The Swing Spring: A Search for Classical Monodromy

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Abstract

This experiment was designed to demonstrate classical monodromy in the swing spring system. A swing spring in 1:2 resonance shows a stepwise precession in swinging planes. Monodromy is manifested as a discontinuity in the precession angles. We have been successful in showing the stepwise precession of the swing plane; however, we have been unable to show agreement between the experimental and theoretical step angles. Work on the experiment continues, and we soon hope to demonstrate theoretical and experimental agreement, as well as to show monodromy.

1 Introduction

1.1 Motivation

Quantum monodromy has been well documented and shown for some time. Monodromy in molecules like carbon dioxide and the imidogen (NH) radical has implications for molecular spectroscopy due to its effect on the allowable energy transitions. However, classical monodromy, although a well-developed theoretical concept, has never been demonstrated experimentally. We hope that by understanding classical monodromy in the swing-spring system, we will be further able to understand the implications of monodromy in the quantum systems that our research is largely focused upon.

1.2 Purpose

This project was intended to continue the work of a past REU student, Sarah Anderson. Her project resulted in the construction of an apparatus suitable for data taking, as well as work on some of the theoretical basis behind classical monodromy. The continuation of this project focused mainly on the improvement of the accuracy of data taking and further theoretical improvements.

2 Theory

2.1 Resonance

The swing spring is a system simple enough so that most first- or second-year physics undergraduates are capable of solving it. However, the system has the potential to display some interesting behaviors that are disphase space. A successful demonstration of monodromy would show the discontinuity in step angle in the first circle but the smooth, continuous nature of the step angle in the latter.

3 Experiment

3.1 Experimental Setup

The experimental setup was mostly constructed and functional when I began this project, so not much description will be given to the construction of the setup. Essentially what was necessary was a black enclosure, constructed from foam board and felt, to provide a dark background against which the cameras could capture the moving ball. Two cameras were included in the setup, one for the xy-plane, another for the yz-plane. This way, the motion of the illuminated ball could be captured in three dimensions. Due to a noticeable blur in the motion of the ball, the shutter speed on the cameras was set high, about 1/60 to 1/100 of a second. This allows less light to get into the camera, reducing blur but also brightness of the ball. For this reason, bright halogen lights were mounted near each camera, with the intention of saturating the image captured by the camera. This renders the ball as bright as possible on the black background. This high contrast is necessary for the computerized image processing tools to recognize the ball. Gain and saturation can also be adjusted with the computer by adding light to each pixel in the grayscale image (a higher pixel value gives a brighter picture). The swing spring itself is by far the most important part of the whole setup, given that the resulting behavior depends highly on the $\nu = 2$ condition. To construct this, brass swivels are glued to the ends of an appropriately loose spring. A spring must be chosen such that it is loose enough to provide long enough transition times to be noticeable, but not so loose that damping becomes an issue due to the long times necessary for data taking. A stiffer spring is generally to be avoided, how-

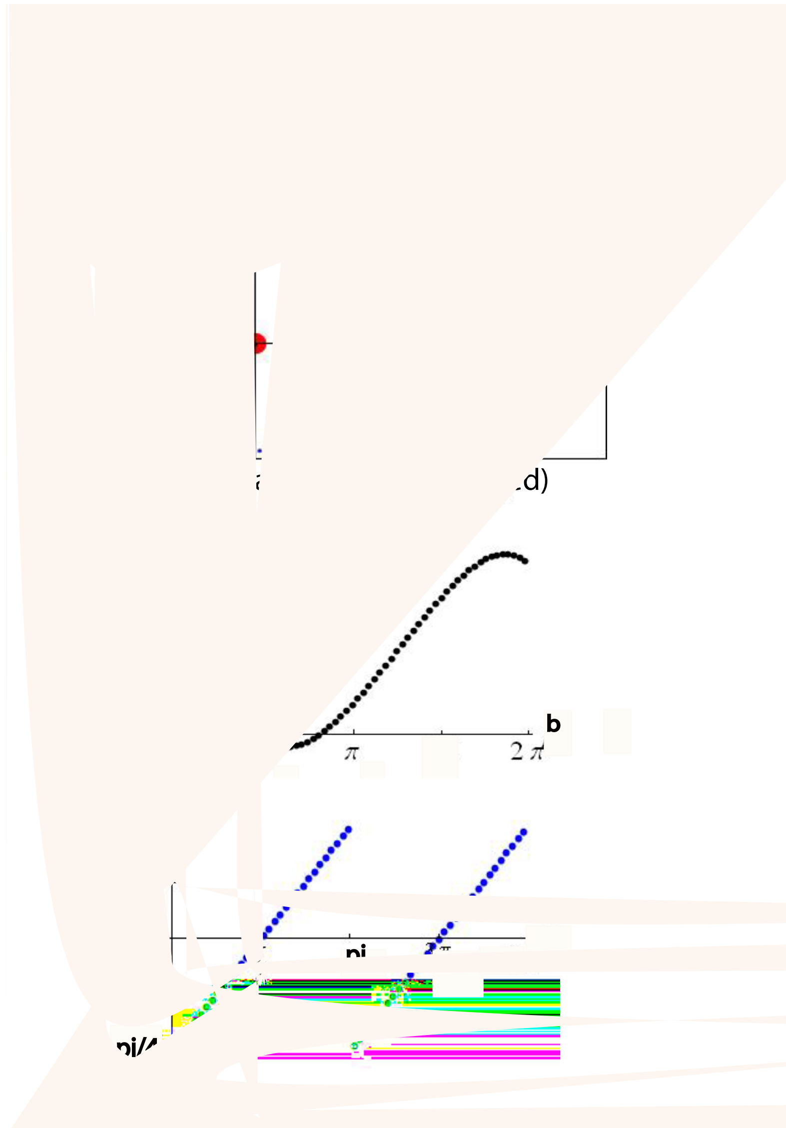

Figure 1: (a) Energy and Angular Momentum Phase Space (b) A circle without the singularity shows continuity in precession angles (c) A circle surrounding the singularity shows a clear jump discontinuity in precession angles

ever, since it is difficult to tell when the mass is in pure swinging motion, since the energy transfer between swinging and springing modes happens on such a short timescale. The spring constant k is measured by finding the amount of time it takes a known mass on the end of the spring to oscillate ten times. With this value, the spring is then calibrated to resonance by tuning the length, such that $\frac{mg}{kl} = \frac{1}{4}$, giving the 1:2 resonance necessary to observe monodromy. Once the necessary length has been found, fishing line is added to the spring to give the extra length necessary. Then, the spring is tested to make sure the energy transfer between swinging and springing modes is observed. The current swing spring setup in use has $k = 6.5$ N/m, $l = 1$ m, $m = 0.2246$ kg. The mass in the setup is a stainless steel ball painted white. To maintain the contrast, the experimenter wears a black long-sleeved shirt, black gloves and a black ski cap. For a diagram of the experiment, see Figure 2.

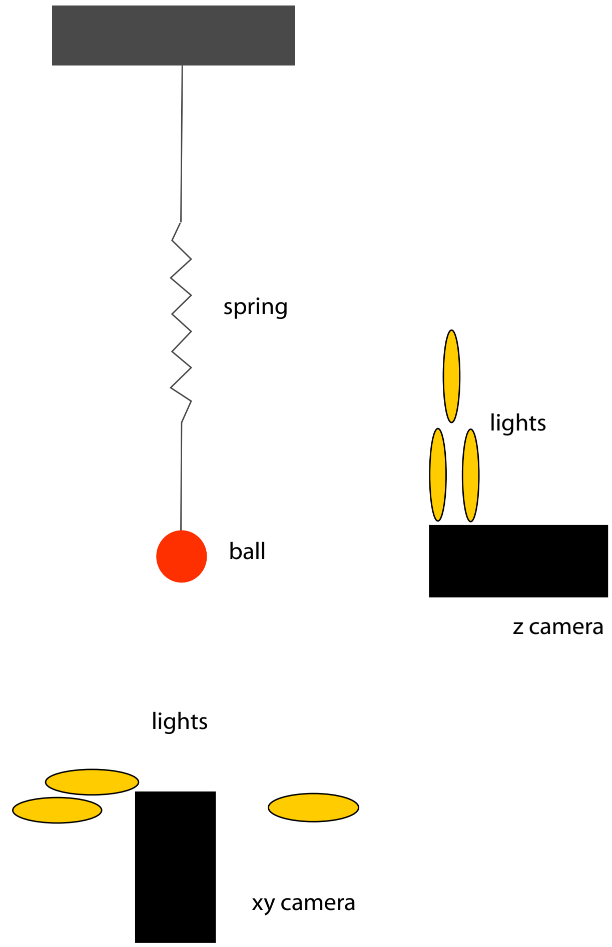

Figure 2: The Experimental Setup

3.2 Data Taking

Once the ball is hanging in the enclosure, both cameras are turned on and manually focused on the ball. An automatic focus is not desirable because the changing focus in the camera lenses changes the pixels to meters calibration factor. Once the cameras are properly focused, footage of the still ball is recorded in all dimensions. From this, the radius of the ball as captured by the camera (in pixels) can be measured and compared to the known radius of the ball. This allows the experimenter to set the pixels to meters calibration, as well as to find the center of the still ball (which corresponds to the origin). Then, the ball is started moving by hand, with ideally low energy and low angular momentum. This is necessary since the scaled energy and angular momentum variables are series expansions about the origin in phase space–remaining close to the origin gives much better approximations. The cameras then capture the motion of the ball, and the resulting movies are input into the computer, where they are deinterlaced (for clarity) and synchronized (so that the z motion and corresponding xy motion line up) in Adobe Premiere Pro CS3. This allows the experimenter to analyze the z motion of the ball and pick out the swinging motion in the xy-plane much more easily. With a non-zero angular momentum, the swinging motion strikes out an ellipse in xy-space, although the ellipse narrows out significantly with decreasing angular momentum. A video of this motion is then fed into MATLAB, where each frame in the video is rendered grayscale, then changed to simply black and white by setting a contrast limit. Pixels above this limit are rendered white, where those at or below the limit are black. This gives the image of a white ball on a black background. Then, the edges of the ball are found using the gradient function in MATLAB, where a nonzero gradient corresponds to the edge of the ball. These points are then fit to a circle, and the center of this circle gives the position of the ball. A sample fit to a ball is shown in Figure 3. In this way, the motion of the ball can be tracked through the video. The data are then fit to ellipses using a fitting scheme given by Halíř [1]. Examples of typical ellipse plots showing a constant precession angle are shown in Figure 4. The axes are in units of pixels, with (0,0) at the lower left-hand corner.

To find the position of the data run in phase space, it is necessary to measure the instantaneous conditions of the ball. That is, we must measure the position and velocity of the ball at any time in the motion. For ease, we choose a point on the ellipse with low curvature, and thus the least acceleration. The position is measured relative to the center of the still ball, and the instantaneous velocity is found by taking the central difference approximation. That is, the position of the ball is taken at the frame before and after the frame at which we measure the velocity. We then divide the distance traveled in these three frames by the time taken to shoot three frames (at frame rates of 29.97 fps). These values are then fed into a Mathematica script that computes the scaled energy and angular momentum of the system and places the run at a point in phase space. The script then calculates the theoretical precession angle based on the experimentally-determined scaled variables. Theory was provided by Holger Dullin in [2].

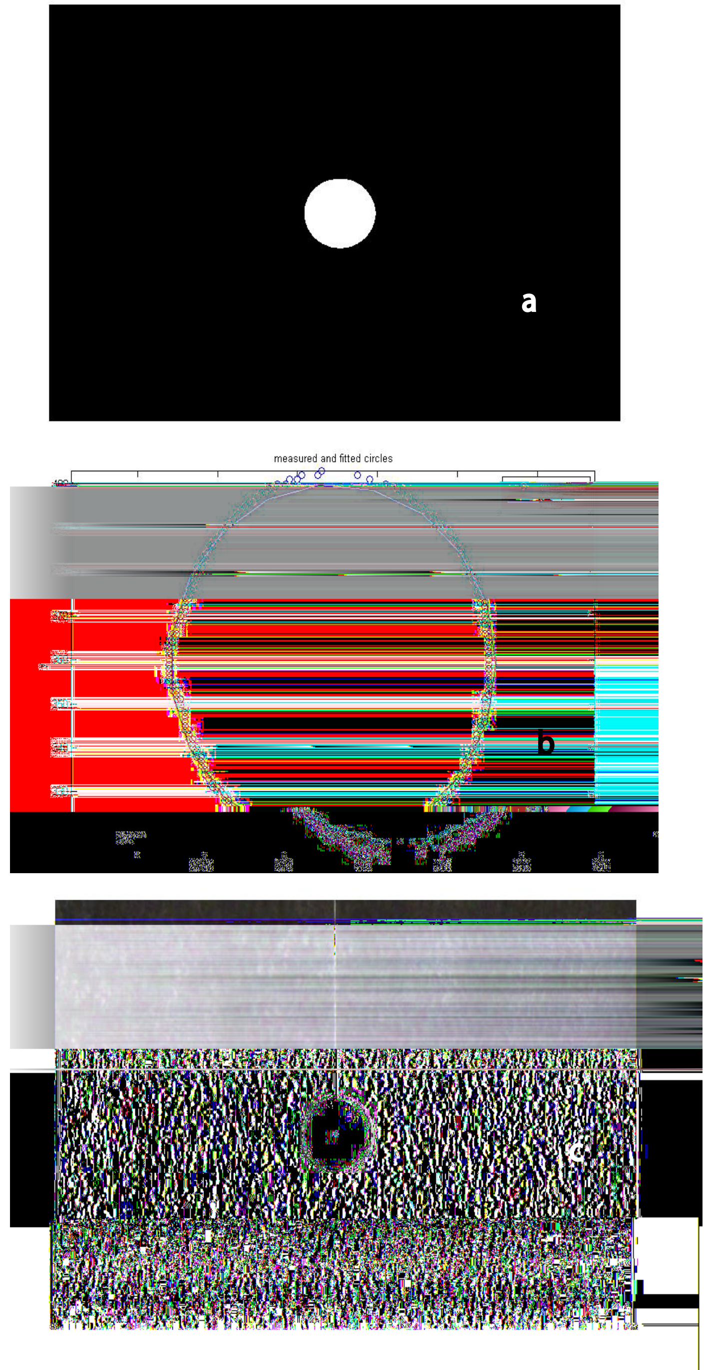

Figure 3: (a) The white ball on a black background (b) Points on a circle and fitted line (axes in pixels) (c) The actual image of the ball and the fitted circle

4 Conclusion

4.1 Future Work

The experiment is fully constructed and running. However, we are unable to present any good data because there is little agreement of experiment with theory. This is most likely due to imprecision when mea-

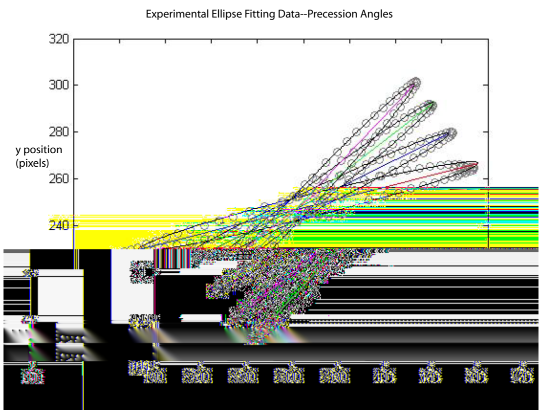

Figure 4: Ellipses Showing a Precession Angle of 5 Degrees

suring the position and velocity of the ball combined with the sensitivity of the theoretical precession angle to the experimental measurements. We must work to measure the position and velocity better. One cause of imprecision is the camera's depth of field. We must work to quantify how the pixels to meters calibration value changes as one moves in the x, y and z directions. Once this is well-quantified and accounted for in the image processing, we hope to have better agreement between the experimental and theoretical precession angles. When we have this agreement, we will proceed in traversing the two circles needed to adequately show the presence of monodromy–that is, the circle around the singularity at the origin and one without this singularity within it. After this has been demonstrated, the resulting paper will be submitted to the American Journal of Physics.

References

[1] Radim Halíř and Jan Flusser, *Numerically Stable Direct Least Squares Fitting of Ellipses*. Department of Software Engineering, Charles University, Czech Republic, 2000.

[2] Holger Dullin, Andrea Giacobbe and Richard Cushman *Monodromy in the Resonant Swing Spring* Physica D 190, pp. 15-37, 2004.