OPTICAL CAVITY FOR THE STRONTIUM LATTICE ATOMIC CLOCK

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ABSTRACT. The Sr lattice atomic clock in Jun Ye's group has been experiencing frequency shifts due to the interaction between the Sr atoms in the optical trap. One possible solution is to change the optical trap in the system such that the Sr atom density, and therefore the interaction between the atoms, is reduced. For this project a test system is necessary to test the vacuum chamber viewports, which involved building a laser diode and an optical cavity. This paper will explore the theory behind both as well as their implementation.

1. INTRODUCTION

With the astounding technological advances that have been achieved in the last couple of decades comes the need for higher precision in measurements in time. Global Positioning Systems (GPS), for example, have become incredibly accurate due to the precision timing of so-called atomic clocks, which are based on the transition frequencies between internal quantum states of atoms or ions. [1] The atomic clock that is used as the standard for time utilizes a radio frequency hyperfine transition in Cesium. Jun Ye's group at JILA currently has an atomic clock that uses neutral strontium (Sr) atoms. The transition used for the atomic clock is in the optical domain, a factor of 10^4 higher in frequency than that of the Cesium primary frequency standard. In general, clocks based on optical transition frequencies have the potential to be at least 100 times more accurate than microwave clocks, and of these, the Sr system is at the forefront of neutral-atom atomic clocks, with fractional frequency uncertainty at one part in 10^{16}. [2]

One important technological advance that has paved the way for optical clocks is Lamb-Dicke confinement in so-called magic wavelength optical lattices for neutral atoms. The magic wavelength for the ^{87}Sr is 813.4 nm. [3] The Lamb-Dicke confinement is a type of extremely tight confinement eliminates Doppler broadening and recoil shifts, which are due to the movement of the atoms. [3], [4] In the JILA system, the Sr atoms are currently confined in an optical lattice created by either one or two standing light waves. One set of standing waves creates traps that are pancake-shaped, while two sets of standing waves in two orthogonal directions create tube-shaped traps. Both of these implementations have been used successfully. [4] While these traps eliminate motional effects, there have

been observations of collisions that cause frequency shifts due to the high density of the atoms in each of the traps, in addition to poorly-understood decoherence mechanisms in the two-dimensional case. [5]

One way to investigate these decoherence mechanisms was to build an optical cavity-based dipole trap that will enjoy the benefits of lower density with higher atom number, while still permitting spectroscopy in the Lamb-Dicke regime. An added benefit of the cavity-based approach is that any spectral impurities in the lattice laser are eliminated, reducing the possibility of detrimental frequency shifts. The focus of the work this summer is to begin work on this trap. This entails building a cavity similar to the one that will be in the Sr clock system.

2. Background

As this project requires some background knowledge, this section will take the opportunity to explain the physics behind portions of the system.

2.1. **Diode Lasers.** Diode lasers are typically made from a semiconductor material. There is an active region in this semiconductor material that generates electrons and holes that recombine to produce photons. The light that is emitted, often in single-mode, as a stripe, with the polarization perpendicular to the direction of the stripe.
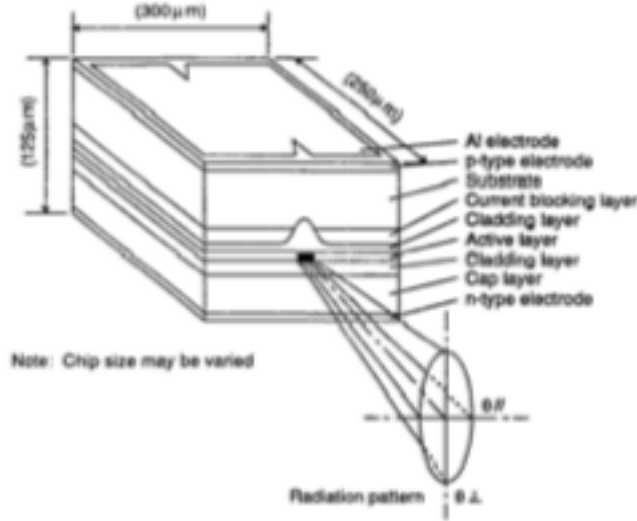

FIGURE 1. This figure shows the diode chip in a diode laser. This figure was taken from Reference [6].

One of the benefits of using a diode laser is that one can tune the laser to produce a particular wavelength. This can be done by changing the temperature of the diode laser as well as using an external cavity. For small temperature changes, the resulting changes

in laser frequency for the same injection current, that is, the current driving the diode, are approximately linear. However, these temperature changes will cause the laser to mode hop relatively frequently. [6]

Another method for tuning a diode laser is to use an external cavity to control the wavelength of the emitted light. This works especially well when the diode has an anti-reflection coating at its output face. An external cavity for the diode laser requires a grating that will reflect some of the light back to the diode. There are two common configurations for external cavity diode lasers (ECDLs): the Littrow and the Littman-Metcalf configurations. In the Littrow configuration, the grating retro-reflects the first order beam back to the diode, while the zeroth order beam is the output beam. In the Littman-Metcalf geometry, the grating is placed at a grazing angle to the output beam from the laser diode. The first order beam is diffracted to a mirror that reflects the beam back onto itself back into the laser. The position of the mirror determines the laser wavelength. As with the Littrow configuration, the zeroth order diffraction beam is the output beam. An advantage to the Littman-Metcalf configuration is that one can change the length of the cavity in order to tune the wavelength of the laser. [7]

2.2. **Optical Cavities.** Optical cavities are formed when two mirrors face each other and light reflects back and forth between the two mirrors. As previously stated, optical cavities can be used to create powerful traps. The most basic optical cavity is known as the Fabry-Perot cavity, which basically consists of two planar mirrors facing each other. For the following derivation, infinitely thin mirrors will be assumed with transmittance t and reflectance r. The incoming beam with frequency ω is given by

$$E = E_0 e^{i\omega t}, \tag{1}$$

where E_0 is the amplitude.

When the beam hits the first cavity mirror with transmittance t and reflectance r, it is reflected with amplitude $E_0 r$, while the rest of the beam is transmitted through the first cavity mirror, with amplitude $E_0 t$. That is,

$$E_{0r} = E_0 r e^{i\omega t + \delta}, \text{and} \tag{2}$$
$$E_{1a} = E_0 t e^{i\omega t}. \tag{3}$$

When E_{1a} reaches the other side, some of it is transmitted through the second mirror such that the transmitted light can be described as

$$E_{1t} = E_0 t^2 e^{i\omega t} \tag{4}$$

In a roundtrip in the cavity, E_{1a} will be reflected and pick up a phase shift of δ with respect to E_{1r} such that when transmitted it can be described as:

$$E_{1r} = E_0 t^2 r e^{i\omega t + \delta}. \tag{5}$$

Meanwhile, the corresponding reflected beam travels back to the second mirror and is transmitted, as well as picking up a phase shift δ with respect E_{1t}:

$$E_{2t} = E_0 t^2 r^2 e^{i\omega t + \delta}. \tag{6}$$

The light continues to reflect within the cavity as well as let some of the light transmit every time it encounters one of the mirrors.

The sum of all of the reflected beams is:

$$
\begin{align}
E_r &= E_{0r} + E_{1r} + E_{2r} + \ldots \tag{7} \\
&= E_0 r e^{i\omega t} + E_0 t^2 r e^{i\omega t} + E_0 t^2 r^3 e^{i\omega t} + \ldots \tag{8} \\
&= E_0 e^{i\omega t}(r + t^2(re^{-i\delta} + r^3 e^{-i2\delta} + \ldots)) \tag{9} \\
&= E_0 e^{i\omega t}(r + t^2 r e^{-i\delta}(1 + r^2 e^{-i\delta} + (r^2 e^{-i\delta})^2 + \ldots)) \tag{10} \\
&= E_0 r e^{i\omega t}\left[1 + t^2 e^{-i\delta}\left(\frac{1}{1 - r^2 e^{-i\delta}}\right)\right], \tag{11}
\end{align}
$$

where we assume that the quantity $|r^2 e^{-i\delta}| < 1$ and use the well-known Maclaurin series substitution. After some more algebra, we arrive at the following result:

$$
E_r = E_0 e^{i\omega t}\left(\frac{r(1 - e^{-i\delta})}{1 - r^2 e^{-i\delta}}\right). \tag{12}
$$

The reflected flux density, I_r is given by:

$$
\begin{align}
I_r &= \frac{E_r E_r^*}{2} \tag{13} \\
&= I_i \frac{2r^2(1 - \cos\delta)}{(1 + r^4) - 2r^2\cos\delta}, \quad \text{where } I_i = \frac{E_0^2}{2} \tag{14} \\
&= I_i \frac{(\frac{2r}{1-r^2})^2 \sin^2(\delta/2)}{1 + (\frac{2r}{1-r^2})^2 \sin^2(\delta/2)} \tag{15}
\end{align}
$$

Similarly, we can derive the equations for the transmitted beam and flux density to be:

$$
\begin{align}
E_t &= E_0 t^2 e^{i\omega t}\frac{1}{1 - r^2 e^{-i\delta}} \tag{16} \\
I_t &= I_i \frac{1}{1 + (\frac{2r}{1-r^2})^2 \sin^2(\delta/2)}. \tag{17}
\end{align}
$$

From Equations 15 and 17, we notice that there is a common factor and define it as:

$$
F \equiv \left(\frac{2r}{1 - r^2}\right)^2, \tag{18}
$$

If we take Equation 17 and divide by I_i we get the so-called Airy function:

$$
A \equiv \frac{1}{1 + F\sin^2(\delta/2)}, \tag{19}
$$

whose graph is shown below $\delta \in (-15\pi, 15\pi)$. [8]

We notice that the light is only transmitted for certain δs, which correspond to when the cavity length is an integer multiple of the wavelength of the light. The distance between the peaks is known as the free spectral range, and is given by:

$$
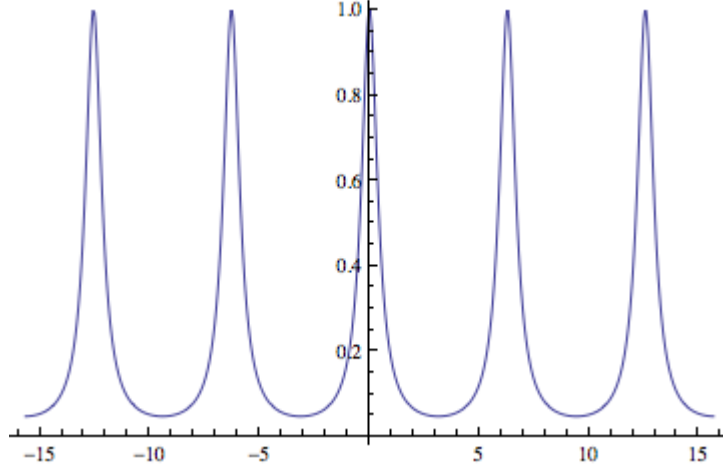\Delta\nu_{\text{FSR}} = \frac{c}{2L}, \tag{20}
$$

FIGURE 2. This graph is the Airy Function, as given in Eq. 19.

where c is the speed of light and L is the length of the cavity.

There is a quantity called the finesse of the cavity, which is defined as,

$$(21) \qquad \mathcal{F} = \frac{\pi\sqrt{F}}{2}.$$

The finesse quantifies the quality of the cavity.

Though the most basic cavity from a theoretical standpoint is the planar-mirror cavity, it is difficult to implement in practice, since it requires that the two mirrors are perfectly parallel with one another and the incoming beam to be perfectly normal to them. In reality, it is easier and more stable to implement two curved mirrors. In order for the cavity to be stable, it must satisfy the confinement condition. The confinement condition for a cavity with curved mirrors with radii R_1 and R_2 is given by:

$$(22) \qquad 0 \leq g_1 g_2 \leq 1,$$

where

$$(23) \qquad g_1 = 1 + \frac{d}{R_1}$$

$$(24) \qquad g_2 = 1 + \frac{d}{R_2}.$$

[9]

2.3. **Frequency Stabilization.** In order to mode-match perfectly into a cavity, there must be something to signify whether one is on resonance or not. This can be done by retrieving an error signal that shows how far off resonance one is and creating a system to steer the laser back to cavity resonance if the laser is not on resonance. This technique to correct and stabilize the laser frequency is known as the Pound-Drever-Hall technique.

When the light is slightly off cavity resonance, it is difficult to tell what side of resonance one is because the signal is symmetric about resonance, just by studying the light that is being transmitted from the cavity. However, by looking at Figure 2, one notices that the slope on either side of resonance is different. Therefore, if one can obtain the derivative of this signal, it would be possible to know which side of resonance one is on. This is the idea behind the Pound-Drever-Hall technique.

In actuality, one dithers the laser frequency very quickly in order to generate this derivative signal. This is achieved by modulating the laser frequency with sidebands of small amplitude. The modulated incident beam is given by the following equation:

$$
E_{inc} = E_0 e^{i(\omega t + \beta \sin \Omega t)}, \tag{25}
$$

where β is known as the modulation depth, and the Ω is the phase modulation frequency. This expression can be expanded in terms of Bessel functions such that

$$
E_{inc} \approx [J_0(\beta) + 2iJ_1(\beta) \sin \Omega t] e^{i\omega t} \tag{26}
$$

$$
= E_0[J_0(\beta) e^{i\omega t} + J_1(\beta) e^{i(\omega + \Omega)t} - J_1(\beta) e^{i(\omega - \Omega)t}]. \tag{27}
$$

The reflected beam is given by:

$$
E_{ref} = E_0[F(\omega)J_0(\beta) e^{i\omega t} + F(\omega + \Omega)J_1(\beta) e^{i(\omega + \Omega)t} - F(\omega - \Omega)J_1(\beta) e^{i(\omega - \Omega)t}], \tag{28}
$$

where F is given by:

$$
F(\omega) = \frac{E_{ref}}{E_{inc}}, \tag{29}
$$

and is known as the reflection coefficient. The J_0 term is the carrier frequency, while the two sidebands have frequencies $\omega \pm \Omega$. The total reflected power is given by

$$
P_{ref} = |E_{ref}|^2 \tag{30}
$$

$$
= P_c|F(\omega)|^2 + P_s|F(\omega + \Omega)|^2 + |F(\omega - \Omega)|^2 \tag{31}
$$
$$
+ 2\sqrt{P_c P_s}(\text{Re}[F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)] \cos \Omega t
$$
$$
+ \text{Im}[F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega) \sin \Omega t]) + (2\Omega \text{ terms}),
$$

where P_c and P_s are the power in the carrier and the sidebands, respectively.

We are interested in the cosine and the sine terms in this last expression, as this will give us the error signal that we require. When the carrier is near resonance, but the sidebands are not, then the reflection coefficient for the sidebands are approximately equal to 1. This means that the cross-terms between the sideband and the carrier frequencies in Equation 32 become imaginary. The error signal is:

$$
\epsilon = -2\sqrt{P_c P_s} \text{Im}[F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)] \tag{32}
$$

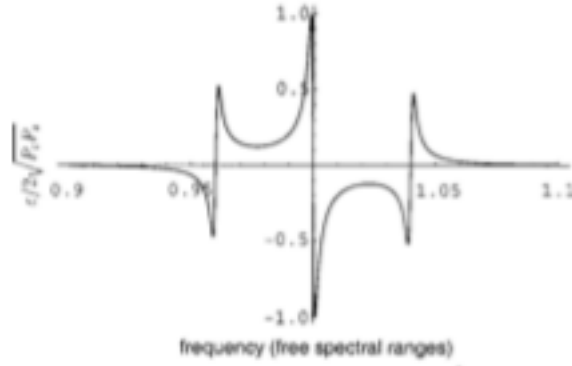Figure 3 shows the Pound-Drever-Hall error signal.
[10]

FIGURE 3. This graph is the Pound-Drever-Hall error signal. This figure was taken from Reference [10].

3. EXPERIMENTAL DESIGN

3.1. **Building the laser diode.** This project began with the construction of an ECDL in a Littman-Metcalf configuration, using an Eagleyard diode and a grating of 1200 lines/mm. In order to achieve optimal optical feedback, a triangle function from a function generator was connected to the servo input of the laser as well as an oscilloscope. At the output of diode laser, there was a power meter that was connected to the oscilloscope. The oscilloscope mode was adjusted such that it plotted the power against the driving current on the diode. Once the location of the threshold current, that is the current at which the diode begins to lase, was determined and it was minimized by using the knobs to adjust the feedback mirror position. Figure 4 shows the actual laser configuration.

3.2. **Constructing the optical cavity.** After building the laser, it was possible to start adding components to the system to test the cavity. First, a set of cylindrical lenses were placed in the beam in order to shape the beam. Next, the beam travels through an isolator, that rotates polarization of the light such that any reflected light cannot return to the diode and cause instabilities. The beam is then coupled into a single-mode fiber. When the beam leaves the fiber, it is reflected with two steering mirrors and two lenses, separated by about 35.5 cm and with focal lengths 200 mm and 100 mm. After passing through a polarizing beam splitter and a quarter-wave plate, the beam enters the cavity. The cavity mirrors each have a radius of curvature of 30 cm, while the cavity length is about 40 cm, thereby satisfying the confinement condition given in Equation 22.

In order to align the beam to the cavity, it was necessary to adjust the tilt of the mirrors such that the subsequent reflections overlap each other completely. Additionally, a CCD camera that images the transmitted light was used to align one of the mirrors, while a photodiode that collects some of the light. A sweep function controls the piezo on the feedback mirror in the ECDL such that the wavelength of the laser changes slightly. As
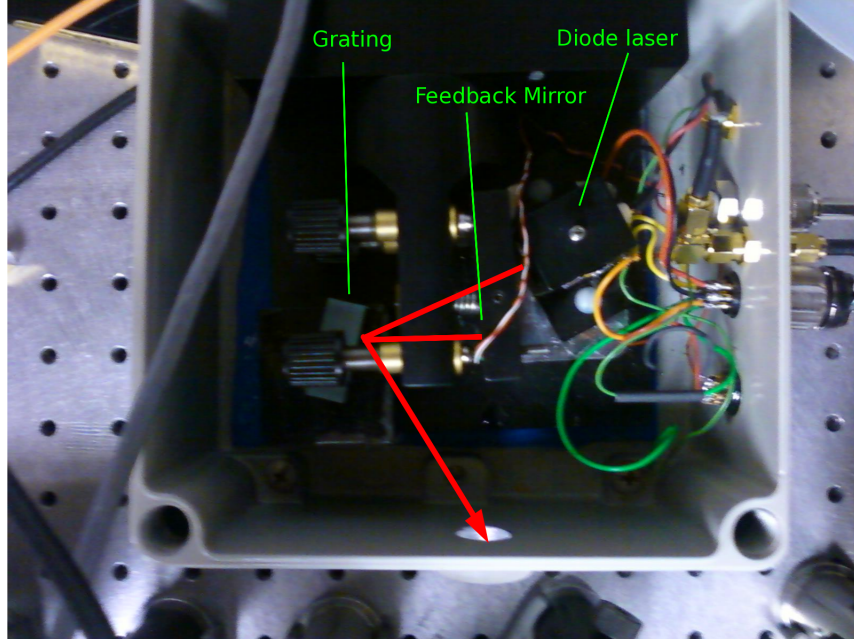
FIGURE 4. This figure shows the layout of the diode laser. The red lines represent the optical paths.

the piezo is being swept, the signal from the photodiode will peak whenever the light is resonant within the cavity. If the cavity is fairly well-aligned, there can be several of these peaks. However, it is preferable that there be less peaks as one scans the piezo, so that it is easier to lock the laser to the cavity.

There are some techniques that one can employ in order to achieve good mode-matching. These include placing two lenses before the cavity to shape the beam waist in the cavity to match the theoretical beam waist of the cavity. Another technique is to use the two mirrors before the cavity to steer the beam such that it is enters the cavity down the center of the axis of the cavity.

3.3. **Pound-Drever-Hall.** Once the cavity was fairly well aligned, it was necessary to generate an error signal using the Pound-Drever-Hall technique. In order to implement the Pound-Drever-Hall technique, one can use the reflected light from the cavity. In order to isolate the reflected light, a combination of a polarizing beam splitter and a quarter-wave plate was implemented. When the light enters the beam splitter, only a certain polarization can continue straight through, while light of the opposite polarization is deflected. The quarter-wave plate changes the polarization of the incoming light from being linearly polarized to circularly polarized. When the reflected light travels through the quarter-wave plate again, it is converted from being circularly polarized to being linearly polarized. However, this time, the light is of the opposite polarization as when it entered the cavity,
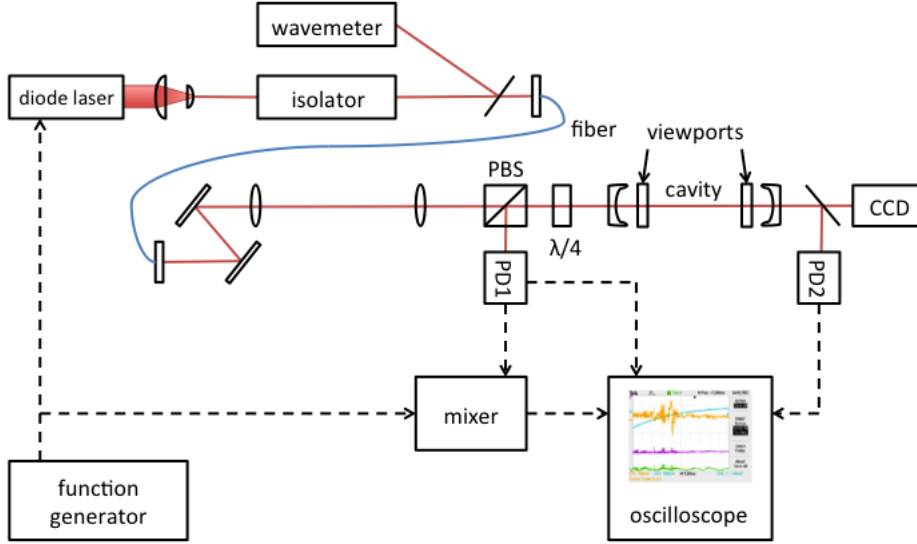
FIGURE 5. This figure shows the layout of the experiment. The solid lines represent the laser, while the dotted lines represent signal connections

so when it reaches the beam splitter, it is deflected into a photodiode. [10] One can generate sidebands on the laser by inputting a signal from a function generator into the RF modulation input on the diode laser controller. The function generator was set to a 24 MHz sine wave with an amplitude of -13 dBm.

The photodetector collects the light and generates a signal based on the incident intensity. This signal and the signal from the function generator are sent to a frequency mixer that will generate a signal containing the sum and the difference of the two frequencies. The signal coming out of the frequency mixer is filtered with a low-pass filter and produces the desired error signal. Often, it is necessary to implement a phase shifter on the signal going from the function generator to the mixer in order to compensate for the delay between the function generator and the photodetector.

4. DATA

The diode laser is being kept at 16.7° C, while the threshold current was minimized to 67.5 mA. The wavelength of the laser was measured to be 814.864 nm and tunes about 2 GHz before mode-hopping. The output power of the laser was measured to be 18.1 mW at an injection current of 150.5 mA.

Figure 6 shows the transmission peaks of the optical cavity when scanning the laser's frequency by controlling the piezo on the diode laser. The large transmission peaks (Channel 4, in green) correspond to the TEM00 mode. One free spectral range of the cavity is shown. The blue trace is the sweep function on the piezo on the feedback mirror in the ECDL.
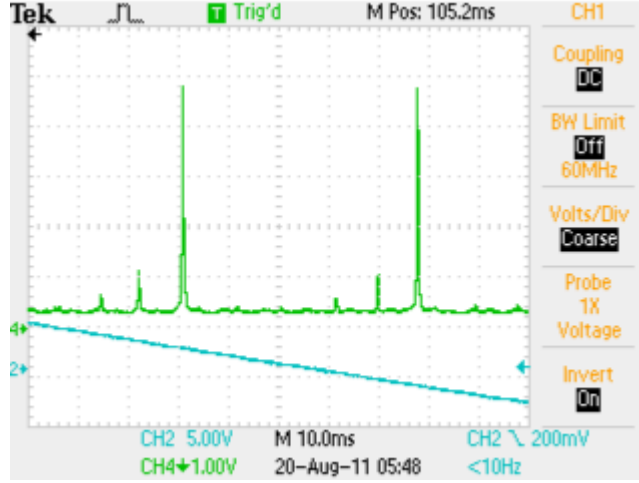

FIGURE 6. This figure shows the transmission peaks of the cavity as the ECDL is being scanned. The large transmission peaks correspond to the TEM00 mode.

Figure 7 shows the error signal generated from the cavity, shown in yellow. The pink trace shows the dips in reflection, which occur with the transmission peaks (in green), as expected.

5. Discussion

For the cavity, it was clear that the quality of one's mode-matching was directly correlated to the quality of one's error signal. In one of the earlier attempts to mode-match to the cavity, the generated error signal was so poor that the desired Pound-Drever-Hall shape was barely recognizable from the immense amount of noise. However, with better mode-matching, cleaner transmission peaks and therefore a better PDH signal were achieved. Although the current quality of mode-matching is acceptable, it is still possible to suppress the higher order modes, that is, the smaller peaks in Figure 6, altogether.

It should be noted that the transmission peaks (and by extension the reflection dips and the typical PDH shape) move from side to side, which is due to both the laser frequency drifting on its own as well as the air moving between the two cavity mirrors.
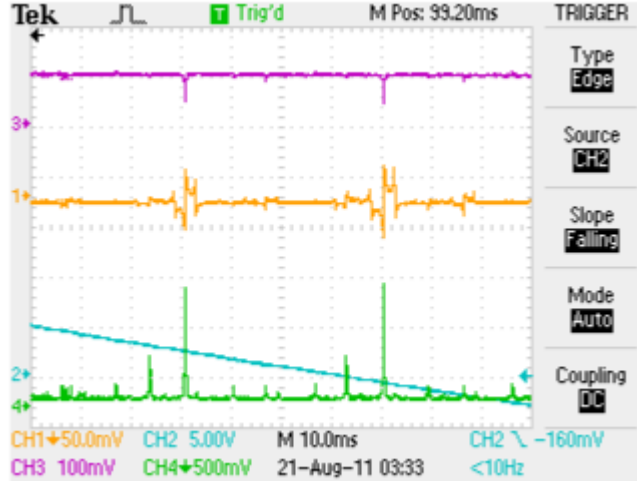
FIGURE 7. This figure shows the PDH error signal in yellow as well as the reflection and transmission peaks in pink and green, respectively.

6. FURTHER WORK

Now that the Pound-Drever-Hall signal has been established, one can frequency lock the laser such that the laser is on resonance with the cavity. Once frequency-locking is achieved it would be possible to test the viewports and to characterize their quality. After this is done, it would be necessary to add the viewports and the cavity to the Sr atomic clock.

7. ACKNOWLEDGEMENTS

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