Collective motion of driven semiflexible filaments tuned by soft repulsion and stiffness

Using simulations of driven filaments with tunable soft repulsion and rigidity, we examined how the interplay between filament flexibility and steric effects can lead to different active dynamic states. Increasing filament stiffness decreases the probability of filament alignment, yet increases collective motion and long-range order, in contrast to the assumptions of a Vicsek-type model. Orientational persistence of the trajectory enhances collective motion and increases with filament stiffness.

Collective motion of driven semiflexible filaments tuned by soft repulsion and stiffness†

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In active matter systems, self-propelled particles can self-organize to undergo collective motion, leading to persistent dynamical behavior out of equilibrium. In cells, cytoskeletal filaments and motor proteins form complex structures important for cell mechanics, motility, and division. Collective dynamics of cytoskeletal systems can be reconstituted using filament gliding experiments, in which cytoskeletal filaments are propelled by surface-bound motor proteins. These experiments have observed diverse dynamical states, including flocks, polar streams, swirling vortices, and single-filament spirals. Recent experiments with microtubules and kinesin motor proteins found that the collective behavior of gliding filaments can be tuned by altering the concentration of the crowding macromolecule methylcellulose in solution. Increasing the methylcellulose concentration reduced filament crossing, promoted alignment, and led to a transition from active, isotropically oriented filaments to locally aligned polar streams. This emergence of collective motion is typically explained as an increase in alignment interactions by Vicsek-type models of active polar particles. However, it is not yet understood how steric interactions and bending stiffness modify the collective behavior of active semiflexible filaments. Here we use simulations of driven filaments with tunable soft repulsion and rigidity in order to better understand how the interplay between filament flexibility and steric effects can lead to different active dynamical states. We find that increasing filament stiffness decreases the probability of filament alignment, yet increases collective motion and long-range order, in contrast to the assumptions of a Vicsek-type model. We identify swirling flocks, polar streams, buckling bands, and spirals, and describe the physics that govern transitions between these states. In addition to repulsion and driving, tuning filament stiffness can promote collective behavior, and controls the transition between active isotropic filaments, locally aligned flocks, and polar streams.

1 Introduction

Active particles exhibit complex and dynamical order at length scales much larger than the scale of a single particle. Living systems with collective dynamics include swimming bacteria, schools of fish, flocks of birds, and crowds of people. The study of active biopolymers is motivated by the activity of cells, because intracellular organization and dynamics are largely governed by the cytoskeleton. Cytoskeletal filaments and motor proteins generate active forces that control the assembly of critical cellular structures and long-range patterns, or active nematics. Collective behavior can also occur in filament gliding experiments, wherein cytoskeletal filaments are propelled by motor proteins bound to a surface. Previous work on filament gliding has reported several nonequilibrium dynamical states. However, our understanding of the physics that controls these phases is incomplete.

Systems of microtubules propelled by kinesin motors often do not interact and align sufficiently for the emergence of collective motion. Recent work showed that adding methylcellulose as a molecular crowding agent can reduce filament crossing and cause microtubules to locally align. This has been proposed to occur due to attractive depletion forces that lead filaments to align and form polar bundles. Such systems may be well-described by Vicsek-type models of active polar particles, wherein collective motion is governed by local alignment interactions. However, previous rheological work reported that depletion forces alone were insufficient to explain the degree of bundling observed in systems of actin and methylcellulose, and proposed that filaments might be kinetically trapped due to the formation of a methylcellulose network. In addition, microtubule-kinesin gliding experiments observed...
the alignment of filaments due to local steric interactions in the absence of any depletants, including the formation of self-interacting, single-filament spirals.\textsuperscript{15,18,19} Therefore, we seek to understand the phenomenology observed in microtubule-kinesin filament gliding experiments with methylcellulose, in the context of purely repulsive filament interactions that are tunable by the methylcellulose concentration. Most previous theoretical and computational work investigating collective behavior of active filaments with repulsive interactions focused on purely rigid rods\textsuperscript{30–38} or flexible filaments with hard-core interactions,\textsuperscript{39,40} but the collective behavior of active semiflexible filaments with tunable steric interactions has not been explored.

In this paper, we study the non-equilibrium phase behavior of active semiflexible polar filaments that interact with a tunable soft repulsive potential. We find that the dynamic state behavior depends on the strength of the repulsive potential, filament rigidity, and activity. Increasing the repulsion leads to a transition between active isotropic filaments and aligned polar streams, matching the alignment behavior observed in experiments with increasing methylcellulose concentration.\textsuperscript{18,19}

However, we also find that increasing filament rigidity promotes collective motion while simultaneously decreasing the probability that two intersecting filaments align (Fig. S3, ESIF). This counter-intuitive behavior is explained by an increase in directional persistence tuned by increasing filament rigidity. Therefore, our results suggest that while filament alignment by collisions is important, the degree of filament directional persistence is an additional key physical effect contributing to the emergence of collective motion.

### 2 Model and simulation

Our simulation model expands our previous work on self-propelled filaments\textsuperscript{38,41,42} by adding semiflexibility and tunable repulsion. Our filaments are modeled as inextensible chains of rigid segments, with neighboring segments subject to bending forces to enforce the filament persistence length $L_p$ (Fig. 1A). The filament equations of motion are implemented using the constrained Brownian dynamics algorithm of Montesi, Morse, and Pasquali\textsuperscript{43} for a semiflexible chain with anisotropic friction. Each segment of the filament experiences random forces so that its dynamics obey the fluctuation–dissipation theorem for slender filaments at thermal equilibrium.\textsuperscript{44} Our choice of filament model has the advantage of being inherently inextensible, and lacks a dependence on large harmonic forces that are typically required to model stiff filaments (see ESIF). This allows us to model microtubules, which are inextensible and have high persistence lengths $L_p/L \approx 100–1000$,\textsuperscript{45} but nevertheless often appear bent in both \textit{in vivo} and \textit{in vitro}, indicating the importance of their flexibility.\textsuperscript{46–48}

The activity of motor proteins is modeled by a polar driving force per unit length $f_{dr}$ tangent to each filament segment. This choice reflects experimental observations that filament velocity is constant and independent of methylcellulose concentration, even during filament crossing events,\textsuperscript{15,20} suggesting that stochastic effects due to the motors (e.g., binding and unbinding, variation in motor stepping) occur at short enough time scales so as to not significantly alter long-time behavior.

Interactions between filaments are repulsive but soft, defined by the generalized exponential potential (GEM-8), $U(r) = e^{-(r/\sigma)^8}$, with cutoff $U(r > \sqrt{2}\sigma) = 0$. Here $r$ is the minimum distance between neighboring segments and $\sigma$ the diameter of a filament (Fig. 1B). The maximum potential value $\varepsilon$ represents the energy required for two segments to overlap. This potential is steep near the edge of a filament. Interactions occur between each filament segment, except for nearest-neighbor segments of the same filament. While filaments experience local drag, we neglect long-range hydrodynamic interactions because previous experiments found these forces to be negligible.\textsuperscript{19}

Filaments are inserted at a packing fraction $\phi = A_{fil}/A_{sys}$, where $A_{sys}$ is the area of the simulation box and $A_{fil} = N(L\sigma + \pi\sigma^2)$ is the total area occupied by $N$ spherocylindrical filaments of length $L$ and diameter $\sigma$. The characteristic timescale is the time for a filament to move the distance of its contour length, $\tau_A = L/v$, with the velocity depending on the total driving force and the coefficient of friction acting parallel to the filament, $v = \zeta_0^{-1}F_{dr}$. The filament driving force per unit length is set by the Péclet number, which is the ratio of active and diffusive transport time scales, $Pe = \tau_{dr}/\tau_A = f_{dr}L^2/k_BT$. We explore the range $Pe = 10^4–10^5$, based on calculations of active forces in experiments\textsuperscript{15} (see ESIF). Simulations were run for $10^3–10^4\tau_A$.

Our results depend on six dimensionless parameters: rigidity $\tilde{\kappa} = L_p/L$, interaction energy $\tilde{\varepsilon} = \varepsilon L_{dr}$, packing fraction...
\( \phi = A_{fil}/A_{sys} \), aspect ratio \( l = L/\sigma = 60 \), system size \( L_{sys}/L = 20 \), and Péclet number \( Pe \). The interaction energy has been rescaled by \( \varepsilon_{dr} \), which is the energy required for the potential to exert a maximum force equal to the driving force of a particle with length \( \sigma \). Our simulation varies \( \tilde{\kappa} \) from 5–1000; previous computational work on active semiflexible filaments examined the range \( \tilde{\kappa} < 20 \), however this is not well-suited for the study of microtubules, which can have \( \tilde{\kappa} \approx 100–1000 \). We varied \( \tilde{\varepsilon} \) from 1–20, corresponding to filament pair-alignment behavior estimated from previous gliding experiments\(^{18-20} \) (Fig. 3). The work here focuses on the low-density regime of \( \phi = 0.05, 0.1 \), but we also explored higher filament density \( \phi = 0.2, 0.4, \) and 0.8 at \( Pe = 10^5 \) (see ESI†). Although the filament aspect ratio is fixed, varying the filament rigidity is analogous to varying the length of a filament with a fixed persistence length.

### 3 Results

Our simulations generated five primary phases: active isotropic, flocking, giant flocking, swirling, and spooling (Fig. 2). To quantify the dynamical phases, we used six global order parameters: the polar order \( P \), nematic order \( Q \), average contact number \( c \), average local polar order \( p \), average spiral number \( s \), and number fluctuations \( \Delta N \) (see ESI†). We also quantified the collective dynamics of the system by characterizing the flocking behavior in terms of the number of flocking filaments \( N_F/N \), and frequency that filaments joined or left a flocking state, respectively \( f_{NF-F} \) and \( f_{F-NF} \). Using a high-dimensional clustering algorithm, we identified 5 clusters in this nine-dimensional order parameter space corresponding to the different phases observed for our range of parameters (see ESI†).

In the active isotropic phase, filaments cross each other in all directions, resembling filament gliding experiments that do not exhibit collective motion (Fig. 2A). The flocking phase is characterized by the coexistence of multiple polar domains of aligned filaments (Fig. 2B). Flocks are dynamic, with filaments continuously joining and leaving the flocking state. The giant flocking phase occurs when all flocks in the system coalesce into a single dominating flock, and exhibits long-range order. At higher density, giant flocks can span the system length (Fig. 2C), and can be either one band \( \phi = 0.1–0.2 \) or many counter-propagating bands \( \phi = 0.4–0.8 \). In the spooling phase,
filaments are flexible enough to self-interact, and many filaments form spirals (Fig. 2D). The swirling phase contains large, swarming flocks that collide, self-interact, and form transient vortices (Fig. 2E).

Collective motion emerges with increasing repulsion or rigidity. When both filament repulsion and rigidity are small, we find the active isotropic phase (Fig. 2F). Self-organization and collective motion can occur by increasing either repulsion or stiffness; increasing $\tilde{e}$ tends to increase collective behavior, while increasing $\tilde{k}$ or $\phi$ tends to increase long-range order. Previous experimental work has found that the transition from an active isotropic phase to polar streams was driven by an increase in alignment events between pairs of colliding filaments.$^{16,18-20}$ Accordingly, we measured the correlation in agreement with trends in experiments for increasing methylcellulose concentration$^{18-20}$ (Fig. 3). In contrast, $P_{\text{align}}$ decreases monotonically with increasing stiffness (see ESI†). Remarkably, we found regions of phase space where increasing stiffness drives the emergence of collective motion, and regions where increasing repulsion inhibits long-range organization and collective behavior.

The negative correlation between $P_{\text{align}}$ and filament rigidity can be understood by considering the additional energy required to apply a torque on a stiff filament. Flexible filaments need only bend marginally at the leading end to align during a collision. Stiff filaments cannot bend as easily and thus require additional torque to align. Higher torque between colliding filaments would increase overall alignment in the case of hard-core repulsive interactions.$^{32}$ However, with a finite repulsivity, the higher energy cost leads to a reduction in the probability of alignment.

The counterintuitive result that increasing stiffness lowers collisional alignment but increases collective motion reflects a trade-off between filament alignment and the directional persistence of filament trajectories. We quantified directional persistence by measuring the autocorrelation of filament orientation $u$, $C(t) = \langle u(0)u(t) \rangle$. The result can be fitted to a decaying exponential with characteristic timescale $\tau$, with a high $\tau$ corresponding to a high degree of directional persistence. The directional persistence was measured for filaments at different rigidity and Péclet number in the absence of interactions (see ESI†). Since driven filaments are readily deflected from their trajectories by small deviations of the leading filament tip, rigid filaments that resist deflection exhibit longer orientation correlation time, and thus have a greater directional persistence than flexible filaments. Flocking filaments that move ballistically have longer-lived alignment and thus a higher flocking lifetime, permitting other filaments to join. Although the frequency of alignment may be lower for stiff filaments, the longer duration of alignment compensates for the low alignment probability. In contrast, flexible filaments have a higher rate of alignment, leading to the rapid creation of polar clusters (Fig. 4), but they are easily deflected away, resulting in short-lived flocks. Thus, flexible filaments require a larger $\tilde{e}$ to cross the active isotropic-flocking boundary. Our results suggest that filament alignment and directional persistence are both important for the emergence of collective motion and stable flocks.

The flocking phase consists of multiple polar domains of aligned filaments, with individual flocks characterized by high local filament density and local polar order $p$. Individual flocking filaments are identified by the criterion $p_i > 0.5$, with filaments that are located in the flock interior exhibiting high contact number $c_i$ as well. The number of flocking filaments increases with filament stiffness and repulsivity. Filaments with lower $\tilde{k}$ have faster switching between flocking and non-flocking states compared to stiff filaments, yet the saturation of the number of flocking filaments occurs at high $\tilde{k}$ and low switching frequency (Fig. 4).

With increasing stiffness, flocks that form in a transient flocking phase can coalesce into a single flock, characteristic of the giant flock phase. In the giant flock phase, the number of flocking filaments saturates. Giant flocks in this phase are long and narrow, allowing them to efficiently capture non-flocking filaments. The rate of switching between flocking and non-flocking states becomes low in the giant flock phase, due to the large number of filaments that are kinetically trapped in the flock interior (see ESI†). Giant flocks are more stable at lower driving, consistent with previous reports that high activity can inhibit collective behavior of filaments with flexibility.$^{30}$ Due to finite-size effects of the simulation, the continued growth of the giant flock at higher density ($\phi \geq 0.1$) will often lead it to span the length of the system, $L_{\text{flock}} > 2L$, ending with the formation of a persistent polar band. At high density ($\phi \geq 0.4$), multiple independent bands can form simultaneously, resulting in coexisting nematic bands. We note that high filament density paired with high $\tilde{k}$ results in a stable giant flock phase even at low $\tilde{e}$. The high local filament density serves to restrict the rotational degrees of freedom of gliding filaments, further increasing directional persistence and the stability of polar bands.

![Fig. 3](A) Plot of the probability for two filaments to align upon collision for $\text{Pe} = 10^5$. Increasing repulsion $\tilde{e}$ increases $P_{\text{align}}$ while increasing rigidity $\tilde{k}$ decreases it. Experimental values taken from Saito et al.$^{19}$ with corresponding methylcellulose concentration as the top x-axis. (B) Simulation snapshots illustrating how increasing filament rigidity increases collective motion. All images show simulations with $\tilde{e} = 5$, $\phi = 0.1$, $\text{Pe} = 10^5$. 

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Our phase diagram shows a limited region of stability for long-range collective motion in the giant flocking phase. This occurs because although increasing $k$ promotes long-range order, increasing $\varepsilon$ too high can break it. At high $k$ and high $\varepsilon$, collisions cause large deformation of flocking and banding filaments, leading to buckling. In the simulation shown in Fig. 2C, the filaments repeatedly form a polar band, which buckles, shears, falls apart, and eventually reforms. Similar dynamic phases have been observed in systems of self-propelled rods and semiflexible filaments driven by motors. Therefore, only intermediate values of $\varepsilon$ facilitate persistent long-range order as giant flocks.

While increasing $\varepsilon$ typically increases alignment, promoting collective motion, for flexible filaments (low $k$), filaments can bend and self-interact via collisions to form spirals (Fig. 5C). Spirals are seen at all Péclet number, but are increasingly stable with higher driving. Stable single-filament spirals can persist until a collision with another filament or flock deforms the filament enough to release it. At high driving, the system can enter the spooling phase, wherein a majority of filaments become kinetically trapped as spirals. The spooling phase resembles the frozen, active steady states found in previous experimental work. The principal mechanism of spool formation has been of significant interest, with suggestions ranging from defects in the motor lattice to thermal activation. Previous experiments with microtubules and dynein have observed stable filament vortices attributed to filament curvature induced by motors, which is absent in our simulations. The spooling phase demonstrated here is governed by steric self-interactions and collisions, as found in previous modeling work. Their overwhelming appearance at high driving may be due to a rescaling of the effective filament rigidity at high activity, which has been reported in other recent work leading to an apparent softening of the filament that may
contribute to a buckling instability\textsuperscript{57} (see ESI†). Therefore, while spool formation in experiments may have contributions from defects in the protein lattice, pinning of filaments by dead motors, or intrinsic curvature, our results indicate that these mechanisms are not necessary for spool formation.

When repulsion and stiffness are both high, even transient polar bands can no longer form, and the system enters the swirling phase. High-energy collisions due to large bending and repulsive forces cause large deformation of flocks, leading to shorter end-to-end flock length and inhibiting the long-range motors, or intrinsic curvature, our results indicate that these states may have applications for drugs targeting cortical cytoskeletal filaments or nanodevices that use cytoskeletal filaments as molecular shuttles.\textsuperscript{58,59}

**4 Conclusions**

We have examined the role of flexibility and repulsivity in the collective behavior of active polar filaments. The phase diagram presented here makes predictions that could guide future experiments seeking to observe collective behavior in systems of active semiflexible polymers. These systems can exhibit a wide variety of active, dynamic states, with transitions that are controllable by tuning repulsive interactions and filament rigidity. The ability to control the transition between these states may have applications for drugs targeting cortical cytoskeletal filaments or nanodevices that use cytoskeletal filaments as molecular shuttles.\textsuperscript{58,59}

**Conflicts of interest**

There are no conflicts to declare.

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**Notes and references**