**Infinitesimals and the Labyrinth of the Continuum**

The continuum concept is one of the oldest in mathematics and philosophy. The usual meaning of the word *continuous* is "unbroken" or "uninterrupted": thus a continuous entity—a continuum—has no "gaps." We commonly suppose that space, time and motions are continua, and philosophers such as Leibniz have maintained that all natural processes occur continuously. (Leibniz's struggles to understand the continuum led him to term it a "labyrinth" .) In modern times the continuum has ben reduced to discreteness by mathematicians such as Dedekind and Cantor and their successors by providing it with set-theoretic formulations. Certain mathematicians such as Brouwer and Weyl, and philosophers, such as Brentano, resisted the "set-theorization" of the continuum, insisting that the continuum cannot be faithfully represented in discrete terms. Today there are a number of essentially different accounts of the continuum in mathematics, both continuous and discrete.

Closely connected with the concept ofc the continuum is that of *infinitesimal.* Traditionally, an *infinitesimal quantity* is one which, while not necessarily coinciding with zero, is in some sense smaller than any finite quantity. For engineers, an infinitesimal is a quantity so small that its square and all higher powers can be neglected. In the theory of limits the term "infinitesimal" is sometimes applied to any sequence whose limit is zero. An *infinitesimal magnitude* may be regarded as what remains after a continuum has been subjected to an exhaustive analysis, in other words, as a continuum "viewed in the small." It is in this sense that continuous curves have sometimes been held to be "composed" of infinitesimal straight lines.

Infinitesimals have a long and colourful history. They make an early appearance in the mathematics of the Greek atomist philosopher Democritus (c. 450 B.C.E.), only to be banished by the mathematician Eudoxus (c. 350 B.C.E.) in what was to become official "Euclidean" mathematics. Taking the somewhat obscure form of "indivisibles," they reappear in the mathematics of the late middle ages and later played an important role in the development of the calculus. Their doubtful logical status led in the nineteenth century to their being discarded in mathematical analysis and replaced by the limit concept. In recent years, however, the concept of infinitesimal has been refounded on a rigorous basis.

In my talk I shall offer a survey of these ideas.