Introduction

Some radioactive isotopes formed billions of years ago have half-lives so long that they are still found in appreciable amounts on Earth today. Thorium 232, with a half-life of $1.41 \times 10^{10}$ years, is one such isotope, and $^{238}\text{U}$, with a half-life of $4.51 \times 10^9$ years, is another. These two isotopes form the heads of long decay chains that eventually terminate after a number of different alpha and beta decays in the formation of the stable lead isotopes $^{208}\text{Pb}$ and $^{206}\text{Pb}$. A simplified version of the $^{232}\text{Th} + ^{208}\text{Pb}$ decay chain is shown in Fig. 1. The alpha decay of $^{224}\text{Ra}$ (radium) leads to the formation of $^{220}\text{Rn}$ (radon) with a half-life of 55.6 seconds. Radon is a gas and during its lifetime some of it will diffuse from the thorium compounds that contain the parent $^{232}\text{Th}$ into the surrounding air. Historically this gas was called thorium emanation, or thoron, but it is in fact an isotope of the element radon ($^{220}\text{Rn}$).

The decay of $^{238}\text{U}$ also leads through a series of alpha and beta decays to a radon isotope, in this case $^{222}\text{Rn}$ with a half-life of 3.82 days. These two radon isotopes, $^{220}\text{Rn}$ and $^{222}\text{Rn}$, are generally present in small amounts in the air, having diffused out of concrete, rocks, earth and so forth that contained the parent isotopes. In uranium mines and in some tightly sealed buildings, radioactive radon gas can be present in the air in sufficient quantity that it can become a health hazard possibly leading to lung cancer.
If sufficient time elapses after a sample of $^{232}$Th is obtained and if no radon diffuses away, the various radioactive species in the $^{232}$Th + $^{208}$Pb decay chain will be present in the thorium deposit in secular equilibrium. That is, if we ignore the extremely long-term declining source strength of the $^{232}$Th sample, in any given time interval there will be as many transitions leading to the formation of some particular isotope as there will be decays of that isotope. At true equilibrium, transition rates into and out of each isotope within the chain must be the same, otherwise the number of those nuclei would be increasing or decreasing and equilibrium would not have been established.

The expression that governs radioactive decay is

$$N = N_o e^{-\lambda t}$$

(1)

where $N$ is the number of nuclei at any time $t$, $N_o$ is the number at $t = 0$, and $\lambda$ is the decay constant, or the probability per unit time that any one nucleus will decay. This expression is directly obtainable from the fact that the decay rate of the sample of radioactive nuclei is directly proportional to the number present at any time

$$\frac{dN}{dt} = -\lambda N.$$  

(2)

When a measurement of radioactivity is made, the intensity (number per second) of emitted alpha particles, beta particles, or gamma rays is usually measured. For this reason we want to obtain an expression for the intensity (number/second) of the emitted radiation, $I = C \frac{dN}{dt}$, where $C$ is a proportionality constant. Then,

$$I = C \frac{dN}{dt} = -C \lambda N_o e^{-\lambda t},$$

(3)

but $C\lambda N_o$ is just the intensity, $I_o$ at $t = 0$. The above expression then becomes

$$I = I_o e^{-\lambda t}.$$  

(4)

The half-life, $T_{1/2}$, is commonly used rather than the decay constant. By definition this is just the time for the intensity (or number of radioactive nuclei) to decrease by a factor of 2. The half-life is related to the decay constant by:

$$T_{1/2} = \frac{\ln(2)}{\lambda},$$

(5)

since $e^{-\ln(2)} = \frac{1}{2}$.

The two objectives of this experiment are to determine the half-life of the radioactive gas $^{220}$Rn that is collected in a bottle over a sample of powdered thorium oxide, and to make an estimate of the half-life of the $^{232}$Th parent activity.
Procedure

Part I

As can be seen in Fig. 1, $^{220}\text{Rn}$ decays by the emission of an alpha particle of 6.4 MeV energy. The intensity of the alpha particles can be measured with the use of a lucite chamber coated on the inside with zinc sulfide powder. The chamber is situated on the face of a photomultiplier tube so that any light emitted by the zinc sulfide in the chamber can proceed to the photocathode surface of the photomultiplier tube. Zinc sulfide is a phosphor that emits light after an ionizing particle passes through it. Fig. 2 shows schematically the electronic components.

1. Turn on the AC power to the high voltage power supply and the NIM module.
2. Set the high voltage supply so that it furnishes +1300 volts DC to the photomultiplier.
3. The recommended settings for the amplifier and discriminator are posted by the equipment.
4. It is convenient to count radioactive decays for a period of 1 second in every 10 seconds so that you have time to write down the scaler (counter) reading and reset it before the next measurement is due. A sweep-second-hand clock is provided for timing the 10 second intervals.
5. Before introducing the radon gas into the chamber, acquire some data on the counting rate due to background radiation. Background radiation can come from cosmic rays, long-lived daughter activities from the radon, and other radioactivity that may be present in zinc sulfide, lucite, etc.
6. After sufficient background data have been accumulated, evacuate the chamber by turning on the vacuum pump and then opening the pump valve on the chamber with the valve to the thorium source closed. Open the hose clamp on the thorium source bottle and leave it open while pumping down the counting (scintillation) chamber. After a few minutes, close the pump valve and the hose clamp. Then open the valve to the thorium source bottle, leave it open for about 5 seconds, and then close it.
7. **Start taking data immediately on the $^{220}$Rn decay.**
8. **Accumulate data until you are down to about background. This should take no more than 5 or 6 half-lives, i.e. about 5 minutes.**
9. **Repeat steps 6, 7, and 8 until you have 3 complete sets of data.**
10. **Subtract the average background count from the total counts for each of your three data sets. Then, for one trial, make a graph of the natural log of the corrected counting rate versus total elapsed time.** That is, plot the natural log of the corrected counts against time in 10 second intervals. To make this plot, use the “Wlinfit” weighted linear least squares fitting program or its equivalent. Wlinfit is available on the desktop in the “Scratch” folder. From the slope of this graph, given by the program, determine the half-life for $^{220}$Rn. The counting uncertainty is expressed by the error bars plotted on the graph.
11. Using the weighted linear regression program (wlinfit.nb), determine the slope of the curve $\lambda$ and the half-life $T_{1/2}$ for each of the two remaining trials of your data.
12. **Compare your results and the estimated uncertainties with the most precise measurement to date of 55.61 ± 0.4 sec for the half-life.**
13. **Daughter activities of $^{220}$Rn are not gaseous and hence are deposited out in the chamber. Do these interfere with your measurements? Explain.**
14. If you are having some trouble with the analysis of Poisson statistics, check out chapter 11 in the error analysis text. Section 11.4 guides you through how to subtract a background count from a measurement.

**Part II**

It is possible to use the data obtained in Part I to calculate an approximate value for the half-life of the parent $^{232}$Th activity. Assuming the $^{220}$Rn activity is equilibrated with the $^{232}$Th activity in the bottle, the radon activity can be extrapolated back to the time the gas was first removed from the bottle to provide a measure of the $^{232}$Th activity, if the efficiency of counting the radon activity is known. From the following expressions

$$\frac{dN}{dt} = \lambda N \quad \text{and} \quad T_{1/2} = \frac{\ln(2)}{\lambda}$$

one can obtain a value of $T_{1/2}$ for $^{232}$Th where $\lambda$ is the decay constant, and $N$ the number of thorium nuclei. To avoid confusion between Part I and Part II of this experiment, please note that $\lambda(^{232}\text{Th}) \neq \lambda(^{220}\text{Rn})$. The two decay constants are entirely different.

The number of thorium nuclei, $N$, can be calculated from the mass of the ThO$_2$ powder. Because weighing a radioactive powder can lead to contamination of various surfaces and because breathing radioactive powder is hazardous, the mass of the ThO$_2$ will not be measured in this experiment. It has been determined by others to be 40 grams. Do not attempt to weigh it yourself.

The absolute counting rate ($I = C\frac{dN}{dt}$) for $^{232}$Th should be equal to the counting rate observed for the $^{220}$Rn measurement at $t = 0$ (the time at which the radon gas was
transferred) after three important factors are considered. The first is the efficiency of counting the $^{220}\text{Rn}$ alpha particles in the chamber. An approximate measurement of the efficiency of the chamber was made by using a calibrated $^{241}\text{Am}$ source (1.05 µ Ci) placed in various locations within the chamber. The efficiency was detected in this way to be about 50% for the alpha particles from $^{241}\text{Am}$. When this efficiency is considered in combination with the presence of the $^{216}\text{Po}$ decay which quickly follows the $^{220}\text{Rn}$ decay, the $^{216}\text{Po}$ half-life relative to the detector resolving time and one-second counting period, and some statistical considerations, it turns out that in this application one will obtain approximately one count for each $^{220}\text{Rn}$ decay.

The second factor that must be taken into account is the fraction of the volume of the radon gas originally in the glass bottle with the thorium that is finally transferred to the counting chamber. Assuming the pressure equilibrium is established throughout the counting chamber, glass bottle, and hose, the radon gas should be distributed over the volumes of the chamber (see Figure 3a), the glass bottle, and the hose, but it is only the radon gas in the chamber that is counted. Thus a correction must be made for the fraction of the chamber volume relative to the total system volume. The volume of the bottle and hose must be estimated after determining their dimensions.

The third factor involves the fraction of $^{220}\text{Rn}$ that actually escapes from the solid $\text{ThO}_2$ to the air in the bottle above it. A reasonable estimate for this escape efficiency would be in the range of 1% - 3%.

By taking these factors into consideration, an estimate of the absolute value for $dN/dT$ at $t = 0$ can be obtained and a value of $T_{1/2}$ for $^{232}\text{Th}$ calculated. The calculation is based on the fact that for every $^{220}\text{Rn}$ decay, there must be a $^{232}\text{Th}$ decay. Use your measured value of $R_o$, the $^{220}\text{Rn}$ counting rate at $t = 0$, the time the transfer was performed, to estimate the $^{232}\text{Th}$ decay rate. $R$, the decay rate at any time $t$, is directly proportional to the number of nuclei, $N$. $R_o$ then is the initial value of the decay rate at $t = 0$. It is determined by finding the best value for the three data sets, in step 11, or from the graph, in step 10.

$$^{232}\text{Th \ decay \ rate} = R_o \times \frac{\text{volume (bottle + hose + chamber)}}{\text{volume (chamber)}} \times \frac{1}{\epsilon}$$

where $\epsilon$ is the probability that $^{220}\text{Rn}$ escapes from the solid thorium oxide into the air in the counting system. Then $^{232}\text{Th}$

$$\lambda(^{232}\text{Th}) = \frac{(^{232}\text{Th \ decay \ rate})}{(\text{number \ of \ }^{232}\text{Th \ nuclei \ present \ in \ the \ thorium \ oxide \ in \ bottle})}$$

and

$$T_{1/2}(^{232}\text{Th}) = \frac{\ln(2)}{\lambda(^{232}\text{Th})}.$$
An estimate for the uncertainty in the half-life can only be based on the approximate methods used. Compare your result with the accepted value of $1.41 \times 10^{10}$ years. This measurement gives you a clue to the age of the earth. Try to estimate the uncertainties as well.

*Figure 3: Detail of the thorium source container and scintillation chamber*