Chapter 30 – Reflection and Refraction
Geometrical optics

• When light or other electromagnetic waves interact with systems much larger than the wavelength, it’s a good approximation to
  • Neglect the wave nature of light.
  • Consider that the light travels in straight lines called rays.
    • Rays are perpendicular to the wavefronts and the rays’ direction is that of the wave propagation.
  • The ray approximation is also known as geometrical optics.
Reflection

• When light reflects from a surface, the incident and reflected rays make the same angle with the normal to the surface (case a below).
  • For smooth surfaces, parallel rays all reflect at the same angle.
    • The surface then looks shiny and can form images.
    • This is called **specular reflection** (case b below).
  • The angles are equal (locally) even for rough surfaces.
    • But because of the roughness, the light comes off in random directions.
    • This is called **diffuse reflection** (case c below).
The Plane Mirror

The rays from P and Q that reach your eye reflect from different areas of the mirror.

Your eye intercepts only a very small fraction of all the reflected rays.
Question

• You would like to buy a full-length mirror which allows you to see yourself from head to toe. The minimum height of the mirror is

A) half your height.
B) two-thirds of your height.
C) equal to your height.
D) depends on distance you stand from mirror
A group of sprinters gather at point $P$ on a parking lot bordering a beach. They must run across the parking lot to a point $Q$ on the beach as quickly as possible. Which path from $P$ to $Q$ takes the least time? You should consider the relative speeds of the sprinters on the hard surface of the parking lot and on loose sand.

1. $a$  
2. $b$  
3. $c$  
4. $d$  
5. $e$  

6. All paths take the same amount of time.
Refraction

- **Refraction** is the bending of light as it crosses the interface between two different transparent media.
  - Refraction occurs because the wave speed differs in different media.
  - For light, the **index of refraction** \( n \) describes the speed change.
    - The speed of a wave in a medium is \( v = c/n \).
  - The angles of incidence and refraction are related by **Snell’s law**:
    \[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]
  - There’s also a reflected ray at the interface.
Clicker question

- The figure shows the path of a light ray through three different media. Rank the media in order of their refractive indices.

A. $n_1 > n_2 > n_3$
B. $n_3 > n_1 > n_2$
C. $n_3 > n_2 > n_1$
D. $n_2 > n_1 > n_3$
Example: Sunlight strikes the surface of a lake. A diver sees the Sun at an angle of 42.0° with respect to the vertical. What angle do the Sun’s rays in air make with the vertical?

\[ n_1 = 1.00; \text{ air} \]
\[ n_2 = 1.33; \text{ water} \]
Example: Sunlight strikes the surface of a lake. A diver sees the Sun at an angle of 42.0° with respect to the vertical. What angle do the Sun’s rays in air make with the vertical?

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

\[ (1.00)\sin \theta_1 = (1.333)\sin 42° \]

\[ \sin \theta_1 = 0.8920 \]

\[ \theta_1 = 63.1° \]
Question 23.8  Gone Fishin’ I

To shoot a fish with a gun, should you aim directly at the image, slightly above, or slightly below?

1) aim directly at the image
2) aim slightly above
3) aim slightly below
(a) A fish out of water

The eye sees the object at distance $d$.

(b) A fish in the aquarium

The eye sees the image at distance $d'$.

Diverging rays appear to come from this point. This is a virtual image.
Example: How much shallower does the fish appear to be?

The angles $\theta_1$ and $\theta_2$ are related by Snell’s Law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

The actual depth of the fish is $y$ and it appears to be at a depth of $y'$. These quantities are related by:

$$y' \tan \theta_2 = y \tan \theta_1$$

$(x = x)$
Example continued:

Dividing the previous two expressions gives:

\[ n_1 y' \cos \theta_1 = n_2 y \cos \theta_2 \]

As long as you are directly above the defect and its image, the angles \( \theta_1 \) and \( \theta_2 \) are nearly 0°. Rays from only a narrow range of angles will enter your eye. The above expression simplifies to:

\[ n_1 y' = n_2 y \]

\[ \frac{y'}{y} = \frac{n_2}{n_1} \quad \text{(general result)} \]
Thin Lens
Mirages

Higher up, the wave fronts travel approximately straight.

The hot air near ground has a smaller $n$ than cooler air higher up, so light travels fastest near the ground. Thus, the secondary wavelets nearest the ground have the largest radii $r$, and the wave fronts tilt as they travel.
Total internal reflection

- The angle of incidence for when the angle of refraction is $90^\circ$ is called the **critical angle**.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_1 \sin \theta_c = n_2 \sin 90^\circ = n_2$$

$$\sin \theta_c = \frac{n_2}{n_1}$$

- If the incidence angle is greater than the critical angle, the beam can not refract but is completely reflected.

Angle of incidence increasing
Transmission getting weaker

Critical angle when $\theta_2 = 90^\circ$
Reflection getting stronger

Total internal reflection occurs when $\theta_1 > \theta_c$. 
Fiber Optics
Clicker question

- The glass prism in the figure has $n = 1.5$ and is surrounded by air ($n = 1$). What would happen to the incident light ray if the prism were immersed in water ($n = 1.333$)?

A. Most of it would emerge into the water through the diagonal face and some would be reflected as shown.

B. Most of it would be reflected as shown and some would emerge into the water through the diagonal face.
Dispersion

- Index of refraction depends on wavelength

If a beam of white light is incident on silicate glass, which color will have a bigger angle of refraction?

1. blue
2. yellow
3. red
4. all refract the same
(e) A secondary rainbow is formed by rays that undergo two refractions and two internal reflections. The angle $\Delta$ is larger for violet light than for red.

(d) A primary rainbow is formed by rays that undergo two refractions and one internal reflection. The angle $\Delta$ is larger for red light than for violet.

(c) Forming a rainbow. The sun in this illustration is directly behind the observer at $P$.

The rays of sunlight that form the primary rainbow refract into the droplets, undergo internal reflection, and refract out.

Water droplets in cloud

Observer at $P$

Angles are exaggerated for clarity. Only a primary rainbow is shown.

The two refractions disperse the colors.

Light from sun

$\Delta = 40.8^\circ$ (violet)

$\Delta = 42.5^\circ$ (red)

$\Delta = 50.1^\circ$ (red)

$\Delta = 53.2^\circ$ (violet)
Chapter 33: DIFFRACTION

Water Waves Spread Out behind a *Small* Opening (small means not much bigger than wavelength)

A wave approaches a narrow opening from this side.

The wave spreads out behind the opening.

The wave moves straight forward.

"Shadow"

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Light Waves Also Spread Out Behind a Very Narrow Slit
Constructive and Destructive Interference

- Principle of Superposition:
  - Consider two waves moving along a string.
  - Where the two waves overlap, the actual displacement of any point on the string at any time is obtained by adding the displacement of each of the waves (at that point and time) together

\[ y(x, t) = y_1(x, t) + y_2(x, t) \]

http://id.mind.net/~zona/mstm/physics/waves/interference/waveInterference1/WaveInterference1.html
http://id.mind.net/~zona/mstm/physics/waves/interference/waveInterference2/WaveInterference2.html
• Similar phenomena occurs for water waves:

  • when two crests (or troughs) meet, overall amplitude of wave increases. This is constructive interference.

  • when a crest meets a trough, overall amplitude of the wave decreases. This is destructive interference.

  http://www.falstad.com/ripple/
• Similarly, the electric fields (and magnetic fields) of electromagnetic waves superpose together (as vectors)

  • For two electromagnetic waves traveling in the $x$-direction, the resulting E-field is given by

    \[ \vec{E}(x, t) = \vec{E}_1(x, t) + \vec{E}_2(x, t) \]

• For the following discussion of interference, we will consider a generic wave (which could be radiation, sound waves, water waves, etc)
Two-Source Interference

- Consider two monochromatic sources of waves producing the same wavelength and amplitude. The sources are also in phase. Several wavefronts from the two sources are shown below.

At which location is the amplitude of the resulting wave equal to zero?

a) A  b) B  c) C  d) none
Example: Music and stereo speakers
  - Don’t want to sit where there is a lot of destructive interference

Placement of subwoofer
Young’s Double-Slit Interference Experiment

(a) Viewing screen

The drawing is not to scale: the distance to the screen is actually much greater than the distance between the slits.

(b) 1. A plane wave is incident on the double slit.
2. Waves spread out behind each slit.
3. The waves interfere in the region where they overlap.
4. Bright fringes occur where the antinodal lines intersect the viewing screen.

Superposition of electric fields produce interference pattern.

http://surendranath.tripod.com/Applets/Optics/Slits/DoubleSlit/DblSltApplet.html
Analyzing the Double-Slit Experiment

\[ d \sin \theta = \Delta r = r_2 - r_1 \]

if \( \theta \ll 1, \quad \sin \theta \approx \theta \)

\[ \Delta r \approx d \theta \]
• Now the magnitude of the electric field of a monochromatic wave is given by

\[ E(r, t) = E_0 \sin(kr - \omega t + \Phi) \]

• wavenumber: \( k = \frac{2\pi}{\lambda} \)
• angular frequency: \( \omega = 2\pi f \)
• phase constant at slit: \( \Phi \)
• The phase of this wave is given by the argument of the sinusoidal function. Thus, since one beam travels a distance \( \Delta r \) further than the other, they will be out of phase by an amount \( 2\pi \Delta r / \lambda \)
• How much does phase have to change for constructive interference?
Bright and Dark Fringes in the Double-Slit Experiment

constructive interference occurs when \( \Delta \Phi = \frac{2\pi \Delta r}{\lambda} = 2m\pi \)

Thus \( \Delta r = d\theta = m\lambda \)

\[
\theta_m = \frac{m\lambda}{d} \quad m = 0, 1, 2, 3, \ldots \tag{17.7}
\]

Angles (in radians) of bright fringes for double-slit interference with slit spacing \( d \)

\[
y_m = \frac{m\lambda L}{d} \quad m = 0, 1, 2, 3, \ldots \tag{17.8}
\]

Positions of bright fringes for double-slit interference at screen distance \( L \)

\[
y_m' = \left( m + \frac{1}{2} \right) \frac{\lambda L}{d} \quad m = 0, 1, 2, 3, \ldots \tag{17.11}
\]

Positions of dark fringes for double-slit interference
For Young’s double-slit experiment, what happens to the spacing of the bright fringes if the frequency of the radiation is increased?

A) The spacing would increase.
B) The spacing would decrease.
C) The spacing would not change.

For Young’s double-slit experiment, what happens to the spacing of the bright fringes if the separation of the slits increased?

A) The spacing would increase.
B) The spacing would decrease.
C) The spacing would not change.
Example

Two narrow slits 0.04 mm apart are illuminated by light from a HeNe laser ($\lambda = 633$ nm).

A. What is the angle of the first ($m = 1$) bright fringe?
B. If the screen is 2 meters away, how far apart are the bright fringes?
Thin-Film Interference
1. Incident wave is transmitted through the thin film and the glass.

2. Part of the incident wave reflects from the first surface.

3. Part of the transmitted wave reflects from the second surface.

4. The two reflected waves overlap and interfere.
Phase Changes Due to Reflection

Index $n_1$  

Index $n_2$

Boundary

Incident wave

Transmitted wave

Flip this around the boundary to get the reflected wave with no phase change.

Case 1: $n_1 > n_2$. The reflected wave does not have a phase change.

Case 2: $n_1 < n_2$. The reflected wave does have a phase change.

The reflection with the phase change is one half of a wavelength behind, so the effect of the phase change is to increase the path length by $\lambda/2$. 
Analyzing Thin-Film Interference

\[ 2t = m \frac{\lambda}{n} \quad (m = 0, 1, 2, \ldots) \]  
\hspace{1cm} (17.15)

Constructive interference for 0 or 2 reflective phase changes  
Destructive interference for 1 reflective phase change

\[ 2t = \left( m + \frac{1}{2} \right) \frac{\lambda}{n} \quad (m = 0, 1, 2, \ldots) \]  
\hspace{1cm} (17.16)

Destructive interference for 0 or 2 reflective phase changes  
Constructive interference for 1 reflective phase change
Example: A thin film of oil (n=1.50) of thickness 0.40 µm is spread over a puddle of water (n=1.33). For which wavelength in the visible spectrum do you expect constructive interference for reflection at normal incidence?

Consider the first two reflected rays. $r_1$ is from the air-oil boundary and $r_2$ is from the oil-water boundary.

$r_1$ has a 180° phase shift ($n_{oil} > n_{air}$), but $r_2$ does not ($n_{oil} < n_{water}$).
Example continued:

To get constructive interference, the reflected waves must be in phase. For this situation, this means that the wave that travels in oil must travel an extra path equal to multiples of half the wavelength of light in oil.

The extra path distance traveled is $2d$, where $d$ is the thickness of the film. The condition for constructive interference here is:

$$2d = \left( m + \frac{1}{2} \right) \lambda_{\text{oil}} = \left( m + \frac{1}{2} \right) \left( \frac{\lambda_{\text{air}}}{n_{\text{oil}}} \right)$$

$$\therefore \lambda_{\text{air}} = \frac{2dn_{\text{oil}}}{\left( m + \frac{1}{2} \right)}$$

Only the wavelengths that satisfy this condition will have constructive interference.
Example continued:

Make a table:

<table>
<thead>
<tr>
<th>m</th>
<th>$\lambda_{\text{air}}$ (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.40</td>
</tr>
<tr>
<td>1</td>
<td>0.80</td>
</tr>
<tr>
<td>2</td>
<td>0.48</td>
</tr>
<tr>
<td>3</td>
<td>0.34</td>
</tr>
<tr>
<td>4</td>
<td>0.27</td>
</tr>
</tbody>
</table>

All of these wavelengths will show constructive interference, but it is only this one that is in the visible portion of the spectrum.
The Diffraction Grating

- \( N \) slits with spacing \( d \)
- Spreading circular waves from each slit overlap and interfere.
- Plane wave approaching from left
- The wave from each slit travels \( \Delta r = d \sin \theta \) farther than the wave from the slit above it.
Bright Fringes for a Diffraction Grating

\[ d \sin \theta_m = m \lambda \quad m = 0, 1, 2, 3, \ldots \] (17.12)

Angles of bright fringes due to a diffraction grating with slits distance \( d \) apart

\[ y_m = L \tan \theta_m \] (17.13)

Positions of bright fringes due to a diffraction grating distance \( L \) from screen
The Intensity Pattern Due to a Diffraction Grating

\[ I_{\text{max}} = N^2 I_1 \]  

Maximum intensity of a bright fringe for a diffraction grating with \( N \) slits
The Fringes Become Very Narrow as the Number of Slits is Increased

As the number of slits in the grating increases, the fringes get narrower and brighter.
A Diffraction Grating Splits Light into the Wavelengths That Make It Up

Blue light has a longer wavelength than violet, and thus diffracts more.

All wavelengths overlap at $y = 0$.

Used for spectroscopy
Single-Slit Diffraction

Light passing through a narrow slit spreads out beyond the slit.
Analyzing Single-Slit Diffraction

The wavelets from each point on the initial wave front overlap and interfere, creating a diffraction pattern on the screen.

The wavelets going straight forward all travel the same distance to the screen. Thus they arrive in phase and interfere constructively to produce the central maximum.

Each point on the wave front is paired with another point distance $a/2$ away.

These wavelets all meet on the screen at angle $\theta$. Wavelet 2 travels distance $\Delta r_{12} = (a/2)\sin\theta$ farther than wavelet 1.

$$\theta_m \approx 2m\lambda/a \text{ (constructive interference)}$$
Single-Slit Diffraction: Positions and Intensities

For dark fringes:

\[ \theta_p \approx p\lambda/a \] (destructive interference)
\[ y_p \approx pL\lambda/a \] (destructive interference)

width of central maximum:

\[ w \approx 2L\lambda/a \]
Question

In a single-slit experiment, the width of the slit is slowly decreased. The diffraction pattern

A) remains unchanged.
B) spreads out.
C) shrinks together.
D) fades away.
Question

Violet light with wavelength $l$ is incident upon a single slit and produces a diffraction pattern. Red light with wavelength $2l$ will produce the same pattern if the slit width is

A) $a/4$.
B) $a/2$.
C) $a$.
D) $2a$.
E) $4a$. 
Circular-Aperture Diffraction

\[ w = 2y_1 = 2L \tan \theta_1 \approx \frac{2.44\lambda L}{D} \]
Resolution of Optical Instruments

(a) Stars completely resolved

(b) Stars just resolved

(c) Stars not resolved
For a circular aperture, the Rayleigh criterion is:

\[ \Delta \theta \geq \frac{1.22 \lambda}{a} \]

where \( a \) is the aperture size of your instrument, \( \lambda \) is the wavelength of light used to make the observation, and \( \Delta \theta \) is the angular separation between the two observed bodies.

Example: Hubble Space Telescope: \( D=2.4 \) m. For greenish light, (550 nm), what is the angular resolution?

\[ \Delta \theta = \frac{1.22 \lambda}{a} = 2.8 \times 10^{-7} \text{ Rad} = 1.6 \times 10^{-5} \degree = 0.06 \text{ ArcSec} \]
X-Ray Diffraction
DNA Diffraction Pattern
Maxima: \(2d \sin(\theta) = m\lambda\)
Michelson Interferometer

In the Michelson interferometer, a beam of coherent light is incident on a beam splitter. Half of the light is transmitted to mirror $M_1$ and half is reflected to mirror $M_2$.

Example (text problem 25.12): A Michelson interferometer is adjusted so that a bright fringe appears on the screen. As one of the mirrors is moved 25.8 µm, 92 bright fringes are counted on the screen. What is the wavelength of the light used in the interferometer?
Example (text problem 25.12): A Michelson interferometer is adjusted so that a bright fringe appears on the screen. As one of the mirrors is moved 25.8 µm, 92 bright fringes are counted on the screen. What is the wavelength of the light used in the interferometer?

Moving the mirror a distance $d$ introduces a path length difference of $2d$. The number of bright fringes ($N$) corresponds to the number of wavelengths in the extra path length.

\[ N\lambda = 2d \]

\[ \lambda = \frac{2d}{N} = 0.561 \mu m \]