Engineering Plus
UNIVERSITY OF COLORADO **BOULDER**

Spatial Visualization

Workshop 3: One-Axis Rotations

Overview

Grade level: undergraduate engineering students

Estimated time required: 2 hours

Expendable materials: isometric paper printouts, scrap paper and pencils

Reusable materials: workbooks, software, computers, snap cubes (used for all workshops)

See Materials List (below) for details.

Summary

In this workshop, students learn about one-axis rotations with a focus on how to rotate objects physically and mentally. They begin by building an object with cube blocks, defining a one-axis rotation, and then drawing the isometric views of the object before and after the rotation. Then they individually apply the same rotation concepts to workbook problems. Next, they pair up with a partner to test their exclusively mental spatial visualization abilities at a station with no rotating built objects. Lastly, students further solidify one-axis rotation concepts via computer software exercises.

Engineering Connection

Rotating objects is a spatial visualization technique that is essential when performing many tasks in engineering. Engineers use one-axis rotation drawings to better visualize 3D objects to make sure their ideas represented on 2D screens and paper work in the real (3D) world. It is the responsibility of engineers to make sure their designs, products and inventions function and operate correctly, and visualizing their designs is the first step. For example, mechanical engineers use CAD to build, rotate and confirm that parts function and fit together before manufacturing. Mechanisms typically have components that rotate relative to one another; to be able to visualize these rotations in space while the mechanism is functioning requires an understanding of one-axis rotations. Spatial visualization is an essential skill for engineers to be able to take ideas to the next level, that is—ideas that initially only exist in the mind to something that can be communicated clearly to other engineers, and eventually turned into products, structures and systems.

Pre-Requisite Knowledge

Before taking part in this spatial visualization lesson, students should have learned about spatial visualization in the previous two workshops. They should have a firm knowledge about isometric drawing, how to use isometric paper and coded plans (as taught in the previous lesson, *Isometric Drawings and Coded Plans*), as well as orthographic views (as taught in the previous lesson, *Orthographic Views*). Students should be familiar with the Cartesian coordinate system (x-, y-, z-axes) and degrees of rotation.

Learning Objectives

After this workshop, students should be able to:

✓ Rotate objects about one axis.

✓ Translate the rotation of one block to a second block.

✓ Use the right-hand rule to explore rotations of objects.

Materials List

Item	Description & Amount	Source/Supplier/Cost	Station #s
Isometric paper	Isometric, three-dot paper for sketching, 2+ per student	Isometric Paper (pdf); print double-sided as needed	1, 2, 3
Snap cubes	Interlocking cubes, 8 cubes per student	Such as: Higher Ed Services ($3 for 15 cubes) or Amazon ($85 for 200)	1, 3
Pencil with eraser	Any erasable pencil; pens are not recommended	School/students provide	1, 2, 3
Colored pencils	To associate drawings to the snap cubes; to share among all students	Such as: Crayola colored pencils at Amazon ($8 for 50)	1, 3
Developing Spatial Thinking *Workbook & Instructional Software*	1 per student pair; includes web-based software used with workbook	$40 for workbook/software from Higher Ed Services	2 (workbook) 4 (software)
Computer or laptop	1 per student pair	School/students provide	4
Workbook answers	For the instructor and for students to check their work	Workbook Answer Key (pdf)	2
PowerPoint® presentation	For the instructor, slides 22-27	Spatial Visualization Presentation (pptx)	NA

Introduction & Motivation

Have the slide presentation up and displayed to the class, starting with slide 22.

Today, we are going to learn about rotations. Rotating objects is an essential skill in the engineering world and it's something we most likely already do every day.

We use object rotation to understand the world around us—what objects are and how they fit together. For example, how do you know which direction to put a key in a lock? How do you organize your room to optimize space? How many times do you rotate a USB stick before getting it to fit into a computer slot? These tasks require the ability to visualize the one-dimensional rotations of objects.

In this lesson, we are going to rotate both isometric drawings and blocks. Objects can be rotated about the three axes—x, y and z—and any *combination* of the three. We'll start out rotating about one-axis.

Show students slide 23, which is the same as Figure 1.
Look at the top row of this slide. Notice that the first block on the left is the same block as on the right; the object has simply been rotated about an axis. Now try to picture in your mind how the block rotates. What are some strategies you used to decide how it rotates? Which answer in the second row follows the same rotation as the first? *(Answer: B)*
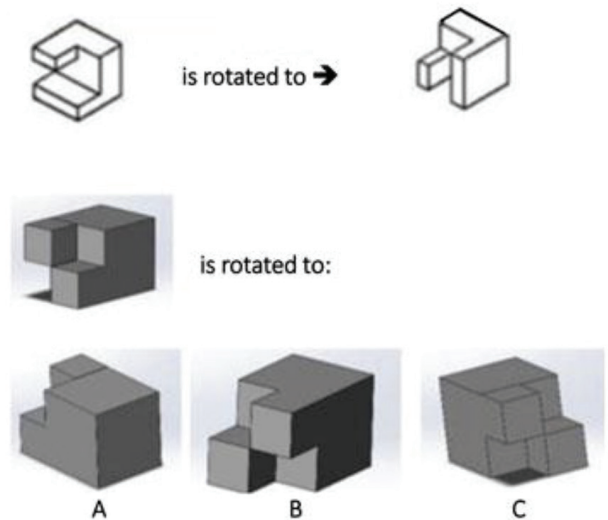
Show students slide 24, which is the same as Figure 2.
Throughout the workshop, this is how we will define our axes: the x-axis runs along the horizontal direction, the y-axis follows the vertical direction (out of the page), and the z-axis is also along the horizontal direction but orthogonal to both the x and y-axes. The axes shown are the positive axes.

Looking back at the previous slide *(slide 23)*, what axis is this object rotated about? *(Answer: Z)* And how many degrees is it rotated? *(Answer: 90 degrees)*

Now we need to define the difference between a positive and a negative rotation using the right-hand rule *(show slide 25)*. The **right-hand rule** is often used in physics and math to calculate vector cross products. The right-hand rule works as follows: Point your thumb along the positive axis you are rotating about and curve your fingers naturally towards the palm of your hand. The direction your fingers curl is the direction of positive rotation about the axis. The axes shown in the previous slide *(slide 24)* are the positive axes, and if you flip the axis 180 degrees, you get the negative axes.
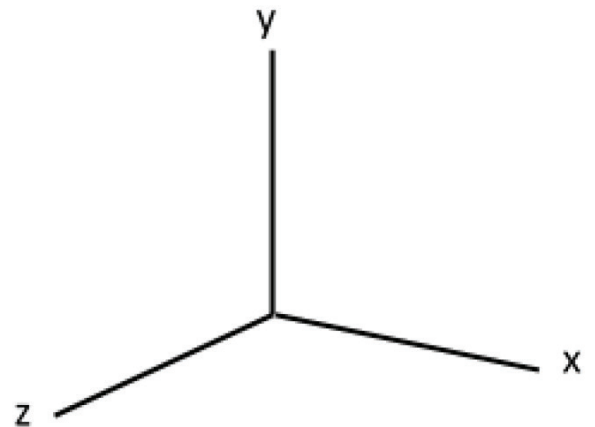

Figure 1. *Which rotation of the gray object is analogous to the rotation of the white object?*

Figure 2. *Three positive axes, x, y and z.*

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Introduction & Motivation Continued

Everyone: Give your neighbor a right-handed thumbs up! Notice that your thumb is along the positive y-axis and your other fingers are rotating counter-clockwise. Now let's try a thumbs-down. Notice that your thumb is now along the negative y-axis and your fingers are rotating clockwise.

Let's look back at our example *(slide 23)*. When we point our right thumbs along the z-axis to rotate this object, we can point towards ourselves or away from us. To rotate this block, which way would we point our thumbs? *(Answer: Towards us)* So, this is a positive rotation.

We will use the following tips *(slide 26)* throughout the workshop today. Remember to only use your RIGHT hand when practicing the right-hand rule; using your left hand will only cause confusion. Also, keep in mind that clockwise implies a negative rotation since counter-clockwise implies a positive rotation. Another useful method is to think of your thumb, or the axes, as a *fixed* flagpole and your object, or fingers, rotating around it like a flag. Using these methods, we will now practice applying one-axis rotations.

Slide 27 recaps the four workshop stations. Leave slide 27 up so students know what to accomplish at each station.

Vocabulary

Term	Definition
coded plan	A two-dimensional depiction of an object that defines the volume based on the top view of the object.
isometric	Of or having equal dimensions. The isometric view of an object is the angle at which an equal angle (120°) exists between all axes (such as looking down a corner of the object).
isometric paper	A grid of dots arranged equidistant from one another. Used in making isometric sketches. Also called three-dot paper.
one-axis rotation	The result of rotating an object solely about a single axis. In this workshop, the rotations only occur in 90º increments. Also called single-axis rotation.
orthographic view	A way to draw an object that shows three views of an object from the three planes in an orthogonal (right-angle) coordinate system. The views represent the exact shape of an object as seen from one side at a time as you are looking perpendicularly to it. Depth is not shown. An orthographic drawing is also called a multi-view drawing.
right-hand rule	A useful memory tool in the rotation of objects that uses a person's right hand and fingers to help in understanding orientation conventions for vectors in three dimensions. Often used in physics and math.

Lesson Procedure

Before the Workshop

✅ Gather materials.

✅ Make copies of the Isometric Paper. Print double-sided to reduce paper use and increase drawing area per page.

✅ Prepare a way to digitally get students the web-based software link they will need for this lesson such as by email or by preparing a workshop website with the link.

✅ Prepare to project the Spatial Visualization Presentation, a PowerPoint® file, and use its content to aid in your instruction, as makes sense for your class. Slides 22-27 support this lesson. The slides are animated so a mouse or keyboard click brings up the next graphic or text.

✅ Prepare the four stations, as described in slide 27 and Figure 3.

Station 1: Block Relay	Station 3: Frozen
• Build an object with 6 blocks • Define the **axes** and a **one-axis rotation** • Draw the **isometric views** before and after rotation • Then pass the object to a neighbor and repeat • Check drawings for accuracy with neighbor	• In pairs, build an object of any size • Define the **axis** and a **one-axis rotation** • Switch objects with partner and draw the **isometric view** before and after rotation without moving the object - **it's frozen!** • Compare drawings with partner and repeat
Station 2: Workbook Drill	**Station 4: Computer-Aided Visualization**
• Individually, answer questions on **ODD** pages rot1-5 to rot1-17 on scrap paper • Check your answers with the answer key	• Read *Module 7 - Rotations about a Single Axis* • Complete the **rotations about a single axis** exercises

Figure 3. *The four stations for Workshop Three.*

With the students: Introduction

✅ Present to the class the Introduction & Motivation content, supported by slides 22-27.

✅ Divide the class into four groups that will rotate through the four stations.

✅ Give each student 2 pieces of isometric paper and 8 cubes.

✅ Explain that students are to primarily work independently on their drawings, but feel free to share and troubleshoot with their partners.

✅ Direct students to work on one piece of paper until it is full.

Lesson Procedure Continued

Station 1: Block Relay
Students begin by each building a shape with six blocks. Next, they define the axes and a one-axis rotation about the desired axis. Then they draw the isometric views before and after the defined rotation. Then they pass the object to a neighbor and repeat. After students have completed drawing both their own objects and their neighbors' objects, have them collaborate and check their drawings for accuracy.

Station 2: Workbook Drill
Have students individually answer the workbook questions on odd pages rot1-5 to rot1-17, recording their answers on scrap paper. After they have completed the problems, have them check their answers with the answer key. Then have them communicate with neighboring students about answers they may have gotten wrong and collaborate on ways to understand and solve the problems.

Station 3: Frozen
Working in pairs, have students each build an object of any size (best to start off with objects composed of 5-8 cubes and then add more). Then have them define the axes about the shape and a one-axis rotation. Then switch objects with a partner and draw the isometric view before and after the rotation *without moving the object*—consider it frozen! Then have them compare drawings with their partners and repeat.

Station 4: Computer-Aided Visualization
Have students individually read *Module 7 – Rotations about a Single Axis* from the *Developing Spatial Thinking Software* and complete all rotations about a single axis exercises.

Instructor Tips
- When first learning about one-axis rotations, students sometimes try to rotate the axis itself rather than the shape about the axis. Emphasize that the axis is fixed by reiterating the flag pole example. Use a pencil as an example axis and demonstrate an object rotating around it.

- Point out to students that they are *rotating* the object about the axis, not "tipping" it.

- The Station 2 workbook pages are more difficult than those in the previous workshops. For additional help, have students draw the problems on scrap paper to aid in solving them. It may also help to build the shapes for the first few problems as they get used to rotations.

- For complex shapes, have students focus on just one part of the shape throughout a rotation and visualize where that part ends up.

- During Station 3, because the object is frozen, it is crucial that the axis it is being rotated about is well defined. To avoid confusion, have students define the axes to their partner visually and out loud. Place colored pencils next to the object to visually distinguish the various axes of rotations.