Learning Goals
Phys 3220 – Quantum I

Phys 3220 introduces students to the formal theory of quantum mechanics. Most of the class focuses on problems in one dimension although the class also covers problems such as the hydrogen atom, angular momentum, and spin. References below to "quantum mechanics problems" refer only to problems at this level as defined more clearly in the subject-specific goals.

Course Scale Learning Goals

- **Math/physics connection:** Students should be able to translate a physical description of a junior-level quantum mechanics problem into the mathematical equation necessary to solve it. Students should be able to explain the physical meaning of the formal and/or mathematical formulation of and/or solution to a junior-level quantum mechanics problem. Students should be able to achieve physical insight through the mathematics of a problem.
- **Visualization:** Students should be able to sketch the physical parameters of a problem (*e.g.*, wave function, potential, probability distribution), as appropriate for a particular problem. When presented with a graph of a wave function or probability density, students should be able to derive appropriate physical parameters of a system.
- **Knowledge Organization:** Students should be able to articulate the big ideas from each content area, and/or lecture, thus indicating that they have organized their content knowledge. They should be able to filter this knowledge to access the information that they need to apply to a particular physical problem. This organizational process should build on knowledge gained in earlier physics classes.
- **Communication:** Students should be able to justify and explain their thinking and/or approach to a problem or physical situation, in either written or oral form.
- **Problem-solving Techniques:** When faced with a quantum mechanics problem, students should be able to choose and apply appropriate problem solving techniques. They should be able to transfer the techniques learned in class and through homework to novel contexts (*i.e.*, to solve problems which do not map directly to those in the book). They should be able to justify their selected approach (see "Communication" above). In addition to building on techniques learned in previous courses (*e.g.*, recognizing boundary conditions, setting up and solving differential equations, separation of variables, power-series solutions, operator methods), students are expected to develop specific new techniques as listed in concept-scale learning goals below.
  - **Approximations:** Students should be able to recognize when approximations are useful, and to use them effectively (*e.g.*, when the energy is very high, or barrier width very wide). Students should be able to indicate how many terms of a series solution must be retained to obtain a solution of a given order.
  - **Symmetries:** Students should be able to recognize symmetries and be able to take advantage of them in order to choose the appropriate method for solving a problem (*e.g.*, when parity allows you to eliminate certain solutions).

- **Problem-solving Strategies:** Students should be able to draw upon their knowledge and skills to attack a problem even when a process leading to a correct solution is not yet clear. Students should continue to develop their ability to monitor their progress towards a solution by learning how to:
  - Backtrack and try a new approach when necessary
  - Recognize when a solution has been reached and be able to articulate why this solution is valid (see "Expecting and Checking Solution" below)
  - Persist through to the solution of problems requiring many steps

  Students should be able to articulate what it is that needs to be solved in a particular problem and to know when they have solved it.

- **Expecting and Checking Solution:** When appropriate for a given problem, students should be able to articulate their expectations for the solution to a problem, such as:
  - The general shape of the wave function
  - Dependence on coordinate choice
  - Behavior at large distances
  - Problem symmetry

  For all problems, students should be able to justify the reasonableness of a solution they have reached, by using methods such as:
  - Checking solution symmetry
  - Verifying boundary conditions
  - Order of magnitude estimates
  - Dimensional analysis
  - Limiting or special cases (*e.g.*, checking the solution for correct behavior in limiting or known cases)

- **Intellectual Maturity:** Students should accept full responsibility for their own learning. They should be aware of what they do and do not understand about physical phenomena and classes of problem. They should learn to ask sophisticated, specific questions. Students should learn to identify and articulate where in a problem they experienced difficulty and to take appropriate action to move beyond that difficulty. Finally, they should regularly check their understanding against these learning goals and seek out appropriate help to fill in any gaps.

- **Coherent Theory:** Students should recognize that the material covered in this course sets a framework for a consistent and complete understanding of quantum mechanics.

- **Build on Earlier Material:** While the material in the course represents a significant departure from earlier course work both mathematically and conceptually, students should recognize and make use of connections to prior work, techniques and understanding gained in classes in classical physics as well as in their modern physics class.

- **Examples from Recent Research:** Whenever possible, examples and homework problems should be drawn from recent research results (nominally defined as the last thirty years).

Subject Scale Learning Goals

The goals below pertain to specific areas in the study of quantum mechanics which are to be learned in this course. They are organized by subject and thus do not follow any textbook. The subject categories are:

- Mathematics
- Measurement and the quantum state
- The Schrödinger Equation
- Formalism
- Important Systems
- Scattering
- Angular Momentum and Spin

Prerequisites for Mathematics Students should already be able to . . .

- **Differential Equations:**
  - solve straightforward first and second order differential equations using a variety of methods.
  - recognize when separation of variables will simplify a differential equation and correctly apply the technique.
- **Complex Numbers:** Students should be thoroughly familiar with complex numbers and be able to find the real part, the imaginary part, the complex conjugate and the norm of any complex expression.
- **Linear Algebra:** Given a matrix operator, students should be able to find the eigenvalues and eigenvectors of the operator. Students should not only be able to diagonalize the matrix but be able to explain the physical significance of the procedure and the result.
- **Hamiltonian Formalism:**
  - Students should be able to set up the Hamiltonian for a classical system.

Subject Goals for Mathematics

- **Statistics:** Due to the statistical nature of quantum mechanics, students should be adept at computing probabilities and standard deviations.
- **Dirac Delta Function:** Students should be able to correctly compute integrals which contain one or more Dirac delta functions.
- **Vector Spaces:**

- Given a set of real or abstract (*e.g.*, Hilbert space) vectors, students should be able to determine whether the set constitutes a vector space.
- Given a set of real or abstract (*e.g.*, Hilbert space) vectors, students should be able to determine whether or not they form a basis of a given vector space.

- **Hilbert Space:** Students should be able to compute the correct coefficients of a Hilbert space vector given a basis.

- **Operator Theory:** Students should be able to compute the expectation value of an operator in a given state. More generally, they should be able to compute all the matrix elements of an operator in a given basis. They should be able to identify a Hermitian operator.

Subject Goals for Measurement and the Quantum State

- **The State Vector:**
  - Students should be able to correctly normalize a (normalizable) quantum state.[1]
  - Students should be able to describe and calculate different representations of a quantum state (*e.g.*, position space, momentum space).[1]

- **Observable Operators:**
  - Students will know that observable quantities are represented by Hermitian operators.
  - Given a wave function and an observable operator, students will be able to calculate that operator's expectation value.
  - For simple systems (*e.g.*, 1-D infinite square well), students will be able to find the eigenvectors and eigenvalues for the energy operator.

- **Measurement Predictions:**
  - Given the eigenstates of an operator, students will be able to compute the possible results of a measurement of the observable which corresponds to that operator.[1]
  - Given a quantum state and the eigenbasis of an observable operator, students will be able to compute the probabilities of obtaining the possible values which would result from a measurement of the corresponding observable quantity.
  - Given the results of a repeated measurement of an observable on a quantum state, students should be able to construct a plausible quantum state as a superposition of the eigenstates of the operator associated with the observable.[1]

- **Measurement Effects:**
  - Students will be able to describe what is known about the state of a system immediately after a measurement, including the significance of the measured value.[1]

[1]Targeted in QMAT

- **Time Evolution:**
  - Given an initial wave function and a basis of energy eigenstates, students should be able to find the time-dependent wave function.
  - Given an initial wave function and a basis of energy eigenstates, students should be able to deduce when the probability distribution of an operator will be time dependent.[1]
- **Operator Commutation and Compatibility:**
  - Students should be able to describe the relationship which must exist between two operators in order for a common eigenbasis to exist.
  - Students should be able to compute the commutator of the position and momentum operators as well as the commutation relationships between angular momentum operators.
  - Students should be able to describe the effect of following the measurement of an observable with the measurement of an incompatible operator.[1]
  - Given two non-commuting observables, A and B and the result of a measurement of A, students should be able to compute the possible outcomes of a subsequent measurement of B along with the appropriate probabilities.

Subject Goals for the Schrödinger Equation

- **Time Dependent Schrödinger Equation:** Students will be able to use the time dependent Schrödinger Equation to compute the time evolution of a wave function.
- **Time Independent Schrödinger Equation:** Students will be able to describe the conditions under which separation of variables can be used to create a time independent Schrödinger Equation and will be able to use this equation to:
  - solve for the energy levels of the system
  - apply boundary conditions and solve for the stationary states (energy eigenstates) of the system
  - apply the Hamiltonian and boundary conditions to determine whether the energy eigenstates are discrete or continuous.
  - specify the evolution in time of a system when both an initial state and the energy eigenstates known

Subject Goals for Formalism

[1] Targeted in QMAT

- **Normalization:**
  - Students will be able to explain the relationship between the normalization of a wave function and the ability to correctly calculate expectation values or probability densities.
  - Students should be able to correctly normalize any wave function which represents a physically realizable state.

- **Hamiltonian:**
  - Students should be able to set up the Hamiltonian for a quantum mechanical system when they can calculate the potential energy for the corresponding classical system.[1]
  - Students should be able to use commutation relations to be able to determine which operators have eigenstates which are time independent.[1]

- **Uncertainty Principle:**
  - Given a quantum state and an observable, students should be able to compute the uncertainty (standard deviation in the measurement) of the observable.
  - Given two observables, students should be able to compute the minimum uncertainty of measuring both observables on any quantum state.

- **Probability in Quantum Mechanics:**
  - Given a (time-dependent) wave function, students should be able to compute the time-dependent probability density.[1]
  - For a given quantum state, students should be able to compute the probability of measuring any particular value for any common observable.[1]

Subject Goals for Important Systems

- **Infinite Square Well:** Students should be thoroughly familiar with all aspects of the one dimensional infinite square well.
  - Given the size and position of the potential, students should be able to compute the energy eigenvalues and the energy eigenstate position-space wave functions.
  - Students should be able to be compute the time evolution of a superposition of energy eigenstates as well as the expectation value of common observables for a superposition state.

- **General One-dimensional Systems:**
  - Given a one-dimensional potential, students should be able to sketch the first few energy eigenstates.

[1] Targeted in QMAT

- **Finite Square Well:** Students should be able to sketch the wave function for a system with one or more finite square wells. They should be able to qualitatively predict the time evolution of the wave function given an initial state.
- **Harmonic Oscillator:**
  - Given a specific harmonic-oscillator potential, students should be able to compute the energy eigenvalues.
  - Given the raising and lowering operators, students should be able to find the lowest energy eigenstate.
  - Given the raising and lowering operators and an energy-eigenstate wave function, students should be able to find the energy eigenstates on either side.
  - Students should be able to sketch the first few energy eigenstates of the harmonic oscillator.
  - Students should be able to compute position and momentum expectation values using the raising and lowering operators.
- **Free Particle:** Students should be adept at using the position-space and momentum-space wave functions of the free particle. In particular, they should be able to use them to construct wave packets.
- **Two-State Systems:**
  - Given a two-dimensional Hamiltonian, students should be able to find its eigenstates and eigenvalues.
  - Given a two-state system in a superposition state, students should be able to correctly compute the probabilities of measuring each eigenvalue.
- **Hydrogen Atom:**
  - Students should be able to set up the Schrödinger equation for a hydrogen-like atoms.
  - Students should be able to perform variable separation on the SE for hydrogen.
  - Students should be able to describe the energy eigenstates for hydrogen-like atoms including the significance and use of their quantum numbers.

Subject Goals for Angular Momentum and Spin

- **Angular Momentum in Quantum Mechanics:**
  - Students will be able to compute the angular momentum of a system in a known eigenstate of an angular momentum operator (*e.g.*, $\hat{L}^2$, $\hat{L}_z$)
  - Given a system in a known state, students will be able to compute the probabilities of the possible results of measuring an angular momentum observable (*e.g.*, L^2, L_z, L_y)

- **Spin:**
  - Given a system in a known state, students will be able to compute the probabilities of the possible results of measuring a spin observable (*e.g.*, S^2, S_z, S_y)