

Spin-Squeezing using the Pound-Drever-Hall Measurement Scheme

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August, 2012

Abstract

A new measurement scheme has been proposed for making quantum nondemolition spin-squeezing measurements on an ensemble of 10^6 laser-cooled Rubidium atoms. The atoms will be probed using a Pound-Drever-Hall (PDH) frequency stabilization measurement technique to measure the populations in a two-level system with more certainty than inferred by the Standard Quantum Limit. A necessary component of this PDH measurement scheme is an avalanche photodiode (APD) circuit. The noise properties of this circuit must be well-understood and tested so it is able to resolve the photon shot noise associated with the spin-squeezing.

Introduction

Precisely resolving the populations in a two-level atomic ensemble is a key component of high resolution atomic sensors which measure time, rotation and gravity. [1] James Thompson's group works with 10^6 laser-cooled Rubidium atoms inside of an optical cavity with two available ground states. The atoms' population count is measured inside of an optical cavity because of the effective increase in optical depth which comes with cavity use, which will increase readout sensitivity. The term optical depth refers to how many photons are absorbed in a medium, and an atomic ensemble with high optical depth simply implies that a high amount of photons are absorbed. Because the photons are in a cavity with mirrors, the photons are able to pass through the ensemble many times, which implies an increased optical depth. Additionally, the readout scheme is considered to be nondestructive to the ensemble, changing the state as little as possible with each measurement. The nondestructive method will relate the resonant frequency of the cavity with the atoms inside to the number of atoms in the atomic state which will be referred to as spin up, and the atomic state referred to as spin down. [1]

Theoretical Background

Quantum Nondemolition Measurements

Making a quantum nondemolition measurements is the key to spin-squeezing. In quantum mechanics, certain variables don't commute. Although position and momentum are not the relevant variables in our experiments, a familiar example is $[x, p] \equiv xp - px = i\hbar$. This particular relation leads to the Heisenberg Uncertainty Principle, which states that $\Delta x \Delta p \geq \frac{\hbar}{2}$. This type of fundamental uncertainty leads to quantum noise. [1]

Consider a Bloch Vector representation of the two-level system as pictured below in **Figure 1**:

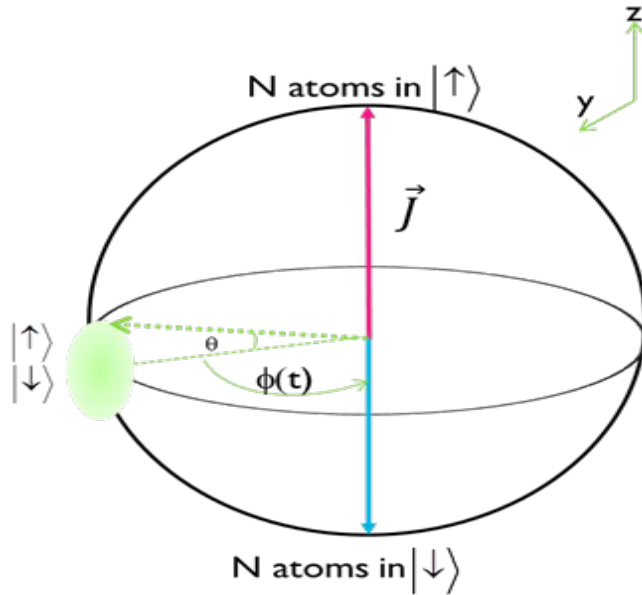


Figure 1: A Bloch Sphere representation of the two-level system. The length of the vector \hat{J} is N , the number of atoms. The azimuthal angle, θ , represents the number of atoms in spin up and spin down. The phase of the system is represented by $\phi(t)$. At the end of the vector is an uncertainty disk. The angle from the vector at the center of the disk to the vector at the end of the disk is $\frac{1}{\sqrt{N}}$

Let the vector \hat{J} represent the state of the two-level system. The length of the vector is N , the number of atoms. The azimuthal angle, θ , represents the number of atoms in spin up and spin down. If the vector aligns with north pole, all of the atoms are in spin up, and similarly, if the vector aligns with the south pole, all of the atoms are in spin down. If the vector is on the equatorial plane, then half of the atoms are in spin up and half are in spin down. The phase of the system is represented by $\phi(t)$. At the end of the vector is an uncertainty disk. The angle from the vector at the center of the disk to the vector at the end of the disk is $\frac{1}{\sqrt{N}}$. [1] This scaling represents the Standard Quantum Limit. In spin-squeezing experiments, it is desired for θ to be “squeezed” to a value smaller than the Standard Quantum Limit. As a consequence, the uncertainty in phase will grow, which is referred to as anti-squeezing. [1] This can be visualized by squeezing the disk as if it were a balloon from a sphere into an ellipse.

The key to spin-squeezing is making a nondemolition measurement. [1] In a simplified model, if an experiment can measure but not change the quantity $N_{\uparrow} - N_{\downarrow}$, then a second measurement can be made which is correlated with the first. This is the nondestructive aspect of measuring which leads to squeezing. Additionally, if the measurement tells how many atoms are in spin up and how many are in spin down without giving information about which atoms are in each state, then an entangled and coherent state is preserved which also ultimately improves readout sensitivity. [1]

Experimental Background

As stated in the introduction, these experiments are performed in a cavity, however, the reason is not immediately obvious. A property of cavities which makes them an appealing place to study atoms is the fact that they have resonant frequencies. These resonant frequencies can be measured, and give information about what is happening inside the cavity. [2]

Fabry-Perot Interferometer

Consider a cavity consisting of two partially reflecting mirrors. As coherent light is incident on the exterior of the cavity, some of it will be transmitted through the cavity. As some of this light is reflected back and forth between the mirrors, it will interfere with light coming into the cavity from the outside of the mirrors. This multiple-beam interference leads to a ratio of the incident beam flux density to the transmitted beam flux density from the opposite side of the mirror, I_t/I_i [2]:

$$\frac{I_t}{I_i} = \left(1 - \frac{A}{1-R}\right)^2 Ai(f) \quad (1),$$

where A is the absorption associated with the cavity, R is a measure of the reflectivity of the cavity, and Ai is defined as

$$Ai \equiv \frac{1}{1 + F \sin^2(\delta/2)} .$$

F is proportional to the finesse of the cavity and is defined in terms of a reflection coefficient, r:

$$F \equiv \left(\frac{2r}{1-r^2}\right)^2 .$$

Finally, the quantity δ is the phase shift added to the wave as the beam is reflected inside the cavity. This quantity depends on the frequency of the light. Thus, tracing δ back to equation 1, the transmission depends on the light frequency, and thus associated with the cavity are resonant frequencies which correspond to maximum transmission.

Below in **Figure 2** is a picture of the Rb atoms inside of a cavity which will ultimately be used in the spin-squeezing experiment:

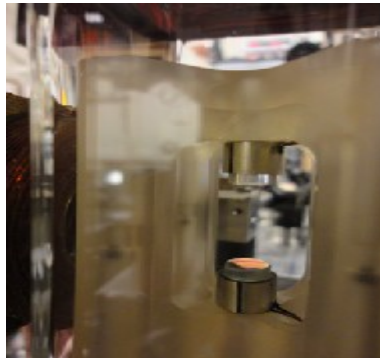


Figure 2: Around one million Rubidium atoms will be measured inside of a cavity with a length of about 2 cm.

Coupled Atom Cavity Modes

The next section will explain the way in which placing atoms in a cavity will influence the associated resonant frequencies. As will be shown, it will finally be possible to count the number of atoms in spin up and correlate this with a cavity's resonant frequencies.

Let two ground-states of a system of N atoms in a cavity be $|\uparrow\rangle$ and $|\downarrow\rangle$, spin up and spin down, respectively. The atoms in $|\uparrow\rangle$ can absorb cavity probe photons while the atoms in $|\downarrow\rangle$ cannot. Certain approximations are made to formulate a Hamiltonian, H for the system. First, the ratio of the number of atoms in the excited state to the number of atoms in the ground state is much less than 1 and second, the coupling between each atom and the cavity field is uniform. [1]

$$H = \hbar \delta_c \hat{c}^\dagger \hat{c} + \hbar \sqrt{N_\uparrow} g (\hat{a} \hat{c}^\dagger + \hat{a}^\dagger \hat{c}) .$$

δ_c is the cavity detuning defined as the difference between the cavity frequency and the angular frequency associated with the transition from $| \uparrow \rangle$ to the excited state: $\delta_c \equiv \omega_c - \omega_{\uparrow e}$. The operators \hat{c} and \hat{c}^\dagger are creation and annihilation operators associated with the cavity field, with a commutator of 1 which define the photon number operator, $\hat{M}_c \equiv \delta_c \hat{c}^\dagger \hat{c}$. The quantity g describes the coupling of each atom to the cavity field. The operators \hat{a} and \hat{a}^\dagger are creation and annihilation operators associated with raising a spin up atom into the excited state. This quantum system acts somewhat analogously to a classical coupled harmonic oscillator. In this case, the coupling occurs between the atoms and cavity. [1]

The coupled Heisenberg equations of motion derived from this Hamiltonian are

$$\begin{aligned} \frac{d \langle \hat{c} \rangle}{dt} &= -i \delta_c \langle \hat{c} \rangle + i \sqrt{N_\uparrow} g \langle \hat{a} \rangle \\ \frac{d \langle \hat{a} \rangle}{dt} &= i \sqrt{N_\uparrow} g \langle \hat{c} \rangle \end{aligned}$$

Solving these equations gives the coupled system's resonant eigenfrequencies ω_\pm ,

$$\omega_\pm = \frac{\delta_c \pm \sqrt{\delta_c^2 + \Omega_\uparrow^2}}{2} ,$$

where $\Omega_\uparrow \equiv \sqrt{N_\uparrow} 2g$.

Thus, there is a mapping between the number of atoms in spin up and the frequency of light which is resonant in the cavity. [1]

To measure how nondestructive the measurement is, m_s^{proj} , the number of photons scattered into free space normalized to N atoms is to be measured. In principle, free space scattering is gaining information about individual atoms, and thus does not treat the atoms as indistinguishable. The more that free space scattering occurs, the more destructive the measurement is. This quantity is to be minimized in order to improve measurement sensitivity [1]:

$$m_s = \frac{1}{4qN_\uparrow C} \left(\frac{\kappa'}{\kappa} \right)^2 \left(1 + \frac{\delta_c^2}{\Omega_\uparrow^2} \right) \frac{\Omega_\uparrow^2}{\omega^2} \quad (2).$$

This quantity depends on the quantum efficiency, q , and a cooperativity parameter, $C \equiv \frac{(2g)^2}{\kappa \Gamma}$, where Γ is the excited population decay rate, and κ is the cavity decay rate. Additionally, when the cavity is damped or driven, a dressed cavity power decay rate is defined as [1]

$$\kappa' = \frac{4 \omega_\pm^2 \kappa + \Omega_\uparrow^2 \Gamma}{4 \omega_\pm + \Omega_\uparrow^2} .$$

In equation 2, m_s decreases as δ_c increases, but as δ_c is increased, more probe photons are required, which contributes to destructiveness. Thus, the degree of nondestructiveness and extent to which a state can be squeezed is ultimately set by $qN_\uparrow C$.

Pound-Drever-Hall

A set-up for the proposed Pound-Drever-Hall scheme to measure the cavity's resonant frequency is shown and explained below in **Figure 3**:

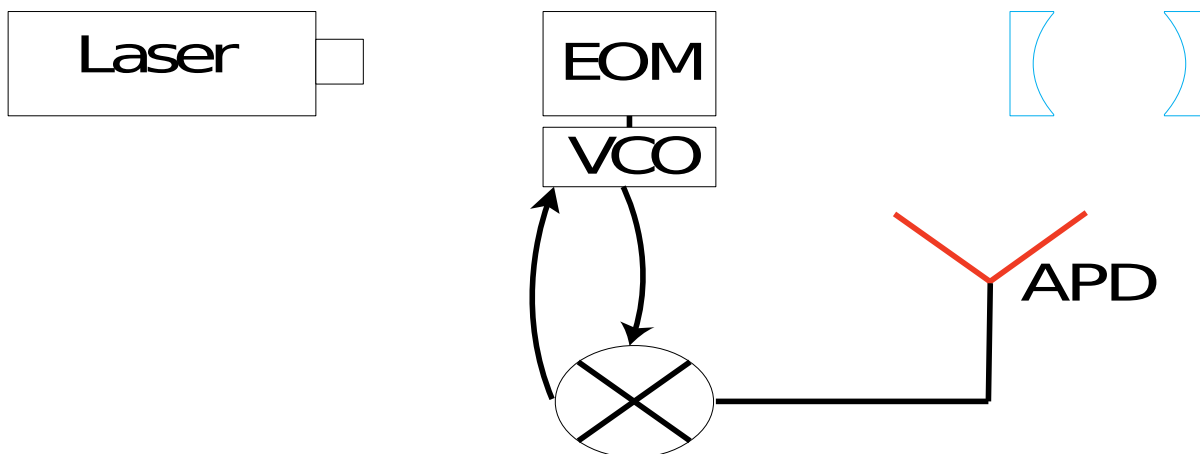


Figure 3: A simplified layout of the PDH measurement scheme which will be used in the experiment is shown. A beat note between two lasers will enter the Pound-Drever-Hall (PDH) set-up. An EOM will add sideband frequencies to the beam which will next enter the cavity with the atoms. A reflected signal will be isolated using a polarizing beam splitter and quarter-wave plate and sent to an avalanche photodiode. The optical signal will be converted into an electrical signal, as denoted by the black lines. The electrical signal will be mixed with a local oscillator coming from the VCO and from this mixing an error signal will be read. This will be fed back to the EOM through the VCO. [3]

A beat note between two lasers will enter the Pound-Drever-Hall (PDH) set-up. A beat note is used because the frequency of a laser (on the order of terahertz) is too high for the instruments used in the PDH measurement scheme, and a beat note is a sinusoidal signal whose frequency is the difference of two beating beam sources, which is on the order of megahertz.

An electro-optical modulator (EOM) will add sideband frequencies to the beam which will next enter the cavity with the atoms. An EOM can add a variable amount of phase to the laser beam. These work through the Pockels Effect, which occurs when the refractive index of a crystal changes in response to a change in the applied electric field. [4]

After exiting the EOM, the beam enters a polarizing beam splitter and a quarter-wave plate. The polarizing beam splitter allows a certain polarization to continue through to the cavity. The quarter-wave plate makes the beam circularly polarized. After the beam is reflected from the cavity, it travels through the quarter-wave plate again. At this point, when the beam reaches beam splitter, it is not at the same polarization as it was when it exited it. Thus, it is deflected into the avalanche photodiode (APD) to be measured and not allowed to continue back to the EOM. [3]

This signal is sent to the APD circuit. A photodiode converts an optical signal into an electrical signal by taking advantage of the photoelectric effect. Incident photons excite electrons in a p-n junction which creates electron-hole pairs and thus a current. An avalanche photodiode is more sensitive and can detect smaller signals. An electric field on the order of 100V will reverse-bias the photodiode, and the excited electrons will excite other electrons to ultimately increase the signal strength. This amplified optical signal will thus be converted into an electrical signal, as denoted by the black lines. [3]

The electrical signal will be mixed with a local oscillator coming from a Voltage-Controlled Oscillator (VCO). A VCO is an oscillator whose frequency is controlled by a voltage input. From this mixing, an error signal will be read. This will be fed back to the EOM through the VCO until the error signal is zero. At this point the sidebands created from the EOM will be locked to the cavity resonant frequency. Thus, since they are locked, the cavity resonant frequency can be deduced. [3]

APD Construction

A photodetector using an APD was constructed. The circuit diagram is shown below in **Figure 4**:

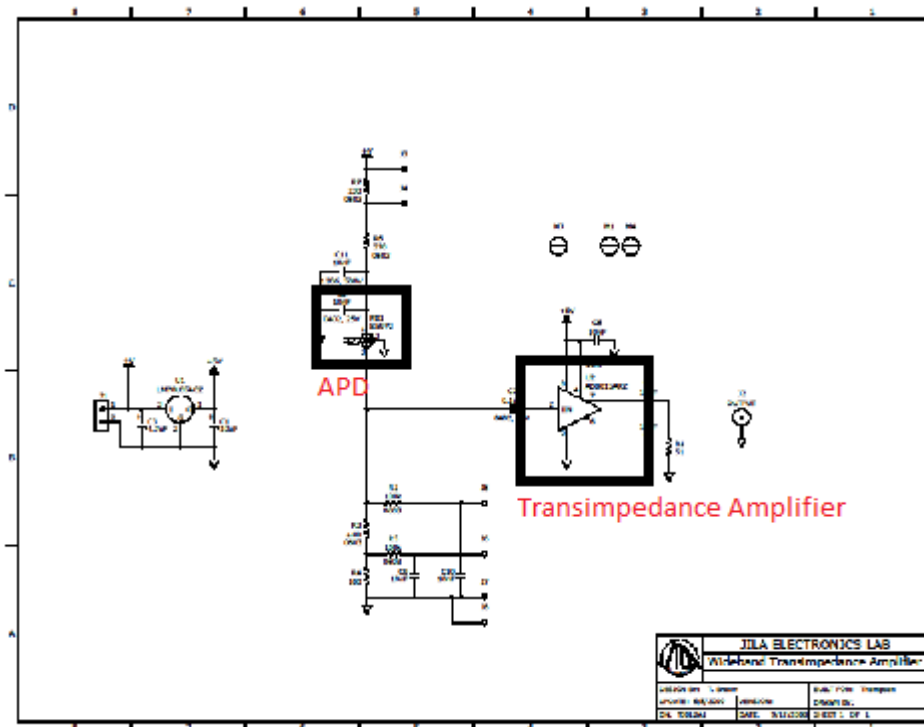


Figure 4: A schematic for the APD circuit used in the PDH measurement is shown above. Boxed on the left is where the APD is placed on the circuit. This is where the focused incident light will be detected. To the right is the transimpedance amplifier, which will amplify the electrical signal from the APD.

Data

Noise Measurements on the APD circuit:

The following noise measurements are taken to determine a minimum frequency and power of photons to send to the APD circuit such that the photon shot noise is stronger than noise from the circuit. Photon shot noise is a noise in current which comes from the discrete nature of particles. Recall that the EOM will add sidebands to the wave at frequencies detuned from the carrier frequency by δ . The carrier frequency is simply the frequency of the light before it enters the EOM. The frequency in all of the following measurements will refer to δ .

A transfer function relates an input signal to an output signal such that the output is equal to the transfer function multiplied by the input. Below in **Figure 5** is the measured transfer function for our circuit:

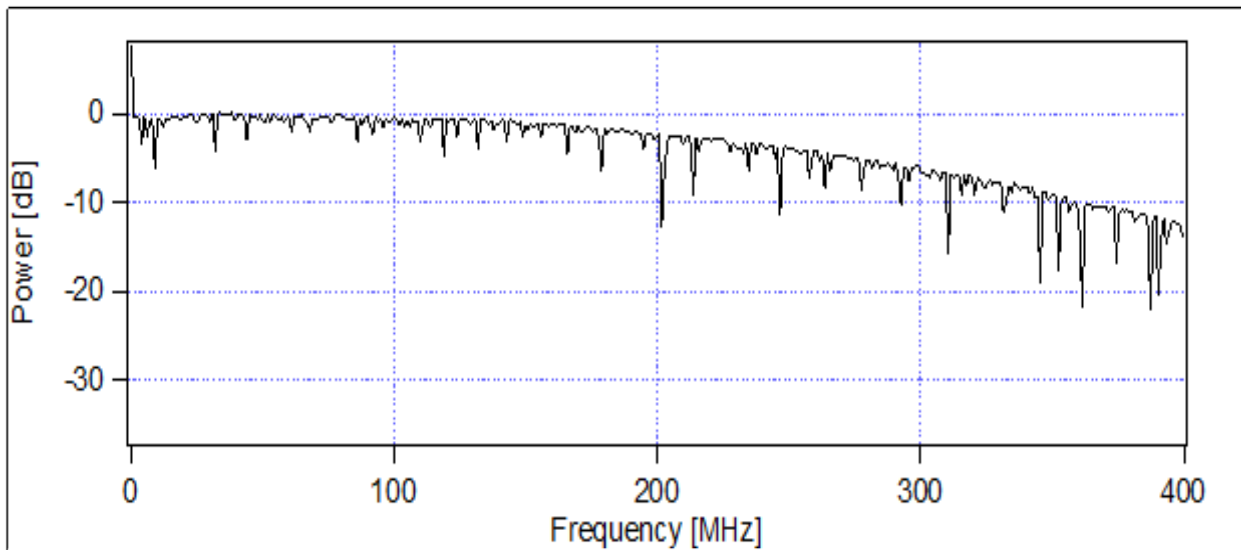


Figure 5: A transfer function for the APD circuit. The power was measured as a function of detuning frequency and normalized to 0.

Outlier fluctuations were deleted, and the transfer function was given a fit in **Figure 6**.

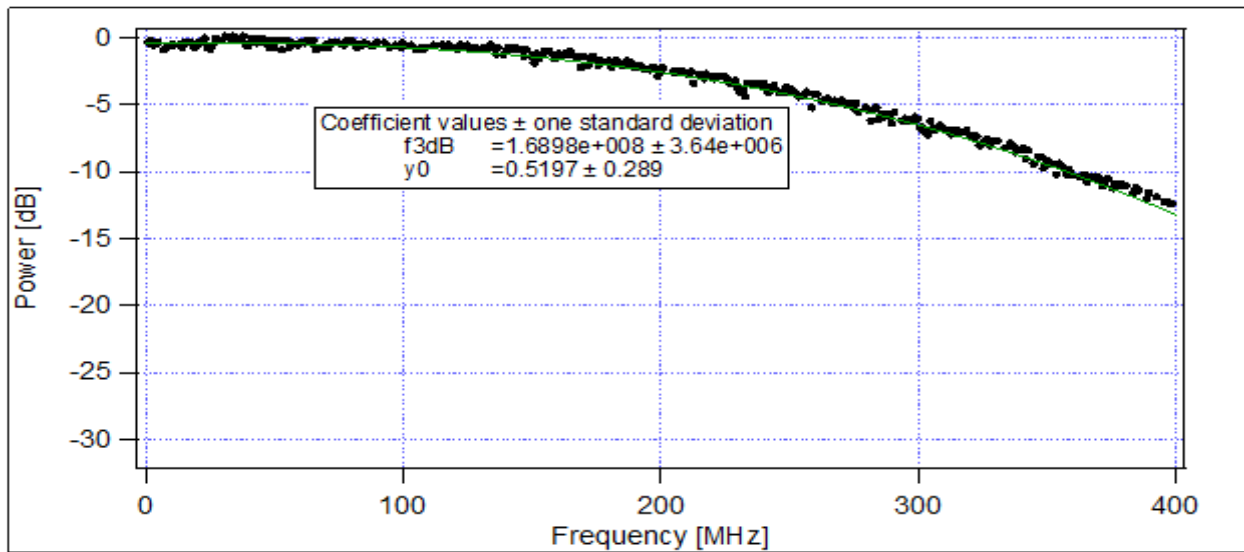


Figure 6: The transfer function from **Figure 5** was fit with a function of the form $y_0 + 10 \log\left(\frac{1}{1 + \left(\frac{\delta}{f_{3dB}}\right)^2}\right)$

. Calculated coefficients are shown above.

Since the input of the transimpedance amplifier is current, and the output is voltage, the transfer function is in units of Ohms. The transfer function's measured units of dBm are converted into a signal amplitude. It is known that the maximum gain is 7.15 k Ω and thus the 0 point was fixed at that point by normalization. Thus, as seen below in **Figure 7**, transimpedance gain (R_{trans}) units of Ohms is calculated as a function of frequency.

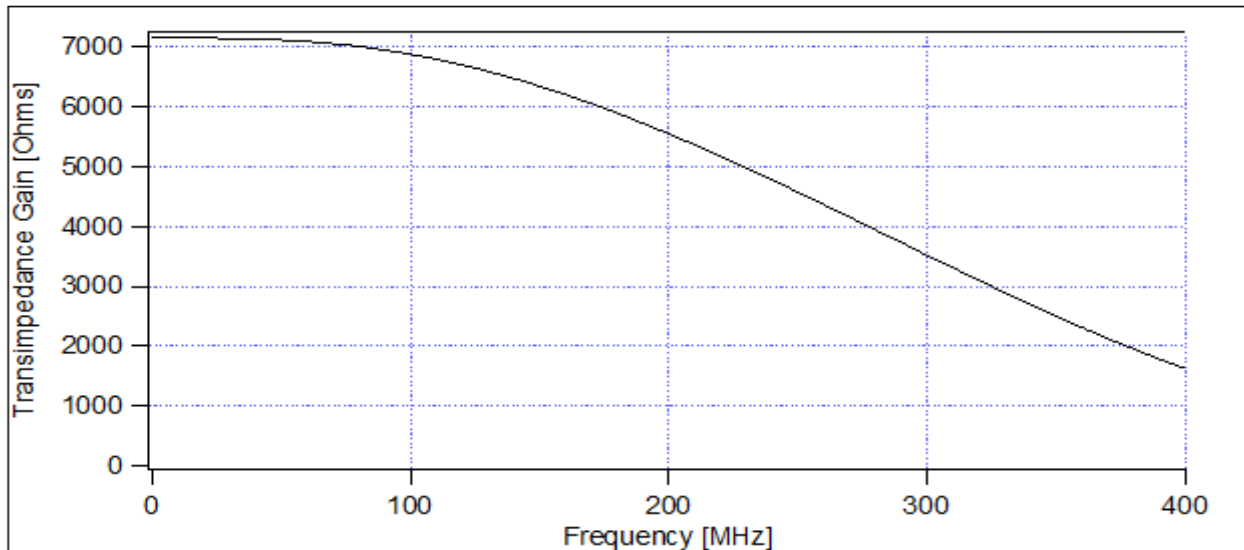


Figure 7: The transfer function was put in appropriate units of ohms and recalculated and graphed above.

With no light incident on the APD, a spectrum analyzer was used to measure the noise in the circuit. The spectrum in **Figure 8** was obtained:

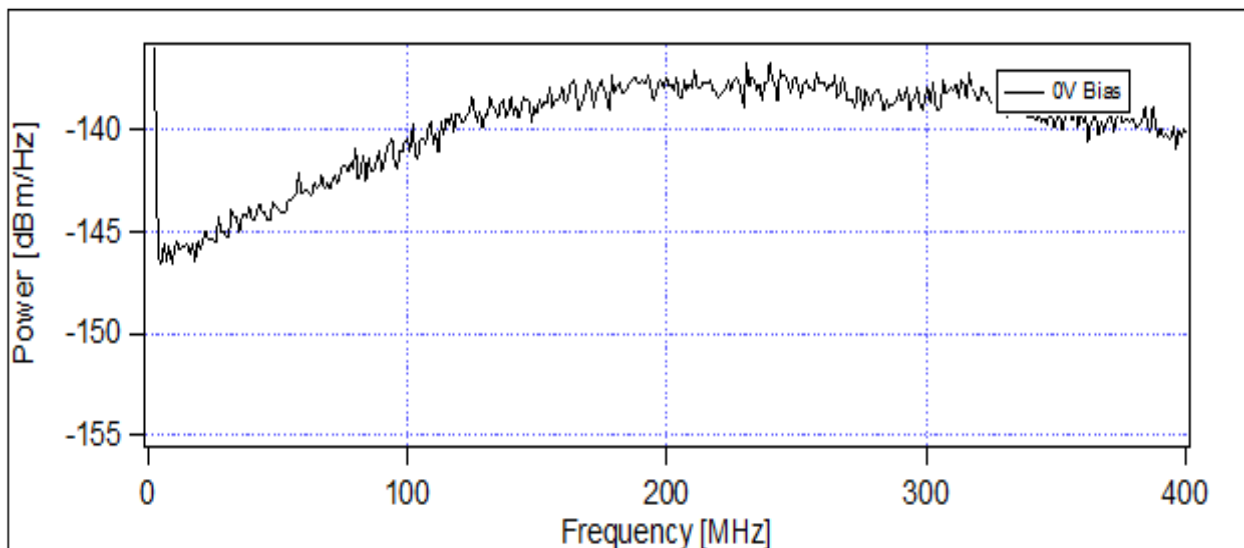


Figure 8: The noise spectrum of the circuit with no light on the APD as measured by a spectrum analyzer.

From this spectrum it is possible to infer a noise threshold for the input photocurrent such that it is ensured that noise measurements in the experiment will come from photon shot noise. The following calculations will determine at what frequency we will minimize this threshold as well as the minimum

optical power associated with this frequency.

First, the power measured in the spectrum analyzer in dBm must be converted into Watts:

$$P_{Watts} = 10^{P_{dBm}/10}$$

Since $P_{Watts} = \frac{V_{RMS}^2}{R}$ and $V_{RMS} = \frac{V}{\sqrt{2}}$, the voltage measured is

$$V = \sqrt{2RP}$$

where R is the resistance inside the spectrum analyzer. Its value is 50Ω.

It is now possible to find the current across the transimpedance amplifier as a function of frequency.

Since $V = IR_{trans}$, with R_{trans} defined as the transimpedance gain across the amplifier, the equivalent input current noise is given by

$$I = \frac{V}{R_{trans}} = \frac{\sqrt{2RP(\delta)}}{R_{trans}(\delta)}$$

This quantity is graphed below in **Figure 9**:

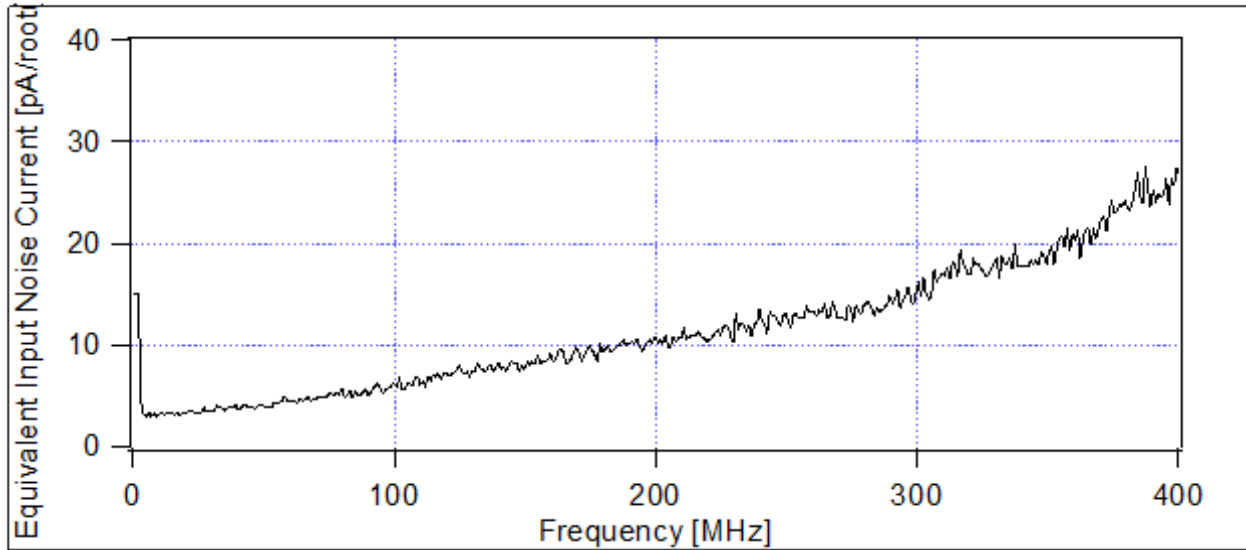


Figure 9: The calculated noise current as a function of detuning frequency in units of pA/root(Hz)

Since photon shot noise current is given by

$$\tilde{I}^2 = 2GqI_0$$

where G is the gain in the amplifier, q is the charge of an electron, and $I_0 = SP_{opt}$, is the product of the sensitivity of the photodiode, S and the optical power of incident light, P_{opt} .

A critical P_{opt} can be calculated and graphed as a function of the measured \tilde{I} in **Figure 10**:

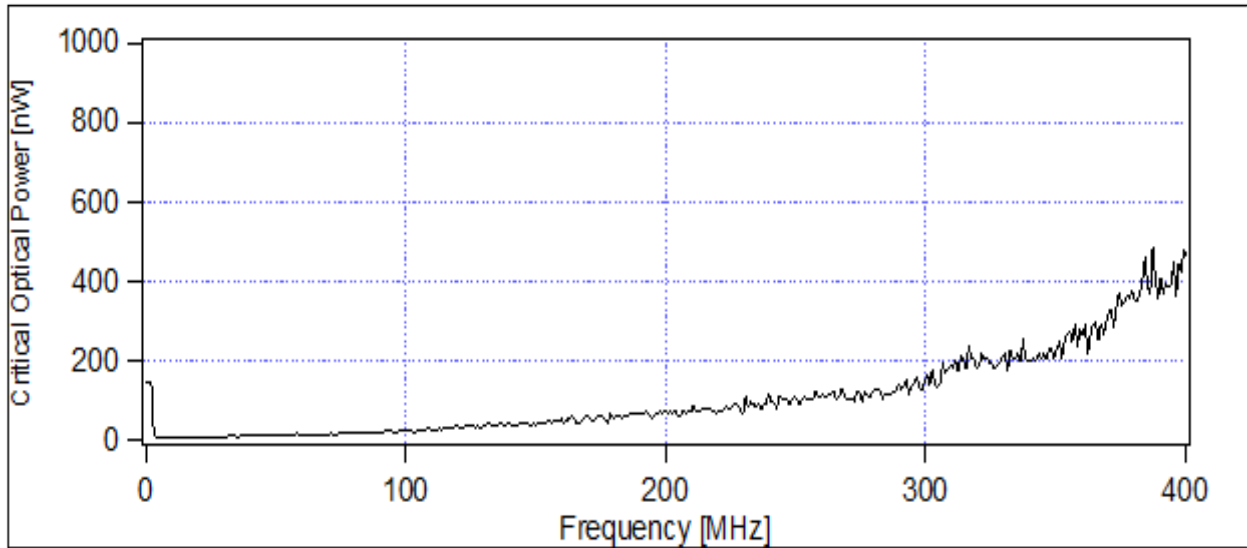


Figure 10: The critical optical power P_{crit} corresponding to the measured noise current.

Keeping in mind that the optical power threshold is to be minimized, the above graph of critical optical power versus frequency implies that a low frequency would be ideal for minimization. However, at low frequency detunings, there is significant leakage from the carrier frequency Lorentzian curve. Thus, the product of the leaked power, P_{leak} and the critical optical power P_{crit} should be minimized. The curve in **Figure 11** shows these quantities' product as a function of frequency:

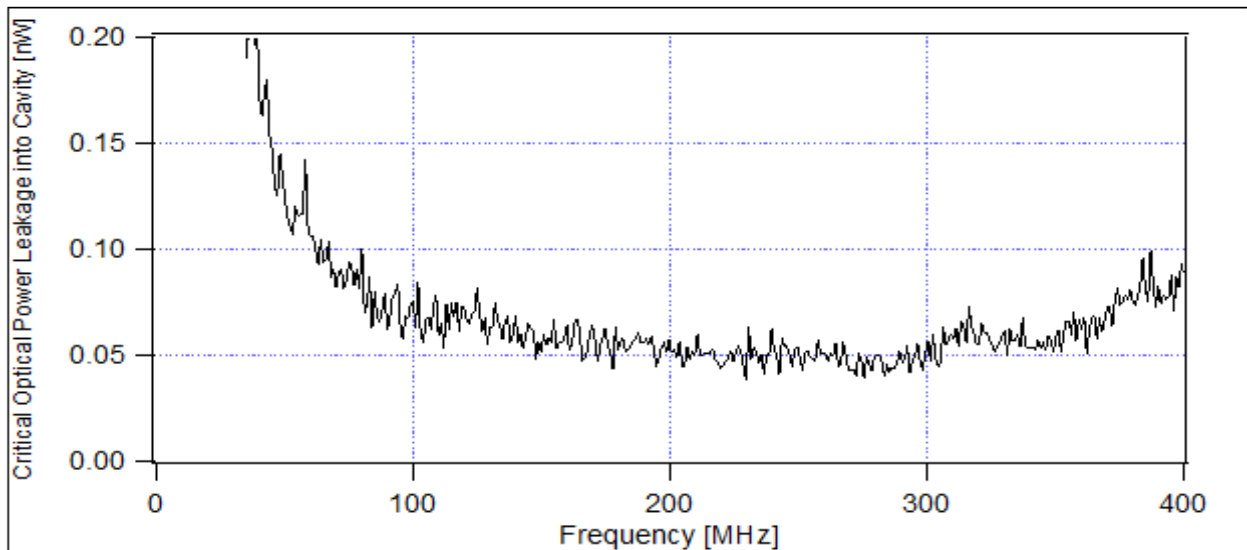


Figure 11: The product of the leaked power from the carrier, P_{leak} and the critical optical power, P_{crit} is shown. This quantity is to be minimized, and this occurs at around 200 MHz.

From the above graph, it can be concluded that an optimal frequency is near 200 MHz.

The critical power can be converted into a minimum photon shot noise-limited photon flux, Φ_{min} . This calculation is done by dividing the power, in units of energy/second by the energy in one photon per second:

$$\phi = \frac{P_{opt}}{\frac{hc}{\lambda}}$$

where h is Planck's constant, c is the speed of light, and λ is the wavelength of light, which is 780 nm. This quantity is graphed as a function of frequency in **Figure 12**:

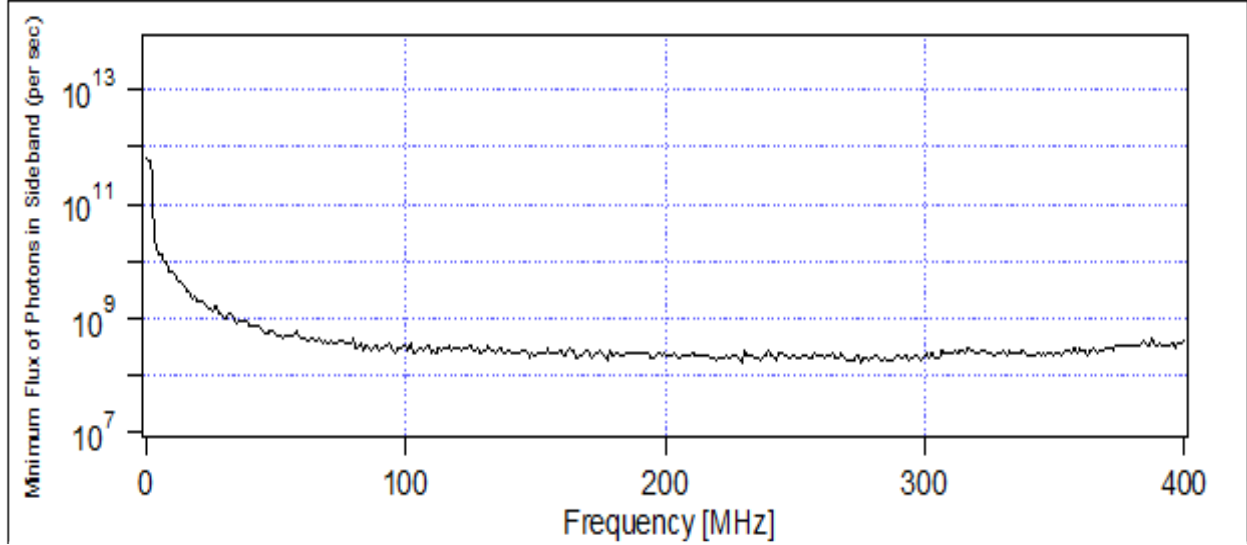


Figure 12: The photon shot noise-limited photon flux is graphed above. At 200 MHz, the threshold is at around 200×10^8 photons/second.

At 200 MHz, the minimum flux of photons in the sideband per second, ϕ is equal to around 200×10^8 photons/second. This is a physically realizable number.

Additionally, a threshold for a quantity relating the ratio of the power in the sideband to the power in the carrier is found. Let $\beta \equiv 2\sqrt{\frac{P_{side}}{P_{carrier}}}$, where P_{side} is the power in a sideband created from the EOM, $P_{carrier}$ is the power in the carrier frequency. β should be determined under the condition that the leakage from the carrier power is equal to the power in the sideband:

$$P_{leak} = P_{side}$$

Since the curves are Lorentzian, the leakage power is given by

$$\frac{P_{leak}}{P_{carrier}} = \frac{1}{1 + \frac{4\delta^2}{\kappa^2}}$$

where κ is the FWHM linewidth of the carrier.

Thus, solving $\beta \equiv 2\sqrt{\frac{P_{side}}{P_{carrier}}}$ for P_{side} and setting it equal to P_{leak} implies that

$$\beta_{crit} = \frac{2}{\sqrt{1 + \frac{4\delta^2}{\kappa^2}}}$$

This critical value of β as a function of frequency is graphed below in black (**Figure 13**). Graphed in

orange is the power $P_{\beta, crit}$ associated with β_{crit} , $P_{b, crit} \equiv \frac{\beta^2}{4}$.

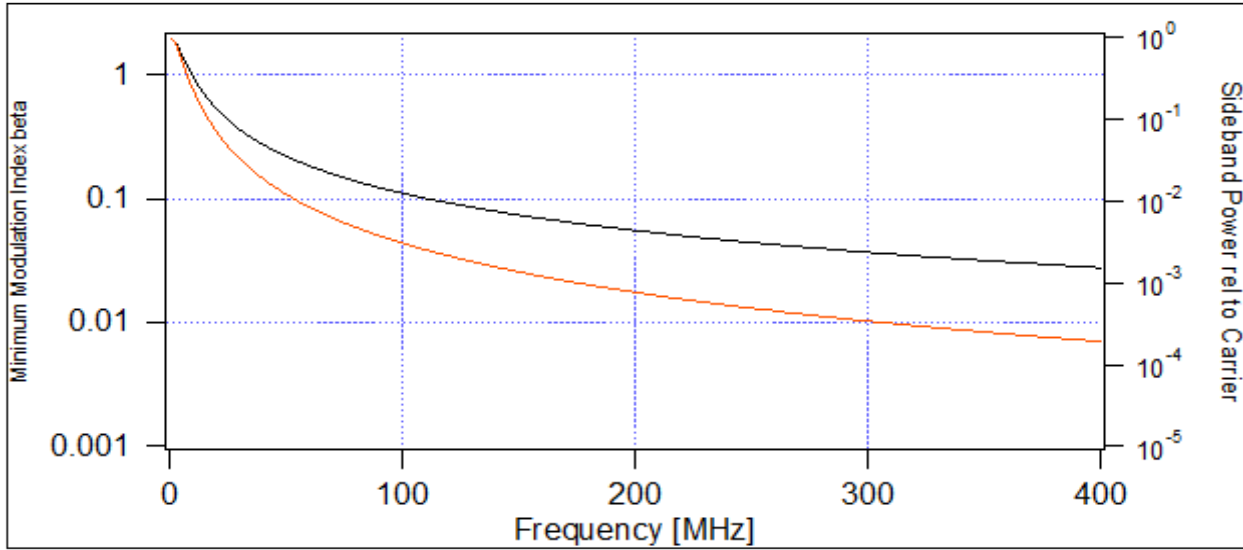


Figure 13: In black is the critical value for β such that $P_{leak} = P_{side}$. At 200 MHz, this critical value for β is .05. In orange is the associated power in one sideband at this critical β . It is calculated by squaring β , and dividing it by 4.

At 200 MHz the power in the sideband is comparable to the leakage power at $\beta = .05$. The corresponding relative power is 7.8×10^{-4} .

Additionally, an equivalent input Johnson Noise Resistance is calculated. Johnson current noise is intrinsic to all resistors and its value is given by

$$I = \sqrt{\frac{4k_b T}{R}}$$

where k_b is Boltzmann's constant, T is the temperature of the resistor, and R is the value of the resistor. An equivalent resistance was calculated as a function of the current noise, \tilde{I} , and graphed below in

Figure 14:

$$R = \sqrt{\frac{4k_b T}{\tilde{I}^2}}$$

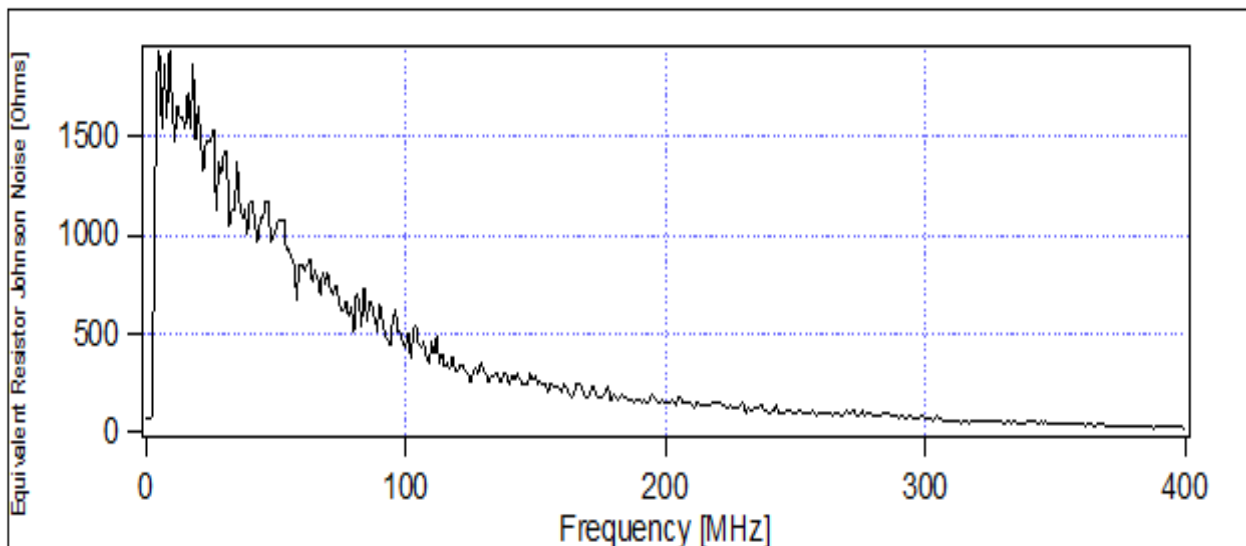


Figure 14: The calculated equivalent Johnson Noise resistance is graphed. At 200 MHz, the equivalent Johnson Noise resistance is $\sim 150\Omega$.

At 200 MHz, the equivalent Johnson Noise is 150Ω . This means that by using the transimpedance amplifier, the current noise is lower than if a mini-circuit with a 50Ω resistor had been used. In that case, it would be expected that there is a greater Johnson current noise. Thus, this confirms that it was a good choice to use the transimpedance amplifier in the APD circuit.

Discussion and Paper Conclusion

The noise properties of the APD circuit were studied and a threshold frequency and optical power for input light was calculated. It was confirmed that the numbers are physically realizable and thus the circuit can be used in the experiment. In the PDH set-up, the cavity should be detuned such that the frequency of the light measured by the APD circuit should be around 200 MHz. Thus, this stage in preparation for making a spin-squeezing measurement using the Pound-Drever-Hall technique is complete.

References

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