**ELECTRIC FIELDS:**
Electrostatic forces are (like gravity) "Action at a Distance". It's a strange idea! One way people have developed to help get more comfortable with this idea is another concept: force fields. (Michael Faraday invented this in the 1800's.) It's strange too, but very useful, and very powerful!

A charge $Q$ produces forces on ANY other charges, anywhere in the universe. Imagine putting a tiny charge $q$ somewhere, a distance $r$ away. Little $q$ is pushed (by Coulomb's law), it feels a force

$$F_{qQ} = kq Q/r^2.$$  
(Read that as "the Force on $q$, by $Q$")

Faraday argued there's an "electric force field" surrounding $Q$. It's like a "state of readiness" to push any other charge that happens to come along. The field isn't exactly physical; it's not something you can taste, or smell, or see. It just manifests itself if you put a charge (any charge) $q$ somewhere (anywhere!) (Thus, Faraday doesn't quite think about electric forces as "action at a distance": it's not so much that $Q$ is pushing on $q$, far away, it's more like $Q$ produces an electric force field everywhere, and it's that field right wherever $q$ is, that finally pushes on $q$.) So

Electric Fields are vectors (they have magnitude and direction)
Electric Fields exist in empty space (think of fields as a property of space!)

Suppose you have a bunch of charges. If you bring in one more little charge, "$q$", anyplace you like, say the little black spot, it will feel an electric force in some direction.

(You could figure it out by using Coulomb's law, adding the four separate forces, as vectors. You'll get SOME answer.)

But again, you can instead think of an "electric force field" at the point, which tells you exactly which way a little "test charge" $q$ will be pushed IF you put it there. The E-field is present whether or not you bother putting $q$ there. It is present at any (and every) point in space.

I'm going to first just define the E-field mathematically. (I'll justify and explain this definition on the next page) It should certainly be a vector (i.e. it has a size and a direction). It should tell you the force on ANY test charge $q$. We define $E = F/q$, or more carefully:

$$E(\text{at point } p) = \frac{F(\text{on test charge } q, \text{ at point } p)}{q}.$$  
From this def., the units of $E$ will be Newtons/Coulomb, or N/C
I did NOT define $E = F$, instead I divided out the "test charge" $q$. Why? Because the $E$ field is a property of space at the point $p$. It shouldn't matter how much charge I use to test for it. If I bring in "$q"$, I'll feel some force $F$. If I bring in "$2q"$, Coulomb's law says I'll feel exactly twice the force, $2F$. But since $E = F/q$, in the second case (twice the force, twice the charge) the factors of two cancel, and $E$ comes out the exact *same* no matter WHAT "$q"$ is! That's what we want: $E$ has some value at every point in space, whether or not there's any charge physically at that spot - and you can use it to *figure out* the force on any test charge of any size that you bring to that spot.

**Analog/Interlude, to help motivate $E$ fields:**

Go to King Sooper's and buy some sugar. On day 1 you buy 2 pounds, and pay $4. On day 2 you buy 3 pounds, and pay $6. What you pay depends on how much you buy. So, it might seem complicated to try to predict how much you'll have to pay tomorrow, when you buy yet a different amount. But notice: $\text{Spent}/(\text{amount bought}) = 4/(2 \text{ lbs}) = 6/(3 \text{ lbs}) = 2/\text{lb}$. There is a simple, underlying, universal, common UNIT PRICE. So now you immediately know how much you'll pay, no matter *how* much you buy: 

$$\text{Price} = (\$2/\text{lb})\times(\text{amount you buy})$$

It's basically the same with $E$ fields: $E$ is like the "unit price per pound" (only here it's really "unit force per charge") Price per pound is a "universal property of sugar", no matter how much you buy (even if you buy none!)

The force (price) on a test charge $q$ seems complicated at first: different if you put in different $q$'s. But then you notice that 

$$\text{Force} = (\text{unit price})\times(\text{amount}) = E\times q$$

Knowing $E$ you can easily figure out the force on ANY $q$ now!

That's one reason why $E$ is useful - it's like knowing the "unit price" at the store.

Bottom line: if you know what $E$ is at any point in space, you can immediately figure out the force on ANY charge "$q" placed at that point, because

$$F = qE$$

(Just multiply both sides of the equation defining $E$ by "$q" to get this.)
Electric Field Lines

*Suppose we put a positive charge at the origin. How might we represent (draw) the E field?*

We might pick a few (randomly chosen) points: the E field points radially outward (radially means directly away from the Q, like a "radius" of a circle.)
The farther away you get, the weaker it is (because force, and thus E, drops off like $1/r^2$)

This kind of drawing is a little tedious, and picking points at random doesn't seem like the best way of drawing an E field. But alas, E is defined everywhere, and it's a vector, so you really can't draw the field in any easy way!
There is a neat pictorial trick that people use to try to "visualize" E-fields. You draw "lines of force". This means, instead of drawing vectors at points, you draw lines, called field lines.

<table>
<thead>
<tr>
<th>Field lines start and end at charges (always!)</th>
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| The direction of the lines (really, the tangent to the lines) at any point tells you the direct of E. (The lines have arrows, to eliminate any ambiguity in direction) |

| The more lines you have (the denser they are), the stronger the E field. (If you double the charge, you double the "density of lines".) (Technically, this is the number of lines per unit area perpendicular to the field) |

| Lines never cross (if they did, E wouldn't be defined at the crossing point. But there must always be a unique force on any test charge!) |

Here are several examples.

**Example 1:**  *A single + charge.*
The location of the arrows is not significant.
The arrows all point away from the + charge (E fields go AWAY from positive charges)
The lines are all radially outward.
Notice that the lines are less dense further away from the charge, which tells you the E field is weaker out there.
(this is just Coulomb's law, E drops like $1/r^2$!)

**Example 2:** A single charge.

The arrows all point towards the charge (E fields go towards negative charges).
The lines are all radially inward.
Notice that the lines are again less dense further away, (this is just Coulomb's law, E drops like $1/r^2$!)

**Example 3:** A single charge of -2.

Just like the last one, but the density of lines is twice as much, because the charge is twice as big.

**Example 4:** A dipole (that means "two poles": + on left, - on right)

The lines are curved now. At any point in space, the E field is given by the tangent to the line. (Knight Fig 26.10b is better drawn version of this same fig!)
As always, the arrows tell you the direction of E, which is the same as the direction that a positive test charge would move, if released at that point. (A negative test charge would go the opposite way!)

Look at the picture and convince yourself that the directions make sense: think about which way a "+" test charge would want to go given those two charges +Q and -Q.

**Example 5:** Field lines between two infinite "lines" of uniform charge.

The lines are uniform density, the E field here is the same size and direction everywhere! Wherever a test charge may be, it is pushed to the right with the same force anywhere... (It may not be totally obvious to you that the E field is uniform here, but it is. This is a case where working it out from Coulomb's law is hard, because there are an infinite number of charges in the story! There are fancier tricks that can be used to deduce E fields which we'll get to next chapter!!)
Computing the $E$ field is pretty much the same as finding the force, just remember to divide out $q$. E.g., if you have a single charge $Q$ somewhere, and want to know the $E$ field at some other point, just imagine putting a test charge "$q$" at that point, and use Coulomb's law: $F$ (on $q$, at point $p$) = $k \frac{Q q}{r^2}$, so $E = \frac{F}{q}$, or

$$|E| = \frac{k Q}{r^2}.$$  
(For a single charge $Q$, this tells $|E|$, a distance $r$ away)

More formally, \[ E = \frac{k Q}{r^2} \hat{r} \]

The direction of $E$ is radially outwards, away from $Q$, if $Q>0$. 
The direction of $E$ is radially inwards, towards $Q$, if $Q<0$. 

(That's just the statement that like charges repel, opposites attract)

Since $E = F/q$, the direction of $E$ is the same as the direction of the force on a positive test charge.

(To ponder: if $E$ is to the left at some point in space, what is the direction of the force on a + charge there? Now, how about on a - charge? Answers on next page)

$E$ is defined at all points in space, and depends ONLY on $Q$, not on the test "$q$". Regions of space have a "possibility" of providing electric forces, even if no test "$q$" happens to be there at the moment. Similarly, gravitational fields exist at all points. There's a "g-field" in this room. It's downwards everywhere. If you DO release a pebble at some spot, the pebble immediately starts to accelerate down because of the "g-field". But the g-field is still there whether or not you put a "test pebble" at some spot to check.

By the way, you can't "see" E-fields, but you CAN see time-varying E-fields - that's precisely what light is! We'll talk about this more later.

If you have many charges, finding $E$ is little more than an exercise in adding vectors: you just imagine putting a test charge $q$ down, and then, as before:

$$E = \frac{F_{\text{net}} \text{(on } q)}{q}.$$  
(i.e. E fields superpose too!)

Finding $E$ (or equivalently $F_{\text{net}}$) can be a chore if there are lots of charges. But in the lab, finding $E$ is fairly easy -just put a test charge there, and measure the force on it! Then divide out the charge... That's it.
**Directions:** $\mathbf{F} = q\mathbf{E} \Rightarrow$ the direction of $\mathbf{F}$ is the same as $\mathbf{E}$ (if $q > 0$)

If e.g. $\mathbf{E}$ points west (and if $q$ is positive) $\mathbf{F}$ is also west.

However, if $q$ is negative, $q\mathbf{E}$ would be east (recall, multiplying a vector by a negative number flips the DIRECTION of the vector!)

**Example:** Put a $+10$ uC charge at the origin.
Look at a point $p$ some distance to the LEFT of the origin.

*What is the direction of $\mathbf{E}$ there?*

*What is the direction of force on a $+$ test charge there?*

*What is the direction of force on a $-$ test charge there?*

**Answers:** Remember, $E$ fields point away from $+$ charges. At $p$, $E$ points left.

The direction of force on a "$+$" test charge is the same as $\mathbf{E}$, left.

(Like charges repel. $Q$ and "$q$" are both "$+$", they repel, it all makes sense)

The direction of $\mathbf{F}$ on a "$-$" test charge opposes $\mathbf{E}$, i.e. to the right

(Makes sense. The $q$ and $Q$ are now opposite, so they attract)

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**If** the charge "$Q$" had been $-10$ uC in the previous example, **ALL** of the answers would be reversed:

In that case the $\mathbf{E}$ field at $p$ would point right, TOWARDS the "$-$" charge.

The force on a "$+$" test charge would be the same direction as $\mathbf{E}$

(that's always true!), i.e. to the right.

(Makes sense. The test charge is now different from $Q$, so the force is attractive.)

The force on a "$-$" test charge opposes $\mathbf{E}$ (always true!) In this case, left.

(Again, makes sense. The test charge is now the same as $Q$ here, both are "$-$", the force between them is repulsive. At $p$, that means it points left, away from $Q$.)

This is all a bit confusing to write, but it's really not that bad if you just think about the above examples a little! (Please do so! If this doesn't make sense, it's going to be very hard as we move on - think about it, read the book, ask about it, talk to your TA, or me...)
Example:
A -4 uC charge is at the origin, as shown. What is the E field at a point 2 m to the right of the origin?

Answer:
The force on a test charge q at point p would be k Q q/r^2
(that's just Coulomb's law)
E=F/q, so
E = k Q/r^2 = (9E9 N m^2/C^2)(-4E-6 C)/(2 m)^2 = 9E3 N/C
The direction of the E field is towards the ("-" charge; to the left.
(A "+") test charge at p would move to the left, opposites attract)

Example: Two -4 uC charges are set up as shown. What is the E field right in the middle, between them?

Answer: The force on a "+" test charge at point p would be to the RIGHT from the right-hand charge, and to the LEFT from the left hand charge (opposites attract, the test charge I'm imagining is +.) Since I picked p in the middle, the forces exactly cancel, F_net=0, so E=0 at p. If I shift a little away from the center, however, the forces won't exactly cancel, and there will be a nonzero E field. To find it at any point "off-axis" (anywhere besides on the line shown between the charges) I'd have to add the two forces as vectors, a bit of a pain.

In the end, the field lines look exactly like the ones shown in your textbook Fig 26.11b (except my 2 charges are ", so the arrows would all be reversed.) Notice in that figure there is zero density of lines right in the middle, which corresponds to the answer E=0 we just got.

Think about what you get a little away from the center, and see if the field lines in the book's picture make physical sense to you.

Field lines are very useful - they're a pictorial way to "visualize" the force that any test charge would feel anywhere in space. But, if you want to be quantitative, you usually need to compute (or be told) the value of E fields, it's hard to get numerical values from field line graphs.
One more example:

You have charge #1 (-Q) located r from the origin on the +x axis. You have a charge +Q located r away, but 45 degrees from the +x axis. What is the E field at the origin due to those charges?

Here are the two relevant unit vectors FROM the given charges TO the origin, located at the origin.

Mathematically, we need to add the two E vectors arising from the 2 charges:
\[ \vec{E}_1 = k \frac{(-Q)}{r^2} (-\hat{i}) = -k \frac{Q}{r^2} \hat{i} \] (The sign is sensible, E1 points to the RIGHT!)
\[ |E_2| = kQ/r^2 \] (direction should be "down and left") Let's find the components:
\[ \vec{E}_2 = k \frac{Q}{r^2} (-\frac{1}{\sqrt{2}} \hat{i}) + k \frac{Q}{r^2} (-\frac{1}{\sqrt{2}} \hat{j}) \]

The 1/Sqrt[2]'s come from cos and sin of 45°. Do you agree with the signs?!

So Etot(x) = \[ \frac{kQ}{r^2} \left(1 - \frac{1}{\sqrt{2}}\right) \]
Etot(y) = \[ \frac{kQ}{r^2} \left(-\frac{1}{\sqrt{2}}\right) \]

The x component is positive, the y component negative, the field points down and right. This makes sense if you think about it... That's the way a positive test charge released at the origin would move: attracted to the -Q on its right, repelled from the +Q. (The -Q "wins" in the x direction: do you see why?)
Continuous Charge Distributions:
In real life, you often have collections of charges which you can approximate as a 3-D continuum (a smear, or blob) of spread out charges. If you want to find the E field at some point of interest (outside the blob) produced by this distribution of charge, can you just still use Coulomb's law? No! Coulomb is only valid for POINTLIKE charges!

Instead, you have to add up (as vectors) the $E$ produced by all the little "chunks" of charge throughout the entire volume of charge.

$$ \mathbf{E} = \sum \Delta \mathbf{E} \quad \text{which we might think of it as } \int d\mathbf{E} \text{ in the limit of small chunks} $$

$$ = \int k \frac{dq}{r^2} \hat{r} $$

This is very formal! I'm saying "add up the $dE$ produced by each chunk of charge $dq$", and adding lots of tiny things is just what "integrate" means.

What is the small $dE$ produced by a little chunk $dq$? It comes right from Coulomb's law: $k \frac{dq}{r^2}$, where $r$ is the distance from the chunk to the point of interest. (Now we can use Coulomb, because a tiny chunk acts like a point) The $\hat{r}$ just tells you the direction: along the line FROM each chunk TO the point of interest... (Sometimes this integral is do-able, sometimes it's tough!)

Quick reminder of some basic ideas about integrals:

$$ \int_{0}^{1} x^2 dx = \frac{x^3}{3} \bigg|_{0}^{1} = \frac{1}{3} $$

I think of this formula as telling me about the area under the $x^2$ curve: (Which is 1/3).

In general, for a curve $f(x)$, the area of one little rectangle under the curve would be $f(x) \Delta x$, and then when you add up all these little rectangles, you get the total area under the curve,

$$ \text{area} = \sum_{i} f(x_i) \Delta x $$

$$ = \int_{x_i}^{x_f} f(x) \, dx $$
If your charge is just spread over a line, then we'll need to think about the "linear charge density" \( \lambda \), where \( \lambda \) tells you the "charge per meter". Then \( dq = \lambda \, dx \).

If your charge is spread over a surface, then we'll need to think about the charge density in 2-D, called "surface charge density", \( \sigma \), which is "charge/area" or "Coulombs/m^2". Then \( dq = \sigma \, dA \) (\( dA \) is the AREA of a tiny square on the surface).

If the charge is spread out over a volume, then we'll need to think about the charge density, usually called \( \rho \), which tells you "charge/m^3". Then \( dq = \rho \, dV \) (\( dV \) is the VOLUME of a tiny cube).

Let's do an example!

1) Suppose we have a total charge \( Q \) smeared out evenly over a rod of length \( L \). So, we have a charge density \( \lambda = \text{charge/length} = Q/L \).

Let's figure out \( E \) at a point a distance "a" to the right of the end of the rod. (It is NOT just \( kQ/a^2 \), because most of the charge is farther away than \( a \), the rod is not a point, we can't just use Coulomb's law!) Let's work it out:

\[
E(\text{over at } x=a+L) = \sum_{x=0}^{x=L} \sum_{x=a+L} \frac{k \, dq}{(\text{dist from } x \text{ to point})^2} \hat{i} = \sum_{x=0}^{x=L} \sum_{x=a+L} \frac{k \, dq}{(L+a-x)^2} \hat{i}
\]

(Do you see how I got \((L+a-x)^2\) there? Look, figure it out from the picture!) We already said that \( dq = \lambda \, dx = (Q/L) \, dx \), so we can put that in, and convert this sum of "tiny chunks of E" into a sensible integral:

\[
E = \int_{0}^{L} k \frac{Q \, dx/L}{(L+a-x)^2} \hat{i} = \frac{kQ}{L} \left[ \frac{1}{L} - \frac{1}{L+a-x} \right]_0^L \hat{i}
\]

(Do you see how I got the last step? Think about how do we do the integral, and especially about the signs!) So now I finish up the math:

\[
E = \frac{kQ}{L} \left[ \frac{1}{a} - \frac{1}{a(L+a)} \right] \hat{i} = \frac{kQ}{a(L+a)} \hat{i} \quad \text{(Don't take my word for it - check!)}
\]

Let's stop and see if this makes sense. It has the correct units, \( kQ/\text{distance}^2 \). If \( L=0 \), it reduces to Coulomb's law, \( kQ/a^2 \), which makes sense. If \( a>>L \), this formula still reduces to \( kQ/a^2 \), i.e. if you're very far away, the "rod" begins to look like a point, as we'd expect!! Your text has many more examples - you may want to work through them.
Earlier we talked about water molecules, and saw they are neutral "dipoles" which we might model (simplify, approximate) as a simple dipole:

\[ \begin{align*}
+q & \quad \text{A dipole has no net charge, but is made of charges } +q \text{ and } -q \text{ separated by distance } d. \\
-\frac{d}{2} & \quad \text{What happens if you put a dipole in an E field? You might think "nothing", because } F = Q E, \text{ and the total charge } Q \text{ of the dipole is zero. And indeed, there is zero NET force on a dipole....}
\end{align*} \]

But if you look at it closely, you'll see the dipole can get twisted, it can experience a torque: (recall, \( \tau = r \times F \))

Here, we get TWO torques, one on each charge, that both add up the same way (clockwise, in the picture shown):

\[ \tau = \left( \frac{d}{2} \right) q E \sin \theta + \left( \frac{d}{2} \right) (-q) E (-\sin \theta) \]

\[ = \frac{d}{2} q E \sin(\theta). \] It's not zero, the dipole wants to "line up" with the field.

Some people like to define \( d \times q = \text{(separation} \times \text{charge)} \) as the "dipole moment". Technically, \( p = \text{"dipole moment"} = q d \) is a vector pointing from the - to the + charge, in which case \( \tau = p \times E \).

This is the physics behind microwave ovens: an \( E \) field in the oven oscillates in time, and the water molecules get twisted by the resulting torque, back and forth as \( E \) varies. The twisting molecules dissipate the rotational energy they get, which heats the food!

If the \( E \) field is not uniform (i.e. is different strength in different places), then the + and - charges on the dipole can feel different forces, and then there IS a net force on the dipole! So, you can move dipoles around, even though they have zero net charge, using \( E \) fields!

If you have symmetric neutral atoms or molecules, you might think you've got no dipole moment, and hence nothing to work with... but if you put such objects in an \( E \) field, you can induce a dipole moment. (Because the + parts of the molecule will be pushed in the direction of the external \( E \), and the - parts get pushed the other way. They spread apart, CREATING a dipole, which then behaves as described above!

If \( E \text{(external)} \) is large enough, you can pull the - and +'s apart so hard you rip the molecule or atom apart. This is called "dielectric breakdown". It takes an \( E \) field of roughly \( 3 \times 10^6 \text{ N/C} \) in air to do this. (That'll make sparks, or lightning bolts!)
An end of chapter comments:

Another point of interest: DNA molecules, like any molecules, are made up of + and - charges, interacting via electrostatic forces. Biophysics is an incredibly interesting and rapidly growing field of study, and ultimately can be understood by the simple physics introduced in this chapter! The interplay of basic laws of physics in biological systems allows us to understand, and control, living systems in ways that were unimaginable just a few years ago. Electric fields make forces on charges, shaping and controlling E field allows you to manipulate charged objects (which ultimately means just about everything!)
Aside: a little lightning physics: (http://thunder.msfc.nasa.gov/primer)

The story of lightning is pretty complicated, but here's a summary of some of the basic ideas of "ordinary" lightning: In a thundercloud, "+" charges tend to get swept up towards the top, "-" are down low (the mechanism for this charge separation is not yet fully understood - nice to know that the physics of something as elemental as lightning still has some scientific mystery associated with it!) and the large electric fields near those charges begin to break down the air molecules, forming paths of "ionized" (broken down) air, which is a much better conductor of electricity than normal air. This process is called forming a "stepped leader", because it tends to occur in steps or jumps of 50-100 m. The path is pretty random, although it tends to work its way down towards the ground (which is inductively charged the opposite way). Meanwhile "positive streamers" (which are like the stepped leaders only they're working their way UP from the ground, usually from tall or sharp objects - we'll see why later) are forming near the ground. When a "-" stepped leader path meets a "+" streamer path, there's now a continuous path of good conductivity between ground and cloud, and BOOM there's a lightning bolt.

The "flash" arises from the high temperatures associated with the energy being dumped into the air from all those charges passing by, and also from electrons getting re-attached to molecules after they're stripped off. The flash usually works its way UP from the point where the leaders joined the streamers, up towards the cloud (so in this sense, lightning goes UP from the ground to the cloud!). There can be one flash, or a couple of quick strokes through one path (typically 4, each lasting about 30 microseconds). The "boom" of thunder is the shock wave of the air as this superheated column (20,000 C, hotter than the surface of the sun!) expands and then recontracts. Since sound travels so much slower than light, you hear the boom later than you see the flash. If you count, you can estimate how far away the bolt was - sound goes about 1 mile in 5 secs...