D_{\mu\mu} + \alpha_2 n_\beta N_\alpha + \alpha_3 n_\alpha N_\beta \quad (4)$$

$$+ \alpha_4 D_{\alpha\beta} + \alpha_5 n_\beta n_\mu D_{\mu\alpha} + \alpha_6 n_\alpha n_\mu D_{\mu\beta}. \quad (5)$$

Here $\alpha, \beta \in (x, y, z)$ and summation over repeated indices is assumed. α_n ($n = 1 \dots 6$) are the Leslie viscosities. The vector $\mathbf{N}$ describes the relative rotation rate of the director field:

$$\mathbf{N} = \frac{\partial \mathbf{n}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{n} - \mathbf{W} \cdot \mathbf{n} \quad (6)$$

whereas the corresponding strain rate $\mathbf{D}$ and vorticity $\mathbf{W}$ tensors are

$$D_{\alpha\mu} = \frac{1}{2} (\partial_\alpha u_\mu + \partial_\mu u_\alpha), \quad (7)$$

$$W_{\alpha\mu} = \frac{1}{2} (\partial_\mu u_\alpha - \partial_\alpha u_\mu). \quad (8)$$

The elastic stress tensor $\boldsymbol{\sigma}^e$ is specified as

$$\sigma_{\alpha\beta}^e = -\partial_\alpha n_\gamma \frac{\delta \mathcal{F}}{\delta (\partial_\beta n_\gamma)}, \quad (9)$$

where $\mathcal{F}$ is the Frank–Oseen free energy and defined as

$$\mathcal{F} = \int dV \left(\frac{K_{11}}{2} (\nabla \cdot \mathbf{n})^2 + \frac{K_{22}}{2} (\mathbf{n} \cdot [\nabla \times \mathbf{n}] + q_0)^2 + \frac{K_{33}}{2} [\mathbf{n} \times [\nabla \times \mathbf{n}]]^2 \right). \quad (10)$$

Here K_{11} , K_{22} and K_{33} are the Frank elastic constants, $q_0 = 2\pi/P$ and P is the cholesteric pitch. For torons, we adopt rigid perpendicular boundary conditions for the director field at the confining surfaces. When considering skyrmion tubes, we use free boundary conditions and add a bulk ordering term $-W_0 n_z^2/2$ to the free energy density in equation (10), with a small coupling constant W_0 . This mimics, in an effective way, weak perpendicular anchoring conditions. The second set of equations describes the evolution of the director field as

$$\frac{\partial \mathbf{n}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{n} = -\frac{1}{\gamma_1} \left(\frac{\delta \mathcal{F}}{\delta \mathbf{n}} + \gamma_2 \mathbf{D} \cdot \mathbf{n} \right) - \mathbf{n} \times \boldsymbol{\omega}, \quad (11)$$

where $\gamma_1 = \alpha_3 - \alpha_2$ is the rotational (twist) viscosity determining the rate of relaxation of the director, $\gamma_2 = \alpha_3 + \alpha_2$ is the torsion coefficient characterizing the contribution to the viscous torque arising from gradients of the velocity field⁴⁰ and $\boldsymbol{\omega} \equiv \frac{1}{2} \nabla \times \mathbf{u}$ is the vorticity field. The ratio γ_2/γ_1 is known as the aligning parameter, where $|\gamma_2/\gamma_1| > 1$ represents flow-aligning systems and $|\gamma_2/\gamma_1| < 1$ represents flow-tumbling ones.

The simulations used a hybrid numerical method. The velocity field is solved using the LBM^{41,42} with the elastic and viscous (except by the term proportional to α_4) stress tensors being introduced as a

force term. The equation for the director field, equation (11), is solved using a predictor–corrector finite-differences algorithm. On the solid plates, we apply infinite homeotropic anchoring and no-slip boundary conditions, which is done using the bounce-back condition with an imposed plate velocity⁴¹. In simulations, we normalized the director to unity at each time step of the numerical integration of equation (11).

The simulations are initialized with the liquid at rest and directors pointing mostly perpendicularly to the plates, except in the proximity of the toron whose configuration is obtained by minimizing the free energy starting from the ansatz in ref. 36. The material parameters are close to those of 4-methyl-4'-cyanobiphenyl at 22 °C (ref. 43), except the absolute viscosity (or, equivalently, α_4), which is two times larger in the simulations to run in reasonable times. The chosen parameters, including the lattice spacing, are a compromise between simulation speed and spatial discretization. A typical simulation of 2 s of physical time takes about 10 days to run on an NVIDIA RTX 3080 graphics card. The code is parallelized in CUDA-C and the simulations are run on graphics processing units.

Perturbation analysis: no solitons

Consider a cholesteric LC confined in the z direction by two parallel surfaces placed at $z=0$ and $z=L$, and unconstrained in the lateral x and y directions. We assume that the plate at $z=L$ moves in the $+x$ direction with the speed $\dot{\gamma}L_z$. We find it convenient to introduce ε in the following form:

$$\varepsilon = \frac{\alpha_4 \dot{\gamma} L_z^2}{K}, \quad (12)$$

where α_4 is one of the Leslie's viscosities (Table 1) and K is the averaged elastic constant.

We assume $\varepsilon \ll 1$ and perturbatively determine the leading-order terms and next-to-leading-order corrections to the steady-state solutions $\mathbf{n}(z)$ and $\mathbf{u}(z)$ to equations (2), (3) and (11). To deal with the constraint $\mathbf{n} \cdot \mathbf{n} = 1$, we add a term $\Lambda(z)\mathbf{n}(z)$ to the right-hand side of equation (11), where Λ is a scalar Lagrange multiplier. Additionally, we adopt the Frank–Oseen free energy elastic constant $K = K_{11} = K_{22} = K_{33}$, which renders the following expression for the molecular field:

$$\mathbf{h} \equiv -\frac{\delta \mathcal{F}}{\delta \mathbf{n}} = K \nabla^2 \mathbf{n} - 2Kq_0 [\nabla \times \mathbf{n}], \quad (13)$$

and the corresponding elastic torque driving the director field towards equilibrium (global or local) has the form

$$\Gamma^{\text{el}} = [\mathbf{n} \times \mathbf{h}]. \quad (14)$$

Additionally, the viscous force and torque are given by the following expressions:

$$\mathbf{g} = -\gamma_1 \mathbf{N} - \gamma_2 \mathbf{D} \cdot \mathbf{n}, \quad (15)$$

$$\Gamma^{\text{v}} = [\mathbf{n} \times \mathbf{g}]. \quad (16)$$

The unperturbed zeroth-order configurations $\mathbf{u}^{(0)} = (0, 0, 0)^T$ and $\mathbf{n}^{(0)} = (0, 0, 1)^T$, and the boundary conditions read $\mathbf{u}(z=0) = (0, 0, 0)^T$ at the lower boundary and $\mathbf{u}(z=L_z) = (\dot{\gamma}L_z, 0, 0)^T$ at the upper one; we set $\mathbf{n}(z=0, L_z) = (0, 0, 1)^T$ at the boundary planes. We non-dimensionalize the equations by setting L_z , $\dot{\gamma}^{-1}$, $\dot{\gamma}L_z$ and KL_z^{-2} as the units of length, time, velocity and pressure, respectively. In the expressions below, all Leslie viscosities are rescaled by α_4 and to keep the notation simple, we use the same symbols α_i for the dimensional and dimensionless viscosity parameters. Next, we expand $\mathbf{n}(z)$, $\Lambda(z)$ and $\mathbf{u}(z)$ in powers of ε as

$$n_x(z) = \varepsilon n_x^{(1)}(z) + \varepsilon^2 n_x^{(2)}(z) + \varepsilon^3 n_x^{(3)}(z) + \mathcal{O}(\varepsilon^4), \quad (17)$$

$$n_y(z) = \varepsilon n_y^{(1)}(z) + \varepsilon^2 n_y^{(2)}(z) + \varepsilon^3 n_y^{(3)}(z) + \mathcal{O}(\varepsilon^4), \quad (18)$$

$$n_z(z) = 1 + \varepsilon^2 n_z^{(2)}(z) + \varepsilon^3 n_z^{(3)}(z) + \mathcal{O}(\varepsilon^4), \quad (19)$$

$$\Lambda(z) = \varepsilon \Lambda^{(1)}(z) + \varepsilon^2 \Lambda^{(2)}(z) + \varepsilon^3 \Lambda^{(3)}(z) + \mathcal{O}(\varepsilon^4), \quad (20)$$

and the velocity as

$$u_x(z) = u_x^{(1)}(z) + \varepsilon u_x^{(2)}(z) + \varepsilon^2 u_x^{(3)}(z) + \mathcal{O}(\varepsilon^4), \quad (21)$$

$$u_y(z) = u_y^{(1)}(z) + \varepsilon u_y^{(2)}(z) + \varepsilon^2 u_y^{(3)}(z) + \mathcal{O}(\varepsilon^4). \quad (22)$$

The selected velocity scale $-\varepsilon$ dictates the expansion for the flow field $\mathbf{u}(z)$ as in equations (21) and (22). Recall that the fluid velocity terms in equations (2) and (11) are multiplied by ε after rewriting the equations in the dimensionless form. $n_z^{(1)}(z) = 0$ due to the condition $(\mathbf{n} \cdot \mathbf{n}) = 1$, and the incompressibility condition equation (3) renders $u_z = 0$. The only non-zero boundary condition is $u_x^{(1)}(z=1) = 1$, with all the other terms in equations (17)–(22) set to zero at both bounding surfaces. As shown below, the second-order terms $u_\mu^{(2)}(z) = 0$, $n_\mu^{(2)}(z) = 0$ with $\mu = x, y$; therefore, the first non-trivial corrections to the flow field are $\mathcal{O}(\varepsilon^3)$. We stress that $n_z^{(2)}(z) \neq 0$, but this component is not of interest here. We also do not discuss the behaviour of the hydrostatic pressure P . Finally, we demand that the normalization condition $(\mathbf{n} \cdot \mathbf{n}) = 1$ is satisfied at each order of ε .

At the first order in ε , we find

$$\partial_z^2 n_x^{(1)} + 2q_0 \partial_z n_y^{(1)} - \alpha_2 \partial_z u_x^{(1)} = 0, \quad (23)$$

$$\partial_z^2 n_y^{(1)} - 2q_0 \partial_z n_x^{(1)} - \alpha_2 \partial_z u_y^{(1)} = 0, \quad (24)$$

$$\Lambda^{(1)} = 0, \quad (25)$$

$$\partial_z^2 u_x^{(1)} = 0, \quad (26)$$

$$\partial_z^2 u_y^{(1)} = 0, \quad (27)$$

where ∂_z denotes a differentiation with respect to z . The first-order $\mathcal{O}(\varepsilon)$ contributions $u_{x,y}^{(1)}(z)$ to the velocity field are not affected by the director field, and obey Laplace's equation. With the considered boundary conditions, we find $u_x^{(1)}(z) = zL_z^{-1}$, $u_y^{(1)}(z) = 0$. This flow configuration gives rise to a dynamic viscous torque exerted on $\mathbf{n}$. The y component of the torque, which acts on n_x , can be written as $\Gamma_y^{\text{v}} = (\alpha_3 n_x^2 - \alpha_2 n_z^2) \partial_z u_x = -\alpha_2 \partial_z u_x + \mathcal{O}(\varepsilon^3)$, where α_2 and α_3 are two other Leslie's viscosities specified in Table 1. The $\mathcal{O}(\varepsilon)$ component of the viscous torque $\propto -\alpha_2 \partial_z u_x^{(1)}$ drives the x component of director away from the uniform state imparting positive ($\alpha_2 < 0$) contributions to $n_x^{(1)}(z)$. By contrast, the x component of the viscous torque is $\Gamma_x^{\text{v}} = -\alpha_3 n_x n_y \partial_z u_x = \mathcal{O}(\varepsilon^3)$ and, therefore, does not contribute to the first-order term $n_y^{(1)}(z)$. The y component of the director field is driven out of the ground state solely by the chiral component of the elastic torque ($\propto [\mathbf{n} \times [\nabla \times \mathbf{n}]]_x = n_z \partial_z n_x = \mathcal{O}(\varepsilon)$).

In summary, the solutions to equations (23)–(27) read

$$n_x^{(1)}(z) = -\frac{\alpha_2}{2q_0} \csc[q_0] \sin[q_0(1-z)] \sin[q_0 z], \quad (28)$$

$$n_y^{(1)}(z) = \frac{\alpha_2}{4q_0} (2z - 1 + \csc[q_0] \sin[q_0(1-2z)]), \quad (29)$$

$$u_x^{(1)}(z) = z, \quad (30)$$

$$u_y^{(1)}(z) = 0. \quad (31)$$

Equations (28) and (29) show that $n_x^{(1)}(z)$ and $n_y^{(1)}(z)$ diverge at $q_0 = m\pi$ for any nonzero integer m . This artefact arises from the fact that at the first order in ε , the equation $(\mathbf{n} \cdot \mathbf{n}) = 1$ reduces to $n_z^{(1)} = 0$, that is, it imposes any condition on $n_x^{(1)}(z)$ and $n_y^{(1)}(z)$. As such, this divergence can be absorbed into the director normalization ‘constant’. Both first-order director perturbations are plotted in Extended Data Fig. 2 for two different values of P/L_z .

The second-order equations are

$$\partial_z^2 n_x^{(2)} + 2q_0 \partial_z n_y^{(2)} - \alpha_2 \partial_z u_x^{(2)} = \gamma_1 \Lambda^{(1)} n_x^{(1)}, \quad (32)$$

$$\partial_z^2 n_y^{(2)} - 2q_0 \partial_z n_x^{(2)} - \alpha_2 \partial_z u_y^{(2)} = \gamma_1 \Lambda^{(1)} n_y^{(1)}, \quad (33)$$

$$\partial_z^2 n_z^{(2)} - \alpha_3 n_x^{(1)} = \gamma_1 \Lambda^{(2)}, \quad (34)$$

$$(n_x^{(1)})^2 + (n_y^{(1)})^2 + 2n_z^{(2)} = 0, \quad (35)$$

$$\partial_z^2 u_x^{(2)} = 0, \quad (36)$$

$$\partial_z^2 u_y^{(2)} = 0. \quad (37)$$

Considering vanishing boundary conditions at the confining surfaces and equation (25) and assuming q_0 different from $m\pi$, the only non-trivial solutions of the above system are

$$n_z^{(2)}(z) = -\frac{(n_x^{(1)})^2 + (n_y^{(1)})^2}{2}, \quad (38)$$

and

$$\Lambda^{(2)}(z) = \gamma_1^{-1} (\partial_z^2 n_z^{(2)} - \alpha_3 n_x^{(1)}). \quad (39)$$

$n_z^{(2)}(z)$ is shown in Extended Data Fig. 3 at specific values of α_2 and α_3 for several P/L_z values.

Finally, collecting the terms $\mathcal{O}(\varepsilon^3)$ renders the equations

$$\partial_z^2 n_x^{(3)} + 2q_0 \partial_z n_y^{(3)} - \alpha_2 (\partial_z u_x^{(3)} + n_z^{(2)}) = \gamma_1 (\Lambda^{(1)} n_x^{(2)} + \Lambda^{(2)} n_x^{(1)}), \quad (40)$$

$$\partial_z^2 n_y^{(3)} - 2q_0 \partial_z n_x^{(3)} - \alpha_2 \partial_z u_y^{(3)} = \gamma_1 (\Lambda^{(1)} n_y^{(2)} + \Lambda^{(2)} n_y^{(1)}), \quad (41)$$

$$\partial_z^2 n_z^{(3)} = \gamma_1 (\Lambda^{(1)} n_z^{(2)} + \Lambda^{(3)}), \quad (42)$$

$$n_x^{(1)} n_x^{(2)} + n_y^{(1)} n_y^{(2)} + n_z^{(3)} = 0, \quad (43)$$

$$\partial_z^2 u_x^{(3)} + \mathcal{A} \partial_z (n_x^{(1)} n_x^{(1)}) + \mathcal{B} \partial_z n_z^{(2)} = 0, \quad (44)$$

$$\partial_z^2 u_y^{(3)} + \mathcal{A} \partial_z (n_x^{(1)} n_y^{(1)}) = 0, \quad (45)$$

where

$$\mathcal{A} = \frac{2\alpha_1 + \alpha_3 + \alpha_6}{1 + \alpha_5 - \alpha_2}, \quad (46)$$

$$\mathcal{B} = \frac{2(\alpha_5 - \alpha_2)}{1 + \alpha_5 - \alpha_2}. \quad (47)$$

Taking into account the solutions $n_x^{(2)}(z) = n_y^{(2)}(z) = 0$ (q_0 different from $m\pi$), equation (43) shows that $n_z^{(3)}(z) = 0$; next, from equation (42), it follows that $\Lambda^{(3)}(z) = 0$ (considering $\Lambda^{(1)}(z) = 0$).

Extended Data Fig. 4 compares $u_y^{(3)}(z)$ with $u_x^{(3)}(z)$ and Extended Data Fig. 5a depicts $u_y^{(3)}(z)$ for several values of P/L_z . Interestingly, at intermediate values of P/L_z , $u_y^{(3)}(z)$ exhibits a layered structure ($P/L_z = 1.45$; Extended Data Fig. 5a (dashed blue curve)), where the fluid occupying the central region of the cell flows in the $-y$ direction, whereas the outer layers flow in the opposite $+y$ direction. The relative flow magnitudes in these flow layers, and as a result, the net transverse mass flow $U_y^{(3)} = \int_0^{L_z} u_y^{(3)}(z) dz$, can be adjusted by varying L_z/P (Extended Data Fig. 5b). $U_y^{(3)}$ exhibits oscillatory behaviour as a function of L_z/P , and vanishes as P^{-1} for $P \rightarrow \infty$ (pure nematic phase).

The third-order corrections to the flow field can be written as

$$u_x^{(3)}(z) = \frac{a_2^2 \csc[q_0]^2}{384 q_0^3} (2\mathcal{B}(2z-1)(3+2q_0^2 z(z-1))(1-\cos[2q_0]) + 3\mathcal{A}q_0((3-6z)\sin[2q_0] + \sin[2q_0(1-2z)] + 4\sin[2q_0 z]) + 6\mathcal{B}(\cos[2q_0 z] - \cos[2q_0(z-1)] + 2q_0(z-1)\sin[2q_0 z]) - 12q_0(\mathcal{A} - \mathcal{B}z)\sin[2q_0(1-z)]), \quad (48)$$

and

$$u_y^{(3)}(z) = \frac{A a_2^2 \csc[q_0]}{128 q_0^3} (4\cos[q_0](2q_0^2 z(1-z) - 1) + 4\cos[q_0(1-2z)] + q_0 \csc[q_0](3\cos[2q_0] + \cos[2q_0(1-2z)] + 4(z-1)\cos[2q_0(z-1)] - 4z\cos[2q_0 z])). \quad (49)$$

The Thiele equation

We start from equation (11) governing the dynamics of the director field $\mathbf{n}(\mathbf{r})$ in the presence of a time-independent flow field $\mathbf{u}(\mathbf{r})$. For the shear flow considered here, $\mathbf{u}(\mathbf{r}) = \dot{\gamma} z \hat{\mathbf{e}}_x$. The last term in equation (11) can be rewritten as

$$-\mathbf{n} \times \boldsymbol{\omega} = -\mathbf{n} \times (\hat{\mathbf{e}}_z \times \mathbf{J}), \quad (50)$$

where $\mathbf{J} \equiv \frac{\dot{\gamma}}{2} \hat{\mathbf{e}}_x$ is an analogue of the charge current density $\mathbf{j}_{\text{hm}}$ flowing in the heavy metal layer underneath the ferromagnetic film. The LC analogue of the Thiele equation is obtained from director relaxation (equation (11)) by applying to it the following three operators:

1. $\mathbf{n} \times$, that is, carrying out a cross product by $\mathbf{n}$;
2. $\frac{\partial \mathbf{n}}{\partial x_i}$, $i \in (1, 2)$, that is, carrying out a dot product by $\frac{\partial \mathbf{n}}{\partial x_i}$;
3. Integrating the result over the LC volume.

Here we consider only translationally invariant (along the z -direction) skyrmion tube configuration $\mathbf{n}_{\text{drift}}(\mathbf{r}, t)$, which drifts without changing its intrinsic structure with a constant velocity $\mathbf{v} = (v_1, v_2, 0)^T$. The spatiotemporal director configuration in this case can be written as $\mathbf{n}_{\text{drift}}(\mathbf{r}, t) = \mathbf{n}(\mathbf{r} - \mathbf{v}t)$, where $\mathbf{n}(\mathbf{r})$ is the static configuration of the skyrmion tube, from which it follows that $\frac{\partial \mathbf{n}}{\partial t} = -(\mathbf{v} \cdot \nabla) \mathbf{n}$. The skyrmion velocity $\mathbf{v}$ then obeys the following coarse-grained Thiele-like equation:

$$G_{ij} (\langle u_j \rangle_z - v_j) + T_{ij} J_j = 0; i, j \in (1, 2), \quad (51)$$

where $G_{ij} = -4\pi N_{\text{sk}} \varepsilon_{3,ij}$ is the gyrotropic tensor with $N_{\text{sk}} = \frac{1}{\int_{R^2} \mathbf{n} \cdot (\frac{\partial \mathbf{n}}{\partial x_1} \times \frac{\partial \mathbf{n}}{\partial x_2})} \text{ being the skyrmion number (topological charge)}^{24}$ and $\varepsilon_{3,ij}$ is the Levi-Civita symbol. We also introduced the z -averaged flow field component as

$$\langle u_j \rangle_z = \frac{1}{L_z} \int_0^{L_z} u_j dz = \begin{cases} \frac{\dot{\gamma} L_z}{2}, & j = 1 \\ 0, & j = 2. \end{cases} \quad (52)$$

The torque tensor $\mathbf{T}$ in equation (51) depends on the skyrmion configuration and is defined as²⁴

$$T_{ij} \equiv - \int_{R^2} \left(\varepsilon_{2\mu\nu} \frac{\partial n_\mu}{\partial x_i} n_\nu n_j - \varepsilon_{\mu\nu j} \frac{\partial n_\mu}{\partial x_i} n_\nu n_z \right) d^2 r. \quad (53)$$

Solving equation (51) for v_1 and v_2 , we obtain equation (1). The LC Hall effect, thus, is driven by the coupling of the LC director $\mathbf{n}(\mathbf{r})$ to the flow vorticity $\mathbf{w}(\mathbf{r}) = \frac{1}{2} \nabla \times \mathbf{u}$ given by the last term in equation (11).

The terms in brackets on the right-hand side of equation (11) do not contribute to equation (51) due to translational invariance in the lateral directions assumed here.

Data availability

All data generated or analysed during this study are included in the Article and its Supplementary Information. Data are also available from the corresponding author on reasonable request. Source data are provided with this paper.

Code availability

The computer codes used to produce the results presented in the Article are available from the corresponding author upon reasonable request.

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Author contributions

R.C.V.C. and M.M.T.G. conceived the idea. R.C.V.C. developed the numerical method and implemented the computing codes. G.N.C.A.

parallelized the code and carried out the numerical simulations. M.T. conducted the perturbation analysis and analysed the results. H.Z. and I.I.S. conducted the experiments. All authors contributed to writing the paper and discussions.

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Competing interests

The authors declare no competing interests.

Additional information

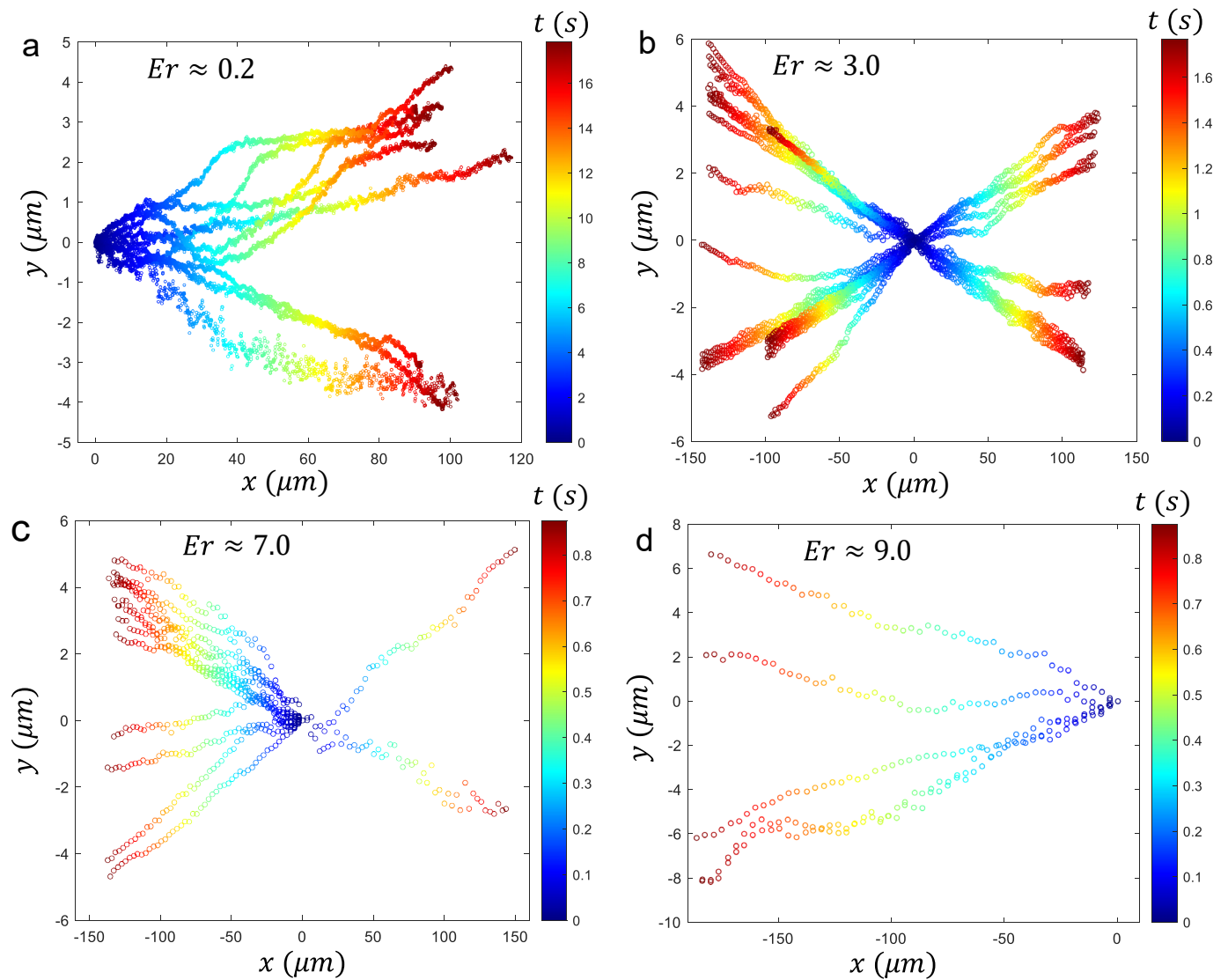
Extended data is available for this paper at <https://doi.org/10.1038/s41567-026-03379-0>.

Supplementary information The online version contains supplementary material available at <https://doi.org/10.1038/s41567-026-03379-0>.

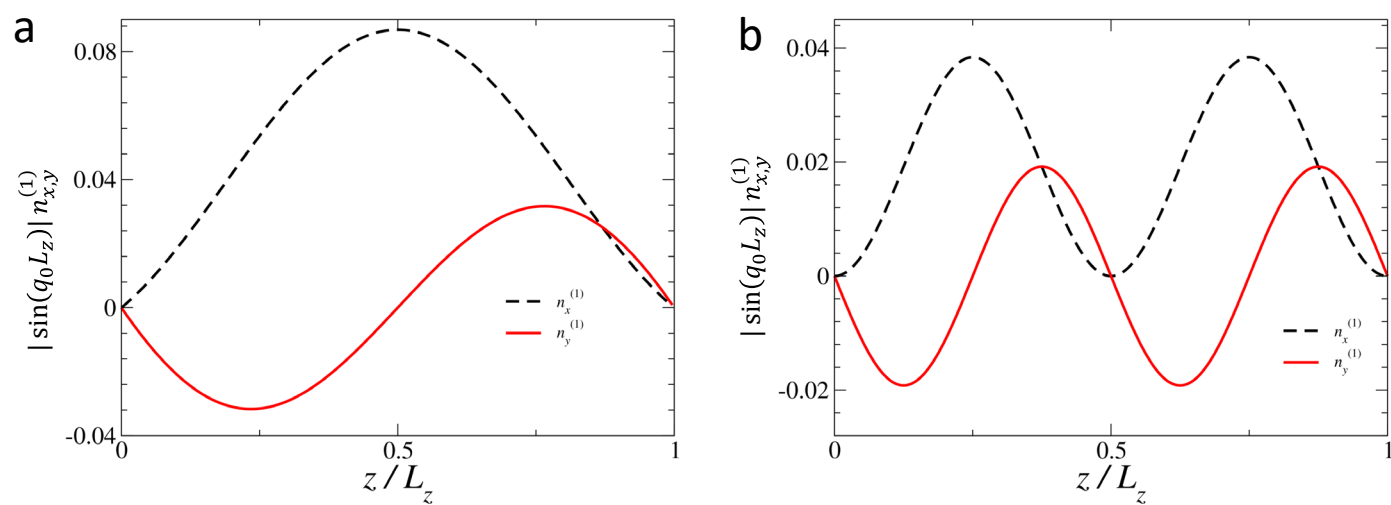
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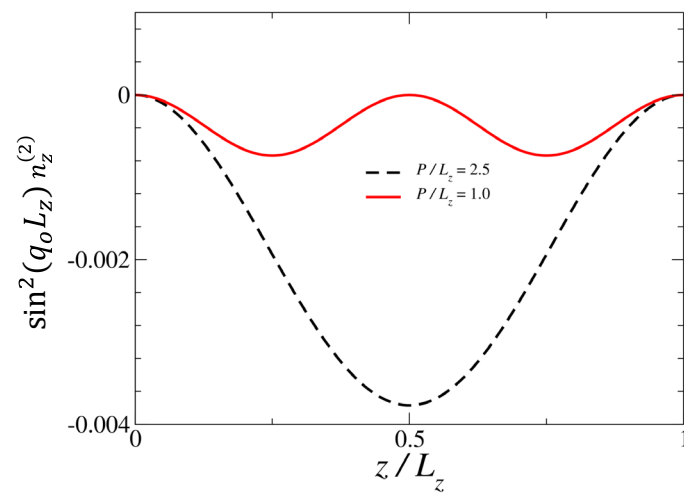
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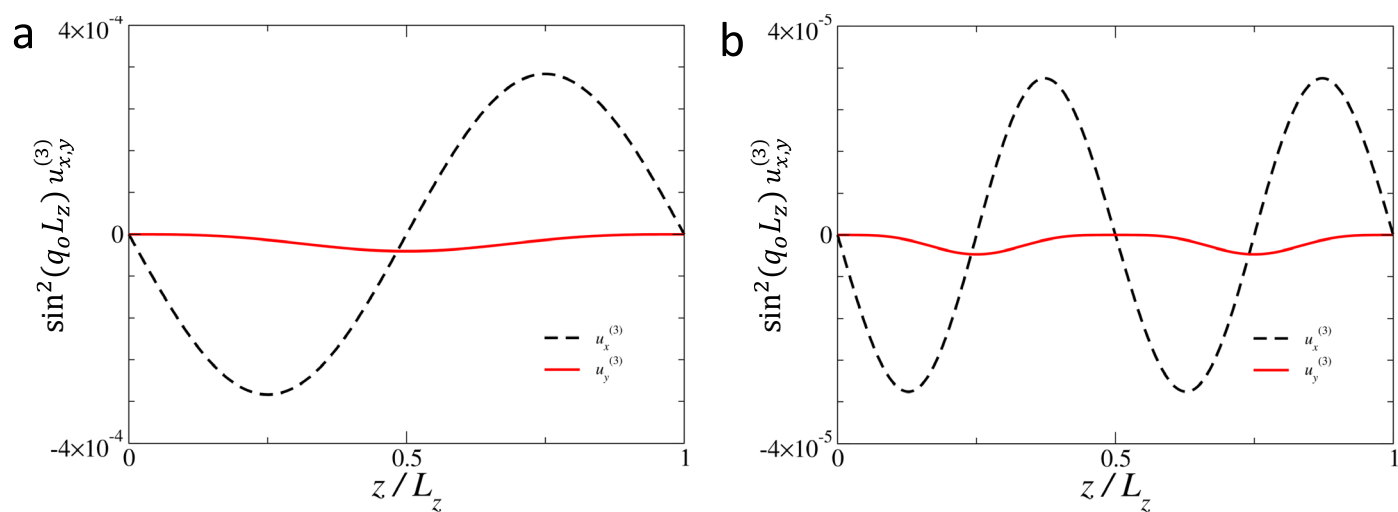
Extended Data Fig. 1 | Toron trajectories. **a–d**, Experimentally tracked toron trajectories in a parametric representation $x(t), y(t)$, where t is time, at values of the Ericksen number $Er \approx 0.2$, **a**, 3.0 , **b**, 7.0 , **c**, and 9.0 , **d**. Trajectories are colour-coded by time according to the scale on their right.



Extended Data Fig. 2 | $\mathcal{O}(\varepsilon)$ contributions to the director field. $n_x^{(1)}, n_y^{(1)}$ contributions to the director distortions as functions of the distance z to the lower surface. $n_x^{(1)}(z)$ and $n_y^{(1)}(z)$ are obtained from (28) and (29). The curves are computed at $P/L_z = 2.5$, **a**, and $P/L_z = 1$, **b**, where P is the cholesteric pitch, L_z is the thickness of the LC cell and $q_0 = 2\pi/P$.

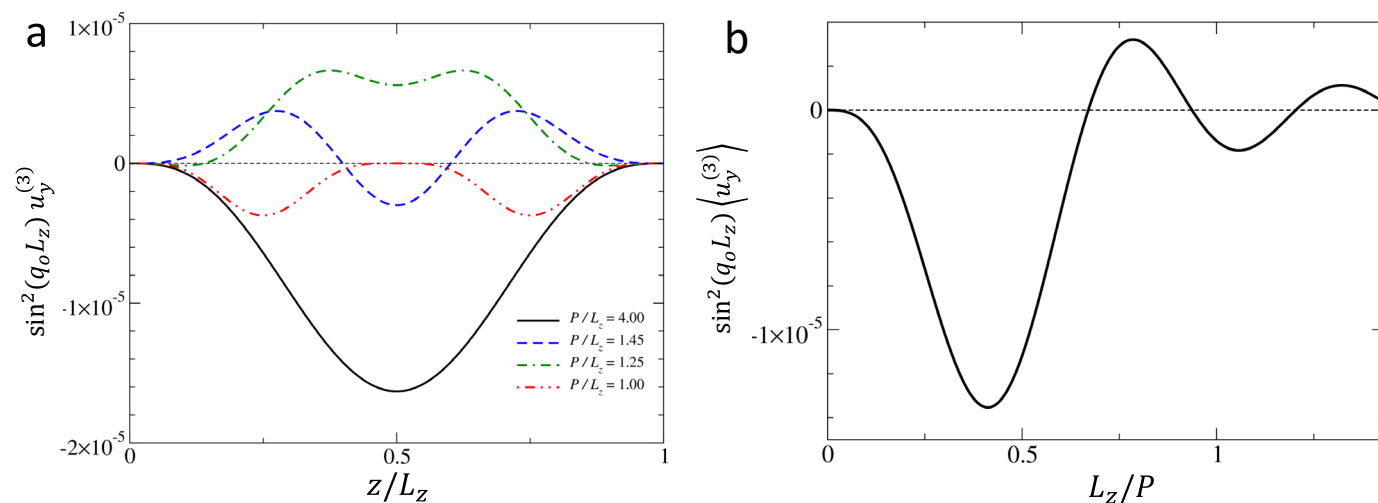


Extended Data Fig. 3 | $\mathcal{O}(\varepsilon^2)$ contribution to the director field. $n_z^{(2)}$ as a function of the distance z from the lower plate. $n_z^{(2)}(z)$ is given in Eq. (38). The curves are computed at different values of P/L_z as indicated in the legend, where P is the cholesteric pitch, L_z is the thickness of the LC cell and $q_0 = 2\pi/P$.



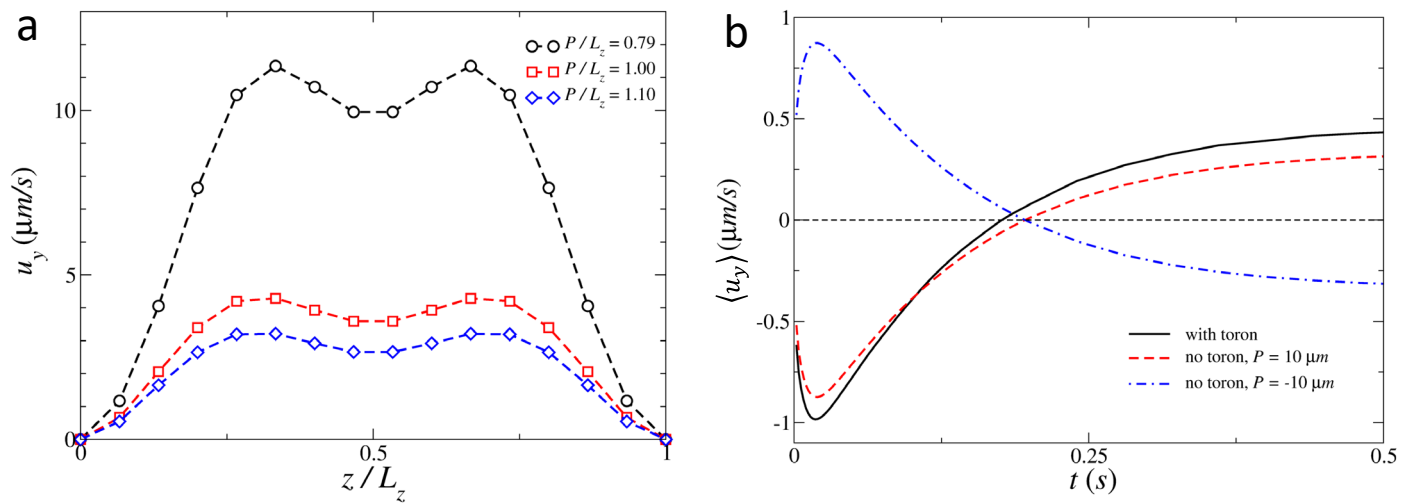
Extended Data Fig. 4 | $\mathcal{O}(\varepsilon^3)$ contributions to the flow field. $u_x^{(3)}(z)$ and $u_y^{(3)}(z)$, obtained from (48) and (49). The curves are computed at $P/L_z = 2.5$, **a**, and $P/L_z = 1$, **b**, where P is the cholesteric pitch and L_z is the thickness of the LC cell. Values for the Leslie viscosities $\alpha_1, \dots, \alpha_6$ are provided in Table I in Methods. Here P is the

cholesteric pitch, L_z is the thickness of the LC cell, $q_0 = 2\pi/P$ and the velocity is in units of $v_p = \dot{\gamma} L_z$, where $\dot{\gamma}$ is the shear rate and v_p the speed of translation of the upper glass plate relative to the lower one.



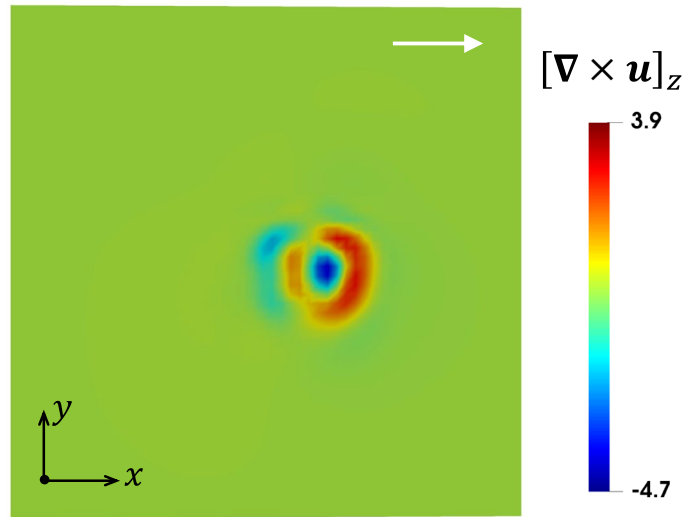
Extended Data Fig. 5 | Transverse fluid currents from perturbation analysis. **a**, $\mathcal{O}(\varepsilon^3)$ dimensionless contributions $u_y^{(3)}(z)$ to the out of the shear plane component of the flow field, obtained from (49) at different values of the pitch to cell thickness ratio P/L_z . Values for the Leslie viscosities $\alpha_1, \dots, \alpha_6$ are provided in Table 1 in Methods. **b**, Space averaged $\langle u_y^{(3)} \rangle = L_z^{-1} \int_0^{L_z} u_y^{(3)}(z') dz'$ transverse mass

current as a function of the cell width to pitch ratio L_z/P , obtained from the perturbation theory. For $q_0 \rightarrow 0$, $\langle u_y^{(3)} \rangle \propto q_0$. Here $q_0 = 2\pi/P$ and the velocity is in units of $v_p = \dot{\gamma} L_z$, where $\dot{\gamma}$ is the shear rate and v_p the speed of translation of the upper glass plate relative to the lower one.

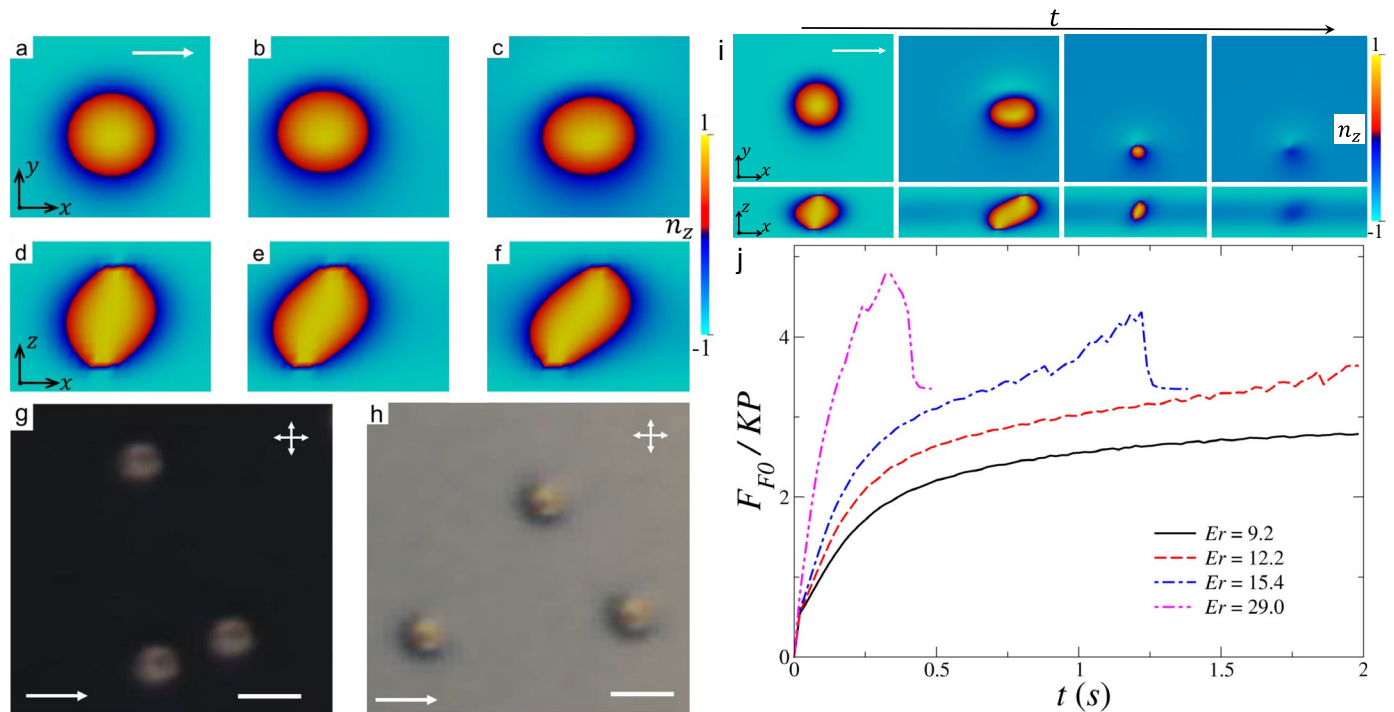


Extended Data Fig. 6 | Transverse fluid currents from the Lattice Boltzmann simulations. a, The y -component u_y of the steady state flow velocity as a function of the distance z from the lower plate for the case of no toron. The plate speed $v_p =$

$80 \mu\text{m/s}$ which renders the Ericksen number $Er \approx 11.2$. **b**, Space averaged transverse current $\langle u_y \rangle$ as a function of time, for systems with and without a toron. Results for systems with opposite handedness are shown when no toron is present; $Er \approx 3.1$.



Extended Data Fig. 7 | Flow vorticity around a toron. Cross sectional view, in the plane $z = L_y/2$, of the z-component of the vorticity $\propto [\nabla \times \mathbf{u}]_z$, corresponding to the steady state solution of the Ericksen-Leslie equations. The numerical calculations are carried out for $L_x = L_y = 35 \mu\text{m}$; $L_z = P = 8.75 \mu\text{m}$.



Extended Data Fig. 8 | Toron deformations in shear flows. Torons moving under different plate speeds. **a**, **b** and **c** depict the steady state at $t = 2$ s in the mid plane for the Ericksen number $Er \approx 3.1$, $Er \approx 6.1$ and $Er \approx 9.2$. **d**, **e** and **f** show the corresponding figures in the plane $y = y_{TCM}(t)$, where $y_{TCM}(t)$ is the y-component of toron's center of mass. Experiments at low ($Er \approx 0.3$), **g**, and high ($Er \approx 7.0$), **h**, plate speeds illustrate the changes in the toron configuration. The scale bars in **g**

and **h** represents $10 \mu\text{m}$. **i**, Unstable toron at high shear rate demonstrated by the time evolution of the director field shown in the cross sections $z = L_z/2$ (top) and $y = y_{TCM}(t)$ (bottom), $Er \approx 15.4$. **j**, Time evolution of the Frank-Oseen free energy for different Ericksen numbers; K is the average elastic constant. The white arrows in **a**, **g**, **h** and **i** indicate the direction of the moving plate. The numerical calculations are carried out for $L_x = L_y = 35 \mu\text{m}$; $L_z = P = 8.75 \mu\text{m}$.