Statistics + Geochronology

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AGeS WORKSHOP 2022

Interpreting data is core to accurate and precise scientific interpretations.

"The uncertainty of a date is as important as the date itself."
– Ken Ludwig

photo credit: nps.gov

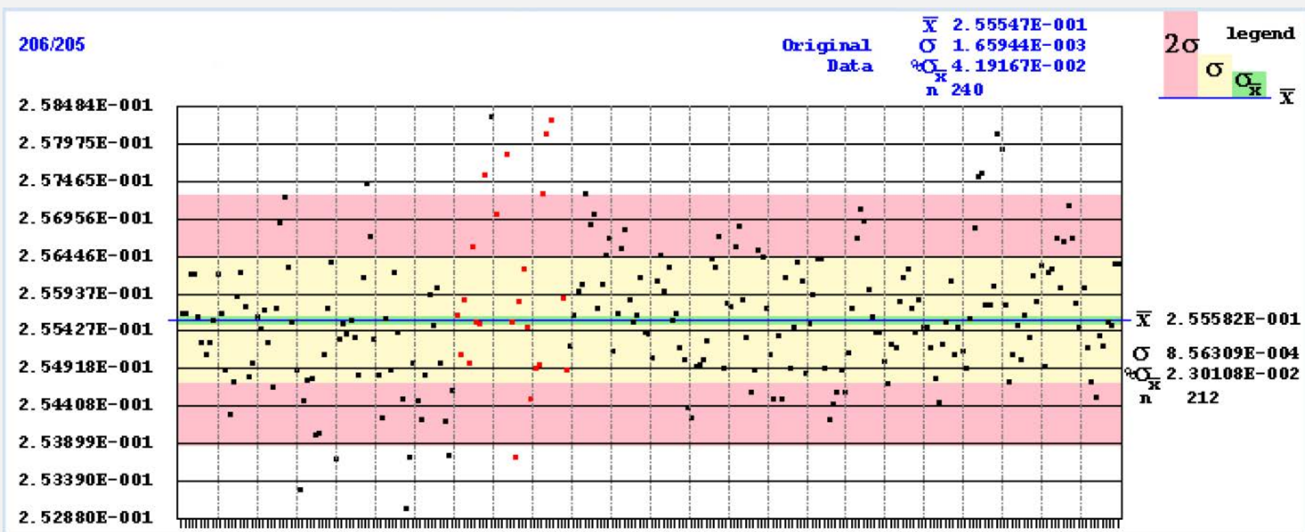
Statistics is a cross-cutting theme!

Where do statistics enter the picture?

1. Making measurements (isotope ratios, fission track counts, luminescence, etc)
2. Calculating dates
3. Interpreting dates as ages
4. Comparing ages
5. Putting together age models

Making measurements

- How do we best interpret the data we have now?
- Can we make better (more, faster, more precise) measurements with the samples we have?

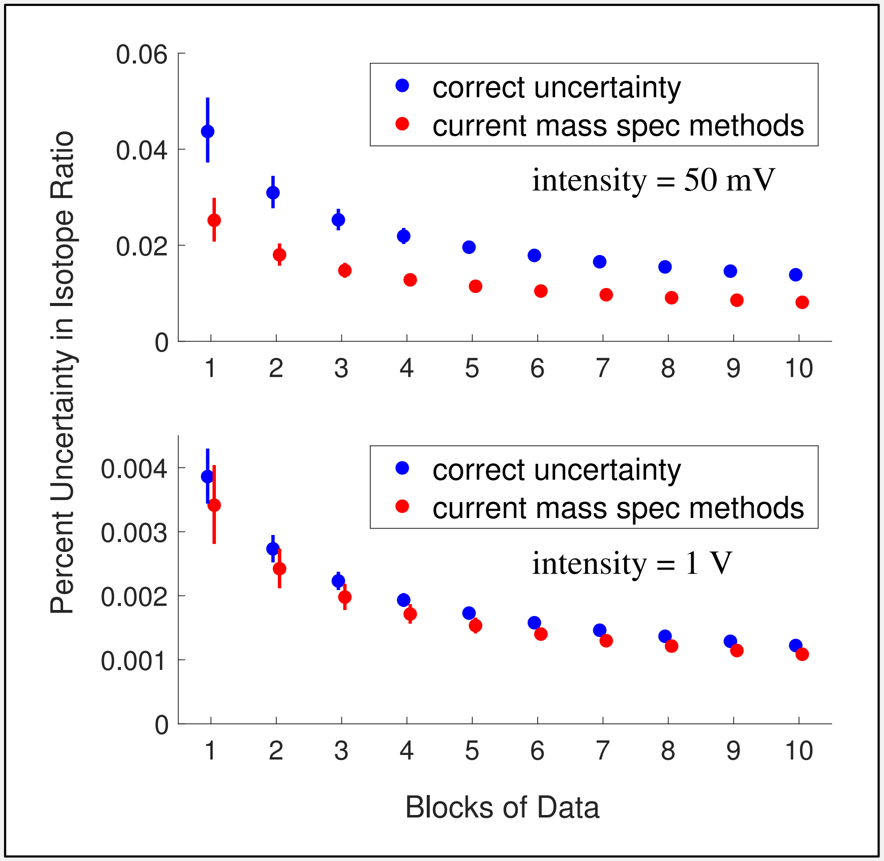


Current data 1: What's the average ratio?

intensities		ratios	
a	b	a/b	b/a
3.70	2.32	1.59	0.63
1.61	1.89	0.85	1.17
2.25	3.74	0.60	1.66
0.91	2.67	0.34	2.93
1.06	2.95	0.36	2.78
1.17	2.02	0.58	1.73
2.24	1.75	1.28	0.78
1.48	1.57	0.94	1.06
2.09	1.95	1.07	0.93
2.07	1.10	1.88	0.53
arith. mean:		**0.950**	**1.421**

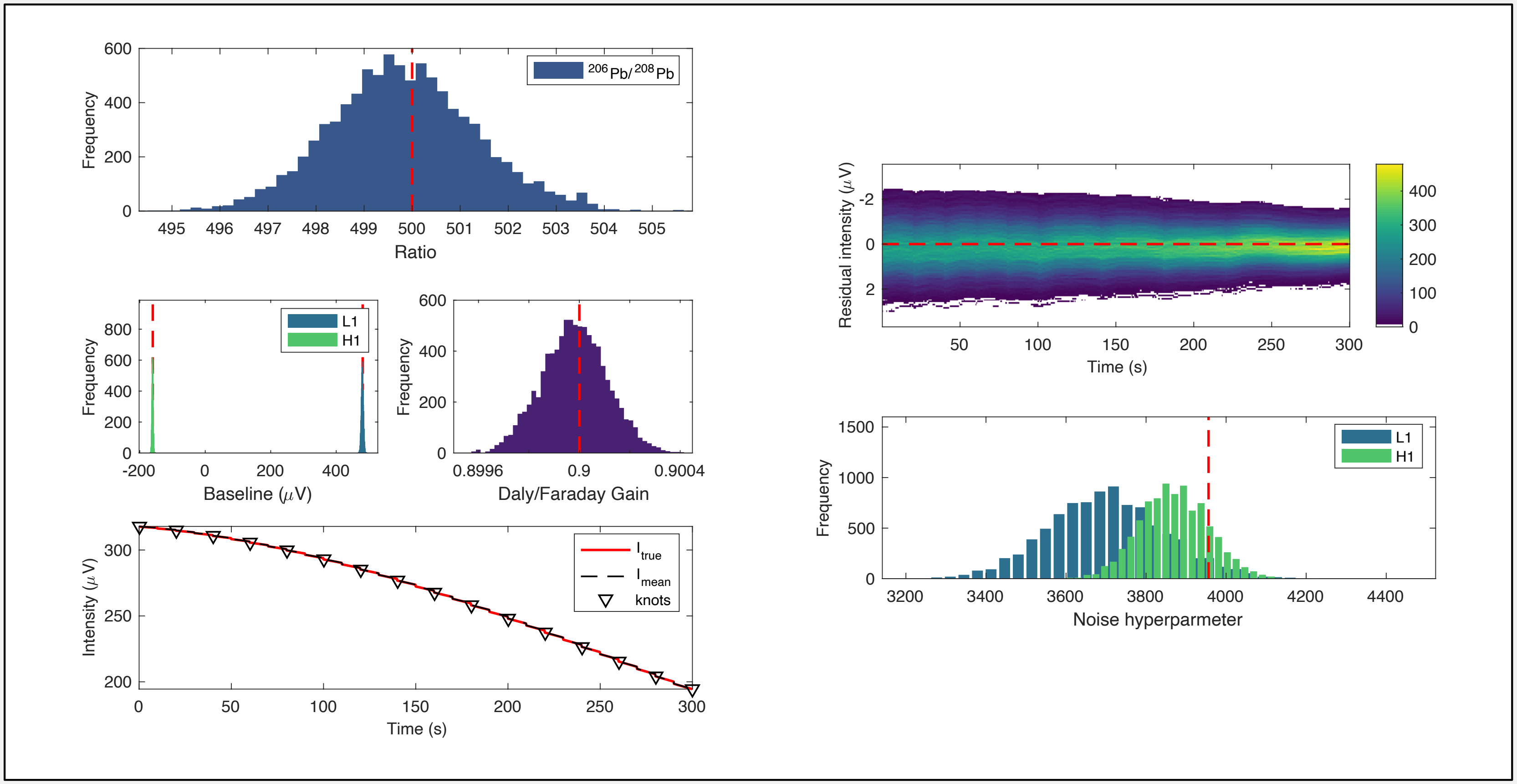
- Traditional method: use measured intensities to calculate an isotope ratio
- Take the average of the ratios
- Problem!!! $a/b \neq b/a$
- Solution: use a different mean

Current data 2: Correcting for baselines



- Traditional method: subtract an average baseline from each ion beam intensity
- Average the resulting ratios
- Problem!!! Violate assumption of independence
- Solution: use a new calculation

Better data?
Mass spectrometer as a seismic network



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Calculating Dates 1: What's the equation?

<u>data</u>

$ratio_1$
$ratio_2$
$ratio_3$
$ratio_4$
$ratio_5$
$ratio_6$
$mass_7$
$mass_8$
$\vdots$

→ data reduction

$$z = f(x,\, y)$$

<u>model</u>

$date_1$
$date_2$
$ratio_3$
$ratio_4$
$\vdots$

Calculating Dates 2: Uncertainty Propagation

- A. Linear uncertainty propagation

$$z = f(x,\ y)$$

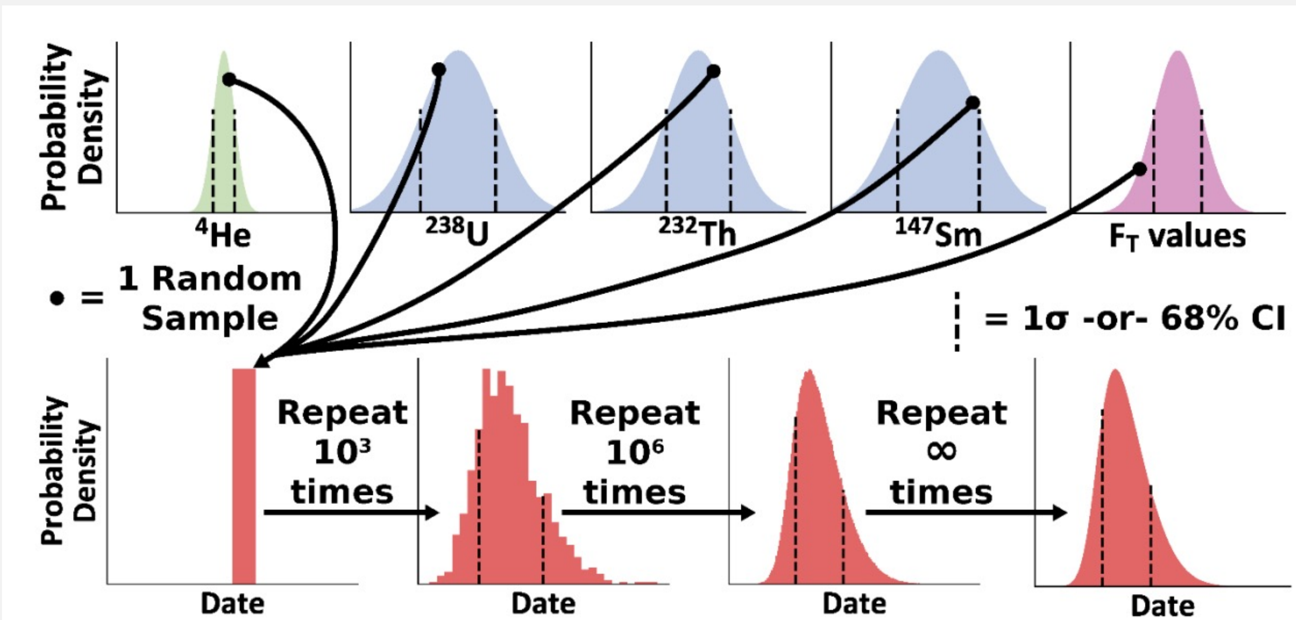
$$\sigma_z^2 = \sigma_x^2 \left(\frac{dz}{dx} \right)^2 + 2\sigma_{xy}^2 \left(\frac{dz}{dx} \right) \left(\frac{dz}{dy} \right) + \sigma_y^2 \left(\frac{dz}{dy} \right)^2$$

$$\sigma_z^2 = \begin{bmatrix} \frac{dz}{dx} & \frac{dz}{dy} \end{bmatrix} \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{xy}^2 & \sigma_y^2 \end{bmatrix} \begin{bmatrix} \frac{dz}{dx} \\ \frac{dz}{dy} \end{bmatrix}$$

McLean et al., 2011, G[3]

Calculating Dates 2: Uncertainty Propagation

- B. Monte Carlo uncertainty propagation



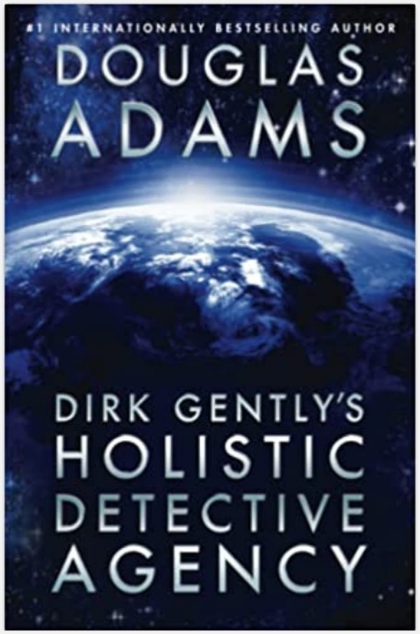
Martin et al., Gchron 2022, preprint

Calculating Dates 3: Correlation

- There are often deep connections among your measurements/variables!

$$\sigma_z^2 = \sigma_x^2 \left(\frac{dz}{dx}\right)^2 + 2\sigma_{xy}^2 \left(\frac{dz}{dx}\right)\left(\frac{dz}{dy}\right) + \sigma_y^2 \left(\frac{dz}{dy}\right)^2$$

DIRK GENTLY'S HOLISTIC DETECTIVE AGENCY
We solve the *whole* crime
We find the *whole* person
Phone today for the *whole* solution to your problem
(Missing cats and messy divorces a specialty)



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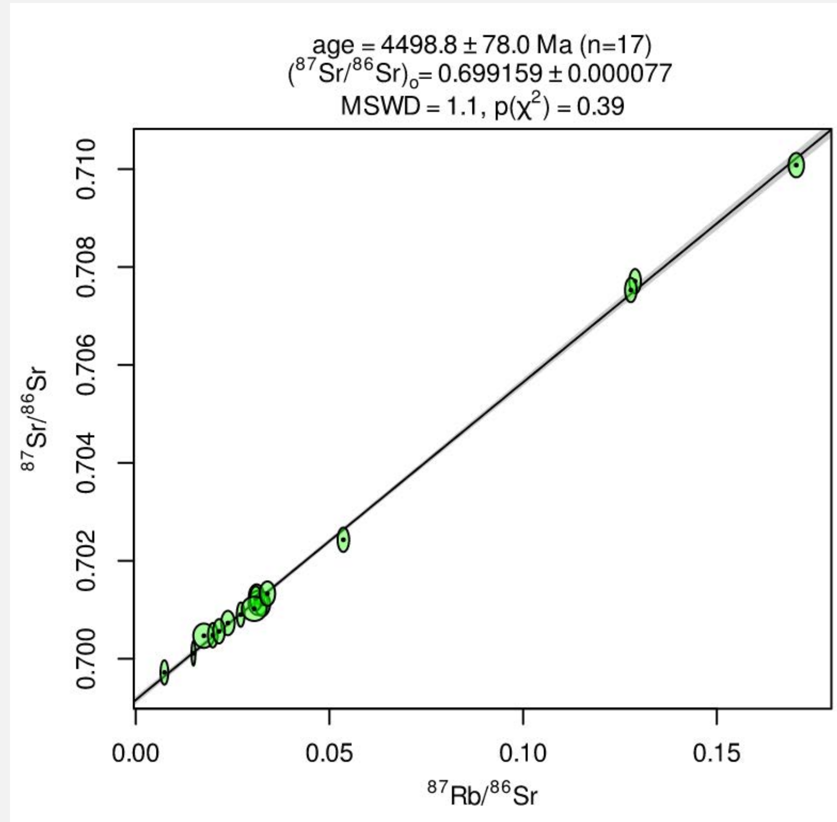
Age interpretation: Combining measurements

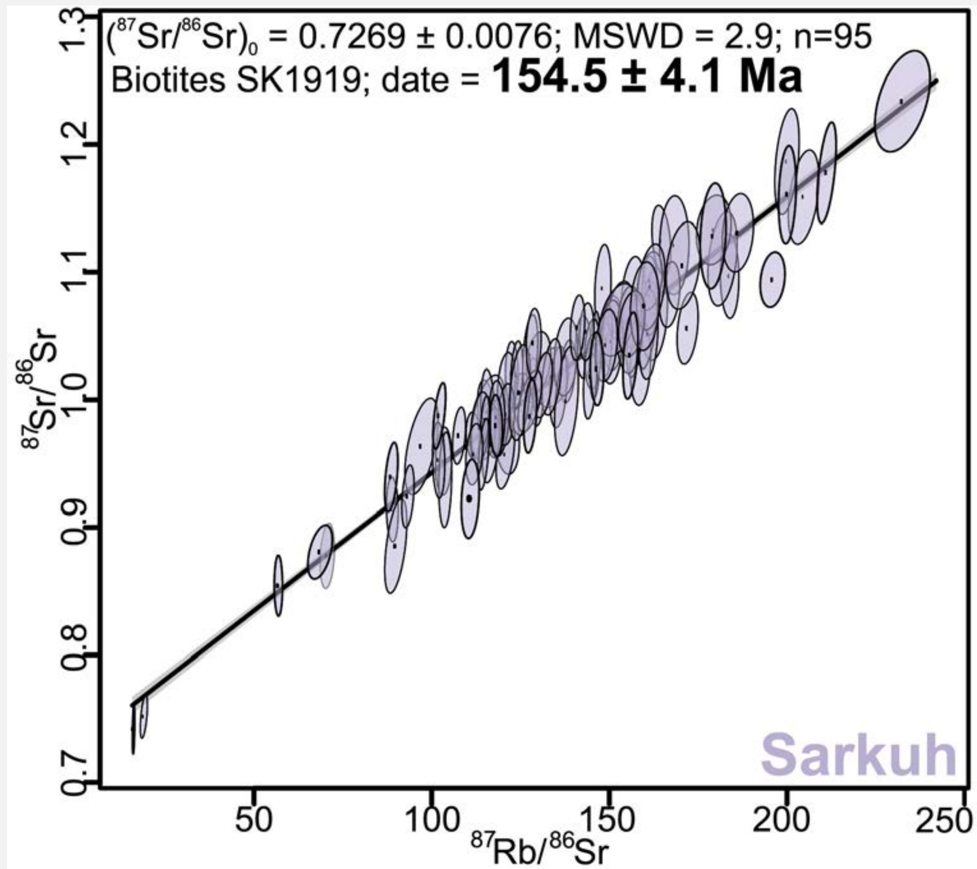
- Weighted means, regression
- Are your assumptions valid?
- What do you do if not?

Vermeesch, P., 2018, IsoplotR: a free and open toolbox for geochronology. *Geoscience Frontiers*



Age interpretation: robust methods

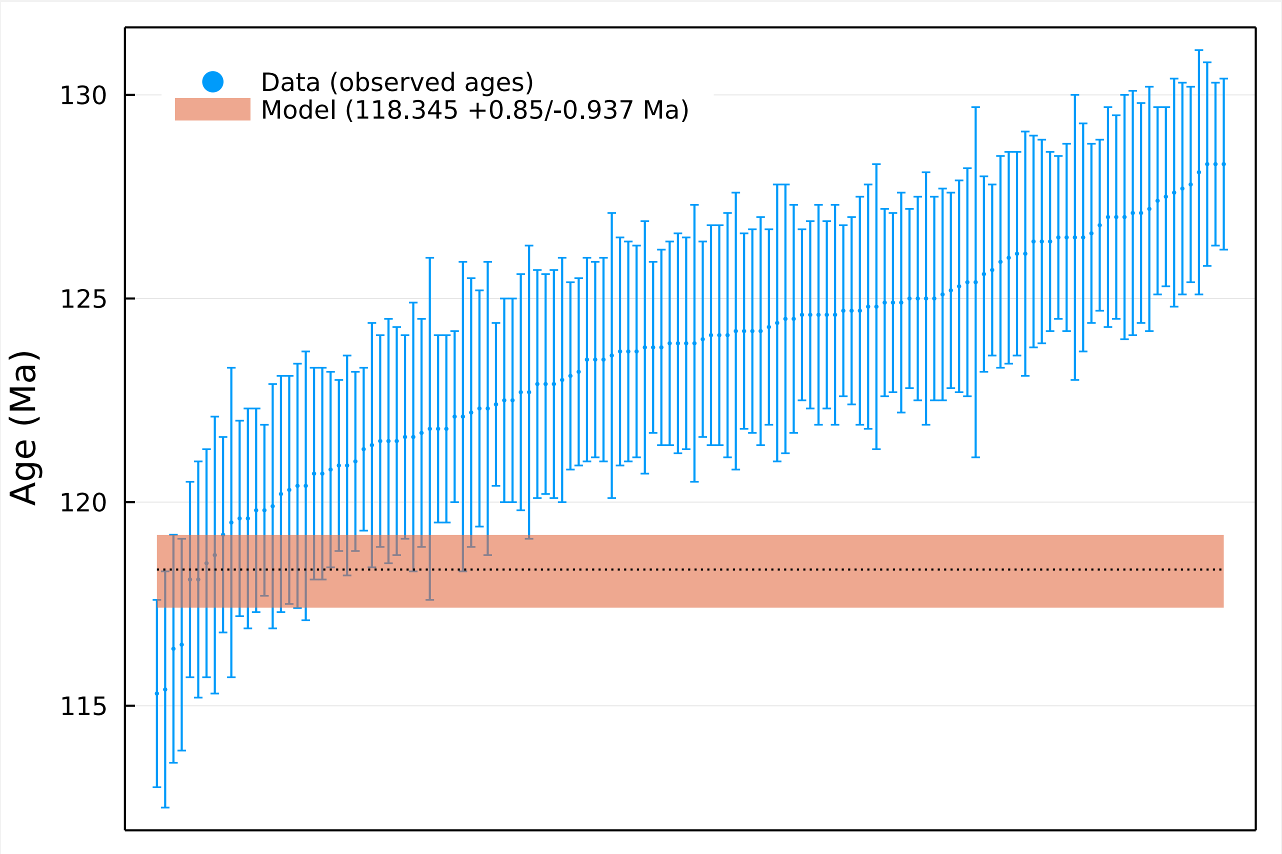
- Scatter is greater than analytical uncertainty
- SPINE regression (Powell et al., 2002, 2020)



Gyomlai et al., 2022, Journal of Asian Earth Sciences

Age interpretation: New approaches

- Chron
  Keller et al., 2018
- Does not assume all dates measure the same age
- Markov Chain Monte Carlo



Quick aside: what is Bayesian?

- Not too much!
- Different tools for different problems
- Bayesian is basis for MCMC methods

Art credit: Agoston Torok, https://agostontorok.github.io/

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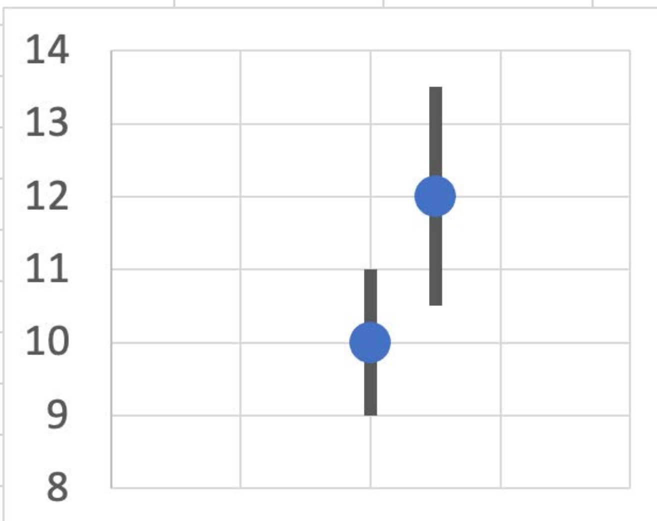
Do these two dates agree?

	sample1	sample2
date	10	12
± 2s	1	1.5



- Best way to tackle this problem: ask a different question: what is $t_1 - t_2$?
- This lets us use the uncertainty propagation equations, including terms for correlation.

difference	2
± 2s	1.80

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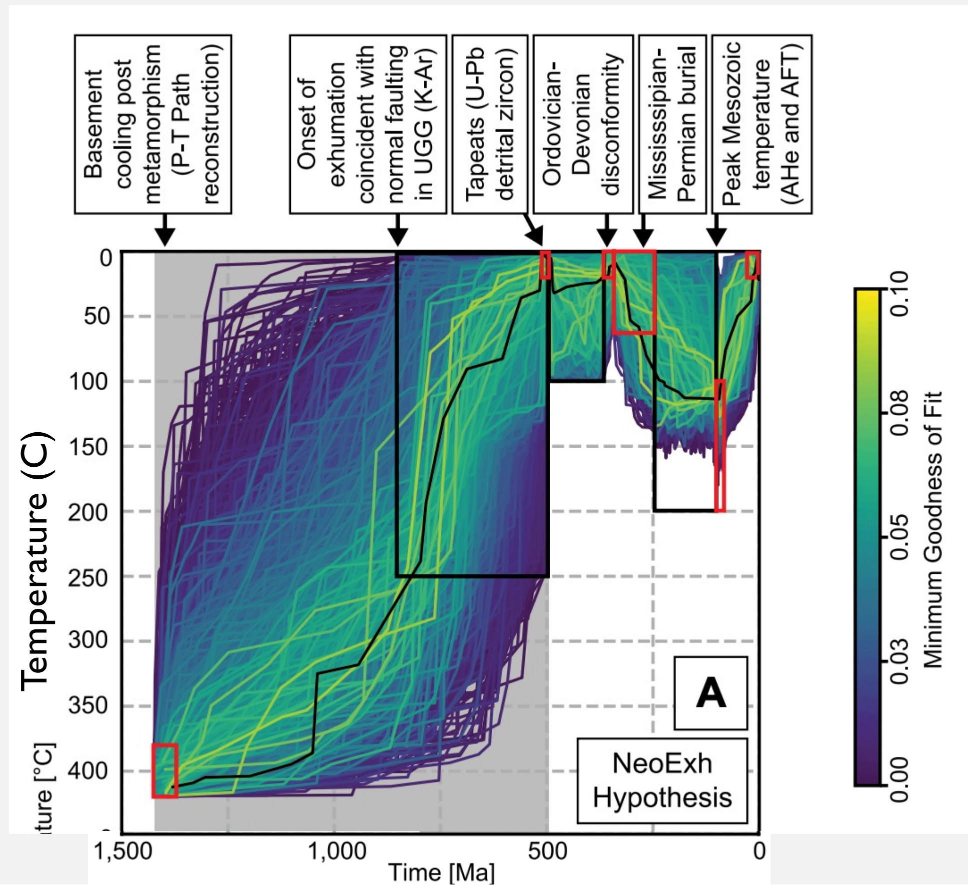
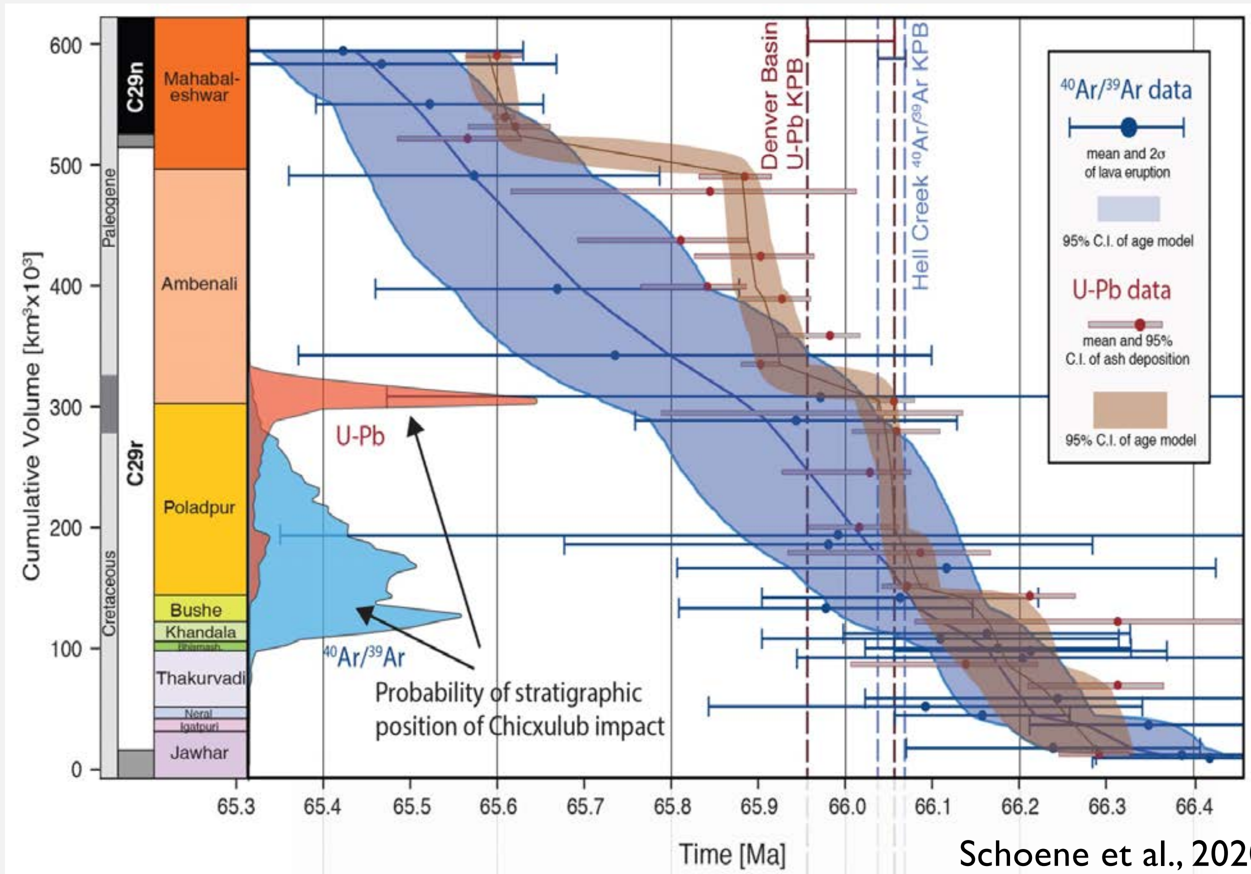
Age Models 1: T-t paths

- HeFTy and QTQt
- Model measurements plus observations and assumptions



Age Models 2: Age-depth models

- Chron
- Bacon/rbacon
- Bchron
- Clam
- Many more



Schoene et al., 2020, GChron

Age Models 3: Catchment erosion rates

- Modeling catchment-level erosion rates with cosmogenic nuclides

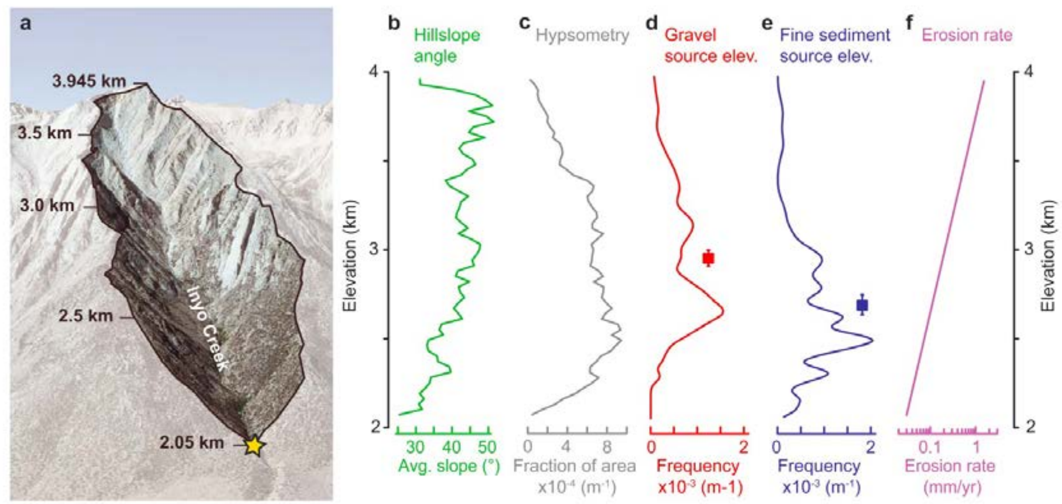


Figure 3. (a) In the steep, 2 km relief catchment drained by Inyo Creek, California, vegetation cover decreases and (b) hillslope angle increases markedly with elevation. (c) The distribution of elevations across the catchment does not closely match the distribution of source elevations for either (d) gravel or (e) finer sediment, suggesting spatial variability in sediment size across the catchment [*Riebe et al.*, 2015]. Source elevations in Figures 3d and 3e are inferred from thermochronometry in stream sediment collected from the sample location (star in a) (fine sediment: *Stock et al.* [2006] and gravel: *Riebe et al.* [2015]). On average, gravel originates from higher elevations than the finer sediment (symbols in Figures 3d and 3e show mean ± s.e.m (standard error of mean)), indicating that sediment size increases with elevation. Together, source elevations and cosmogenic nuclide measurements from previous work [*Stock et al.*, 2006; *Riebe et al.*, 2015] imply that erosion rates increase quickly with elevation across the catchment. (f) An optimization analysis yielded the best fit exponential increase in erosion rates after *Riebe et al.* [2015], which is used in our forward model.

Lukens et al., 2016, JGR

The Status of Statistics in Geochronology

- A cross-cutting theme!
- Can be a steep learning curve, difficult to enter
- Small research community, not very diverse by any metric (e.g., references in this talk!)
- There is plenty of room for everyone here (you?)