Chapter 3: Mirrors and Lenses

• The Lens Equation
  – Calculating image location
  – Calculating magnification

• Multiple Lenses
  – Ray tracing
  – Lens equation

• Special Lenses
  – Ball lens retroreflector
  – Fresnel lens

• Aberrations
  – Spherical aberration
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The Lens Equation

• Ray tracing is useful, but kind of tedious for all these different cases, and accuracy requires very precise drawings.

• We can avoid ray tracing by using the lens equation

• However, this will require some algebra.
Focal Length

• Remember we defined the focal length for a lens

![Diagram of focal length (f)](image)

• We also defined the sign of \( f \). The focal length, \( f \), is defined as positive for converging lenses and negative for diverging lenses.

Lens Equation Quantities

• We also need to define some other distances.

![Diagram showing object distance, image distance, and focal length](image)

• The object distance is positive for an object to the left of the lens. The image distance is positive for a (real) image on the right of the lens. These quantities are negative for the reverse situation. Be careful with this.
Lens Equation Quantities

- The image distance is **negative** for a (virtual) image on the left of the lens.

Clicker Question

- Which quantities are **negative** in this example?
  A. Image distance
  B. Focal length
  C. Object distance
  D. A and B
  E. A and C
The Lens Equation

\[ \frac{1}{x_o} + \frac{1}{x_i} = \frac{1}{f} \]

- **Given:**
  - \( f = 10 \text{ cm} \)
  - Object is 15 cm in front of lens: \( x_o = 15 \)

- **Find:**
  - Where is image and is it real or virtual?

- **Solve** equation for \( x_i \):
  - Substitute numbers for letters
  - Subtract 1/15 from both sides
  - Arithmetic on calculator
  - Multiply by \( x_i/0.033 \)

\[
\begin{align*}
\frac{1}{x_o} + \frac{1}{x_i} &= \frac{1}{f} \\
\frac{1}{15} + \frac{1}{x_i} &= \frac{1}{10} \\
\frac{1}{x_i} &= \frac{1}{10} - \frac{1}{15} \\
\frac{1}{x_i} &= 0.033 \\
x_i &= \frac{1}{0.033} = 30 \text{ cm}
\end{align*}
\]

Image is 30 cm to the **right** of the lens and **real** because \( x_i \) is positive.
The Lens Equation

Object distance, 15 cm
Image distance, 30 cm
Focal length, 10 cm

We can verify our result is consistent with the result from ray tracing. Ray tracing does not give an exact numeric answer, because we can’t draw all our lines perfectly. But we can verify our answers.

Lens Equation: Magnification

Object size, $S_o$
Image size, $S_i$
Object distance, 15 cm
Image distance, 30 cm
Focal length, 10 cm

• For objects and images above the axis, $S$ is positive, for those below the axis, $S$ is negative.
Magnification

Object size, $S_o$

Object distance, $x_o$

Focal length, $f$

Image size, $S_i$

Image distance, $x_i$

Magnification is indicated by the letter $M$, and is negative for an image inverted with respect to the object, and positive for an image that is not inverted.

$$M = \frac{S_i}{S_o} = -\frac{x_i}{x_o}$$

Magnification of -2 means the image is twice the size of the object and inverted.
Lens Equation Example

- **Given:**
  - $f = 10 \text{ cm}$
  - Object is 5 cm to left of lens center: $x_o = 5$

- **Find:**
  - Where is image and is it real or virtual?

- **Solve equation for $x_i$:**
  - Substitute numbers for letters
  - Subtract 1/5 from both sides
  - Arithmetic on calculator
  - Multiply by $x_i/0.1$

\[
\frac{1}{x_o} + \frac{1}{x_i} = \frac{1}{f} \\
\frac{1}{5} + \frac{1}{x_i} = \frac{1}{10} \\
\frac{1}{x_i} = \frac{10}{10} - \frac{1}{5} \\
1 \frac{1}{x_i} = -0.1 \\
x_i = -\frac{1}{0.1} = -10 \text{ cm}
\]

Image is 10 cm to the left of the lens and virtual because $x_i$ is negative.

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Lens Equation Examples

- **Magnification**
- **Diverging lens**

![Diverging lens diagram](https://via.placeholder.com/150)

Object distance, $x_o = 10 \text{ cm}$

Focal length, $f = -14 \text{ cm}$
The “Power” of a Lens: Diopters

- Definition of $P$ in terms of $f$ is

$$P \text{ (in diopters)} = \frac{1}{f \text{ (in meters)}}$$

- Meaning of $P$
  - $P$ is a measure of the ray bending power of the lens
  - Your eyeglass or contact lens prescription is usually given in diopters ($P$)
  - $P$ is positive for converging lenses (positive $f$) and negative for diverging lenses (negative $f$)

**Clicker Question**

- The power of the right lens in my glasses is -2.0 diopters. What is the focal length?
  - A. 1 meter
  - B. -1 meter
  - C. 0.5 meters
  - D. -0.5 meters

$$P \text{ (in diopters)} = \frac{1}{f \text{ (in meters)}}$$

-2.0 diopters = 1/f

$$f = \frac{1}{-2.0 \text{ diopters}}$$

$$f = -0.5 \text{ meters}$$
Combining Lenses: Using Diopters

- Add together the powers of each of the two lenses to get the power of the combined lens
- Example:
  - Lens A has \( f_A = 0.5 \) m,
  - Lens B has \( f_B = -1 \) m.
  - What is the power of each lens?
  - What is the power of the combined lens when the two lenses are held together?
  - What is the focal length of the combined lens?

- Power of lens A is 2 diopters
- Power of lens B is -1 diopters
- Power of combined lenses is 2 -1 diopters = 1 diopter
- Focal length of combined lens is \( f = 1 \) meter

**This is only for THIN LENSES held so they are touching each other!**

Multiple Lenses

If we add a second lens, we can find the image produced by the combination of lenses by using the image from Lens 1 as an effective image for Lens 2.
Multiple Lenses

• We know where rays from the original object have to hit Lens 2 because we know where the image is. We can use this to find the special rays for Lens 2, and the final image.
• This is complicated, but we can use the Lens Equation!

Multiple Lenses: Lens Equation

The trick to using the image from Lens 1 as an effective object with the lens equation is knowing what the object distance is for Lens 2. Note that it will be negative, because it is to the right of Lens 2.
Multiple Lenses: Lens Equation

Let’s add some numbers and see how it works.

- The lens equation is consistent with our ray tracing result!
Corrective Lenses

• If you have 20/20 vision, the lens of your eye (Lens 2) produces an image correctly on your retina. Here it produces an image behind the retina. This looks blurry.

• If we add a lens in front, like glasses or contacts, the combination of the two lenses will produce an image correctly located at your retina.

Ball Lens Retroreflector

• A sphere of water, here a droplet on a blade of grass, acts as a very good converging lens.

• The focal point is just past the surface of the glass, so light is reflected off the grass and returns the way it came.

• This is another kind of retroreflector.
A lighthouse wants to collect as much light as possible (large diameter lens) and send it out in a beam. This means the lens must have a short focal length (thick lens), so it can be placed close to the light source. This makes the lens very heavy.
Fresnel Lens

Fresnel, a French physicist, had the idea that because refraction occurs at the curved surface, the rest of the glass is unnecessary. If you remove the non-essential glass and flatten the remaining glass segments, you get a Fresnel lens. There are weird effects at the edges, but most applications do not require precision lenses, so this isn’t a problem.

Fresnel Lenses: Applications