Settlement scaling theory: Bridging the study of ancient and contemporary urban systems

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Abstract
A general explanatory framework for the social processes underpinning urbanisation should account for empirical regularities that are shared among contemporary urban systems and ancient settlement systems known throughout archaeology and history. The identification of such shared properties has been facilitated by research traditions in each field that define cities and settlements as areas that capture networks of social interaction embedded in space. Using Settlement Scaling Theory (SST) – a set of hypotheses and mathematical relationships that together generate predictions for how measurable quantitative attributes of settlements are related to their population size – we show that aggregate properties of ancient settlement systems and contemporary metropolitan systems scale up in similar ways across time, geography and culture. Settlement scaling theory thus provides a unified framework for understanding and predicting these regularities across time and space, and for identifying putative processes common to all human settlements.

Keywords
cities, comparative urbanism, data, pre-modern cities, settlement scaling

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Introduction

There is a general recognition that cities share a number of organisational, social and economic characteristics and play similar functional roles in human societies regardless of size, geography, time or culture. Despite being separated by thousands of years of cultural, social and technological development, settlements of past and contemporary societies seem to share enough in common that the term ‘city’ can meaningfully refer to both (Algaze, 2018; de Vries, 1984; Mumford, 1961). Given this, it stands to reason that there should be empirical regularities for important attributes of cities common to urban systems that arose and evolved independently across time, culture and levels of technology. Explanations for such regularities should invoke social processes that are also common to cities of all times and places. In the same way that Darwin argued that a good theory for the variety of biological forms should apply to the fossil record as well as contemporary life, an adequate explanation for the variety of settlements and urban forms should apply to the archaeological record as well as contemporary cities. But what features of cities and urban systems are common across eras? What can be measured, compared and predicted? What common underlying processes generate these regularities?

Answering these questions necessitates several epistemological components. First, it requires methods for defining socio-spatial units in the contemporary world, and from material remains of the past, in such a way that these units capture similar social functional entities (not just similar physical forms). Second, it pre-supposes an explanatory framework that is general enough to encompass urban systems that vary substantially with respect to social, political and technological details. It cannot simply be a projection of contemporary social and economic arrangements backwards in time as such a framework would break down as soon as modern political institutions and economic markets ceased to apply. Third, such an explanatory framework must identify salient properties of cities that can be captured by common measurements, the empirical foundation of any truly comparative approach. Finally, the explanatory framework, or theory in a manner defined below, must generate specific predictions that can be tested using data from a variety of settlement systems.

Here, we present an approach that meets these requirements and can ground the examination of urban systems across eras.
Although novel in its scope, it builds on the long history of quantitative models in urban economics, economic geography, complex systems and regional science that traces the origin and persistence of population agglomeration to the advantages of concentrating human populations in space after accounting for associated costs (Fujita et al., 1999). These are sometimes referred to as agglomeration effects, and they constitute the foundational concepts for explaining the formation and persistence of population agglomeration (Duranton and Puga, 2004; Storper, 2013). Our framework also provides articulating arguments for the long-standing recognition (in economics, sociology and anthropology) that population size is a determinant of many socio-economic features of human settlements (Johnson and Earle, 2000; Nordbeck, 1971).

Our approach builds on research in urban economics, geography and complex systems that has identified relationships between urban scale and economic productivity, innovation rates, energy use and infrastructure needs (Bloom et al., 2008; Glaeser et al., 2003; Nordbeck, 1971). Such relations are known as scaling relations (Barenblatt, 2003; Chave and Levin, 2003). For this reason, the systematic study of such relationships has come to be known as urban scaling. Over the past decade, a formal theory of urban scaling has begun to emerge (Bettencourt, 2013). The fundamental process at the core of this theory is the concentration of social, economic and political interactions in space and time, subject to constraints imposed by environmental conditions, technology and institutions (Bettencourt, 2014; Schläpfer et al., 2014). These processes, which are the micro-level foundations upon which the explanatory framework is built, are very general and are not tailored to the specific characteristics of modern cities, or more broadly, settlements, of a certain size. As a result, it potentially applies to settlements and settlement systems in any context. Here, we provide an overview of this explanatory framework which we call settlement scaling theory (SST), illustrate the scope of its applicability, and show that its extension to archaeological contexts allows for a truly comparative study of urbanisation.

Our use of the term ‘theory’ is deliberate and justified. The explanatory framework is based on first principles (assumptions whose general validity we dare take for granted). There are many theories of cities, stemming from different disciplinary perspectives and addressing different characteristics and dynamics; SST does not aspire to be a general theory of urbanisation but rather a theory of how the spatially embedded social interactions among agglomerated individuals generate several of the most salient properties of settlements (including cities). SST does not seek to explain all aspects of cities but does seek to account for the effects of scale for their extensive properties. Importantly, the theory makes clear quantitative predictions for the values of posited observables that can be tested (and falsified) against settlement data from any given era. To the best of our knowledge, no other existing model or theory allows for such a practical, unified and predictive framework for human settlements across history.

The city as a unit of study

The initial challenge confronted when building an explanatory framework for any urban phenomena is the seemingly innocuous question of how to define a city. Such a definition must be grounded in principles of what a city is and how it operates. Louis Wirth (1938) proposed that a city is a large and permanent settlement of heterogeneous individuals living and working at high population densities. Richard Sennett (1977: 39) suggested that ‘a city is a human settlement where strangers are likely to meet’. Architectural
historian Spiro Kostof (1991) observed that ‘cities are places where a certain energised crowding of people takes place’. More recently, the urban economist Edward Glaeser (2011) described cities as ‘the absence of physical space between people and companies. They are proximity, density, closeness.’ O’Sullivan (2011) defines cities as geographical areas with concentrations of individuals and activities that are higher relative to the surrounding area. These various characterizations illustrate the general principle that the essence of urbanism is not physical space per se, but frequent and intense social interactions among a diversity of individuals and organizations within a given space (Smith, 2019). Researchers confront different challenges when operationalising this view in contemporary versus archaeological contexts.

**Contemporary cities**

Operationalising a view of cities as spatially embedded social networks requires the self-consistent identification of spatial units that capture their relevant social and economic aspects. Owing to the mobility of urban dwellers, this task is far from trivial, even when large volumes of data are available. Measures of density and interaction are typically used, but each has its associated problems. Measuring population density pre-supposes a relevant physical space within which people are counted. Such spaces can be defined using residential densities, but because of commuting the resulting units may not capture actual patterns of daily interaction – where people work, shop and socialize – that are at the core of the city as a social entity. And it is even more difficult to directly observe and measure the totality of social and economic interactions which generate and define urban life.

We emphasise two observations that facilitate the definition of cities as integrated socio-economic units. First, movement entails costs: social interactions in space have, throughout history, involved travel, which carries monetary, energy and time costs. Second, human effort is bounded – for any given transportation technology, humans can only move so much per unit time and spend so much time in transit. Together, these two assumptions justify drawing the boundaries of urban areas as containing the space around built-up infrastructure (homes, roads, workplaces, shops), which can be traversed within about a day’s movement effort (Marchetti, 1994). The prevalent interpretation of urban areas, or functional cities, in contemporary urban studies is thus of a spatial object whose outlines contain daily flows of people, goods and information within one or more adjacent residential centres (Pumain, 2000). How much distance can be covered in a day, and at what energy and monetary cost, is strongly dictated by available technologies and infrastructure and their local implementations.

This perspective leads to a variety of ways whereby one might delineate a functional city as an entity bounded by density and movement (Bretagnolle et al., 2009). Arguably the most consistent definition is the Metropolitan Statistical Area (MSA), developed by the US Census Bureau in the 1960s and updated annually (Berry et al., 1969). An MSA consists of a core county or counties in which lies an incorporated city (a politico-administrative entity) with a population of at least 50,000 people, plus adjacent counties having a high degree of social and economic integration with the core counties as measured through commuting ties. Essentially, MSAs are unified labour markets revealed by daily commuting flows. These flows are interpreted as reflecting the frequent exchange of goods, labour and
information, which in turn is a proxy for intense socio-economic interaction (Glaeser et al., 1995). Because of its unique socio-economic relevance, the concept of the metropolitan area has been more recently adopted by the OECD, the EU and by various other major national statistical offices including those of China, Brazil, South Africa, Mexico, Chile and Colombia.

Some authors have questioned the emergent global consensus around metropolitan area definitions, expressing concerns that the documented empirical characteristics of cities may be unduly influenced – or even determined – by the choice of spatial unit of analysis (Arcaute et al., 2015; Cottineau et al., 2017). This issue is often referred to as the Modifiable Areal Unit Problem (MAUP): ‘... the areal units (zonal objects) used in many geographical studies are arbitrary, modifiable, and subject to the whims and fancies of whoever is doing, or did, the aggregating’ (Openshaw, 1983: 3). It is of course true that different spatial units will have different analytical consequences. But this only reinforces how essential it is that the chosen spatial units encapsulate the phenomenon of interest, which in the case of a city is a network of social interaction embedded in space. The MSA definition does this, whereas alternative urban spatial units, such as areas defined by local density thresholds or contiguous built areas, need not. Still, a fundamental issue with regard to the definition of contemporary cities is the complex relationship between built space and daily patterns of social mixing. Below, we show that this issue is much less severe for pre-industrial settlements.

Archaeological cities and settlements

Archaeologists conduct fieldwork to locate and study artefacts (objects made or used by humans) and features (fixed constructions) that are the physical remains of past human behaviour. Archaeological ‘sites’ are spatial concentrations of such artefacts and features, ranging in scale from small artefact scatters to large expanses of numerous features. The archaeological concept of the settlement has been codified as ‘[...] the physical locale or cluster of locales where the members of a community lived, ensured their subsistence, and pursued their social functions in a delineable time period’ (Chang, 1968: 3). Archaeologists have subsequently linked the settlement concept to the notion of the place-based community: a group of people who live in close proximity within a geographically limited area, who have face-to-face interaction on a regular basis, and who share access to resources in their local sustaining area (Murdock, 1949; Varien, 1999). Ancient settlements are thus locations where human interactions were concentrated in space. While the most detailed information about individual sites comes from excavation, surface remains and, increasingly, remote sensing and geophysical prospection provide sufficient evidence for studying demography, wealth and other aggregate properties (Drennan et al., 2015; El-Qady and Metawaly, 2018; Johnson and Millett, 2012).

The identification of settlement extent is crucial for studying how humans enact social processes in physical space. In settings where current ground-cover reveals ancient house remains, but not associated artefacts – such as Classic-period Maya sites – archaeologists draw site boundaries based on the distribution of house remains, using conventions such as an empty zone of at least 100 m to mark settlement boundaries (Hutson et al., 2008). More commonly, house remains are buried, but surface artefacts become visible when brought to the ground surface by ploughing and other modern disturbances. In these situations, site boundaries are identified from the spatial distribution of potsherds and other non-perishable artefacts.
deposited by human activity. The density of surface artefacts varies tremendously across sites and regions based on a variety of factors. Thus, site boundaries are rarely defined by absolute density figures. Instead, field-workers typically walk outward from the centre of a site in various directions and define the boundary where the visible artefact density drops off substantially (Drennan et al., 2015: 17–20). In areas with sparse vegetative ground cover, surface artefact densities typically show a clear ‘edge’.

The two primary proxies used to estimate the populations of archaeological sites are house remains and site area. In the case of house remains, the first step is to convert the number of house remains into an estimate of the number of households that lived in a settlement during a given time period. The ‘momentary’ household estimate may also be revised downward to account for the non-contemporaneous occupation of structures within a given time period (Rice and Culbert, 1990). The final step is to multiply the household estimate by an average household size to obtain an estimate for the resident population. Acknowledging that household sizes varied with wealth, context (rural versus urban) and other parameters, archaeologists typically use a household size figure that has historical or sociological validity for a particular area or time period. When house remains are not visible on the modern ground surface, archaeologists typically estimate population by combining the site area with a population density estimate. The simplest procedure is to apply a constant population density to the entire area of the site (Adams, 1981). More sophisticated methods involve adjusting the density based on either surface artefact densities, house densities within excavated areas, and/or the inferred type of settlement (Drennan et al., 2015; Hanson, 2016; Hanson and Ortman, 2017; Sanders et al., 1979).

Measuring material output and wealth presents its own challenges (Morris, 2013; Ortman and Davis, 2019; Stark et al., 2016). The strongest material proxy for household wealth is the size of residences, and there is now a robust literature that uses this proxy to measure wealth distributions in the deep past (Kohler et al., 2017, 2018; Smith et al., 2014). Table S1 of Kohler et al. (2017) lists the ethnographic and historical analogues which demonstrate the strong relationship between house size and household wealth.

Despite the many challenges of working with archaeological evidence, one of its advantages is the intrinsic correspondence between settlement boundaries and spatial patterns of past social interaction. In ancient societies, people walked or, in some rarer cases, rode animals or carts, on paths that were much more uneven than modern roads. Most people who worked within settlements lived close to – or even at – their place of work (Laslett, 1971). In the ancient world, workers rarely lived in one settlement and worked in another (Laslett, 1971; Sjoberg, 1960). In addition, most workers were farmers who regularly walked out to their fields (Christiansen, 1978). In a classic study, Michael Chisholm adapted von Thünen’s central market model to pre-industrial peasant farming, finding that the distance farmers travelled from their homes to their fields was rarely greater than 4 km (Chisholm, 1968: 46). So, for such settlements, if there were ‘commuter flows’ at all they involved farmers commuting between settlements and their fields.

These features of transport, agriculture and movement mean that commuter flows between ancient settlements were minimal. Commuting served to disperse people for individual farm work, with most social interactions being confined to the settlement area itself. This pattern is in strong contrast to contemporary cities, where commuting serves to concentrate people for socio-
Table 1. Empirical support for settlement scaling theory from contemporary urban systems. Unless otherwise noted, the independent variable is settlement population.

<table>
<thead>
<tr>
<th>Case</th>
<th>Dependent variable</th>
<th>N</th>
<th>Exponent $\beta$ (95% C.I.)</th>
<th>$R^2$</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cities and towns, Mexico (1960)</td>
<td>Area</td>
<td>181</td>
<td>0.64 (0.55–0.72)</td>
<td>0.53</td>
<td>(Ortman et al., 2015)</td>
</tr>
<tr>
<td>Urbanised areas, Sweden (1965)</td>
<td>Area</td>
<td>1800</td>
<td>0.65 (0.64–0.65)</td>
<td>0.89</td>
<td>(Nordbeck, 1971)</td>
</tr>
<tr>
<td>Metropolitan Statistical Areas, USA (1980–2000)</td>
<td>Area</td>
<td>329</td>
<td>0.62–0.63 (NR)</td>
<td>NR</td>
<td>(Paulsen, 2012)</td>
</tr>
<tr>
<td>Global cities (2000)</td>
<td>Areal extent</td>
<td>3646</td>
<td>0.83–0.85 (NR)</td>
<td>0.71</td>
<td>(Angel, 2012)</td>
</tr>
<tr>
<td>Urban areas, China (2005)</td>
<td>Built-up area</td>
<td>660</td>
<td>0.82 (NR)</td>
<td>0.84</td>
<td>(Chen, 2010)</td>
</tr>
<tr>
<td>Urbanised prefectural cities, China</td>
<td>GDP</td>
<td>287</td>
<td>1.22 (1.17–1.27)</td>
<td>0.69</td>
<td>(Zünd and Bettencourt, 2018)</td>
</tr>
<tr>
<td>Metropolitan Statistical Areas, USA (2002)</td>
<td>Total wages</td>
<td>361</td>
<td>1.12 (1.09–1.13)</td>
<td>0.96</td>
<td>(Bettencourt et al., 2007a)</td>
</tr>
<tr>
<td>Metropolitan Statistical Areas, USA (2002)</td>
<td>New patents</td>
<td>331</td>
<td>1.27 (1.25–1.29)</td>
<td>0.81</td>
<td>(Bettencourt et al., 2007b)</td>
</tr>
<tr>
<td>Global Metropolitan Statistical Areas</td>
<td>New patents</td>
<td>1530</td>
<td>1.47 (S.E. = 0.03)</td>
<td>0.84</td>
<td>(Lobo et al., 2013a)</td>
</tr>
<tr>
<td>Metropolitan areas (&gt; 500,000 people), EU (2012)</td>
<td>GDP</td>
<td>102</td>
<td>1.17 (1.11–1.22)</td>
<td>0.96</td>
<td>(Bettencourt and Lobo, 2016)</td>
</tr>
<tr>
<td>Urbanised areas, India (2011)</td>
<td>Built area</td>
<td>909</td>
<td>0.8846 (0.84–0.93)</td>
<td>0.61</td>
<td>(Sahasranaman and Bettencourt, 2019a)</td>
</tr>
<tr>
<td>Urbanised areas, India (2011, Districts)</td>
<td>GDP</td>
<td>21</td>
<td>1.12 (0.94–1.30)</td>
<td>0.88</td>
<td>(Sahasranaman and Bettencourt, 2019b)</td>
</tr>
<tr>
<td>Metropolitan areas, Brazil</td>
<td>Personal income</td>
<td>39</td>
<td>1.11 (1.03–1.20)</td>
<td>0.90</td>
<td>(Breslford et al., 2017)</td>
</tr>
</tbody>
</table>

Notes: NR: not reported; SE: standard error.

economic interactions. As a result, the physical settlement (based on the circumscribing or built-up area) and the functional settlement (based on social mixing patterns) were essentially one and the same in the ancient world. In other words, archaeological settlements conform to the model of cities as spatially embedded networks of social interaction more directly than contemporary urban areas often do.

Settlement scaling theory: Background

The manner in which aggregated individuals produce greater value, innovate more rapidly and utilise infrastructure more efficiently is transposable between modern and pre-modern contexts (Glaeser, 2011). What has been lacking until recently is a formal framework that accounts for the varied effects of settlement aggregation in general terms. SST is an explicit attempt to accomplish this (Bettencourt, 2014; Ortman et al., 2014). SST does not pre-suppose that costs and benefits are monetised, nor that costs and benefits are experienced and compared within a market system or a hierarchical polity. Rather, it replaces the market-based utility functions of standard economics models with an even more basic and more fundamental social network whereby individuals
balance interaction benefits with movement costs (production functions, however, do emerge from the SST formalism). The SST framework makes quantitative predictions about the specific values of elasticities (scaling exponents) of a variety of urban characteristics, focusing on aggregate (extensive) as opposed to per capita (intensive) measures; traditional per capita measures conflate scale-driven effects with effects deriving from technology and institutions.

The foundational assumptions of SST are that (a) human interactions are exchanges of material goods and information that take place in physical space; (b) the intensity, productivity and quality of individual-level efforts are mediated and enhanced through interaction with others (social networks); (c) any human activity can be thought of as generating benefits and incurring costs (especially the costs of moving people and things in physical space); (d) human effort is bounded; and (e) the size (scale) of a human agglomeration is both a consequence and a determinant of the agglomeration’s productivity. These assumptions connect settlement scaling theory to several well-established research traditions in anthropology, sociology and economic geography. These assumptions in effect provide the microfoundations for the aggregate scaling phenomena in terms of the behaviour of individual agents and their (economic and non-economic) interactions (Janssen, 2008).

The observation that the size of a human spatial agglomeration is both a cause and an effect of technological and cultural development has become widely accepted within anthropology (Feinman, 2011). The complex cultural and technological systems that humans develop do not stem simply from individual cognition but from distributed knowledge maintained in social networks, with larger-sized groups maintaining larger accumulations of such knowledge (Henrich, 2015). In economics, the non-rivalry of knowledge is taken to encourage new ideas and new combinations of existing ideas in larger-sized groups, thereby making scale a determinant of innovation and output (Jones and Romer, 2010; Simon, 1986). The realisation that individuals’ behaviours and performance are crucially influenced by whom they interact with is also fundamental to sociology (Granovetter, 1973; Watts, 2004). More recently, the realisation that economic behaviour is explicitly mediated by social interactions has seeped into economics and geography (Acemoglu et al., 2015; Easley and Kleinberg, 2010; Jackson, 2014). As an example, Storper and Venables (2004) identify four properties of spatially proximate social interactions that promote increasing benefits for the people and activities involved: they provide effective communications; they generate trust and incentives in relationships; they help in screening and socialising; and they involve personal stimulation and motivation.

These general insights echo ideas that pervade all mathematical models of spatial agglomeration in economics and geography, including those of von Thünen (1966), Alonso (1964), and Krugman (1991). However, in SST there is no representative agent optimising city size: individuals simply experience benefits and accrue costs, and may stay or leave a settlement as they wish. A standard economic approach would be to posit a ‘utility function’ where individuals have preferences in terms of location and consumption. This utility is then varied relative to these quantities (marginal utility) under a cost–benefit constraint. Such an approach requires additional assumptions – about utilities, their global maximisation and the quantities that matter for everybody – that are not necessary to model the observable relationships we are interested in.

The absence of an optimising ‘representative agent’ scaffolding should not blind the reader to the deep connections between
settlement scaling theory and standard urban economics. The ‘five axioms of urban economics’, which O’Sullivan (2011) argues provide a foundation for economics models of locational choice, pertain equally to SST. These axioms are: (1) location-specific costs and benefits balance to generate a locational equilibrium; (2) self-reinforcing effects induce concentration of activities and individuals; (3) externalities are prevalent in the costs and benefits experienced by individuals in cities; (4) production is subject to economies of scale, which in turn engenders specialisation and favours agglomeration; and (5) competition generates zero economic profit, meaning that total revenue balances total economic cost, including the opportunity costs of all inputs. While the language of these axioms is tailored to modern market economies, they represent ‘first principles’ about human behaviour that are also reflected in SST. However, in SST modern markets or political organisation are not necessary: spatially concentrated social networks and associated costs are sufficient, which can take different institutional and cultural forms.

Settlement scaling: Mathematical framework

The basic models of SST have been presented in detail elsewhere (Bettencourt, 2013, 2014; Lobo et al., 2013; Ortman et al., 2014). Here, we provide an overview of the framework, highlighting its points of contact with standard approaches as well as its divergences. We begin by noting that when settlements are relatively small and unstructured spatially, the travel cost of maintaining a mixing population for the average individual within the settlement is given simply by

\[ c = \varepsilon L, \]

where \( \varepsilon \) is the energetic cost of movement and \( L \) is the distance across the area of the settlement. Note that in this circumstance the distance is proportional to the square root of the circumscribed area, \( L \sim A^{1/2} \). The average number of interactions a resident will have with others is given by

\[ i = a_0 l N / A, \]

where \( l \) is the average length of the path travelled by an individual in a day, \( a_0 \) is the distance over which interaction occurs (a cross section), and \( N / A \) is the population density. Different individuals will experience different interactions, of course, and this can be accommodated via statistical extensions of these arguments. However, for computing scaling relations, the average over the population is sufficient. These interactions are mostly intentional so that they can be translated into net benefits, \( y \), by considering that there is some average energetic (or monetary) consequence of an interaction, across all types of interactions (some may be negative, such as crime) that can occur \( \hat{g} \) such that

\[ y = \hat{g} a_0 l N / A; \]

see Bettencourt (2013) for a more developed mathematical argument. Then, by setting costs equal to benefits

\[ c = y : \varepsilon A^{1/2} = \hat{g} a_0 l N / A, \]

and this simplifies to:

\[ A(N) = (G/\varepsilon)^{2/3} N^{2/3}, \]

where

\[ G = \hat{g} a_0 l. \]

One can think of \( G \) as the net attractive ‘force’ (resources per unit time per unit area = power density) an individual exerts on others through his/her interactions. Equation 1 can be simplified further by defining

\[ a = (G/\varepsilon)^{2/3} \quad \text{and} \quad \alpha = 2/3, \]

yielding:

\[ A(N) = a N^\alpha. \]

The pre-factor in equation (2), \( a = (G/\varepsilon)^{2/3} \), varies in accordance with the strength of social interaction and transportation costs \( \varepsilon \), and can change over time with changes in transport and social institutions, but is independent of population.

Equations (1) and (2) apply to small and spatially unstructured settlements, but as settlements grow, the inhabitants must set aside some of the land area for accesses, \( A_n \), of roads, paths and other public spaces so that
residents can continue to move around and mix socially. This is the actual area over which interactions occur, and as a result it is necessary to specify the relationship between the built or ‘network’ area and the circumscribing area. We assume that on average the distance \( b \) between people is set in accordance with the current population density, such that \( b = (A/N)^{1/2} \). This can be justified on theoretical grounds as is observed in modern large and dense cities (Bettencourt, 2013) and observed in some developed settlements in the archaeological record (Ortman et al., 2014). Thus, one can think of \( b \) as the length and width of street-frontage per resident in a city. Under this model, the total area of the access network is:

\[
A_n \sim N b = A^{1/2} N^{1/2}. \tag{3}
\]

From here, one can substitute \( a N^{2/3} \) for \( A \) and simplify, leading to:

\[
A_n \sim a^{1/2} N^{5/6}. \tag{4}
\]

Equation (4) implies that, as settlements in a given context grow, movement and interaction become increasingly structured by the access network and its associated public spaces, and that the area of this network grows with population more rapidly than the circumscribing area; namely, in accordance with the settlement population to the \( \alpha = 5/6 \) power. There is still an economy of scale in space use per capita, but the exponent of the growth rate of the built area with population is slightly higher than it is for the circumscribing area.

Next, we propose that the socio-economic outputs, \( Y \), generated by a settlement are, on average, proportional to the total number of social interactions that occur among its inhabitants per unit time. This notion, that increasing productivity derives from the concentration, intensification and differentiation of social interactions, goes back at least to Adam Smith and is the basic idea behind economics models of agglomeration effects (Glaeser et al., 1995; Hausmann and Hidalgo, 2011; Jones and Romer, 2010). Given this, and the assumption that settlements support as much social mixing as is possible, we can write:

\[
Y(N) = GN(N - 1)/A \approx GN^2/A, \tag{5}
\]

where \( G \) once again represents the net social attraction of an individual’s movements and interactions, and one can compute the expected scaling of outputs relative to population by substituting \( a N^{2/3} \) for \( A \) in the case of circumscribing areas, and \( a^{1/2} N^{5/6} \) for \( A \) in the case of built areas. This leads to:

\[
Y(N) \propto N^{2-\alpha}, \tag{6}
\]

with \( 2/3 \leq \alpha \leq 5/6 \). Equation (6), derived from social interactions, is also a production function, \( Y_i = AN_i^\beta \), typically assumed to be at play in settings where there is free mobility of capital and labour and everyone has access to the same production technology; the output of the \( i \)th city is a function of labour (\( N \)) with \( \beta \) a system-wide production parameter and \( A \) is a Hicks-neutral technology term; see, for example, Glaeser et al. (1995), Jones and Romer (2010). Equation (6) can also be written in a more exact way as

\[
Y(N_i, t) = Y_0(t)N_i^\beta e^{\psi_i(t)} \tag{7}
\]

to emphasise that scaling relations are relative to the transport costs and social institutions of a given context \( Y_0 \) and time \( t \), and each settlement exhibits a deviation from the expectation value \( e^{\psi_i(t)} \) that summarises the effects of a variety of other factors in the realised properties of each individual city. These additional details are implied in the other equations of SST but are omitted here for simplicity of presentation.
From equation (6) an expression for average per capita output can be obtained:

\[ y = Y/N = GN/A \times N^\delta, \]  

with \(1/6 \leq \delta \leq 1/3\). This means that as human settlements increase in population their average per capita socio-economic rates grow proportionately to population raised to the \(\delta\) power, and their total aggregate rates grow proportionately to population raised to the \(\beta = 1 + \delta\) power. In other words, there are increasing returns to scale, such that more populous settlements are more productive per capita (of course they are also more expensive by about the same proportion). Finally, if one assumes that each individual requires access to a certain number of functions, \(F\), to meet his/her needs, and that increasing average connectivity per person \(k\) makes it possible for each individual to specialise in a decreasing range of such functions per person \(d\), then the product \(k(N) \times d(N) = F\), with \(F\) a constant independent of \(N\), and we can see that increasing connectivity enables increasing functional specialisation (i.e. division of labour). Namely, if

\[ k(N) = K(N)/N = k_0N^\delta, \]  

then \(d(N) = (F/k_0)N^{-\delta}\) and the total productive diversity, \(D(N) = d(N)N\) is:

\[ D(N) = (F/k_0)N^{1-\delta}. \]  

This result indicates that as settlements grow, new explicit productive activities are added more slowly than people (not everybody does something different), and people become more connected and occupationally diverse overall, but with each individual becoming also increasingly specialised.

The parameters of the models discussed above are all very general and are not tailored to the specific technologies and institutions of any particular society. They can be derived from a more general statistical theory of settlements, where the exponents can receive corrections from different patterns of mobility and/or from deviations from average behaviour that are scale dependent, if these are strong. Also, by providing a simple and measurable set of relationships through which one can characterise any settlement system, SST effectively controls for the effects of scale in human networks, thus making it easier to identify properties of settlements that are not scalar effects (the fluctuations in equation (7)) and are more likely to reflect their unique characteristics and history (Bettencourt et al., 2010).

**Scaling results for settlement systems: Past and present**

The exponents predicted by the models in section ‘Settlement scaling: Mathematical framework’ represent the variation with population size of expectation values. These expectation values can be estimated through ordinary least squares (linear) regression of log-transformed population and other aggregate measures for any sample of settlements from a given system because \(XN = X_0N^\beta\) and \(\log XN = \beta \log N + \log X_0\) are equivalent. More sophisticated estimation methods, such as fixed-effects regressions, can also be applied.

Tables 1 and 2 present estimated scaling exponents between population, area and a variety of measures of socio-economic outputs and functional diversity for a wide range of sedentary societies dating from several thousand years ago to the present. All of these results are from peer-reviewed publications that can be consulted for the details. Table 1 compiles results for contemporary urban systems, and Table 2 compiles results for societies documented by archaeology and history.

These results derive from a variety of settlement types, settlement systems, cultures,
Table 2. Empirical support for settlement scaling theory from archaeological and historical sources. Independent variable = settlement population.

<table>
<thead>
<tr>
<th>Case</th>
<th>Dependent variable</th>
<th>N</th>
<th>Exponent (95% C.I.)</th>
<th>R²</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ancestral Pueblo villages, southwest Colorado, USA (1060–1280 CE)</strong></td>
<td>Circumscribing area (ha)</td>
<td>278</td>
<td>0.66 (0.51–0.81)</td>
<td>0.22</td>
<td>(Ortman and Coffey, 2017)</td>
</tr>
<tr>
<td><strong>Ancestral Pueblo villages, southwest Colorado, USA (1060–1280 CE)</strong></td>
<td>Total house area (m²)</td>
<td>130</td>
<td>1.17 (1.04–1.29)</td>
<td>0.74</td>
<td>(Ortman and Coffey, 2017)</td>
</tr>
<tr>
<td><strong>Mandan/Hidatsa villages, North Dakota, USA (1200–1886 CE)</strong></td>
<td>Circumscribing area (ha)</td>
<td>35</td>
<td>0.64 (0.48–0.80)</td>
<td>0.65</td>
<td>(Ortman and Coffey, 2017)</td>
</tr>
<tr>
<td><strong>Mandan/Hidatsa villages, North Dakota, USA (1200–1886 CE)</strong></td>
<td>Mean house area (m²)</td>
<td>17</td>
<td>0.16 (0.04–0.29)</td>
<td>0.31</td>
<td>(Ortman and Coffey, 2017)</td>
</tr>
<tr>
<td><strong>Pueblo villages, northern New Mexico, USA (1200–1600 CE)</strong></td>
<td>Total house area (m²)</td>
<td>165</td>
<td>1.15 (SE = 0.02)</td>
<td>0.95</td>
<td>(Ortman and Davis, 2019)</td>
</tr>
<tr>
<td><strong>Pueblo villages, northern New Mexico, USA (1200–1600 CE)</strong></td>
<td>Decorated pottery consumption rate (sherd/yr.)</td>
<td>224</td>
<td>1.20 (SE = 0.04)</td>
<td>0.81</td>
<td>(Ortman and Davis, 2019)</td>
</tr>
<tr>
<td><strong>Pueblo villages, northern New Mexico, USA (1200–1600 CE)</strong></td>
<td>Stone tool production rate (waste flakes/yr.)</td>
<td>64</td>
<td>0.82 (SE = 0.07)</td>
<td>0.67</td>
<td>(Ortman and Davis, 2019)</td>
</tr>
<tr>
<td><strong>Farming/administrative settlements, central Andes, Peru (1000–1532 CE)</strong></td>
<td>Circumscribing area (ha)</td>
<td>57</td>
<td>0.69 (SE = 0.06)</td>
<td>0.68</td>
<td>(Ortman et al., 2016)</td>
</tr>
<tr>
<td><strong>Herding settlements, central Andes, Peru (1000–1532 CE)</strong></td>
<td>Circumscribing area (ha)</td>
<td>39</td>
<td>0.66 (SE = 0.156)</td>
<td>0.32</td>
<td>(Ortman et al., 2016)</td>
</tr>
<tr>
<td><strong>Wanka settlements, central Andes, Peru (1000–1532 CE)</strong></td>
<td>Domestic structure size (m²)</td>
<td>91</td>
<td>0.13 (SE = 0.04)</td>
<td>0.13</td>
<td>(Ortman et al., 2016)</td>
</tr>
<tr>
<td>‘Amorphous’ settlements (pop. &lt; 5000), Basin of Mexico (1150 BCE–1520 CE)</td>
<td>Settled (circumscribing) area (ha)</td>
<td>1510</td>
<td>0.67 (0.65–0.69)</td>
<td>0.74</td>
<td>(Ortman et al., 2015)</td>
</tr>
<tr>
<td>‘Networked’ settlements (pop. &gt; 5000), Basin of Mexico (1150 BCE–1520 CE)</td>
<td>Settled (outlined) area (ha)</td>
<td>22</td>
<td>0.85 (0.59–1.101)</td>
<td>0.71</td>
<td>(Ortman et al., 2015)</td>
</tr>
<tr>
<td><strong>Pre-Hispanic settlements, Basin of Mexico (1150 BCE–1520 CE)</strong></td>
<td>Civic mound volume (m³/yr)</td>
<td>48</td>
<td>1.18 (1.03–1.33)</td>
<td>0.85</td>
<td>(Ortman et al., 2015)</td>
</tr>
<tr>
<td><strong>Pre-Hispanic settlements, Basin of Mexico (1150 BCE–1520 CE)</strong></td>
<td>Mean domestic mound area (m²)</td>
<td>80</td>
<td>0.19 (0.08–0.29)</td>
<td>0.86</td>
<td>(Ortman et al., 2015)</td>
</tr>
<tr>
<td><strong>Medieval Western European cities and towns (1300 CE)</strong></td>
<td>Settled area (ha)</td>
<td>173</td>
<td>0.71 (SE = 0.03)</td>
<td>0.81</td>
<td>(Cesaretti et al., 2016)</td>
</tr>
<tr>
<td><strong>Greek and Roman cities (100 BCE–300 CE)</strong></td>
<td>Settled area (ha)</td>
<td>53</td>
<td>0.65 (0.59–0.72)</td>
<td>0.88</td>
<td>(Hanson and Ortman, 2017)</td>
</tr>
<tr>
<td><strong>Imperial Roman cities (100 BCE–300 CE)</strong></td>
<td>Occupational diversity</td>
<td>210</td>
<td>0.66 (0.61–0.79)</td>
<td>0.66</td>
<td>(Hanson and Ortman, 2017)</td>
</tr>
<tr>
<td><strong>Imperial Roman Cities (100 BCE–300 CE)</strong></td>
<td>Forum/agora area (m²)</td>
<td>80</td>
<td>0.67 (0.57–0.77)</td>
<td>0.68</td>
<td>(Hanson et al., 2019)</td>
</tr>
<tr>
<td><strong>Imperial Roman cities (100 BCE–300 CE)</strong></td>
<td>Street area (m²)</td>
<td>80</td>
<td>0.67 (0.59–0.75)</td>
<td>0.77</td>
<td>(Hanson et al., 2019)</td>
</tr>
</tbody>
</table>

Notes: SE: standard error (when a confidence interval is not reported).
geographies and levels of socio-economic development. Nevertheless, the entries show a striking consistency of scaling exponents. Almost all of the exponents for the population–area relationship are within a 95% confidence interval of \( 2/3 \leq \alpha \leq 5/6 \), with the specific value of \( \alpha \) being determined by the nature of the area measurement used (an ellipse versus an outline of the built area), as predicted by settlement scaling models. This consistent result shows that, across cultures and history, human settlements become denser, on average and at predictable rates, with population. This in turn indicates that humans in larger settlements live in contexts that facilitate faster rates of social interaction, and thus faster socio-economic production rates.

The results in Table 2 also show that a number of measures of socio-economic output obtainable from the archaeological record, especially public works construction rates and house sizes, exhibit increasing returns to scale to the same degree observed with respect to GDP in Table 1, and as predicted by equation (7). These data derive from a range of contexts, from Native American villages to Pre-Hispanic Mesoamerican and Andean settlements. Finally, Table 2 includes one case where an index of the division of labour – the diversity of professional associations mentioned in stone inscriptions recovered from Imperial Roman cities – scales as predicted by equation (10). All these results are in keeping with the expectations of SST. Importantly, the areas, populations, densities and socio-economic indicators of the archaeological settlements were determined using the methods discussed earlier and are not subject to the modifiable areal unit problem.

The fact that nearly identical results have been obtained for contemporary urban systems using metropolitan statistical areas as the spatial units of analysis (see Bettencourt, 2013: table S3) suggests that archaeological settlements are functionally analogous to MSAs; that MSAs and their analogues are appropriate units for urban scaling research; and that the social networking processes revealed through the use of MSAs reflect fundamental aspects of urbanisation as a social process in human history and development. In short, SST appears to account not only for general properties of contemporary urban systems, but for general properties of human settlements and their evolution throughout history.

Still, comparison of the results in Tables 1 and 2 does highlight an important difference between human settlement systems of the past and present. The data analysed in Table 1 are for the most part cross-sectional, representing aggregate properties of cities at a given moment in time. In contrast, most of the data analysed in Table 2 reflect temporal averages that may span centuries. When one examines scaling relations in contemporary systems one typically finds that the pre-factor of the scaling relation, or the intercept of the line fit to the log-transformed values, changes from year to year. According to SST, this changing intercept is driven by changes in transport costs and the productivity of interaction, which are associated with processes of economic growth and technological change. The fact that such temporal dependence is not typical of archaeological cases suggests that rates of change in these parameters were much slower (and indeed in many cases perhaps vanishing in the long run) in pre-industrial societies than they are today.

**Discussion**

We have shown that the contemporary concept of the metropolitan area, and the archaeological concept of the archaeological settlement, both capture the fundamental nature of cities as containers for frequent and heterogeneous social interaction.
Archaeologists have developed a robust set of tools for estimating both the relative and absolute populations that lived within them. These methods have been tested against historical records, ethnographic reports and material patterns in present-day traditional communities and proven to be consistent and reliable. Finally, the archaeological record provides material proxies for a variety of socio-economic measures, including wealth distributions and consumption rates, which are measurable for settlements of any size. Owing to the vagaries of preservation and the complexities of sampling in archaeological sites there are always errors associated with these measurements. However, these errors are typically unstructured relative to site size (i.e. they are not systematic) so that it is feasible to recover the average statistical relationships between population and other quantities using archaeological data, despite their noisy nature.

We have also shown that many aggregate properties of ancient settlement systems are analogous to those of contemporary metropolitan systems. The interdependences between spatial, infrastructural and social facets of cities and settlements seem to be quantitatively and qualitatively consistent across time, geography and cultures. Settlement scaling theory provides a unified quantitative framework for understanding and predicting these regularities and therefore gives urban studies a formal framework for their investigation across time and space. Such comparisons serve to strengthen our confidence in the empirical findings for modern urban systems and on the use of metropolitan areas as the principal spatial units of analysis in urban studies.

The incorporation of archaeological and historical data into scaling research dramatically expands the empirical basis for investigating the ways in which human societies can, in principle, harness the open-ended, exponential properties of spatially embedded social networks. This approach does not merely provide a way of verifying patterns identified in contemporary data, but it actually expands the effort by bringing to light a wider range of ways in which humans take advantage of social networking in space than is apparent in contemporary data. Some of these insights may not be ‘on the radar’ of modern urbanists; but in the context of a general theory that applies to all human networks they can be expected to have practical relevance for the future.

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