A New Resistance Standard
Microwave Measurements of Shot Noise Suppression in an Atomic Point Contact

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Abstract

In this paper we investigate electron transport probabilities by simultaneously measuring conductance and shot noise in an atomic point contact. The results show a characteristic noise spectrum for shot noise at many resistances, with observed suppression of shot noise at integer multiples of the quantum of conductance in accordance with theoretical expectations. We are able to use our shot noise suppression measurements as a standard resistor, from which other unknown resistances can be calibrated.

Introduction and Motivation

Shot Noise as a New Resistance Standard

The noise in the electrical current through an atomically thin gap in a wire is related to the probability that discrete electrons will be able to jump (or rather tunnel through) that gap [17]. This manifestation of the discreteness of charge is known as shot noise, and has many interesting qualities. One demonstrated behavior of shot noise is that it decreases theoretically to zero when the probability of electron transport is 100 percent [17].

This is reasonable–if every electron is tunneling through the gap, then there should be no fluctuations in the current and hence no shot noise. What is particularly interesting is that in an atomic point contact (essentially a wire with a gap) the transmission probability becomes 100 percent at integer values of $2e^2/h$, which equals $1/12.9\,\mathrm{k}\Omega$. Thus, shot noise is suppressed at resistances determined entirely by fundamental physical constants. Measurements of shot noise at a variety of resistances, therefore, should show minima at certain well-defined resistances. If the resistance scale was previously undetermined, then the shot noise minima would be enough to calibrate it. This is how the physical phenomena of shot noise can be used as a resistance standard.

The quantum Hall effect is used worldwide to maintain and compare the unit of resistance, and for good reason: ”the reproducibility reached today is almost two orders of magnitude better than the uncertainty of the determination of the ohm” using the best known values of Planck’s constant h and the unit of charge e [1], about one part per billion. Such remarkable precision cannot be matched by a shot noise resistance standard yet. However, the intense magnetic fields required to measure quanta of resistance via the hall effect are difficult to produce on the small scale of a device. Because of this, using an atomic point contact to measure shot noise suppression has many practical applications for calibrating resistance.

Possible Applications

Instruments that can combine high precision with low maintenance are rare and in high demand. Indeed, a great deal of the cost of micro electro-mechanical devices (MEMS) comes from frequent calibration. Accelerometer MEMS, for example, are used to deploy airbags in cars, inform the inertial guidance systems of missiles, and direct satellites. Such devicies usually consist of little more than a cantilever beam with some type of deflection sensing circuitry. As an accelerometer experiences a jerk, the cantilever beam will deflect a piezo-electric crystal, providing a measurable voltage to the device’s circuit. However, over the course of the device’s life as it bends again and again, resistances will change as the cantilever and piezo change. Instead of calibrating it over and over (a process that is costly for airbags and early impossible to do for missiles or satellites), a shot noise resistor within the device’s digital circuitry could calibrate all the resistances at once.

Noise Due to Electron Tunneling

Electrical current across a conducting wire will fluctuate over time. These fluctuations occur about an average value–at one time it will be at one current, another time at some other value, but over time the average value of the current, $\langle i(t)i(t-\tau)\rangle_\tau$ (where tau is some interval between the present measurement time and another) can be found.

The similarity of the current at t and at $t-\tau$ can be measured via the cross-correlation, a function of the time between measurements used to find features in an unknown signal by comparing it to a known one. For our experiment, we use one measured value of current in order to determine the value at some later time: we cross-correlate the current signal with itself, which is known as an auto-correlation function [2] of the current, $R_I(\tau)$.

The Fourier transform of an autocorrelation function gives the spectral density, which can be written in this context in terms of the current as

$$\langle I^2 \rangle_{\triangle\omega} = \int_{\triangle\omega} S_I(\omega), \quad (1)$$

over a range of frequencies $\triangle\omega$. Electrical current fluctuations resulting in a measurable noise power $S_I(\omega)$ can be caused by a number of physical phenomena [5]. In this paper, the motion of electrons leads to two dominant types of noise : thermal noise and shot noise.

Thermal Noise

A conducting material at a nonzero temperature will spontaneously have an electrical current due to phonon interactions with free electrons. At equilibrium with no applied voltage but a finite temperature, phonons in the metal wire will agitate charge carrying electrons. Movement of electric charge defines a current, so this spontaneous electron movement leads to a measurable noise in the current across the wire without any applied voltage.

This kind of thermal noise is known as Johnson-Nyquist noise after the Bell Laboratory researchers who first discovered and explained it in 1928 [3][4]. They determined that in terms of the noise spectral density, thermal noise is

$$S(\nu)_{JN} = 4k_bTG. \quad (2)$$

Note that thermal noise is linearly dependent with temperature; the higher the temperature, the more thermal noise in a conductor. In order to measure shot noise we must perform experiments at a low enough temperature to minimize this noise.

Shot Noise

The fundamental source of shot noise is the discreteness of charge. Charge being carried by individual electrons is usually not apparent in conductors–the mass of other electrons in the electron sea strongly screen out the current fluctuations from any individual electron. However, when charges are tunneling through a potential barrier or an atomically-thin wire (where the conductor's length is less than the mean free path of an electron), the discreteness of charge manifests itself as shot noise.

Electrons tunnel across a potential barrier with no time or frequency dependence. So for a conducting wire with a small gap in it, current measurements should have detectable fluctuations–noise–due to individual electrons randomly crossing the gap. This can be seen in the classical result for the noise power of shot noise,

$$S(\nu) = 2e\bar{I} \quad (3)$$

where $\bar{I}$ is the average current. Although up to this point we have discussed noise in terms of fluctuations in the current, ohm's law provides an insightful version of the classical shot noise equation, namely,

$$S(\nu)_{SN} = 2eGV \quad (4)$$

In this case, G is conductance and V the applied voltage. So although shot noise occurs randomly, it (and thereby the likelihood of electron tunneling events) can be increased by applying a voltage across the barrier. This is discussed further in the next section, where it is shown mathematically that electron tunneling is the main source of noise in an atomic point contact.

Electron Tunneling

The voltage dependence of noise in an atomic point contact is what makes shot noise so useful. Shot noise has previously been used as a thermometer that doesn't require calibration [16], to determine the gain and noise temperature of an amplifier [10], and to study the fundamental physics involved in electron transport. As this paper tries to shine light on fundamental transport physics allowing for the possibility of a new resistance standard, it follows that electron tunneling must be discussed. Indeed, we will show that this view of noise power is much more enlightening than the classical equations for thermal noise and shot noise.

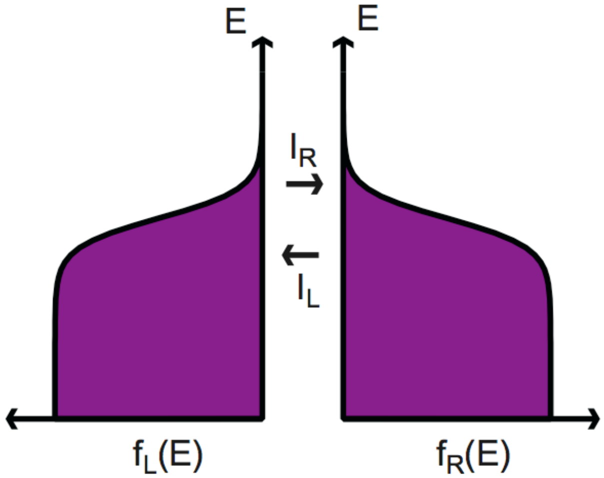

Figure 1: *A diagram of two groups of electrons separated by a barrier in equilibrium. I_R, the current from electrons tunneling from the left to the right, and I_L, the current for electrons going left, are equal. Graphic from [16].*

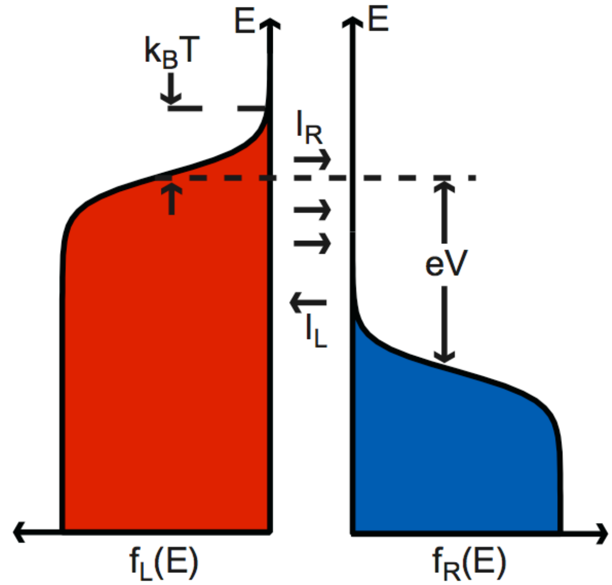

Figure 2: *A diagram of two groups of electrons separated by a potential barrier with an applied bias eV. This applied bias changes the difference in energy levels between the two groups, making electron transport easier and I_R larger than I_L. Each group is represented by a Fermi function at a finite temperature T that causes the spread in values at high energies. Graphic from [16].*

An atomic point contact can be thought of as a tunnel junction with electrons passing from one conductor to another across either an empty barrier or an atomically-thin wire. This arrangement can be represented by two groups of electrons spread across a continuous set of energies separated by an empty space as in Figure (1). Thermal energy, k_BT, smears out the top of both energy distributions. The rate at which electrons can move from one group to another is simply the current, with the average current equal to the difference between the two rates. As shown in Figure (2), when a voltage is applied the energy levels adjust to the potential difference, making electrons more likely to move in one direction than the other.

Because the two groups of electrons distributed evenly across energies are easily represented by Fermi functions, two interesting and important results can be obtained by applying the concepts of solid state physics to this problem. The total current can be written simply as $I = I_R - I_L$, but if we model the electron seas as Fermi functions, the current can also be written as $I =$

$$\frac{2\pi e}{\hbar}|\langle l|M(E_F)|r\rangle|^2 D(E_F)^2 \int [f_r(E) - f_l(E)]dE \tag{5}$$

When evaluated, this integral yields $I = V/R$, showing that a tunnel junction such as an atomic point contact is simply a ohmic resistor.

Furthermore, the noise spectral density of the current can be written as $S_I(V) =$

$$\frac{2}{R}\int \{f_r(E)[1-f_l(E)]+f_l(E)[1-f_r(E)]\}dE \tag{6}$$

which when evaluated [16] gives a second important result: the noise from electrons tunneling is

$$S(\nu) = 2eVG\coth\left(\frac{eV}{2k_BT}\right) \tag{7}$$

This is the general expression for noise–both thermal and shot noise–due to electrons tunneling through a potential barrier. It takes into account the experimental energy limits hinted at in earlier descriptions of noise above, as well as in Figures (1) and (2). In the high-temperature limit where the thermal energy, $k_B T$, is much larger than the electrical energy applied, eV, the noise due to electrons tunneling becomes

$$S(\nu) = 4k_b TG. \tag{8}$$

the classical equation for Johnson noise. If, however, $k_B T << eV$, then equation (7) is dominated by shot noise,

$$S(\nu) = 2eGV, \tag{9}$$

Rolf Landauer was the first to clearly point out that in some systems, ”shot noise and thermal equilibrium noise are special limits of a more general noise formula,” [17], as we have shown.

Neither thermal noise nor shot noise depend on the frequency of the measurement–they are both white noises. However, a fundamental difference between the two, as can be seen from equations (9) and (8) is that shot noise depends on the potential across the barrier while thermal noise does not. This means that, as shown in Figure (3), a plot of noise versus voltage will show a flat level of thermal noise with shot noise linearly increasing with voltage.

Note again that because thermal noise increases linearly with temperature, at higher temperatures the level of thermal noise will overwhelm the shot noise. In order to reduce the amount of thermal noise and successfully measure shot noise we perform our experiments at the low temperature of $4.2\,\mathrm{K}$.

In addition to working at low temperatures, however, we must also make the length of the conductor as short as possible, if not break it outright. This was done by using a mechanical break junction to stretch a gold wire and then form an atomic point contact, as described below.

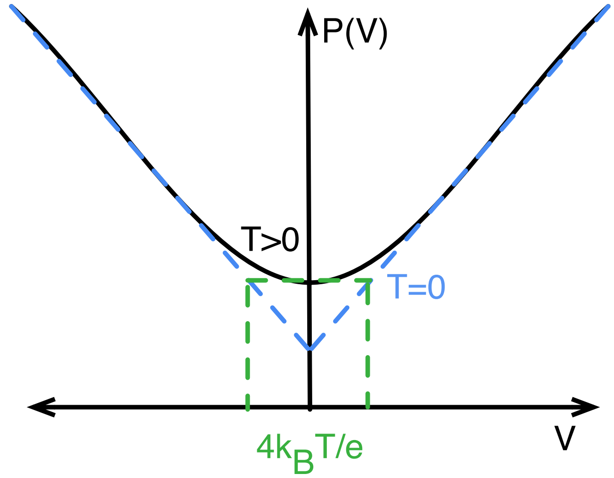

Figure 3: *A theoretical plot of noise power $P(V)$ from electron tunneling in an atomic point contact as a function of voltage V. At the zero temperature limit, theoretical shot noise is plotted as a blue dotted line. The green dotted line indicates the level of thermal noise. Interestingly, the temperature can be determined from the length of the dotted green line. Graphic inspired by [16].*

Methods and Materials

Mechanical Break Junction

Deep within a helium dewar, at the very end of a long stainless steel evacuated probe, is a vibration-isolated apparatus that holds a mechanical break junction, the heart of our experimental setup. This technique has been used widely to create an atomic point contact [10][11], but the purpose of a mechanical break junction (or MBJ) must be made clear and our design is innovative in ways worth noting.

Atomic point contacts can be made by bringing an atomically sharp tip of an atomic force microscope [12] or an STM tip [13] near a substrate covered in nano-wires or GaAs-AlGaAs heterostructures; anything sharp, conducting, and dense enough so that the sharp tip is likely to be atomically-close to one of them, like a javelin over a field of grass. A mechanical break junction, however, creates an APC by by pulling apart a piece of conductor to stretch it until it breaks. The MBJ leaves the conducting wire either in two parts with atomically-sharp points, or with an atomically thin wire between them [14].

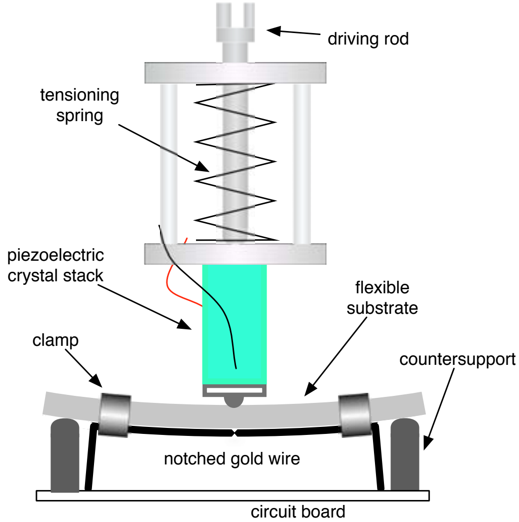

Figure 4: *A diagram of our mechanical break junction setup. During an experiment, the forked driving rod is screwed down, bending the flexible beryllium-copper substrate, which then stretches the clamped-down and notched gold wire until it breaks. Once broken, the piezoelectric crystal is used to finely tune the bend of the substrate and thereby the separation of the gap in the gold wire, which is connected to a circuit board to make measurements. The entire apparatus operates in vacuum at* $4.2\,K$

In our experiments we use a gold conducting wire notched with a razor blade. The internal stresses, reduced connection, and brittleness caused by the liquid helium temperature (4.2 K) makes breaking the wire easier. However, in order to make electron transport measurements, we also need to be able to change the separation of the wires–our quantum tunneling barrier–from open to close and many places inbetween. Controlled bending is accomplished by attaching the gold wire to a appropriately flexible piece of 0.25 mm-thick beryllium-copper, which is electrically isolated from the conducting wire by a layer of insulating kapton tape about 0.08 mm thick. The gold wire is held in place on the beryllium-copper by two clamps, which is pressed against three countersupports by the driving rod and piezoelectric crystal stack.

The separation between the two parts of the broken wire comes is roughly controlled by the extention of the driving rod. At the top of the driving rod, a hand screw extends out of our cryostat probe. It then passes into the probe via an ultra torr fitting, down the 1.5 m long interior, and meets the driving rod at the top of Figure (4). The forked connection allows us to bring the atomically-sharp points of the stretched and broken wire together and then disengage, switching to the finer control of the piezo stack actuator, which is rated to move $11.6 \pm 2.0 \mu m$ when 100 V is applied at room temperature. At 4.2 K we expect perhaps 10 percent of that range of motion, yet even that is enough for our measurements.

Piezoelectric crystals transform electrical energy into precisely controlled mechanical displacements[15]. While making measurements of shot noise suppression we need to take data across a wide range of resistances. Since the separation of the atomic point contact determines the transmission probability of electrons and therefore the resistance of the wire (see equation (5)), we can use a piezo actuator to finely adjust the resistance of the APC.

From there, however, we need to be able to measure the resistance as well as the fluctuations in the APC current. Thus, at the bottom of Figure (**??**), the notched gold wire is attached to a circuit board with the measurement circuit described below.

Measurement Circuit

After using a mechanical break junction held at 4 K to stretch a gold wire and create atomic point contacts, we then made simultaneous measurements of current and noise power as a function of voltage. The current measurements allowed us to find the resistance of the APC, which was then used to investigate the suppression of shot noise.

This is trickier than it sounds, however. Because shot noise occurs due to individual electrons moving across a potential barrier, it occurs very quickly. Moreover, the fluctuations in the current that we are measuring are small. In our experiments we used a microwave amplifier to account for the problems with measuring shot noise and allow for a faster readout.

The circuit inside the helium cryostat along with the

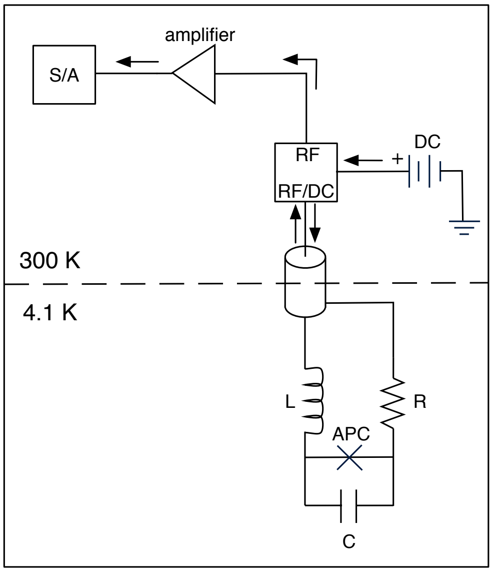

Figure 5: *The circuit diagram for the circuit used to make simultaneous conductance and noise measurements. A DC power supply at room temperature applies a voltage to the atomic point contact and accompanying circuit in the helium dewar. This circuit matches the impedance of the shot noise measurement circuit, which amplifies and then measures the noise power of the APC.*

APC is used to effectively couple the noise power from the high impedance APC to the microwave amplifier at the top of the circuit (see Figure(5)). This is done by carefully matching impedances via selection of the inductor and resistor to account for the capacitance inherent in the system. Because the selection of appropriate resistors and inductor values for impedance matching is a fundamental exercise and is explored more thoroughly elsewhere [10] [21], we will now focus on a direct measurement of a mismatch, the reflectance coefficient Γ.

Reflectance Calibration

The reflectance coefficient, Γ, indicates the amplitude of the reflected voltage and current standing wave on a conducting wire, where the other part of the superposition is the incident signal. To obtain no reflected power, the load impedance must match the characteristic impedance of the tramsission line, which in this experiment are BNC cables. If the load is mismatched, however, the power being measured will be inaccurate [21]. Therefore, in addition to our measurements of conductance and shot noise, we use a slightly different circuit to measure Γ for each APC (Figure (6)). These results for a typical sample are shown in Figure (7).

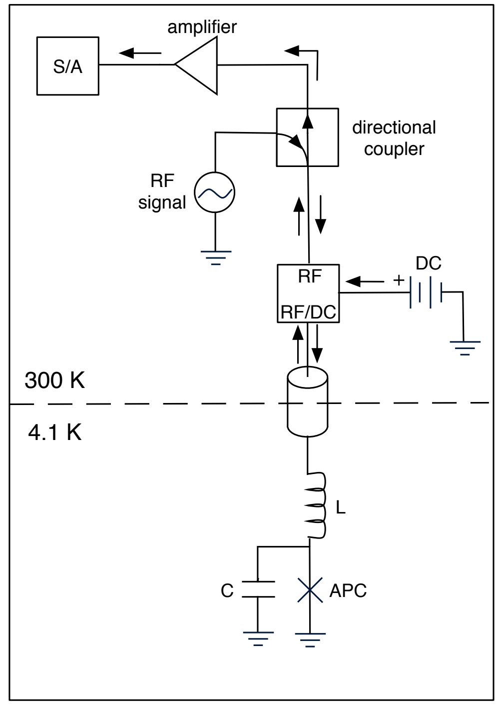

Figure 6: *This circuit diagram used for measuring the reflectance coefficient Γ has a slight difference between the conductance noise measurement circuit: a directional coupler allows us to send a signal down at the APC circuit and then measure the reflected signal. The signal is at the resonant frequency of the APC, as measured with a network analyzer.*

Just as $P = IR$, the total noise power can be expressed as

$$S_P = \frac{S_I}{G} = S_I R \tag{10}$$

If there is a power mismatch in our measurement circuit, then the available power P_{av} is reduced to only the fraction that is coupled into the sample. Since the available power is equal to the total noise power, we can write

$$P_{av}(1 - \Gamma^2) = \frac{S_I}{G} \tag{11}$$

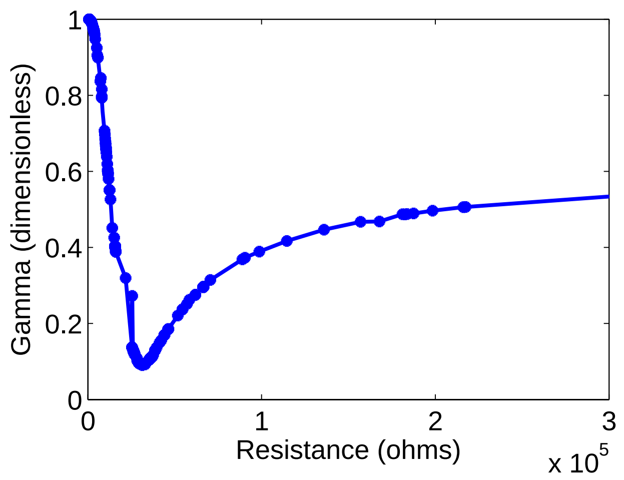

Figure 7: *A plot of the reflectance coefficient Γ as a function of resistance. Interestingly, this APC is best coupled near 30 kΩ, as the minimum indicates.*

which is equivalent to the result that the noise power, instead of being equal to $2eVG$ as before, is really

$$S_I = 2eVG(1 - \Gamma^2) \tag{12}$$

Thus, is is clear that to return the expected value of noise power, we must have to normalize S_I by $(1 - \Gamma^2)$.

The measurements for Γ are taken for a wide range of resistances, allowing us to normalize each measurement of shot noise. One aspect of our shot noise measurements is particularly well-suited for measuring shot noise suppression, and must be briefly discussed.

Shot Noise Measurements

Shot noise increases linearly with voltage. However, even at zero volts the noise can be offset significantly from zero depending on the microwave amplifier being used. This behavior is the reason why, in these experiments, we measured the supression of the *slope* of shot noise, not exactly shot noise itself.

Shot noise, when plotted as a function of voltage, looks like a ”V”. Three such ”V”s are shown in Figure (8), where each was measured at different

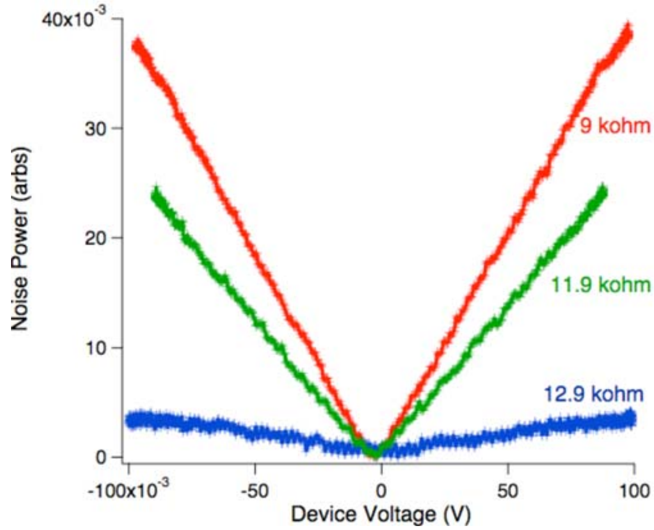

Figure 8: *The noise power from an atomic point contact plotted as a function of voltage for three different resistance values. As the resistance approaches 12.9 kΩ, the first quantum of resistance where the tunneling probability $T = 1$, the slope the noise is reduced.*

separations of the APC and therefore three different resistance values. As the resistance increases, the slope of noise ”V” arms becomes smaller and smaller until, at 12.9 kΩ, the noise curve looks almost flat in comparison with the other two. This is due to shot noise suppression, which will be better explained in the next section.

Results

Suppression of Shot Noise

Measurements of shot noise in two-dimentional electron gases in quantum hall experiments [18] show that the conductance G increases in quantized steps of $G_0 = 2e^2/h$ [18] known as the quantum of conductance. In some of these experiments, shot noise was observed to be suppressed around integer multiples of G_0 [6]. Indeed, previous experiments have shown similar suppression of shot noise in atomic point contacts, most notably from the research by van Ruiteenbeek *et al.* [20].

The suppression of shot noise at multiples of the

quantum of conductance can be explaine
ory about the nature of electron tunneli
atomic point contacts. Rolf Landauer ar
Büttiker give the conductance of a mesosc
ductor as [7]

$$G = \frac{2e^2}{h} \sum_{n=1}^{N} T_i \qquad (13)$$

where T_n are the probabilities of transmission for conducting channels. Note that for one channel the conductance is simply $G = 2e^2/hT$, and when that channel is perfectly correlated i.e. the probability of an electron tunneling through that channel is 1, then the conductance is G_0.

Landauer-Büttiker theory changes the equation for the shot noise spectral density, previously $S(\nu)_{SN} = 2eGV$ in terms of conductance, to

$$S(\nu) = 2e\frac{2e^2}{h}V \sum_{i=1}^{N} T_i(1-T_i). \qquad (14)$$

The extra term $(1-T_i)$ in the sum is due to the Pauli exclusion principle, which states generally that no two fermions can occupy the same quantum state. As electrons are fermions, the Pauli exclusion principle prevents two electrons to tunnel through the same conducting channel at once. It is this $(1-T)$ term–and more broadly the behavior of fermions in conducting channels–that causes the suppression of shot noise shown above in Figure (8) as well as more fully in Figure (9) below.

Our measurements of noise power spectral density across a larger range of conductances are plotted in Figure (10). At larger conductances, although the shot noise still comes to a point at the quanta of conductance (implying that it is still being suppressed), the overall level of shot noise is increasing. This is due to more and more conducting channels opening

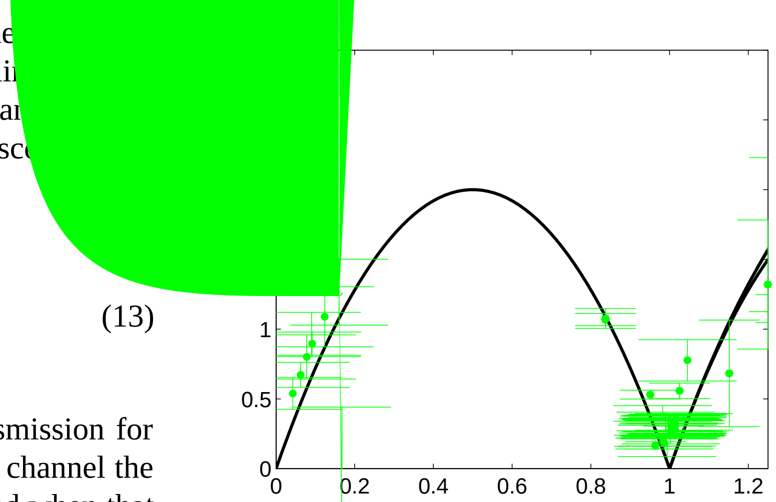

Figure9:

up at higher conductances, which is unaccounted for

[1]Mesoscopic is an intermediate scale between the macroscopic world and the point where the behavior of each individual atom becomes significant, roughly ten nanometers.

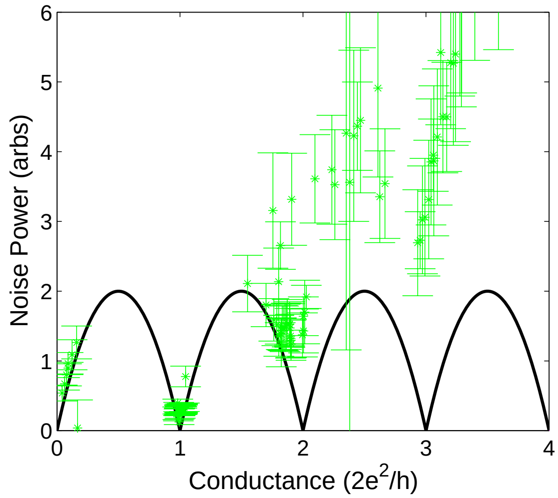

Figure 10: *Shot noise measurements (in green) with minima corresponding to the theoretical prediction from equation (14) in black. Instead of being completely suppressed, however, the noise increases with increasing conductance. This is due to more and more conducting channels opening up at higher conductances, which is unaccounted for in the simple model of equation (14).*

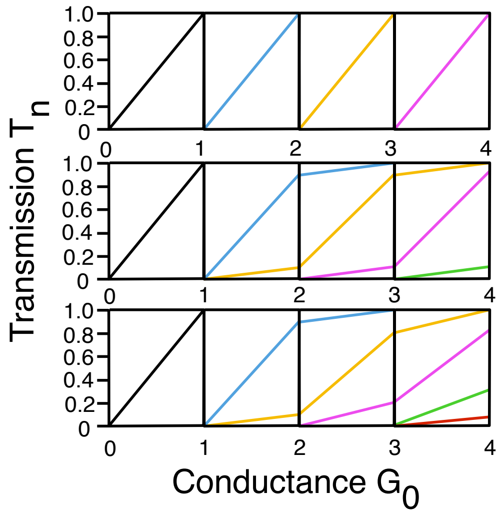

Figure 11: *Three diagrams of the probability of electrons tunneling, T, as a function of conductance, with each line representing a conducting channel in an APC. In the top diagram, onlychannel9.32rtimnce,43453(makcting)-32rthet(gr)12ransmissim, themiddlthe8(dia)12(gr)12(ame)481(hownce)1veamemo[(r)36.1tya6(c)12(hanne ,*

channels opening at once would allow for the overall rise in shot noise seen in Figure (10). Equation (14), when the T_i are as shown in Figure (11), are plotted below in Figure (12).

Determining Unknown Resistance Values

In the previous section we showed that shot noise is suppressed at integer multiples of G_0. Indeed, it seems that our measured data not only confirmed the Landauer-Büttiker theory but showed evidence of multiple conducting channels open at once. Now that the suppression of shot noise in our system has been established, we can finally perform what this paper set out to do in the beginning: use shot noise suppression as a resistance standard.

To provide an adequate test of this standard, we took four resistors between 1.2 kΩ and 120 kΩ, concealed their markings, and labeled each with a randomly-generated number. Keeping their true resistances unknown, we then determined their resistance values by analyzing the shot noise suppression of an connected atomic point contact. This analysis was

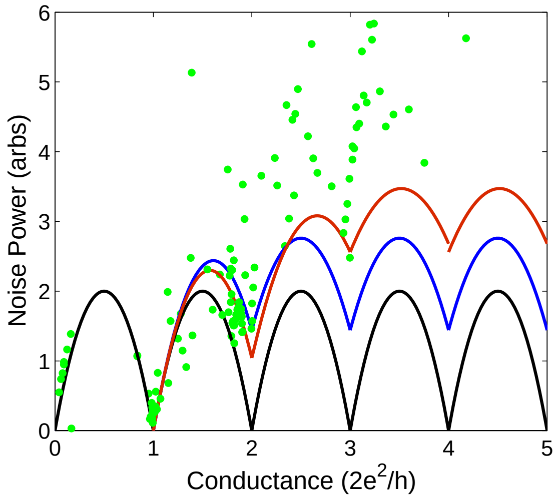

Figure 12: *Shot noise as a function of conductance with three plots of equation (14) in black, blue, and red for the upper, middle, and lower diagrams in Figure (11), respectively. As more and more channels open, the theory better fits the measured data.*

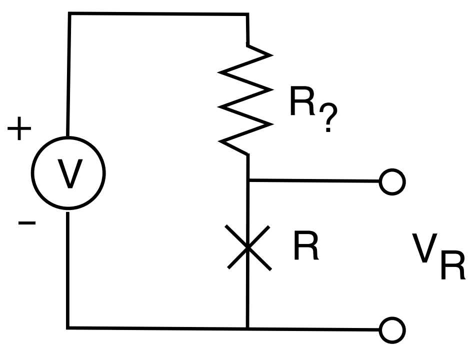

Figure 13: *In this diagram of a voltage divider, $R_?$ represents our bias resistor, R is the atomic point contact, V the applied voltage and V_R the voltage across the APC.*

voltage divider. In this setup, a voltage is applied to two resistors in series, R and $R_?$, where R is the resistance of an APC and $R_?$ that of a resistor with unknown value. The voltage across R can be measured, with the relation between it and the applied voltage and two resistors being

$$\frac{V_R}{V} = \frac{R}{R_? + R} \tag{16}$$

where V_R is the measured voltage across the APC. Using our experimental setup with an unknown bias resistor, we can take measurements of shot noise as a function of conductance. Although this measurement of conductance is necessarily arbitrary[2], we can then find two minima of the shot noise and know, due to theory described above, the resistance of those two minima. This allows us to divide out the applied voltage and write, for two APC-derived resistances R_1 and R_2,

$$R_? = \frac{R_1 R_2 (V_2 - V_1)}{R_2 V_1 - R_1 V_2} \tag{17}$$

For the first and second minima of shot noise, R_1 should be equal to $h/2e^2$ and R_2 equal to $h/4e^2$. This then makes $R_?$ equal to

$$R_? = \frac{h}{2e^2} \frac{(V_1 - V_2)}{(2V_2 - V_1)} \tag{18}$$

Switching to an unknown resistor, measuring shot noise across a wide range of arbitrary conductances, fitting curves to find the minimums at the first and second quanta of resistance, finding the corresponding voltage across the APC, and plugging them into equation (17), we are able to make bridge measurements to determine the resistance of an unknown bias resistor. An example of this curve fitting is shown in Figure (14)

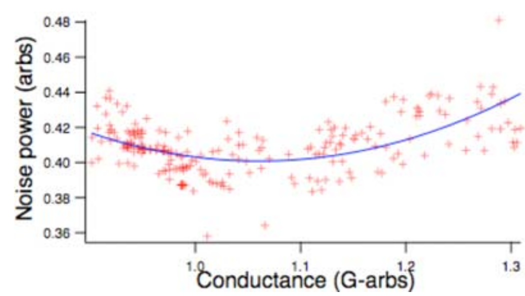

Figure 14: *Noise data near the second quantum of conductance for resistor 225 is plotted as a function of conductance in arbitrary units. A model is fit to the data in order to find the minimum.*

[2] Our bias resistor allows us to determine the resistance of a sample, and therefore if the bias resistor is replaced with a resistor of unknown value, the measured conductance would be uncalibrated. From one conductance to another it would still be relatively correct (presuming that the voltage measurement is linear), just in arbitrary units of conductance.

Method 2: Recalibration

Because our bias resistor allows us to determine the resistance of a sample, when the bias resistor is replaced with a resistor of unknown value, the measured APC conductance–such as the x-axis of Figure (10)–would be uncalibrated. Recall however how simple the calibration is, and moreover, that the calibration of the APC's conductance with a known bias resistor can be just as easily made of the bias resistor if the APC's conductance is known.

Fitting a curve to noise data near a minimum of shot noise (as in Figure (14)) determines the exact arbitrary conductance where Landauer-Büttiker theory predicts the APC's conductance to be G_0. Multiplying the arbitrary conductance by the reciprocal of G_0 gives the bias resistor's value. This is because equation (15) for an unknown resistor makes $R_{bias} = 1$ so that

$$R_{arb} = \frac{V_{APC}}{V_{bias}} \tag{19}$$

Sources of Error and Results

In the bridge measurement, we used two minima of shot noise and assigned them the resistances R_1 or R_2, claiming that because of equation (14) the minima would have to correspond to $R_1 = h/2e^2$ and $R_2 = h/4e^2$. But what if R_1 or R_2 are not integer mulitples of G_0? That is, what if noise power is at a minimum and the APC has a conductance other than G_0? In that case, R_1 and R_2 would no longer be standard resistances. The noise minima are unlikely to shift if, as is usually assumed for mesoscopic conductors such as an APC, there is only one contributing channel [17], and this is almost certainly the case for the first quantum of conductace. However, we have already shown evidence for multiple channels being open simultaneously at the second noise minimum. Not only could these extra channels lead to shot noise not being entirely suppressed, but extra open channels could also make the noise minimum differ from where it would be when the sum in equation (14) is equal to one. The uncertainty in the location of the second minimum is one possible source of error in the bridge measurement, which depends on that second measurement to determine an unknown resistor. Moreover, due to the geometry of most APC samples used, measurements near the second quantum of conductance were difficult to obtain–often the conductance would bounce back and forth from one conductance to another, giving inaccurate noise data.

The recalibration method of determining a resistor's resistance also has inherent sources of error, but most probably in the measurement of voltage–the recalibration method only uses the first noise minimum and is thus free of any uncertainty in the conductance of the second. One source of error in measuring V comes from the voltage source itself. The voltage applied by it is offset a small fraction, perhaps 0.3 microvolts. Of course, since we are applying a voltage of 0.3 milivolts, this error would be near one part in a thousand. Larger sources of error, perhaps as high as one part in ten, exist due to the fact that we are taking a measurement across lines that could act as a thermocouple and add voltage due to the extremely large thermal gradient. Thermocouples are usually made of two different metals that, when across a thermal gradient, have a potential difference between them. In our experiment, the stainless steel cryostat is the ground, while our RF and DC measurements are carried through a copper BNC wire, both of which go from ($4\,\mathrm{K}$) to ($300\,\mathrm{K}$). This could be solved by measuring the voltage across the APC with two wires of the same material.

Despite these sources of error, we found the following results. Using the bridge measurement technique, we obtained the values 6.25 $\pm$ 0.24 kΩ, 18.98 $\pm$ 0.23 kΩ, 13.59 $\pm$0.20 kΩ, and 55.67 $\pm$ 0.48 kΩ for the four unknown resistors labeled 225, 640, 423, and 582 respectively. With the recalibration method the derived resistances for 225, 640, 423, and 582 were 7.11 $\pm$ 0.37 kΩ, 21.98 $\pm$ 0.75 kΩ, 12.78 $\pm$ 0.51 kΩ, and 55.84 $\pm$ 2.84 kΩ. After these resistances were found from the noise, we measured the unknown resistors with a Agilent 34401A 6 $1/2$ digit multimeter. They were 7.06902 kΩ for 225, 21.6989 kΩ for 640, 12.8957 kΩ for 423, and 55.495 kΩ for 582. The three data sets, with error bars, are plotted in Figure (15).

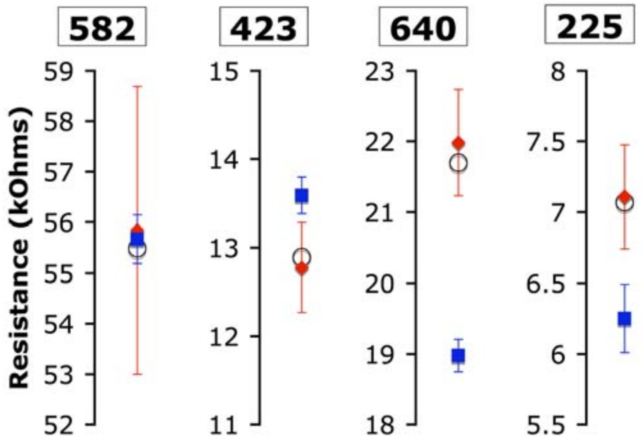

Figure 15: *This figure shows all four test resistors and their measured values as a black ring. The bridge measurements (plotted as blue squares) are off by as much as 13 percent, while the red resistance measurements found via the recalibration method (in red) are off by at most 1.3 percent.*

Conclusion

Although there are important sources of error that could be removed, the general conclusion that can be drawn from the results of this work is that using the shot noise of an atomic point contact as a resistance standard is very promising.

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