Design of a Prototype Low-Grade Heat Engine

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Background

What is "Low-Grade Heat"? Low-grade heat (LGH) can be defined as thermal energy too low to boil water, or, in our case, simply waste hot water from power plants (usually on the order of 200 °C). We know from the Kelvin statement of the Second Law of Thermodynamics that no heat engine is 100% efficient; even the Carnot cycle (the most efficient cycle possible has a low-temperature heat sink (T_C) for venting waste heat.

From an economic standpoint, there is a huge potential demand for an engine that could utilize the latent thermal energy of LGH. It is waste heat anyway (and thus essentially a free resource), and indeed some power plants have to pay to cool or dispose of it, for example with cooling towers or flue gas.

One prevalent competing technology is combined heat and power (CHP), a.k.a. "cogeneration." CHP distributes the waste heat for district heating, i.e. hot water for nearby buildings. Other processes include thermal desalination, the Stirling cycle, and the Organic Rankine cycle. All of these technologies have their drawbacks: While CHP is in principle the simplest of these technologies, it is often undesirable to locate power plants in residential areas. Furthermore, the Stirling and Organic Rankine cycles require relatively high input temperatures (200 °C or higher), and the Organic Rankine cycle requires an organic working fluid that must be carefully sealed to prevent pollution.

The Cornell Power Plant includes several novel features, including a moving heat exchanger device (many, if not all, heat exchangers are stationary), louvers or flaps to control airflow (analogous to diodes in an electrical circuit), and poured concrete construction. A 10 MW power plant would require about 10,000 m^2 of concrete and allots a budget of $20,000,000 for construction.

The Cornell Cycle

The Cornell Cycle has 3 steps or strokes: (1) Intake/Exhaust stroke, (2) Heat Exchange stroke, and (3) Power stroke. See Figures (1) – (3) for diagrams.

In the Intake/Exhaust stroke (Fig. 1), a motor moves the heat exchanger upwards, inducing a draft that opens the top and bottom louvers and closes the heat exchange louvers. Warm air (T_H) in the top of the cylinder is vented to the turbine (see the power stroke), while air at ambient temperature (T_C) is drawn into the cylinder. Once the heat exchanger reaches the top of the cylinder, the motor is turned off, and the louvers close.

In the Heat Exchange stroke (Fig. 2), the top and bottom louvers are closed and locked, perhaps with electromagnets. The motor moves the heat exchanger downward, and the draft opens the heat exchange louvers. Water at T_H (presumably LGH from an adjacent power plant) enters the middle of the heat exchanger while the air at T_C moves up through the heat exchanger fins (louvers). In the ideal conceptualization, the air is heated from T_C to T_H, while the water is cooled from T_H to T_C and released. Since the air is heated at constant volume, the pressure increases from P_{atm} to P_{high}.

In the Power stroke (Fig. 3), the air at T_H and P_{high} is vented to the turbine to transform it into useful electrical energy. Some of the energy of the air does work spinning the turbine blades, while the rest does work pushing aside the ambient air outside the turbine. While the air will likely still be warm exiting the turbine, it can be vented to the T_C reservoir (ambient air) without fear of pollution.
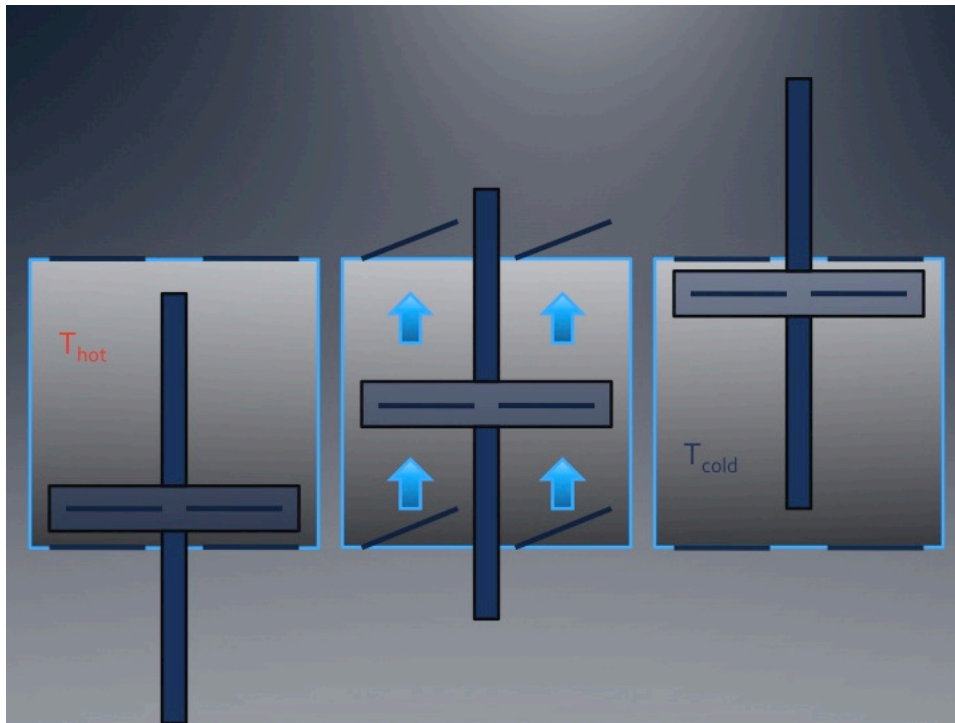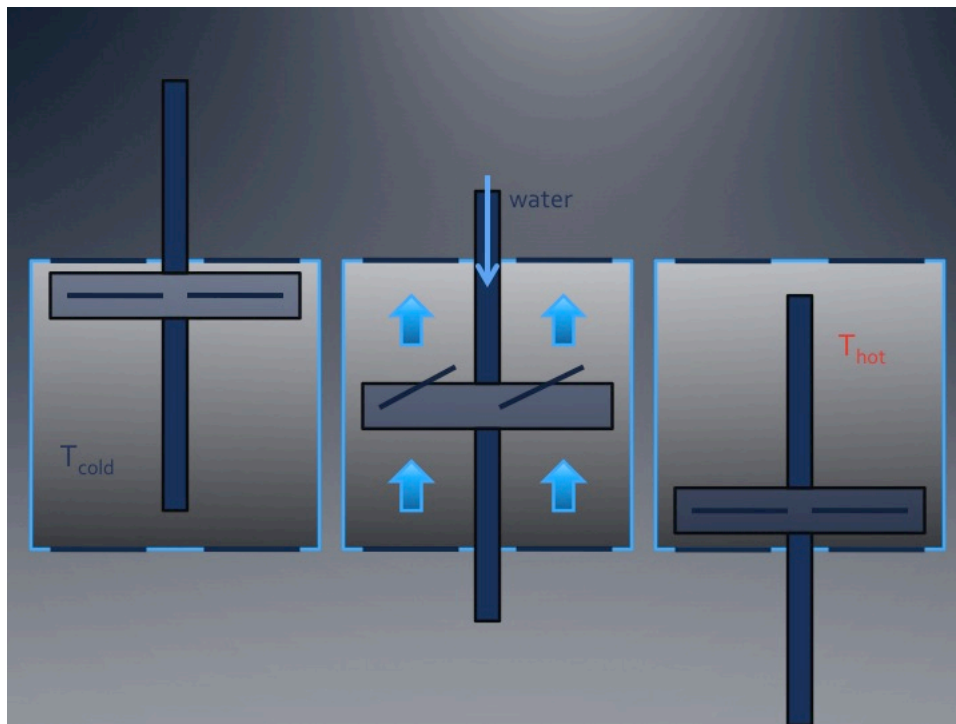

Figure 1. Intake/Exhaust Stroke
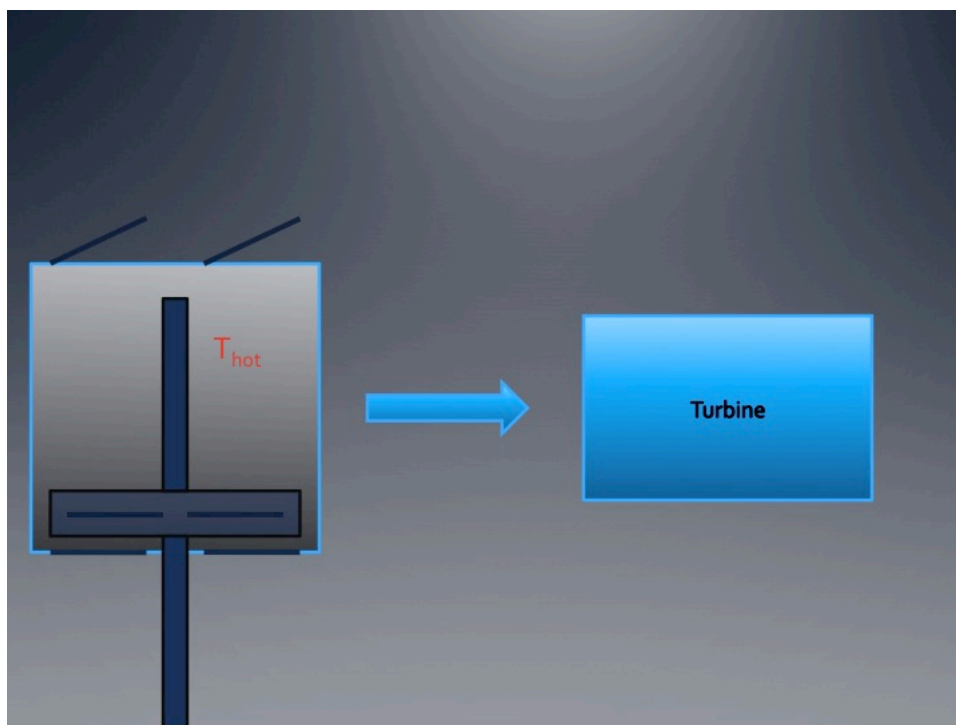
Figure 2. Heat Exchange Stroke

Figure 3. Power Stroke

Experimental Design and Lab Setup

We are assembling a prototype engine, or as Dr. Cornell calls it, a "test bed," in order to model the temperature and pressure conditions of the cycle and test various louver and heat exchanger fin designs. To characterize the engine and its louvers and heat exchanger fins, we require temperature and pressure sensors. We used two MKS Baratron® 226A differential capacitance manometers to measure pressure gradients and numerous Digi-Key NTC 5-kΩ thermistors for temperature measurements.

Since thermistors are glass-encased semiconductors, their resistance varies predictably with temperature and is modeled by the Steinhart-Hart equation.

$$\frac{1}{T} = a + b\ln(R) + c(\ln(R))^3 \qquad\qquad\qquad [1]$$

We assumed that the third term was negligible and could be ignored.

$$\frac{1}{T} = a + b\ln(R) \qquad\qquad\qquad [2]$$

We placed each thermistor in the following circuit (Fig. 4) in water at freezing, ambient, and boiling temperatures. This provided two points to determine the constants *a* and *b* in [2], as well as a third point to validate the expression.
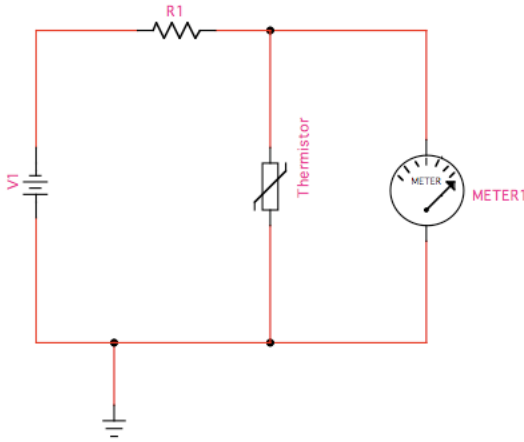

Figure 4. Thermistor calibration circuit

Once the thermistors were calibrated, we could solve for resistance (and thus voltage) in [2] to determine the temperature for a given voltage. The code for calibration and temperature calculation was written in MatLab (see appendix), and the temperature calculation was later ported to LabView.
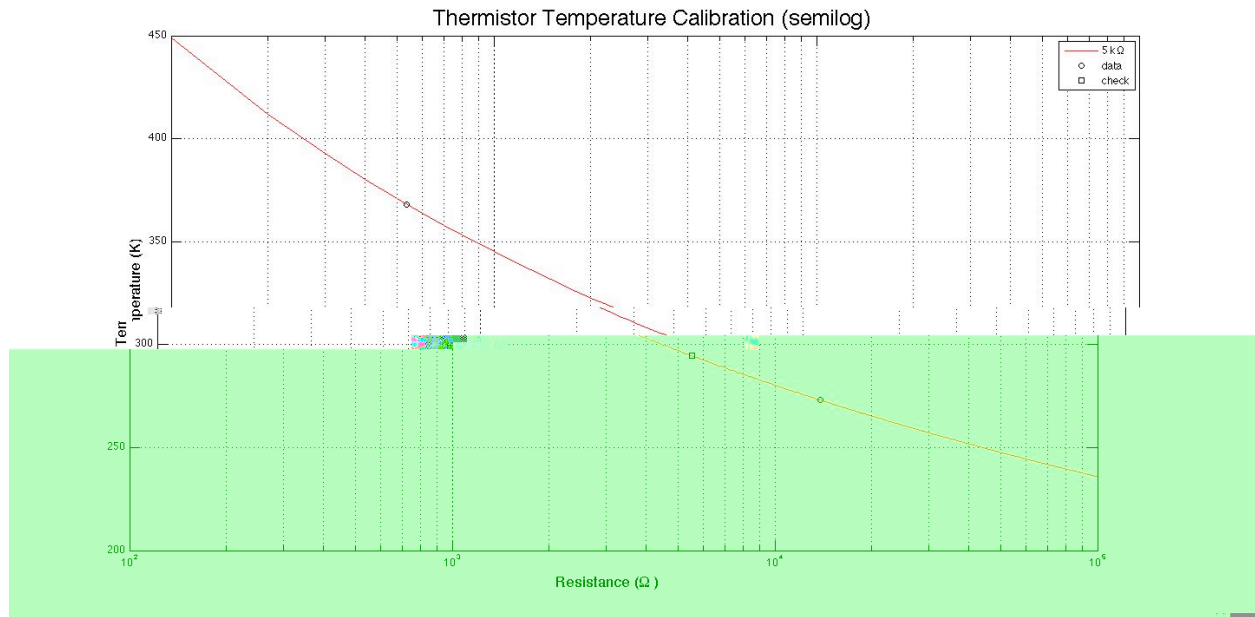
Figure 5. Example thermistor calibration curve

There are 2 pressure and 4 temperature measurements of interest:

Pressure differences between

- The engine and the ambient air pressure
- The top and bottom compartments of the engine (separated by the center plate heat exchanger)

Temperature of

- Air in (before heat exchanger)
- Air out (after heat exchanger)
- Heat exchanger fin in
- Heat exchanger fin out

Thus this required 6 simultaneous channels of data. We used a Keithley PCI-3101 data acquisition board to interface our instruments with LabView.

I wrote LabView programs to accomplish the following functions: collect the voltages from the temperature and pressure sensors and convert them into degrees Kelvin via [2] and psi, respectively; average the data; graph the data; export the data to tab-delimited text file; and view the temperature and pressure in real-time.

Results

We determined the time constant for the thermistors is ~0.5 s by subjecting them to a step voltage and observing the resultant exponential decay to equilibrium temperature. This allows us to characterize the temperature of the airflow on a relatively short timescale.

We first tried to pressurize the can with compressed air and quickly discovered that the original flaps with check valve design (constructed to model the intake and exhaust louvers) don't seal. This is an important finding, since these louvers are crucial to the engine cycle. An alternate (or modified) design could include hinges that are less thick and electromagnetic locks.

We replaced the louvers with sealed sheet metal plates and pressurized the prototype engine to as much as ~1.5 psi. Since the cycle is expected to operate on the order of several psi, this pressure approaches that regime.

Finally, we installed the center plate (which will eventually hold the heat exchanger apparatus) and induced a pressure gradient of ~.01 psi from its motion in the cylinder (Fig. 6).
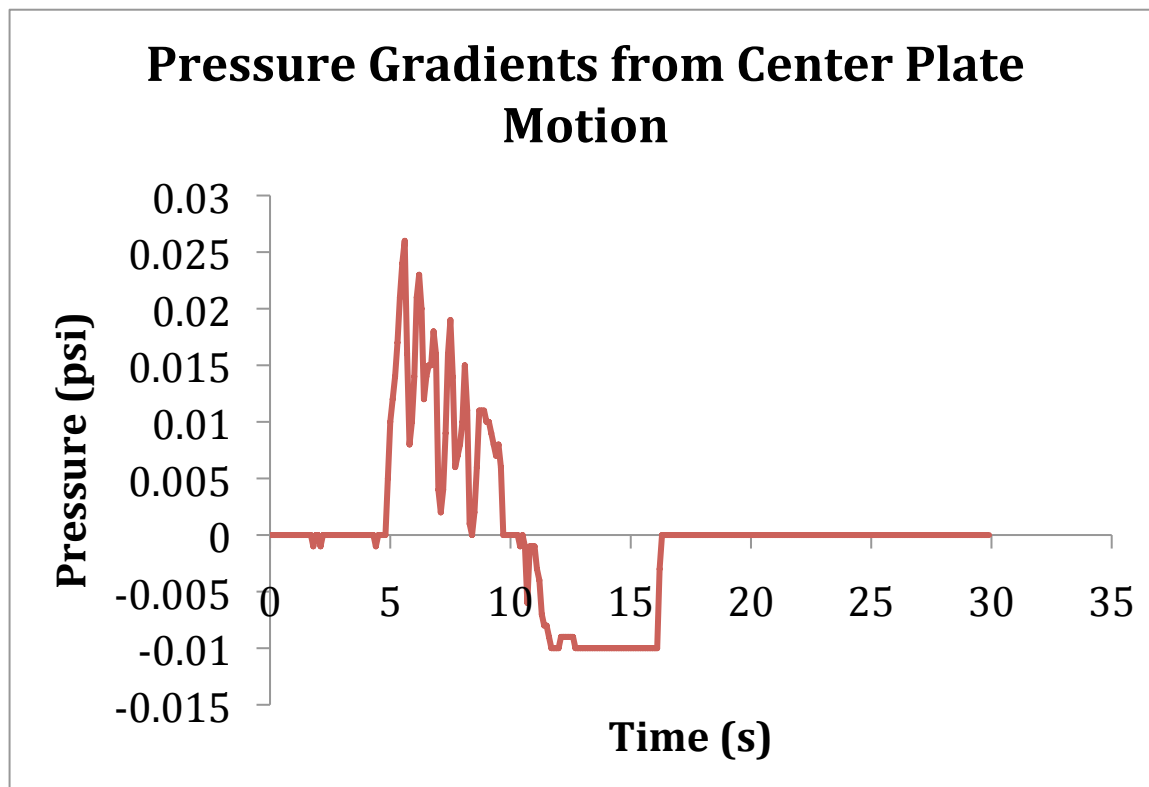

Figure 6. Differential pressure between top and bottom compartments of the engine from center plate motion. The positive data correspond to the plate moving upwards (the jagged edges are when I changed hands on the guide pole); the negative data correspond to the plate moving downwards (the data are more even since I simply let the center plate fall).

Validating the Pressure Measurements

We would like to validate the pressure data from the pressure gauges using a theoretical argument. This requires a geometry in the center plate such that the airflow is well known. The Darcy-Weisbach equation

$$\Delta P = f \frac{L}{D} \frac{\rho V^2}{2} \qquad\qquad [3]$$

(which can be derived from the Bernoulli equation) models fluid flow through a pipe. Here ΔP is the pressure drop across the pipe, f is the dimensionless Darcy friction factor, L is the pipe length, D is the pipe diameter, ρ is the fluid density, and V is the fluid velocity.

For the region of interest, we have turbulent flow, so the friction factor f is not linear (as would be the case for laminar flow) but is related to the roughness of the pipe and the viscosity of the fluid; it is modeled in a Moody diagram.

The Reynolds number R

$$R = \frac{\rho V D}{\mu} \qquad\qquad [4]$$

gives a qualitative measure of the turbulence of a flow. Here μ is the dynamic viscosity of the fluid, and since we are concerned with air (an ideal gas for our purposes), it is given by Sutherland's formula,

$$\mu = \mu_0 \frac{T_0 + C}{T + C} \left(\frac{T}{T_0} \right)^{3/2} , \qquad\qquad [5]$$

where μ_0, T_0, and C are constants. The MatLab program I wrote to perform these calculations is attached in the appendix.

Future Work

The next step is to conduct the pipe pressure experiment to corroborate the veracity of the pressure data. We then will install the thermistors in the prototype engine for temperature measurements. Finally, we will test various heat exchanger fin configurations, and optimize the fins to extract the maximum heat from the air with a minimum loss of momentum/kinetic energy.

```matlab
%Thermistor Calibration
%Joe Lanska
%2009 06 29


clc;
clear all;
close all;



R_load = .98 *10^6;
V_tot = 20.29;

index = input('Enter the thermistor number (0 for old 5k): ');
[T, V] = getThermistor(index);

%calculate resistance from V
R = zeros(3);
for k=1:3
    R(k) = get_R_from_V( V(k), V_tot, R_load );
end

%find parameters for Steinhart-Hart Equation
[a, b] = getParams( R(1), R(3), T(1), T(3) );
disp(['The value of a is ', num2str(a), '.']);
disp([
```

```matlab
%semilog
figure;
semilogx(R2,T2, 'r');
hold on;
semilogx(R(1), T(1), 'ko');
semilogx(R(2), T(2), 'ks');
semilogx(R(3), T(3), 'ko');
title('Thermistor Temperature Calibration (semilog)', 'FontSize', 20);
xlabel('Resistance (\Omega)', 'FontSize', 14);
ylabel('Temperature (K)', 'FontSize', 14);
legend('5 k\Omega', 'data', 'check');
grid on;
```

```matlab
%Joseph Lanska
%2009 06 29

function [ T, V ] = getThermistor( i )
%getThermistor( i )        Returns the calibration temperatures and
%                          voltages for one of the 11 5k thermistors.

%Temperatures are freezing, ambient, and boiling.
%The old 5k thermistor is thermistor #0.

%MatLab array indices start at 1
i=i+1;

T = [0.1 22.3 94.5;
     0.1 21.8 95.3;
     0.1 21.6 95.2;
     0.1 21.5 95.2;
     0.1 21.5 95.2;
     0.3 21.4 95.1;
     0.1 20.9 95.1;
     0.1 20.9 95.2;
     0.1 20.9 95.2;
     0.1 20.9 95.2;
     0.1 20.8 95.2];

V = [.2831 .1090 .0112;
     .2805 .1095 .0105;
     .2808 .1131 .0109;
     .2818 .1140 .0111;
     .2809 .1135 .0108;
     .2808 .1152 .0110;
     .2805 .1151 .0108;
     .2810 .1164 .0108;
     .2806 .1161 .0107;
     .2810 .1134 .0103;
     .2806 .1156 .0108];

T=T(i,:);
V=V(i,:);

T = T + 273;

end
```

```matlab
oseph    nsk


function [    ] = get    from  (  ,   tot,   lo d )
   from       tot      lo d         omputes the resist nce    from    volt ge
                                di vi der ci rcui t


     is the vol t ge  cross the thermi st or
     tot is the vol t ge from the power supply
     lo d is the current limi ti ng resi st or

   = (   / (  tot  -  )) *    lo d;
end
```

```matlab
%Joseph Lanska
%2009 06 26

function [ a, b ] = getParams( R1, R2, T1, T2 )
%getParams( R1, R2, T1, T2 )          Solves the simplified (first order)
%                                     Steinhart-Hart Equation for the 2
%                                     constants a and b.

% The Steinhart-Hart eqn is 1/T = a + b ln(R) + c (ln(R))^3.
% The third term is ignored.

X = [ 1,  log(R1),  1/T1;
      1,  log(R2),  1/T2];
Y = rref(X);
a=Y(1,3);
b=Y(2,3);
end
```

```matlab
%Thermistor Temperature
%Joe Lanska
%2009 06 29

clc;
clear all;
close all;

R_load = .980*10^6;          %load resistance (ohms)
V_tot = 20.29;               %total voltage (V)

index = input('Enter the thermistor number (0 for old 5k): ');
[T, V] = getThermistor(index);

%calculate resistance from V
R = zeros(3);
for k=1:3
    R(k) = get_R_from_V ( V(k), V_tot, R_load );
end

%find parameters for Steinhart-Hart Equation
[a, b] = getParams( R(1), R(3), T(1), T(3) );


%_____

%calculate Temperature
V_meas = input('Enter the measured voltage: ');
R_meas = get_R_from_V ( V_meas, V_tot, R_load );
T_calc = 1 / ( a + b *log(R_meas));
disp(['The temperature is ', num2str(T_calc-273), ' deg C.']);
```

```matlab
%Thermistor Variation
%Joe Lanska
%2009 06 26


%compare new 5k thermistors

clc;
clear all;
close all;


%from '2009 06 22 new 5k thermistors.xlsx'
a = [0.9278 0.8992 0.8857 0.9072 0.8987 0.9085 0.9073 0.9144 0.9467 0.9065]*10^(-3);
b = [0.2869 0.2899 0.2912 0.2890 0.2897 0.2890 0.2890 0.2883 0.2849 0.2892]*10^(-3);


avgA = mean(a);
avgB = mean(b);


%define a standard R
T1= 40 + 273;
R = exp( (1/T1 - avgA) / avgB );   %inverse of 1st-order Steinhart-Hart eqn

T = 1 ./ ( a + b * log(R) );
T_avg = 1 / (avgA + avgB *log(R) );
delta_T = T - T_avg;


%plot variations
figure;
plot(delta_T, 'bo-');
title('Variation in Temperature measured by Thermistors', 'FontSize', 20);
xlabel('Thermistor #', 'FontSize', 14);
ylabel('\DeltaT (K)', 'FontSize', 14);
grid on;
```

```matlab
%Pressure Drop of Air Through Pipes
%Joseph Lanska
%2009 08 04


%assume incompressible flow
%conversion: 1 inch = 0.0254 meters

clc;
clear all;
close all;


%for air
C=120;                          %K
T0 = 291.15;                    %K
mu0 = 18.27 * 10^(-6);          %Pa*s = N*s/m^2

%estimates
T = 23 + 273;                   %temperature of air (K)
rho = 1.2;                      %density of air (kg/m^3)

e = 1.5 * 10^(-6);              %pipe roughness (m)
L = 2.5 * .0254;               %pipe length(m); 10 inches
D = 4 * .75 * .0254;           %internal pipe diameter(m); 4 pipes
D0 = 23.5 * .0254;             %plate diameter; 23.5 inches
V0 = 0.2;                       %velocity of plate (m/s), (5s to drop 1m)
V = V0 * ( D0^2 / D^2 );        %velocity of air through pipe (m/s), 2 feet
display(['The velocity of air through the pipe is ', num2str(V), ' m/s.']);

%Sutherland's Formula (assumes air is ideal gas)
mu = mu0 * ((T0 + C)/(T + C)) * (T/T0)^(3/2);

%Reynold's Number
R = rho * V * D / mu;


display(['R is ', num2str(R)]);
if ( R < 2000 )
    display('Laminar flow');
    f = 64 / R;                 %friction factor (only valid for laminar flow)
else
    display('Turbulent flow');
    display(['The relative roughness of the pipe (e/D) is ', num2str(e/D), ' .']);
    %Colebrook equation
    %f = solve('1/(sqrt(f)) = -2 * log10(e/D * 1/3.7 + 2.51/(R* sqrt(f)) )', 'f');
    f = .02;                    %from Moody diagram
end


%display(['f is ', num2str(f)]);


%Pressure Drop
%Darcy-Weisbach equation (from the Bernoulli Equation)
dP = f * (L/D) * (rho/2) * V^2;         %Pa
dP = dP *14.696 / (101.325 * 10^3);     %convert to psi
display(['The pressure drop in the pipe is ', num2str(dP), ' psi.']);
```

```matlab
%output from 12T pressure gauge (mV)
volts = dP / 0.1934;
display(['The resulting voltage output from the 12T pressure gauge is ', num2str(volts), ' V.']);




D = .1:.01:1;
D = D * 4 * .0254;
dP = .5 * f * rho * L * V0^2 * D0^4 ./ D.^5;
dP = dP *14.696 / (101.325 * 10^3);      %convert to psi

% figure();
% D = D/ (4 *.0254);
% semilogy(D, dP, 'bo-');
% grid on;
% xlabel('Pipe Diameter (inches)');
% ylabel('Pressure Drop (psi)');

volts = dP / 0.1934;

figure();
semilogy(D, volts, 'bo-');
grid on;
xlabel('Pipe Diameter (inches)');
ylabel('Voltage 12T (V)');
```

Joseph Lanska
August 6, 2009

1. Intake
Model air as an ideal diatomic gas.

$$p = p_{amb}$$

$$T = T_C$$

$$pV = N k_B T = nRT \tag{1}$$

$$N = \frac{pV}{k_B T} = \frac{p_{amb} V}{k_B T_C} \tag{2}$$

2. Heat Exchange

$$Q_{water} \qquad Q_{gas}$$

$$V = constant$$

$$T_1 = T_C \qquad T_2 = T_H$$

$$P_1 = P_{amb} \qquad P_2 = P_{high}$$

Since a diatomic molecule has 5 degrees of freedom, then by the Equipartition Theorem,

$$U = \ E \ = N \frac{k_B T}{2} = \frac{5}{2} k_B T \tag{3}$$

and the specific heat per molecule is

$$c_V = \left(\frac{\partial U}{\partial T} \right)_V = \frac{5}{2} k_B \tag{4}$$

$$
\begin{aligned}
Q \ &= \ mc\Delta T \\
&= \ N \left(\frac{5}{2} k_B \right) (T_2 - T_1) \\
&= \ \left(\frac{p_{amb} V}{k_B T_C} \right) \left(\frac{5}{2} k_B \right) (T_H - T_C) \\
&= \ \frac{5}{2} p_{amb} V \left(\frac{T_H - T_C}{T_C} \right)
\end{aligned}
$$

constant volume process $\quad V_1 = V_2$, so by the ideal gas law,

$$p_2 = p_1 \frac{T_2}{T_1} \tag{5}$$

3. Power Stroke
(reversible) adiabatic process, so

$$PV = constant \tag{6}$$

$$TV^{-1} = constant \tag{7}$$

$$P^{-1}T^{-} = constant \tag{8}$$

$$P_1 = P_{high} \rightarrow P_2 = P_{amb}$$

$$V_1 \rightarrow V_2, V_2 > V_1$$

$$T_1 = T_H \rightarrow T_2 = T_C$$

for diatomic gas, $\gamma = \frac{7}{5}$

$$W = \int_1^2 \vec{F} \cdot \vec{dl} \tag{9}$$

$$= \int_{V_1}^{V_2} [(P - P_{amb})\,A]\,dl = \int_{V_1}^{V_2} (P - P_2)\,dV \tag{10}$$

From (6) we have that

$$P = P_1 \left(\frac{V_1}{V} \right) \tag{11}$$

Eqns (10) and (11) yield

$$W = \int_{V_1}^{V_2} P_1 \left(\frac{V_1}{V} \right) \, dV - \int_{V_1}^{V_2} P_2 dV \tag{12}$$

$$= P_1 V_1 \int_{V_1}^{V_2} V^- \, dV - P_2 (V_2 - V_1) \tag{13}$$

$$= \frac{P_1 V_1}{1 - \gamma} \left(V_2^{1-} - V_1^{1-} \right) - P_2 (V_2 - V_1) \tag{14}$$

Using (7), we can rewrite V_2 in (14) as

$$W = \frac{P_1 V_1}{1 - \gamma} \left(\frac{T_2}{T_1} V_1^{1-} - V_1^{1-} \right) - P_2 \left(\left(\frac{T_2}{T_1} \right)^{\frac{1}{1-}} V_1 - V_1 \right) \tag{15}$$

$$= \frac{P_1 V_1}{1 - \gamma} \left(\frac{T_2}{T_1} - 1 \right) - P_2 V_1 \left(\left(\frac{T_2}{T_1} \right)^{\frac{1}{1-}} - 1 \right) \tag{16}$$

Using (8), we can rewrite P_1 in (16) as

$$W = \left[\left(\frac{T_1}{T_2} \right)^{-1} P_2 \right] \left(\frac{V_1}{1 - \gamma} \right) \left(\frac{T_2}{T_1} - 1 \right) - P_2 V_1 \left(\left(\frac{T_2}{T_1} \right)^{\frac{1}{1-}} - 1 \right) \tag{17}$$

$$= P_2 V_1 \left[\left(\frac{T_1}{T_2} \right)^{-1} \left(\frac{1}{1 - \gamma} \right) \left(\frac{T_2}{T_1} - 1 \right) - \left(\left(\frac{T_2}{T_1} \right)^{\frac{1}{1-}} - 1 \right) \right] \tag{18}$$

$$= P_2V_1\left[\left(\frac{1}{1-\gamma}\right)\left(\left(\frac{T_1}{T_2}\right)^{\frac{1}{-1}-1}-\left(\frac{T_1}{T_2}\right)^{\frac{1}{-1}}\right)-\left(\frac{T_2}{T_1}\right)^{\frac{1}{1-}}+1\right] \quad (19)$$

$$= P_2V_1\left[\left(\frac{1}{1-\gamma}\right)\left(\left(\frac{T_1}{T_2}\right)^{\frac{1}{-1}-1}-\left(\frac{T_1}{T_2}\right)^{\frac{1}{-1}}\right)-\left(\frac{T_1}{T_2}\right)^{\frac{1}{-1}}+1\right] \quad (20)$$

$$= P_2V_1\left[\left(\frac{1}{1-\gamma}-1\right)\left(\frac{T_1}{T_2}\right)^{\frac{1}{-1}}-\left(\frac{1}{1-\gamma}\right)\left(\frac{T_1}{T_2}\right)^{\frac{1}{-1}}+1\right] \quad (21)$$

$$= P_2V_1\left[\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{T_1}{T_2}\right)^{\frac{1}{-1}}-\left(\frac{1}{1-\gamma}\right)\left(\frac{T_1}{T_2}\right)^{\frac{1}{-1}}+1\right] \quad (22)$$

At this point, we make the simplifying assumption that

$$(T_1 - T_2) << T_2, \quad (23)$$

or equivalently,

$$T_1 = (1 + \varepsilon)\,T_2 \quad (24)$$

for some small ε. Note that ε is by no means infinitesimal: since the expected range of operation for the heat cycle is from approximately $15°C$ to $90°C$, ε is around 0.26.

The Binomial Theorem,

$$(x + y)^n = \sum_{k=0}^{n}\frac{n!}{(n-k)!k!}x^{n-k}y^k, \quad (25)$$

can be expanded to yield

$$(1 + \varepsilon)^n = 1 + n\varepsilon + \frac{n\,(n-1)}{2}\varepsilon^2 + \ldots \quad (26)$$

$$\left(\frac{T_1}{T_2}\right)^n = 1 + n\left(\frac{T_1}{T_2}-1\right) + \frac{n\,(n-1)}{2}\left(\frac{T_1}{T_2}-1\right)^2 + \ldots \quad (27)$$

Thus, using a second order binomial expansion, (22) can be simplified to $W \approx W'$, where

$$W' = P_2V_1\left(\frac{\gamma}{1-\gamma}\right)\left(1 + \frac{1}{\gamma-1}\left(\frac{T_1}{T_2}-1\right) + \frac{\frac{1}{\gamma-1}\left(\frac{1}{\gamma-1}-1\right)}{2}\left(\frac{T_1}{T_2}-1\right)^2\right)$$

$$\quad - P_2V_1\left(\frac{1}{1-\gamma}\right)\left(1 + \frac{\gamma}{\gamma-1}\left(\frac{T_1}{T_2}-1\right) + \frac{\frac{\gamma}{\gamma-1}\left(\frac{\gamma}{\gamma-1}-1\right)}{2}\left(\frac{T_1}{T_2}-1\right)^2\right)$$

$$\quad + P_2V_1\,(1) \quad (28)$$

$$W' = P_2V_1\left[\frac{\gamma}{1-\gamma}-\frac{1}{1-\gamma}+1\right](1) \quad (29)$$

$$\quad + P_2V_1\left[\frac{\gamma}{1-\gamma}\left(\frac{1}{\gamma-1}\right)-\frac{1}{1-\gamma}\left(\frac{\gamma}{\gamma-1}\right)\right]\left(\frac{T_1}{T_2}-1\right)$$

$$\quad + P_2V_1\left[\frac{\gamma}{1-\gamma}\left(\frac{\frac{1}{\gamma-1}\left(\frac{2-\gamma}{\gamma-1}\right)}{2}\right)-\frac{1}{1-\gamma}\left(\frac{\frac{\gamma}{\gamma-1}\left(\frac{\gamma}{\gamma-1}-1\right)}{2}\right)\right]\left(\frac{T_1}{T_2}-1\right)^2$$

Everything but the second order term cancels, leaving

$$W' = \frac{1}{2}P_2V_1\left(\frac{1}{1-\gamma}\right)\left[\gamma\left(\frac{1}{\gamma-1}\right)\left(\frac{2-\gamma}{\gamma-1}\right) - \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{1}{\gamma-1}\right)\right]\left(\frac{T_1}{T_2}-1\right)^2 \tag{30}$$

$$W' = \frac{1}{2}P_2V_1\left(\frac{1}{1-\gamma}\right)\left(\frac{1}{\gamma-1}\right)^2[\gamma(2-\gamma)-\gamma]\left(\frac{T_1}{T_2}-1\right)^2 \tag{31}$$

$$W' = \frac{1}{2}P_2V_1\left(\frac{1}{1-\gamma}\right)\left(\frac{1}{\gamma-1}\right)^2[\gamma(1-\gamma)]\left(\frac{T_1}{T_2}-1\right)^2 \tag{32}$$

$$W' = \frac{1}{2}P_2V_1\frac{\gamma}{(\gamma-1)^2}\left(\frac{T_1}{T_2}-1\right)^2 \tag{33}$$

For $\gamma \gg 1$, the relation in (33) reduces to Dr. Cornell's result, namely,

$$W' = \frac{5}{14}P_2V_1\left(\frac{T_1}{T_2}-1\right)^2 = \frac{1}{2\gamma}P_2V_1\left(\frac{T_1}{T_2}-1\right)^2 \tag{34}$$

Since $\gamma = \frac{7}{5} = 1.4$ for air, this assumption is clearly not valid. However, we note that the form of both expressions is the same, $W' = cP_2V_1\left(\frac{T_1}{T_2}-1\right)^2$; the expressions differ only in the constant c. It is interesting to note that Dr. Cornell avoided the second order terms in the expansion entirely by dividing by Q to first find the efficiency, $\eta = \frac{W}{Q}$. This, combined with the large value of ε in the expansion, likely accounts for the difference.