# INVESTIGATING AND VALUING THE MESSY NATURE OF LEARNING:

## Ontological, epistemological, and social aspects of student reasoning in quantum mechanics

Jessica R. Hoehn

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M.S., University of Colorado Boulder, 2017

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Noah D. Finkelstein

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Steven J. Pollock

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**ABSTRACT**

Hoehn, Jessica R. (Ph.D., Physics)

Title: Investigating and valuing the messy nature of learning: Ontological, epistemological, and social aspects of student reasoning in quantum mechanics

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Historically, much of physics education research has focused on whether students' answers are correct or incorrect. This thesis presents a complementary perspective that moves beyond a dichotomous view of learning by valuing the messy, or complicated and varied, nature of students' reasoning. We do so by investigating three aspects of student reasoning in quantum mechanics (QM)—ontological (pertaining to the nature of entities), epistemological (pertaining to the nature of knowledge or learning), and social (pertaining to collective reasoning). Through focusing on the kinds of reasoning that students are capable of, we value their creativity, identity, and engagement in our educational environments, in service of supporting and cultivating their learning of physics.

First, we develop and present a framework to describe and distinguish between different ontological structures. We document students' flexible use of ontologies in individual, collective, oral, and written reasoning. The demonstration of this flexible use of ontologies is novel for the PER community which has previously recognized the dynamic nature of ontologies, yet not elaborated on the different forms those dynamics can take. Further, we find that the way we ask questions can impact students' ontological reasoning. These findings suggest that as instructors we should recognize and attend to the ways in which students can engage in flexible use of ontologies. Additionally, we argue that tentativeness and flexible use of ontologies can be productive for student learning. We present an example of how we can work to support students' ontological reasoning through research-based curricular materials.

Next, we conduct a study of students' domain-specific epistemologies and observe that students report "epistemological splits" between classical and quantum physics. Students are more likely to consider quantum physics to be less tangible or less connected to the real world, and to perceive problem solving in QM to rely more heavily on math. We observe these epistemological splits

across multiple institutional and instructional contexts. The existence and prevalence of these splits suggests that when attending to students' views about the nature of knowing and learning physics, we should be cognizant of when we are treating "physics" as a monolithic domain. Further, we identify some of the reasons that students might report epistemological splits, and argue that these stances can reflect epistemological sophistication. We begin to investigate the impacts that individual instructors have on the development of students' domain-specific epistemologies, raising questions for further study.

Finally, we conduct a case study analysis of a group of students engaged in collaborative problem solving and identify *epistemic stances toward group work* as one factor that contributes to the social positioning of the students within the group. The analysis investigates the ways in which epistemology, sense making, and social dynamics are intertwined. The construct of epistemic stances toward group work is a novel contribution to the PER field, and the attention to the fine-grained social dynamics and the intertwining of multiple elements of students' collective reasoning represents a new approach to the study of group work.

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# PART ONE: INTRODUCTION

## Chapter I. Introduction and motivation

Few would argue with the premise that the primary goal of physics education is to teach students physics content. Yet in addition to content mastery, there are many other important goals for students' learning and participation in physics. We might consider goals such as to help students develop scientific practices and learn what it means to *do* physics, to empower students to think critically for themselves by asking questions and looking for answers through evidence and observation, or to cultivate students' identities and sense of belonging. Depending on our particular goals as educators, we might value different elements of students' learning—their abilities to use formal physics language, to connect physics concepts to their everyday experiences, to work collaboratively and productively on a hard problem, or to reflect on their own learning.

Historically, much of physics education research has focused on whether students' answers are correct or incorrect. This approach logically follows from the primary goal of physics education that students develop a *correct* understanding of the physics concepts. By identifying and focusing on the topics or areas where students fall short or get the answers wrong, the PER community has made huge strides in developing new methods of teaching, new curricula, and new theories of learning. In this dissertation, we provide a complimentary approach which moves beyond a binary "get it or don't get it" view of student learning by investigating and valuing the messy nature of learning. In thinking about the "messiness" of learning we draw on the notion of "messing about" [1], think about the *process* of learning as nonlinear and comprised of many intertwining factors, consider the role of tentativeness, and recognize that learning often involves making use of multiple (sometimes seemingly contradictory) ideas simultaneously or across contexts. This notion of messiness is aligned with the idea of knowledge in formation [2, 3] or growth mindset [4]. While someone with a fixed mindset might think that doing physics is related to an innate ability and that you are either good at it or you are not, someone operating from a growth mindset will recognize the areas in which they need to improve and find strategies to work toward that improvement. The "knowledge in pieces" perspective considers learning to be a restructuring of many fine-grained, intuitive knowledge elements which are neither correct nor incorrect [2, 3], and that "being 'wrong' is an inescapable part of learning" [5, p. 32].

Through studies of students' reasoning, we attend to the ways in which the messiness can be productive for learning. Duckworth describes the essence of pedagogy as providing students with opportunities to "have wonderful ideas" [6]. To Duckworth, the having of wonderful ideas means asking your own questions, figuring out how to answer them, and having confidence that your ideas are worth trying out. Providing these opportunities can help to give students' agency over their own learning and to value their creativity, identity, and engagement in the educational environment. Attending to the messy aspects of students' reasoning (in our research and teaching) is *one* way to recognize and value students' ideas. If we hope to empower students and cultivate their learning of physics, we must recognize and value the messy nature of learning. Further, by focusing on the kinds of sophisticated reasoning students are capable of, we can help students recognize the productive elements of their own learning [7].

This is not to say that *all* physics education research must focus on the messiness or that as researchers and teachers we should not care about students getting the correct answers. Certainly a major goal (if not *the* primary goal) of physics education is for students to learn the physics content, and this requires correct identification and application of formal physics knowledge. However, the field has historically focused on whether students "get it" or not. It is the intention of this thesis to provide a complementary view that focuses on the ways in which the messy elements of students' reasoning can be productive for their learning, regardless of whether their understanding of the content in a given moment is canonically correct. We seek to leverage this messiness to further support and advance student learning.

In order to identify the ways in which elements of students' reasoning can be *productive*, we must identify productive *to what ends*. One might consider a student's idea or contribution to be productive if it gets them closer to a correct answer, or to a correct understanding of the physics [8]. We consider this aspect of productivity, along with other dimensions. For example, a student's reasoning might be productive if it reflects expert-like views about what it means to learn and do physics (e.g., learning physics is about constructing your own understanding rather than memorizing formulas), or if it helps a group of students navigate an awkward social interaction. Generally in this thesis, we use the following definition of productivity: an idea, statement, written or oral work, or contribution to a conversation is considered to be productive if it supports progress toward understanding the physics content, sustains engagement in sense making processes, provides opportunities to engage in expert-like practices, or contributes to a cohesive social interaction. However, there will be variation by section in the way we operationalize *productivity*, as each section of this thesis has different objectives and investigates different aspects of students' reasoning. In each of the specific sections, we will provide the specific definition of productivity being used and to what ends we are considering productivity.

Moving beyond a binary view of student learning is not a new idea [5, 7, 9, 10], nor does it suggest that valuing correct answers is necessarily at odds with a constructivist view of learning (i.e., that learners build knowledge based on prior understandings and experiences). When we primarily (or exclusively) value correct understanding or reasoning in our research and teaching, we inadvertently emphasize this dichotomous view of students' ideas as correct or incorrect. Investigating the messy aspects of learning has not been a leading discourse in physics education; here, we bring it to the foreground.

This thesis explores three different aspects of students' reasoning—ontological (reasoning related to the nature of entities), epistemological (reasoning related to the nature of knowledge and learning), and social (the ways in which social dynamics play a role in collective reasoning)—specifically in the context of quantum mechanics (QM). No one study in this thesis centers on the messiness of reasoning *per se*, but it is a general theme that is woven throughout. We begin each study with a fundamental commitment to valuing the messy nature of learning, and thus the research questions we ask have to do with capturing what students are capable of, rather than where they have difficulties. Traditionally, physicists have taught QM from a "shut up and calculate" paradigm [11, 12], focusing on the mathematical formalisms and the predictive power of QM, with the idea that the conceptual understanding will come later. The subject of QM is ripe for thinking about student reasoning, and is particularly relevant for considering ontological and epistemological aspects. Not only do students have to learn new mathematical formalisms, but they have to learn to think about entities in new ways (i.e., an electron, which is classically a particle, now has wave properties). Additionally, students receive messages about the "weirdness" of QM and may tend to view the nature of learning quantum physics different from learning classical physics (i.e., relying on the math more in QM rather than conceptual understanding or intuition).

Within the subject area of QM, we situate most of our work in the sophomore-level modern physics course. This particular course is an important time to reach students during their academic careers. For physics majors, learning QM is a seminal and foundational part of their undergraduate physics career. For engineering or other STEM (but non-physics) majors, modern physics is often the last physics class they take, providing a foundational understanding of quantum phenomena which govern many systems they will encounter in STEM careers. Typically, modern physics courses include a review of physics developments at the turn of the 20th century. Namely, an introduction to QM and sometimes special relativity. Modern physics is an understudied course overall in PER, as many avenues of research focus on the introductory or upper division courses, but leave out the sophomore-level or "middle division" courses in between. We use studies of students' reasoning in this subject area as an exemplar of the kinds of reasoning students are capable of and of the ways we can investigate the ontological, epistemological, and social aspects of students' reasoning.

This thesis falls primarily into the category of theory and methods development, providing proof of concept for ways to investigate students' reasoning and the kinds of reasoning that students can be capable of. Most of the studies demonstrate the utility of a theoretical construct or method, rather than make robust claims of ubiquity. These theoretical constructs and methodological tools are themselves the insights that this work contributes to the PER field, along with a few applications and implications that derive from the core methods and theory work. The thesis is divided into five parts—introduction, ontologies, epistemologies, social dynamics, and conclusions—comprising ten chapters. The three major sections are all interrelated with the general theme of *messiness* connecting them together; the ontologies section consists of three chapters (two publications and one curriculum development application), the epistemologies section consists of three chapters (two publications, one an extension of the other, and a tentative exploration chapter), and the social dynamics section consists of one chapter (a forthcoming publication).

In Part Two, we investigate students' *ontological* reasoning in QM—their reasoning about the nature of quantum entities (i.e., *what kind of a thing is a photon?*). In Chapter III, we develop and present a preliminary framework for investigating the dynamics of students' ontological reasoning. Through qualitative analysis of collective discussions among seven modern physics students, we identify two different ways in which ontologies can be dynamic. This framework advances the state of the art in the PER field which has considered whether ontologies are dynamic or not, but has not distinguished between different kinds of dynamic ontologies. We also provide evidence of messiness in students' ontological reasoning (including flexible use of ontologies and tentativeness) and argue that it can be productive for students' learning. Chapter IV builds on this preliminary framework by extending its application to individual and written work from a representative sample of a modern physics class. We find that students' flexible use of ontologies is not limited to collective oral reasoning in group work settings, but is also observed in individual and written work. In Chapter IV we present the refined framework which describes unitary, parallel, and blended ontological structures. Further, attending to the situated nature of learning, we investigate some of the ways in which the questions we ask impact students' use of ontologies. Chapter V provides a curricular application of the prior studies around ontological reasoning. We present a suite of ten modern physics tutorials (and associated instructor guides) designed, in part, with the goal of supporting students' reasoning around quantum entities.

In Part Three, we investigate *epistemological* aspects of students' reasoning in QM, specifically how their stances toward quantum physics and learning quantum physics may differ from those in classical physics. Chapter VI presents our initial exploratory study of students' domain-specific epistemologies. On a survey consisting of six pairs of bifurcated items that ask about quantum and classical physics, we observe that on some items students respond differently to the classical and quantum versions. We identify these differences as "epistemological splits" and challenge the tendency of treating "physics" as monolithic when measuring or attending to students' epistemologies. This contribution advances understanding in the PER field which has historically treated epistemology as a fixed construct, or when attending to context dependence of epistemology has not considered sub-domains within "physics". Chapter VII is an extension of the initial study that expands the data set of survey responses to investigate not only the existence of epistemological splits but also the prevalence. We find that these splits are prevalent across multiple institutional and instructional contexts. Further, we conduct a focus group study with upper division QM students; analysis of the group conversations helps us identify themes that provide insight into *why* students report these splits, or what they might mean for student learning. We also argue that these epistemological splits can reflect sophisticated epistemological stances. In Chapter VIII, we present a preliminary investigation into the impact of instructors on students' domain-specific epistemologies. This is a tentative exploration that raises questions for future investigation. The preliminary analyses suggest that an instructor's personal epistemological views, and their stated and enacted emphases on these epistemological topics in class may contribute to the epistemological splits reported by students.

In Part Four, we dive deeper into the *social* aspects of learning. Chapter IX presents an exploratory investigation of the interactions between epistemology, sense making, and social dynamics in a group work problem solving environment. While our community has valued group work as being beneficial for students' conceptual mastery, little has been done to understand how students think about group work, how students interact with one another, and how these elements impact students learning. In a case study analysis of one group of four modern physics students, we observe that some students are positioned as less knowledgeable despite steering the group's sense making. We identify differences in students' epistemic stances toward group work—their sense of how knowledge should be generated in the group—as one factor that contributes to the social positioning of the students.

Chapter X includes a list of conclusions and overall summary.

# Chapter II. Overall theoretical framing and literature review

This chapter includes a generalized review of literature relevant to this thesis overall, with the purpose of situating our work within the broader physics education community. Each of the particular sections include their own literature review as relevant to the specifics of each chapter.

The early decades of PER attended primarily to students' *misconceptions*—students' prior knowledge that contradicts correct formal physics knowledge, and that must be replaced with the correct conception. Smith, diSessa, and Roschelle, among others in the field, critiqued this line of research by pointing out that while recognizing that students come to a learning environment with prior knowledge, the identification of this prior knowledge as "wrong" and needing to be replaced contradicted a fundamental premise of constructivism—that learning involves building knowledge based on prior understandings or experiences [9]. An alternative approach considers learners to have multiple *resources*, which are neither right or wrong, that can be activated in a given moment or context [13]. We consider that students may have conceptual, ontological, epistemological, or other types of resources [7, 14], and that the activation of certain resources will depend on the specific conceptual, epistemological, and social contexts. In many instances the notion of a "concept" has been demonstrated to be too coarse a grain size for capturing the nuanced and rich nature of student reasoning [15, 16]. Furthermore, building on the "knowledge in pieces" perspective which views learning as the creation of and coordination among many diverse and fine-grained knowledge elements [2], it has been demonstrated that student epistemological stances can be better explained by examining finer-grained commitments than static, robust beliefs [13]. We draw from these perspectives but do not always consider the resources an *individual* student is using because we also attend to learning in groups and social environments. The resources model is particularly helpful in considering the flexibility, context-dependence, and messiness of students' reasoning.

Sociocultural perspectives of learning foreground the situated nature of learning [17, 18], and consider the ways in which both the objects and processes of learning are social [19]. We approach our work from a sociocultural perspective, considering context to be an integral part of learning as something that shapes and is shaped by the physics content, individual students, and interactions among students [20]. The objects of learning are social in the sense that the discipline of physics (specifically the domain of QM in this work) continues to evolve, and the community of physicists, including physics educators, defines the consensus knowledge [21]. Furthermore, it is increasingly recognized that physics education is not simply about the transfer of a body of knowledge but also about engaging people and developing their capacities in the discourse, practices, and community norms of physics (e.g., [20, 22–26]). The process of learning is similarly social; sociocultural perspectives consider learning as the act of internalizing social norms and practices [27, 28]. To such ends, valuing the social practice of physics and learning physics means valuing the interactions of individuals in collective engagement in the processes of physics—talking about, reasoning about, and solving physics problems. Indeed, it is the doing of physics that supports, and can often be considered, its learning [17, 18]. Such a perspective shapes not only our pedagogical approaches in modern physics (and physics more generally), but also frames our research questions and methodologies. Because we consider the process of learning to be a social act, we look at students' individual as well as their collective reasoning. In doing so, we value collective discourse as a cognitive tool, and use it to help us understand the reasoning structures students use as they learn QM. We value both individual ideas and associated inferred reasoning structures as well as the negotiated collective meanings (inferred from collective discourse) that students develop as they solve physics problems. Not only do socially and collectively developed tools (like language) mediate thought [28], but other people and their use of tools also mediate cognitive processes, including those involved in doing physics and solving physics problems [29, 30]. For this reason, in our analysis we attend to both individual and collectively developed reasoning, but do not always seek to distinguish the two.

Student learning in QM is an active area of research in the PER field, and there remain many aspects of student learning yet to be investigated [31]. Researchers have developed conceptual assessments [32, 33], investigated student difficulties in a variety of QM topic areas (e.g., [34–38]), developed tutorials and simulations (e.g., [39, 40]), begun to investigate mathematical sense making [41], investigated student epistemological framing of QM [42], assessed the impact of attending to questions of physical interpretation in a QM class [43], and explored student difficulties with QM concepts as connected to linguistic difficulties surrounding conceptual metaphors [44], among other things. The subject area of QM provides a rich arena for investigating ontological and epistemological aspects of student reasoning, and this thesis work contributes to our understanding of these aspects in the QM domain of PER. Furthermore, we develop new methods and bring a perspective of valuing the messiness of student reasoning which contrasts the emphasis on student difficulties.

Research on *student difficulties*, in QM and physics more generally, identifies areas where students struggle, or aspects of students' reasoning that impede learning (e.g., [38, 45, 46]). While historically much of the field has focused its efforts here, and been quite successful in improving physics education, other researchers have called for complementary work that attends not to whether students are right or wrong but identifies the productive elements of students' reasoning (e.g., [7, 8, 47]). In line with this work, we seek to focus on the types of sophisticated reasoning that students are capable of and move beyond the dichotomous view of student thinking as "right" or "wrong". Otero proposes a model of how to teach preservice teachers about formative assessment in a way that shifts away from a "get it or don't get it" view of students' prior knowledge [10], and suggests that this is particularly important for teachers because recognizing their own knowledge as "knowledge-in-formation" repositions the teacher as a learner rather than a knower. This positioning is beneficial for student learning, in part, because it sends an epistemological message that learning does not consist of absorbing information from an authority, but is an iterative and ongoing process of refining and building on the knowledge and experiences you already have. In this thesis, we seek to move beyond the "get it or don't get it" view by valuing not only students' prior knowledge or whether they end up with the correct answer, but also the messiness in between. Hawkins proposes the idea of "messing about" as a phase of learning in which the activities are prompted or guided only by the students themselves. In describing a situation in a fifth-grade classroom where the instructors allowed the students time to discover, ask questions about, and design experiments with pendulums, uninhibited by instructions from a teacher, Hawkins states that "discoveries were made, noted, lost, and made again...When the mind is evolving the abstractions which will lead to physical comprehension, all of us must cross the line between ignorance and insight many times before we truly understand." [1, p. 2] Hammer describes the role of messing about for young students as a time in which they can develop resources that they will later build on to construct expert-like understanding, and states that “Learning science cannot end with ‘messing about,’ but it may need to begin there, just as learning to draw must begin with scribbling.” [7, p. 59] We bring this idea to undergraduate learning of QM, and consider “messing about” as one part of the messiness of grappling with ontological and epistemological questions within QM. Much like we know that conceptual change is complicated and cannot be captured purely with a student-centered or content-centered approach [20], we begin from a fundamental position that learning is messy and that the physics education community benefits from attending to the messiness and to the intertwining of several factors that contribute to student learning.

# PART TWO: ONTOLOGIES

> "I think I can safely say that nobody understands quantum mechanics."—Richard P. Feynman [48]

The fact that learning and teaching quantum mechanics (QM) is difficult has been known for a long time. Rather than just accept this or focus on the difficulties as is often done in PER [49], we examine students' capabilities for negotiating meaning of difficult QM topics in productive ways. That is, rather than focus on student difficulties we ask, (in line with [9] and [7]) what kinds of sophisticated reasoning do the students engage in? In this section, we investigate students' knowledge structures in QM, through a fine-grained analysis of collective student reasoning (Chapter III), a study of individual work (Chapter IV), and a curricular application in the form of tutorials that emphasize development of reasoning around the nature of quantum entities (Chapter V). In improving quantum physics education, our goal is not only to promote and document pre- to post-shifts on conceptual surveys (e.g. QMCS [33] or QMCA [32]), but to value and encourage students' engagement in scientific discourse and reasoning about complex interpretive phenomena. Developing this capacity for meaning-making and discussions of interpretation are part of our explicit goals of instruction in the modern physics course in which these studies take place. In understanding what our students are capable of within the current system, we can value what they are doing productively and ultimately build on curricula supportive of such ends. We seek to understand student capacity to further advance and support student learning. We observe that many studies in physics education research focus on pre-post assessments (often multiple choice) of student learning. This approach to understanding student reasoning, while a good tool for triangulation, is limited and when used alone reinforces a binary view of student learning, characterizing student responses as either correct or incorrect, with no nuance in between. One interpretation of the famous Feynman quote (which captures a common sentiment among many scientists) is that there is a monolithic understanding of quantum physics, aligning with a binary "get it or don't get it" view—either you understand QM completely, or not at all. We choose an alternative interpretation, that everyone understands QM incompletely, in their own messy ways. We find value in taking a broader look at students' learning experiences and conceptual development [50, 51], and in emphasizing the productive nature of students' responses even when their ideas may be faulty or scientifically incorrect in some ways [9]. We find that student reasoning is nuanced and it may be the case that, on a given multiple choice question, students are answering favorably for the wrong reasons or that they may not exhibit consistent views across conceptual, epistemological, or social contexts [13, 52]. Focused on the goal of obtaining a more nuanced description of student reasoning, we shift away from this binary view and seek a finer-grained analysis of students' reasoning about these types of questions.

*Ontologies* describe humans' categorization of the kinds of entities[1] in the world—grouping them categorically by fundamental properties or characteristics. In thinking of ontologies for a given entity, one is answering the question "What kind of a thing is it?" A flower might be grouped in a category with other living things (or more specifically, a category of plants), while a chair might be grouped in a category of inanimate objects (or more specifically, furniture items). This categorization based on fundamental characteristics of entities is *ontological*. We might also think about ontologies for entities or concepts within physics—what kind of a thing is an electron, or electric current, or heat? One way to understand patterns and processes of student reasoning is to investigate students' use of ontologies—what properties and behaviors are they assigning to the entity they are reasoning about, and how does their ontological reasoning help them make sense of the concept or phenomenon at hand? Students' ontologies influence the way they learn physics concepts [55, 56], and as such, studies of ontologies used by both students and professional physicists, have been common in physics education research (e.g., [47, 53, 57–65]). Researchers have studied students' ontologies for energy [62, 65], wave pulses [66], and force and motion [47], among other topics. In this dissertation, we focus on students' ontological reasoning around quantum entities (e.g., photons and electrons), while drawing from prior work in other subdomains of physics.

Energy is one relatively well-studied area of physics ontologies. We draw on the example of energy specifically because it is common in physics education, and it provides an example outside the scope of our studies of quantum entities, illustrating that our approach to thinking about ontological

[1] Following Chi [53] and Gupta, Hammer, and Redish [54], we use the word "entity" to denote all possible objects of thought (e.g., processes, material things, concepts, mathematical objects) even though "entity" is not ideal because it connotes matter.

reasoning can be applicable to a wide array of physics concepts. Here we review some of the studies on ontologies of energy, and then later return to this example in Chapter IV to situate it within our framework for describing different kinds of dynamic ontologies. Scherr, et al. identify three different ontologies for energy among learner and expert discourse [63]: energy as a quasi-material substance, stimulus to action, or vertical location. The *substance* ontology considers energy as stuff and objects as containers that have or get energy. Scherr et al. describe it as “quasi-material”, because the “substance” has some material properties (e.g., conservation and localization) but not others (e.g., mass and volume). Although some researchers argue that substance ontologies for physics concepts such as energy, force, and electric current impede student learning [56, 57, 59], many argue for the productivity of such substance-based ontologies [47, 58, 63]. For example, treating energy as a substance-like quantity that can be stored and transferred provides students (and instructors) with conceptual resources [13] that can be drawn upon when thinking about concepts like energy conservation [58]. The *stimulus to action* ontology of energy involves thinking of energy as something that acts on an object, as in the statement “Leaves in the street are *pushed* by energy” [63]. Considering energy as something that acts on, or has an effect on, objects lends to the idea that energy is the ability to do work, yet it does not clearly differentiate energy from forces [63]. The *vertical location* ontology involves thinking of energies as rungs on a ladder, where higher means more energy. For energy levels in an atom, transitions are often represented as arrows between the rungs. The vertical location ontology for energy draws on similar resources as with the concept of gravitational potential energy, which increases with height, and where the motion of objects in a potential energy gradient tends toward the lowest potential energy [63, 65]. In Scherr et al.’s study, eighth-grade students made use of multiple ontologies in a given reasoning episode, sometimes applying a *substance* and sometimes a *stimulus to action* ontology. In addition to the student reasoning, each of the three ontologies (substance, stimulus, location) were present in expert discourse [63]. Dreyfus, Geller, et al. demonstrated that both experts and undergraduate life sciences students can productively coordinate (and combine) substance and location ontologies for energy in the context of chemical bond energy [67], and Dreyfus, Gupta, and Redish explore *how* this blending can happen in the context of energy absorption and emission by atoms [65].

Historically, there has been some debate about the nature of ontologies, which can be broken up into two aspects: (i) the nature of the organization, or underlying structure, of ontologies and (ii) the nature of the movement (if any) between categories. One framework for characterizing ontologies describes discrete ontological categories and assumes a single correct ontology for every entity [53, 55, 57, 68, 69]. According to this view, for two categories to be ontologically distinct, there can be no overlapping ontological attributes.[2] In this line of thought, students come to physics instruction with a pre-existing tree-like structure of ontological categories (e.g. matter, processes) and then the physics concepts they learn are assimilated into the existing structure. According to this view, the underlying organization of ontologies is rigid in structure, but the assumption of ontological correctness necessitates movement between categories, described by Chi's *Incompatibility Hypothesis* [55], a key feature of this framework. When a student assigns a concept to the "wrong" ontological category (e.g. thinking of electrical current as a material substance instead of a process), their conception must be re-assigned through "radical conceptual change". This mismatch between a student's ontology and the correct (or scientifically accepted) ontology presents a conceptual barrier that is difficult to cross, or can be "resistant to instruction" [69]. This perspective supports the argument that substance-based ontologies for things like energy can be a barrier to student learning.

Another line of research challenges the "attribution of stable, constraining ontologies" [54, p. 286], and posits an ontological structure which is more flexible [60, 67, 70]. This perspective does not include the assumption of ontological correctness, but argues that learners' use of a substance-based ontology for a concept that scientists typically conceptualize as a process or interaction can be productive [47, 58, 63]. In this view, not only is the underlying organization of ontological categories flexible, but the movement between ontologies is described as a dynamic process. Gupta, Hammer, and Redish argue that both novices and experts reason across ontological categories, and that these dynamic processes are ubiquitous and productive [54]. They suggest that a person's ontological categorization of a given entity is context dependent and can vary moment to moment. In recent work on the historical development of metaphors for energy, Harrer demonstrates that

[2] An *ontological attribute* is defined by Chi as a property that an entity belonging to the given category *may potentially* have. This is in contrast to a *defining attribute*, which is a property an entity *must* have in order to belong to the category or a *characteristic feature* which is an attribute that an entity in the given ontological category *most frequently* has [68]

expert physicists *necessarily* use multiple ontological metaphors [71]. Additionally, more recent work argues that ontologies can be blended to form new categories [65, 67]. The student reasoning presented in Scherr et al.'s study that identified three ontologies for energy used by experts and students illustrates how students can make use of multiple ontologies [63]. Dreyfus et al. [65] present an example of students blending the substance and vertical location ontologies in order to reason about emission and absorption of energy by atoms. Along with the research we present in the following two chapters, these examples contribute to a prominent line of thinking that considers ontologies to be *dynamic* in nature, where dynamic could mean using multiple context dependent ontologies [54, 60, 70] or developing new ontologies in the moment [61, 65]. In the following two chapters, we provide further evidence for the dynamic nature of ontologies, but refine the notion of "dynamic" to include varying degrees of stability of ontological categories and different types of dynamic and flexible use.

Coming from a resources perspective [13], we treat ontologies as flexible structures that can be cued in certain contexts and that a reasoner can move between or construct in the moment. This view of flexibility in both underlying ontological structures and movement between ontologies influences our research questions and methodologies. However, we avoid an either-or mentality in terms of the theoretical debate (static versus dynamic), because while we adhere to the flexible nature of ontological structure and investigate how students construct ontologies in the moment, we also find utility in the part of Chi's approach which suggests stable and robust notions. We echo Niedderer and Schecker's description of cognition as something that includes stable elements as well as those constructed in-the-moment [72].

The goal of our research project is to study how students reason when learning QM, for both theoretical interest and to ultimately further curricula and student capabilities in the subject areas of Modern Physics and Quantum Mechanics. We focus on students' ontological reasoning in these domains—their reasoning about the nature of *quantum* entities—as one way to understand student reasoning in QM. Modern physics and quantum mechanics (QM) are rich contexts in which to investigate ontologies. In addition to learning new mathematical formalisms, learning QM involves learning how to think about entities in new ways, e.g., light comes in quantized particlelike units called photons, and electrons (matter) can be thought of as waves. A typical modern physics class focuses on introducing quantum physics concepts and formalisms, and is often the first place students formally learn about entities such as photons and electrons exhibiting both particlelike and wavelike behavior. Thinking about the nature of photons and electrons is fundamental to understanding quantum mechanics. In the particular modern physics course in which most of the following studies take place, one of the course goals is that students develop *meta-ontological competence*—the ability to think about how they are thinking about an entity and to determine when it is best to use a given ontology (e.g., when it is best to think of a photon as a particle and when it is best to think of it as a wave, or both, or some other type of entity entirely). The modern physics and QM contexts provide a rich arena in which to explore the dynamics and nuances of students' ontological reasoning as they grapple with these ideas. When students arrive at a modern physics classroom to begin learning about QM, we expect that they will have conceptions of a classical particle and classical wave that could be considered stable or robust (perhaps more so for particles than for waves), and these ontologies will influence the way they learn and reason about QM. Ontologies of quantum entities, like electrons and photons, often include properties or behaviors which resemble those of classical particles and classical waves in specific contexts. However, the resemblance remains piecemeal in the sense that attribution of one property does not always imply attribution of others (e.g., a photon in a double slit experiment is detected at a single point on the screen like a classical particle would be, but a photon is not *always* a localized entity like the classical particle). The modern physics curriculum is a unique place to investigate students' ontologies because "wave-particle duality"[3] is a central theme, but we also believe that ontology is important in other classes and we do not expect the flexible use of ontologies documented in the following two chapters to be limited to the modern physics context.

In our investigations of students' ontologies, we identify "messy" elements of students' ontological reasoning and argue that they can be productive for student learning. In general, we consider an idea or contribution to be productive if it supports progress toward understanding the physics content, sustains engagement in sense making processes, provides opportunities to engage in expert-like

[3] Through our studies of students' ontological reasoning, we will begin to question the use of the common phrase "wave-particle duality" as it may restrict the manner in which students engage in flexible use of ontologies.

practices, or contributes to a cohesive social interaction. Specifically, in the context of reasoning about quantum entities, expert-like practices can include: using multiple ontologies within and across contexts [54, 63], being aware of which ontologies you are using and why (*meta-ontological competence*), putting forth ideas you are unsure about in order to grapple with them, metacognition, and entertaining questions of interpretation of QM[4].

The research described in the following three chapters all takes place in the same modern physics class. Since this course context is common across the three chapters, we present a description of the course in some detail here. Further details relevant to the individual studies are presented in the individual chapters. There are two modern physics classes offered at CU, one primarily for physics majors and one for engineering majors (although students can choose to take the other course if they wish). This course is the third semester of the introductory physics sequence; prior to modern physics, most students have completed two semesters of introductory calculus-based physics (Physics 1 and 2) or occasionally received credit from AP physics in high school. Traditionally, the course for physics majors has more of an emphasis on mathematical formalisms, but this is largely instructor dependent. A typical modern physics class includes a few weeks on special relativity followed by an introduction to quantum physics concepts. The course in which our research takes place is *not* a typical modern physics class. It is a transformed version of the modern physics for engineers course taught by Noah Finkelstein (advisor of this thesis and co-author of this work). The course enrolls primarily engineering majors, most of whom are mechanical engineering, but also includes a fair amount of electrical engineering as well as a variety of other engineering and science majors. The class meets two and half hours (either two or three sessions) per week and provides many weekly optional, but strongly recommended, informal help sessions. There is heavy use of interactive engagement (primarily through clicker questions/peer instruction), the grading policies emphasize homework and participation in and out of class, there are three midterm exams and one final exam, and an optional final project. The curriculum is a result of several years of transformation [43, 73], and continues to be modified each semester; it focuses on the conceptual foundations and real world applications of QM [73], and explicitly addresses physical interpretation

[4] Not only is this a specific instructional goal of our environments, but these are questions that experts are actively negotiating (e.g., what *is* a wave function?). Critically thinking about these questions which in some cases have no single "correct" answer is part of learning QM, and is an activity of expert physicists.

of quantum phenomena [43] (excluding special relativity). Due to the emphasis on interpretation, we often ask questions that have no single correct answer and we hold students accountable for backing up their answers with evidence and reasoning. Beyond the interpretive elements, the course generally emphasizes reasoning over correct answers, through grading policies and messaging to the students in lecture.

Part two of this thesis consists of three chapters. In Chapter III, through qualitative analysis of group conversations around canonical topic areas of quantum mechanics, we provide evidence for dynamic ontologies, and present a preliminary framework for describing and understanding the nature of the dynamics. The framework describes three different types of ontological structures: parallel, blended, and fixed. In this chapter, we also present examples of how messiness (flexible use of ontologies and tentativeness) can be valuable for students' learning. Additionally, we demonstrate a new use of a methodological tool (conceptual blending) for investigating the nuances of collective discourse. This chapter describes the methods and results of a one-semester focus group study with modern physics students, and is published in Physical Review: Hoehn & Finkelstein (2018) [61].

While the preliminary framework was developed in the context of collective oral reasoning in focus groups, in Chapter IV we triangulate the findings of Chapter III by applying the framework to both individual and written work from a representative sample of a modern physics class. We developed a coding scheme and coded individual student responses for ontology and ontological structure, refining the preliminary framework in the process. The resulting framework describes unitary, parallel, and blended ontological structures, and can be applied to collective, individual, oral, and written reasoning. We demonstrate students' flexible use of ontologies in these additional modalities, and investigate the way in which the questions we ask impact student use of ontologies. Chapter IV is a publication under review at Physical Review: Hoehn, Gifford, & Finkelstein (2019) [74].

With an understanding that we can influence students' use of ontologies through curricular prompts, Chapter V describes the development and use of a suite of ten modern physics tutorials designed to support students' reasoning around quantum entities (or their meta-ontological competence). Many of the tutorials stemmed from prompts from the focus group study described in

Chapter III, and their design was further informed by the analysis of collective student conversations. We annotate the tutorials to describe the ways in which they work to support students in dynamic and messy ontological reasoning, and we present examples of student reasoning which illustrate how students engage with these materials. These materials exist on the curated PhysPort website [75].

## Chapter III. Developing a framework to capture the dynamics of ontologies

*This chapter appears in print in a Physical Review Physics Education Research article: Hoehn and Finkelstein 2018 [61].*

While prior work has examined student conceptual mastery and interpretive skills by documenting pre-post shifts [43, 70], we seek to extend such outcomes and to better understand how students organize and understand ideas in QM. Consider the following prompt for students: *When not being observed, an electron in an atom still exists at a definite (but unknown) position at each moment in time.* Students from a sophomore-level Modern Physics course are asked to respond to this statement on a five-point Likert scale ranging from Strongly Disagree to Strongly Agree. This particular question gives us an idea of how students are (or are not) thinking about indeterminacy in the context of atoms. If a student agrees with the statement, they might say "the electron does exist at a certain location but we do not know where it is until we measure it", which we would classify as a hidden variable interpretation.[5] If a student disagrees with this statement, they might say "the electron is not in a definite position until measured".[6] These responses provide valuable information about students' reasoning and interpretation of quantum phenomena. Ultimately, we would like our students to shift toward Disagree after a semester of Modern Physics. We value asking these questions of our students, have incorporated these questions of interpretation into our learning objectives, and see these pre-post results as meaningful for our course and our research [43, 70]. However, we must consider the limitations of this pre-post multiple choice survey approach. Figure III.1 shows the results from this question during one semester of a Modern Physics course. Although we see a statistically significant shift from the beginning to end of the semester, in a direction that we define as "more sophisticated", these coarse-grained, aggregate survey data do not tell us much about students' actual reasoning. On its own, this approach lends to, or derives from, a binary view of student learning—students either get it right or they don't. Noting our commitment to go beyond the binary view, we investigate the nuances and dynamics

[5] A hidden variable (or deterministic) theory argues that QM is incomplete because of its indeterminacy. Einstein, Podolsky, and Rosen argued that hidden variables were "elements of the physical reality" [76, p. 777] that need to be added to QM in order to explain things like entanglement without "spooky action at a distance." Local hidden variable theories have been disproven through experiments such as Wheeler's delayed choice experiments [77].

[6] These are actual student responses to an online survey administered in the same Modern Physics class as the students whose group conversations will be presented in Section III D below.

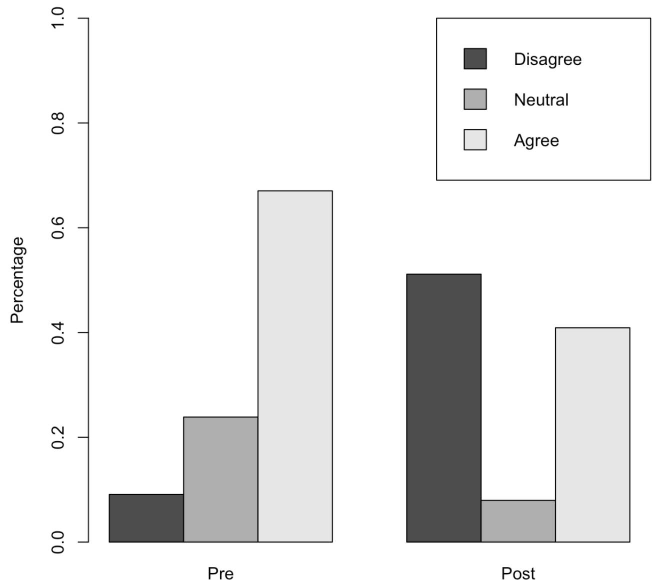



FIG. III.1. Sophomore level Modern Physics class (N=88) responses to: *When not being observed, an electron in an atom still exists at a definite (but unknown) position at each moment in time.* (Following the work of Baily [43, 70]) Highly statistically significant ($p << 0.001$) shifts from pre- to post-survey with the Bhapkar test. These data were collected in the same course and semester as the focus group data presented in Section III D.

of students' reasoning on these types of questions, valuing the messiness rather than the correct answer.

Within the historical debate about the nature of ontologies, there is mounting evidence [65, 70, 78] for the perspective that considers ontologies to be flexible or dynamic in nature. We provide further evidence, arguing from our data that underlying ontological structures and movements between them can be flexible. Pre-post results like the those in Figure III.1 can imply that there are unambiguous [7] before and after states of the students' ontological commitment, and they are either correct or incorrect. Using this measure alone does not acknowledge the nuance of student reasoning, and thus can reinforce a static or fixed view of ontologies.

This chapter makes two theoretical contributions, and one methodological contribution. First, we provide evidence of the dynamic nature of ontological structures, and go further to identify *different types* of dynamic ontologies. In our data, we see students flexibly switching back and forth between parallel stable structures as well as constructing and negotiating new ontologies in-

[7] The question presented in Figure III.1 provides *some* room for ambiguity because of the option for a Neutral response. In order to unpack the nuance contained in a Neutral response, we must look more closely at student reasoning.

the-moment. Our new framework of dynamic ontologies thus includes some stable or robust notions that can be moved between flexibly, in addition to the ability for reasoners to construct and refine ontologies on-the-fly.

Next, we argue that it can be productive for students to reason in a tentative or messy way when learning and grappling with quantum ideas. In making this argument, we shift away from a binary view of student learning [9]. There are many within the Physics Education Research field who study and value the messiness of student reasoning (e.g. [7, 8, 54, 79–81]) and who argue for moving beyond reductionist metrics [9] because we know that nuance supports efforts for things like understanding student reasoning and building inclusive environments [82]. We contribute to this framing and philosophical commitment that values the complexities of student reasoning, and we bring it to a different area of research—Modern Physics/Quantum Mechanics. In the context of ontologies, we refer to the "messiness" of student reasoning as having two parts: (1) students' flexible use of ontologies (whether that means moving back and forth between parallel structures trying to test when each is appropriate to use, or constructing and negotiating ontologies in-the-moment, or a little bit of both, or something different all together), and (2) students' tentativeness around ideas they contribute to group conversations (this comes along with negotiating in-the-moment and flexibly playing with ontological structures). We argue that this "messiness" can be productive for students. Some researchers have addressed the notion of productivity even when students' ideas are not canonically correct [8, 83, 84]; Scherr and Robertson define an idea as productive if it "supports, initiates, or sustains progress" [8, p. 3]. Our notion of productivity is consistent with this definition, but also includes the existence of discourse and practices similar to those of professional physicists. Here "progress" could mean progress toward a correct answer or understanding of the physics, or toward engaging in expert-like practices. Even if students' ideas are not "correct", engaging in *practices* that reflect those of experts is beneficial for their learning and development as scientists. Reasoning across ontological categories and flexibly playing with ideas they are unsure about are practices of experts that we hope to support our students in developing.

Methodologically, we develop a system of studying the nuances of collective reasoning, thereby contributing to the research community that values the messiness and complexities of student reasoning. We employ conceptual blending [85] as a tool for describing student reasoning and elucidating the dynamics of ontological negotiation. Although conceptual blending is a theory of cognition, traditionally referring to what goes on in an individual's mind, we demonstrate how the framework can be used to model and analyze *collective discourse*, thus treating conceptual blends as conceptions that are socially constructed and distributed. We do not claim that blending is the cognitive mechanism by which students reason, but use the framework as a descriptive tool for understanding student reasoning.

In summary, this chapter addresses two research questions:

*1) In learning and reasoning conceptually in QM, do students use dynamic ontological structures, and what does it mean for ontologies to be dynamic?*

*2) In what ways (if any) can reasoning in a messy and tentative way be productive for students learning QM?*

### A. Theoretical framing

There are three theoretical perspectives which together explain why we ask the questions that we ask in our research study: (1) We view learning as a social process and understand that learning environments are situated within larger social environments [17, 19], thus positioning both individual reasoning and collective discourse as cognitive tools; (2) We draw on the resources framework of cognitive structure [13]; (3) We study the nature of ontologies to inform a theoretical view of cognition, and we understand ontological categorization and movement between ontologies to be flexible [54]. We discuss each of these briefly as they relate to and inform our research, and then we discuss how these perspectives combine with the theory of conceptual blending [85] to form a theoretical foundation for our method of analysis, which uses conceptual blending as a tool to analyze group discourse.

### 1. Sociocultural perspectives

As noted in Part One, we approach our work from a sociocultural perspective of learning. Such a perspective shapes not only our pedagogical approaches, but also frames our research questions and methods. Because we consider the process of learning to be a social act, we look at students' individual as well as their collective reasoning. In doing so, we value collective discourse as a cognitive tool, and use it to help us understand the reasoning structures being used by students as they learn QM. We value both individual ideas and associated inferred reasoning structures as well as the negotiated collective meanings (inferred from collective discourse) that students develop as they solve physics problems. In the present piece, we do not always seek to distinguish between individual and collectively developed reasoning. Not only do socially and collectively developed tools (like language) mediate thought [28], but other people and their use of tools also mediate cognitive processes, including those involved in doing physics and solving physics problems [29, 30]. Aligned with our pedagogical approaches grounded in sociocultural framing, we look at collective student discourse, document the complex and dynamic nature of reasoning structures, demonstrate the utility of such reasoning, and broaden the application of a tool that has historically only been used for analysis of individual reasoning.

### 2. Resources perspective

We approach our analysis from a resources perspective of cognition [13], understanding students to have multiple resources available to them in a given moment and that certain resources will be activated depending on the conceptual and/or social context. In short, in many instances the notion of a 'concept' has been demonstrated to be too coarse a grain-size for capturing the nuanced and rich nature of student reasoning [15, 86]. Furthermore, building on the knowledge-in-pieces perspective, it has been demonstrated that student epistemological stances can be better explained by examining finer-grained commitments than static, robust beliefs [13, 87]. We draw from this work to consider the ontological reasoning that students use [54]. The resources perspective lends to a view of student reasoning that is flexible and context dependent. Although we consider organizational structures to be flexible, we also find utility in the notion of fixed or robust ideas. We echo Niedderer and Schecker's description of cognition as something that includes stable elements as well as those constructed in-the-moment [72]. We use the resources perspective to document and catalog fine-grained elements of students' ontological reasoning, but we do not go as far as to argue for a mechanism of ontology development.

### 3. Ontologies

Coming from a resources perspective, we treat ontologies as flexible structures that can be cued in certain contexts and that a reasoner can move between or construct in-the-moment. This view of both underlying ontological structures and movement between ontologies influences our research questions and our methodologies. However, we avoid an either-or mentality in terms of the theoretical debate (static versus dynamic), because while we adhere to the flexible nature of ontological structure and investigate how students construct ontologies in-the-moment, we also find utility in the part of Chi's approach which suggests stable and robust notions. In our data, we see students constructing ontological structures for quantum entities in specific contexts by borrowing from and combining canonical particle and wave language. We see the students flexibly using these already stable conceptions as well as constructing new ontologies in the moment. In this sense, the ontological framework we use here sharply departs from that of Chi [55, 69] (or its precursors from linguistics [88, 89]), where the assumption is that new knowledge is assimilated into the existing structures and a "mix and match" model based on context-specific needs is not suitable.

A slightly different scenario of how classical particle and wave ontologies are used in QM is one that involves switching back and forth between the two ontologies depending on the specific context. This reflects the way we often teach students to think about wave-particle duality (i.e. we model an electron as a particle in some contexts and as a wave in others). Following the work of Charles Baily, we refer to this type of structure as *parallel ontologies* [70]. Figure III.2 depicts this quantum ontological structure for an electron compared to a classical ontological structure which would list 'electron' as one example of an entity in the 'particle' category. In a quantum context though, the electron might be the umbrella category and within that one would switch back and forth between

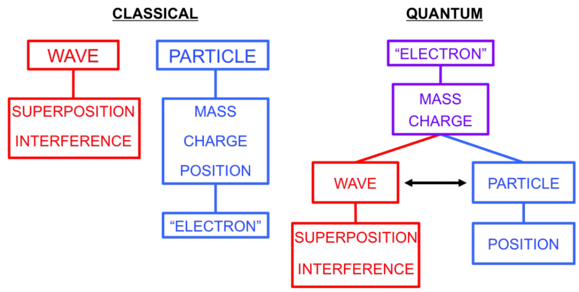



FIG. III.2. Charles Baily's [70] representation of classical versus quantum ontologies of an electron. The double headed arrow represents movement back and forth between parallel ontologies of particle and wave, which are both subsumed within a broader ontology of "electron".

particle and wave. Moving between parallel ontologies aligns with the context dependence (that can happen moment-to-moment or across broader contexts) of the Gupta et al. framework of dynamic ontologies [54], but also aligns with the aspect of Chi's framework that describes stable notions [69]. In our data, we see examples of students switching back and forth between parallel categories for a given entity in this manner, as well as of students constructing blended ontologies in-the-moment. These examples help us to describe the ontological flexibility we see students engaging in as they learn QM, and to refine our framework for dynamic ontologies.

#### 4. Conceptual blending

We use a conceptual blending framework as an analytic tool to capture the dynamics and nuances of students' reasoning. Conceptual blending [85] is a theory of cognition developed by Fauconnier and Turner (F&T) that describes the dynamic process of creating *mental spaces*. F&T define mental spaces as "small conceptual packets constructed as we think and talk, for purposes of local understanding and action" [85, p. 102]. Through conceptual blending, two (or more) input spaces merge in some way to create a blend space. Select elements from each input space are projected to the blend space where new elements that do not occur in either input space also emerge. Using blending as a tool to get at the nuances of students' ontological reasoning, our present work responds, in part, to a call from Brookes and Etkina for conceptual blending analysis that "may better account for "local" or "personal" ways of expression observed among individual

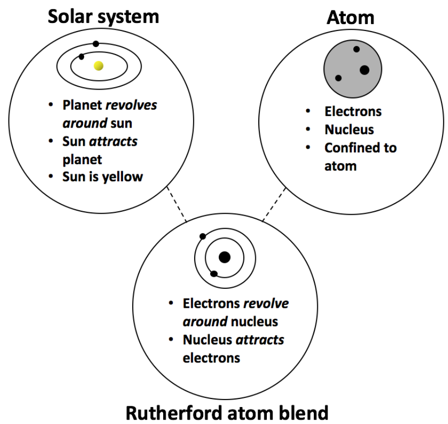



FIG. III.3. Adapted from Podolefsky & Finkelstein [90]. Example of a common conceptual blend in physics. Solar system and atom form the input spaces for the Rutherford model of the atom in which electrons revolve around the nucleus and the nucleus attracts the electrons.

professors and students. The dynamics of blending may also be useful for answering questions about how we can make students more aware of the myriad of models encoded by the metaphors in physicists' language" [44, p. 15].

F&T describe blending as a ubiquitous and unconscious activity involving dynamic processes that result in emergence of new meaning. To illustrate the process of conceptual blending, we present an example from Podolefsky's work on analogical scaffolding [90], which describes the conceptual blend behind the Rutherford model of the atom (Figure III.3). Podolefsky's description assumes students' prior knowledge of atoms as consisting of a nucleus and electrons, which could be arranged in a variety of ways. This knowledge forms one of the input spaces. The other input space is the solar system, which consists of the sun, and planets in concentric orbits around the sun. The sun is counterpart to the nucleus, and the planets are counterpart to the electrons. In the blend space that emerges, electrons orbit the nucleus at fixed radii. Just as the sun attracts the planets in the input space, the nucleus in the blend space attracts the electrons.

As the structure of a blend space emerges, F&T identify three processes: *composition*, *completion*, and *elaboration* [85]. The *composition* of elements from both input spaces creates the possibility for new relations between said elements that would not have occurred had the elements remained in distinct spaces. *Completion* refers to bringing a familiar structure to the blend, or assigning properties based on prior knowledge of one or more of the inputs. *Elaboration* is a dynamic process of mental simulation, and is also referred to as the "running of the blend". As the blended scenario plays out, new ideas come up, along with new meaning of the elements in the blend space. This emergent meaning is a cornerstone of the conceptual blending theory. F&T emphasize that the standard blend diagrams (e.g. Rutherford atom blend diagram in Figure III.3) are static representations of a *dynamic* and imaginative process, which leads to emergent meaning.

Coming from the sociocultural perspective that considers collective discourse as a cognitive tool, we take conceptual blending beyond the analysis of an individual's cognition and describe collectively constructed blends [78]. That is, we model group conversations as conceptual blends, paying attention to shared meaning around ontologies for quantum entities as constructed by the group. Individual students' ideas or contributions to the conversation are modeled as elements in the input or blend spaces. The blending framework then allows us to map out processes within the group that lead to emergent meaning. In using conceptual blending as a tool for analyzing group discourse, we are not claiming that the blends and mental spaces exist within one individual's mind, but rather that they are socially distributed conceptions constructed and utilized by the group to make sense of the phenomenon at hand. This *collective* conceptual blending framework is a useful methodological tool for capturing collective construction of meaning and dynamics of group discourse.

## B. Methodology

### 1. Course context

Our data for this study come from one semester of the transformed Modern Physics for Engineers course described above, a large lecture-style course enrolling approximately 130 students. A result of several years of ongoing course transformation, the curriculum focuses on the conceptual foundations and real world applications of QM [73], and explicitly addresses physical interpretation of quantum phenomena [43]. Physical interpretation of QM was explicitly addressed in lectures and on homework assignments, and many times students were told that there was not necessarily a "right" answer to these interpretation questions but that they were expected to back up their answers with evidence. This curricular approach was informed in part by prior research in this course which found that when instructors do not explicitly attend to interpretation when teaching QM, students will arrive at their own interpretations anyway, which most often rely on intuitive classical views [43].

### 2. Methods of data collection

We use qualitative methods to elicit and analyze student reasoning. Expanding on the historical debate about the nature of ontologies [54, 55], we dive into what it looks like for students to reason in-the-moment and flexibly negotiate ontologies around quantum phenomena. We recruited students from the Modern Physics course, and ran biweekly focus groups throughout one semester. The students volunteered to be a part of the focus group study and were paid for their time each week. The recruitment process involved making announcements in class and sending out class-wide emails; the study was presented to the students as a paid opportunity that would likely benefit the participants' learning in the class, but was in no way connected to course grades, and participation in the group would be kept anonymous for the instructor until after the course was over. From the volunteers, we formed two focus groups—referred to here as Group A and Group B—organized by scheduling constraints. Each group met once every other week for a total of six hour-long sessions. We also conducted brief individual interviews with each participant at the middle and end of the semester to gauge how the social dynamics and learning of the groups were progressing, and to ask about their overall experiences in learning QM. Within each weekly session, the students were presented with prompts directly designed to investigate student reasoning on topics drawn from the class. Some prompts involved multiple choice questions or structured problems (with correct answers) that we asked them to discuss, while others were more open-ended and interpretive. Most prompts were designed to facilitate conceptual understanding of quantum phenomena. The students were encouraged to think aloud and discuss with their peers, and two interviewers were in the room to facilitate and probe the conversation with questions. The focus group sessions were video and audio recorded, and after all sessions and interviews were completed we collected background information from the participants (e.g. race/ethnicity, gender, prior math experience, prior exposure to QM).

### 3. Participants

Group A comprised three students, all freshmen mechanical engineering majors: Eric (White male), Tara (female; chose not to identify race/ethnicity), and Bryan (White male). Group B comprised four students, three of whom were present for the data clip we will present here: Fernando (junior astrophysics major; Hispanic and White male), Zach (junior geophysics major; Japanese and White male), and Jacob (sophomore mechanical engineering major; White male). These six students were top-performing students in the class, receiving either As or Bs for their final grade. At the first focus group session, these students did not know each other (other than having seen one another in class). By the end of the semester, some of the students reported talking with the other focus group participants about coursework outside of class and the focus group sessions.

### 4. Methods of analysis

Upon collecting the twelve hours of video data between the two focus groups, we identified specific clips to analyze. Selection of these initial clips was based on existence of rich conversations around ontologies, including but not limited to: a) particlelike or wavelike language; b) a phrase or sentence combining characteristics from multiple ontological categories (e.g. a hybrid phrase like "blob of electromagnetic wave"); or c) an analogy relating quantum and classical ideas. We then looked at the surrounding discussion in attempt to determine how this language, phrase, or analogy came into existence. For the present analysis, we chose video clips in which students were thinking about photons and/or electrons, because ontological conceptions of these entities are fundamental for students in building a strong conceptual understanding of QM, and thus the topic areas in the selected episodes could be widely recognizable across many different types of QM and Modern Physics instruction. Because we selected episodes of conversation rich in ontological negotiation, the examples of reasoning we present here are mostly conceptual. We acknowledge that mathematical reasoning is essential to understanding QM, but choose to foreground the conceptual understanding, which we believe to be an important aspect of learning QM (and is aligned with the emphasis of the course). We also note that a robust understanding of QM includes *both* mathematical and conceptual understanding, and our overall approach seeks to link these (such linkages are the subject of future work). In this chapter, we present three episodes chosen from the video data collected between the two focus groups. The three episodes were selected with the above-mentioned criteria, but there are many such discussions between the two groups; the type of reasoning we see in the selected clips is not unique to just a small subset of the data.

Our analysis of the selected episodes is what we consider to be a coarse-grained discourse analysis. Attending to language used by the students, but also paying attention to how the conversation is constructed and owned by the group as a whole, we go through the transcript line by line and map out the ontological structures we see the students using. We draw from the grammatical analysis of Brookes and Etkina [44, 64] as we use linguistic cues to guide our analysis. Paying attention to the words students use helps us determine what ontologies they might be using or how individual students are contributing to the collectively constructed ontologies.

In the transcriptions, ellipses (...) represent pauses longer than those natural in speech; gestures or non-verbal actions are indicated in [square brackets]; square brackets also contain information added to the transcript by the researchers for clarity; interruptions in conversation are indicated by an em dash (—).

Paying attention to how a particular entity (photon or electron) is being treated or characterized within a group conversation, we map the conversation (or a chunk of it) onto the conceptual blending framework. Individuals' ideas or turns of talk become elements in mental spaces. As such, the mental spaces constructed in the analysis do not belong to any one student. Rather, the ontologies or conceptions modeled by the blends are socially constructed and distributed. Often in the analysis, we model the blend space first and then work backwards to define the input spaces. This happens in practice by attending to language used by the students that gives us a sense of the ontological properties the individual students and/or the group as a whole are assigning to a given entity. In the specific case of electrons and photons, we look for particlelike and wavelike language and ask how that language is being contrasted or combined to form an understanding—whether shared or not—of the given entity in the group. The conceptual blending framework is not always useful for modeling the group discourse. Sometimes, as we will see in Episode 3, one or more students (or the group as a whole) hold parallel ontologies and move between them in a form of ontological negotiation different from the in-the-moment construction of blended ontologies. This is an example of where some of the data lend themselves to a conceptual blending analysis and others require a different structure to map out the ontological reasoning.

We use the conceptual blending framework as a tool for discourse analysis. This tool is well suited to our research questions for the following three reasons: First, it was designed (originally as a theory of cognition) to investigate the connection of distinct ideas and thus is a model suited for meaning making through analogy or comparison. In our case we sought a tool that would allow us to identify *multiple* ontological resources that students were using and describe how they were being connected together. Second, the framework is inherently dynamic. It allows us to examine the dynamics of students' ontological negotiation in-the-moment, and describes a mechanistic process for emergent meaning. Finally, conceptual blending also allows us to move fluidly between facets of the "static" and "dynamic" views of ontologies, and in fact helped us to develop the refined typology of ontologies we present below. These unique aspects of the conceptual blending framework as a methodological tool make it well suited for our research purposes. We could have conducted similar analyses using other tools, but none were as well suited to our research questions. For example, many forms of discourse analysis do not highlight the conceptual spaces (and instead tend to favor reasoning strategies of individuals). Concept mapping [91] and metaphor theory [92–94] tend to emphasize static concepts and metaphors, and not the dynamic development and reorganization of ideas. Our use of conceptual blending could be considered as one approach to either concept mapping or metaphor theory.

Each of the episodes in this paper exist as an example of student reasoning about a quantum phenomenon. This work provides evidence of dynamic ontologies, while working towards refinement of an ontological framework. We demonstrate the potential for tentative reasoning on the part of the students to be productive for students' engagement in sense making, and argue that it is important to value the messiness of students' reasoning. Additionally, these analyses demonstrate the utility of conceptual blending as an analytic tool for understanding the nuances and complexities of group discourse. The arguments laid out here are based on three examples of student reasoning, including a total of six students. We do not attempt to generalize the findings outside of our population of students in this specific course context, or to make broad claims about patterns of student reasoning or impacts of a curriculum. These episodes—each a conversation among three students—provide evidence for the types of reasoning that can be used by, and valuable for, students in learning QM.

### C. Towards a common nomenclature

Building on the theoretical and methodological tools above and putting them into practice to understand and characterize student reasoning requires the development of a refined nomenclature. The definitions that we operationalize here emerge from and are tested in the episodes of student reasoning in our data. These definitions serve as a refinement of previous approaches to thinking about the dynamic nature of frameworks [54], and allow us to distinguish among different forms of dynamics in developing and applying ontological categories. The framework we rely upon for this study (captured in Figure III.4) comes from adapting these prior frameworks to the data and analysis in Section III D. This represents our first pass at a new framework to describe the dynamic nature of ontologies, which will be further refined and updated in the following study (Chapter IV) which triangulates between the results of collective student reasoning presented in this chapter and individual work.

In the episodes below, we use the terms *blended ontology* and *parallel ontology.* A *blended ontology* is a new ontological structure that is constructed or emerges in-the-moment and draws on prior (usually stable) ontologies. The new ontological structure cannot be fully mapped onto the structures it draws from, as new meaning emerges in the blending of the prior structures. In our data, these blended ontologies are locally sustained. They can be temporary structures, used by a student in a given moment to make sense of a quantum entity or phenomenon, or perhaps they can become compiled into robust and stable structures to be used again in other contexts. In our analyses, we do not make claims about the ontological structures used by students *beyond* the local moments in the given episodes.

As described above in Section III A 3, our notion of parallel ontologies stems from Baily's work [70]. A student who holds a *parallel ontology* moves back and forth between two (or possibly more) stable structures. This is often seen in the context of wave-particle duality, when students (or expert physicists) may think about an electron as a particle in some situations and a wave in others. We see this in Episode 3 below. This particular episode includes both parallel and blended ontologies, and we refer to the parallel stable structures as the *input spaces* of a blend. When there is no blended ontology, and only parallel, we do not use the language of input spaces.

We describe both blended and parallel ontologies as dynamic in nature, but they are dynamic in different ways. Figure III.4 depicts a framework that further describes the nature of blended and parallel ontologies. In a given reasoning moment, ontologies can be dynamic in their *construction* or their *application*. By 'construction', we refer to *when* the ontologies are developed for the reasoner. Ontological structures can be constructed in-the-moment, which would be a dynamic and perhaps messy process, or they could be already compiled as stable structures. That is, sometimes we see students bring robust ontological conceptions (i.e. of a classical particle) with them into a reasoning episode, and other times we see new, *blended ontologies* being constructed in-the-moment. When we think about the *application* of ontologies, we are thinking about *which* ontologies are applied in a given reasoning episode. A student could apply a single ontology (static application), or they could apply multiple ontologies in a juxtaposing or complementary manner (dynamic application). We see parallel ontologies when the ontological structures are stable in construction (i.e. students bring stable notions with them into the reasoning episode), and multiple ontologies are applied within a single reasoning episode. We note that this parallel application could occur on short or long time scales (switching back and forth within one or a few sentences, or a broader context-dependence between different conceptual situations). We choose to focus on single reasoning episodes, thus honing in on ontological flexibility at the shorter time scales.

One goal of our educational environments is for the stable construction and single application of an ontology in a reasoning episode, noting that experts exhibit context-dependence of ontologies, but have the ability to apply a single stable quantum ontology for an electron in a given moment. At the same time, such ontological flexibility may arise from the construction and application of

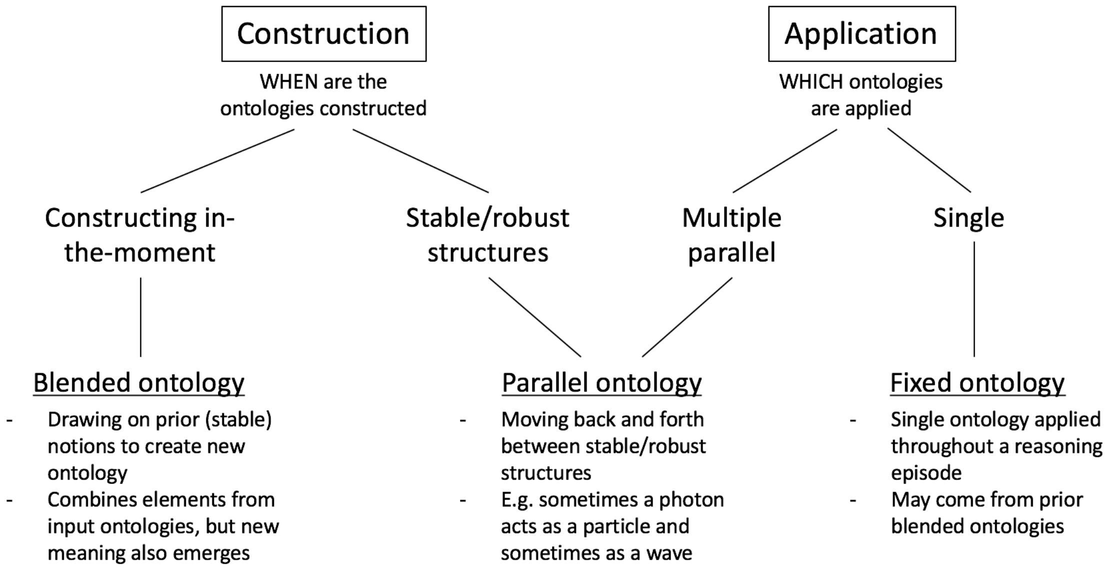



FIG. III.4. Framework to describe the nature of ontologies. *Within a given reasoning episode*, ontologies can be dynamic in their construction or application. Blended ontologies are new structures constructed in-the-moment, as opposed to stable structures that a reasoner brings with them to a reasoning episode. Parallel ontologies occur when multiple stable structures are applied.

blended and parallel ontologies, which while messy can be valuable for students learning and making sense of QM.

### D. Data and analysis

We present three episodes from focus group sessions in which students reason about a canonical topic area of Modern Physics:

Episode 1: double slit experiment with a single photon

Episode 2: Mach-Zehnder interferometer with a single photon

Episode 3: tunneling of an electron in a wire

In Episode 1, we demonstrate the utility of the collective conceptual blending method of analysis with a simple example of three students collectively negotiating an ontology of a photon. In Episode 2, we illustrate how students use a common blended ontology of photon to come to different interpretations of superposition. In Episode 3 we see different types of dynamic ontologies within the group: moving back and forth between parallel ontologies and negotiation of a new blended ontology. All three examples provide evidence of the dynamic nature and students' flexible use of ontologies, and help us to refine our framework for the nature of ontologies. Each episode also speaks to the value of students' tentative reasoning, especially as they learn about and grapple with quantum phenomena. We note that different problem statements, representation and simulation use, and conceptual contexts likely cue different types of reasoning [95] and potentially invoke different mechanisms of ontology development. Investigating these impacts is not the purpose of this paper, but will be a subject of future study. Rather, the purpose of the present study is to provide evidence of different types of dynamic ontological reasoning, and investigate the value of students reasoning in a messy and tentative manner.

#### 1. Episode 1: "Blob of EM wave"

The first episode occurred in the second week of the focus group sessions in Group A with Eric, Tara, and Bryan. The students were given a screenshot from the PhET Quantum Wave Interference simulation [96] which shows a double slit experiment with a single point on the detection screen (Figure III.5). Accompanying the picture was a multiple-choice question: *A single photon is shot towards the slits and detected at the point shown on the screen. What is the most reasonable interpretation of where the photon was just before it was detected? (a) it was located just in front of where it was detected, (b) it was spread out evenly in the space in front of the screen, (c) it was spread out in a non-even pattern in the space in front of the screen, or (d) it was spread out evenly through all space.* This question highlights both the particle and wavelike natures of a photon (energy quanta of light). When a single photon is fired at the apparatus, it is detected at a single point on the screen. When this is repeated many times, the individual photons form an interference pattern on the screen—there are some places with many dots (measurement of a photon), and others with very few. The correct answer to this question is (c) because there are some points on the screen that are more likely to detect a photon than others. The photon is not localized until it is detected. The three students agreed that (c) was the answer and then proceeded to set up the PhET simulation on a laptop. A reductionist framing of student learning that focuses

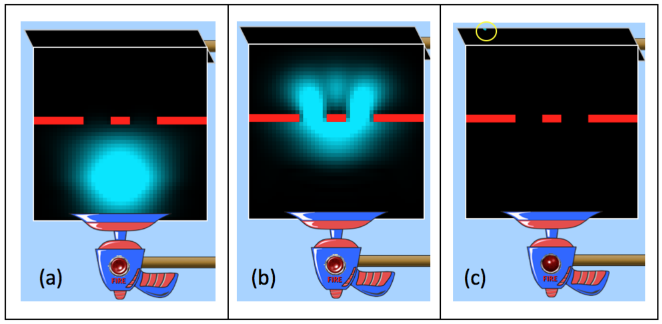



FIG. III.5. In Episode 1, students were given the screenshot in (c) which shows a single photon as a point on the detection screen of a double slit experiment. (a) and (b) show what the PhET simulation [96] looks like as a single photon is fired and travels toward the screen.

on whether or not students got the answer "correct" would stop here, but we believe there is so much more to learn about student thinking.

Looking at the simulation, Eric initiates the conversation by saying:

*Eric: It's different from how I would otherwise think about it because it's a big blob of...electromagnetic...stuff, instead of like a single point [gestures a small point with his fingers] that's like flying through space...And because of that it can interfere with itself and make the interference pattern*

We believe it is reasonable to assume that the simulation, which represents the single photon as a circular blob-like entity traveling toward the slits, cues the word "blob" in Eric's explanation. The students were then prompted by the interviewer to consider how the simulation helped them think about where the photon was just before detection:

*Tara: I mean you can see it like as the photon, like it's just this big blob of light, and it hits the screen, and there's—it kind of spreads out into the interference pattern sort of [gestures fingers spreading out]. And so right before a dot appears, you can see, there's like the [gestures horizontal lines]...spread out photon [mutters] makes sense.*

*Eric: When you think of the photon as–like this blob of electromagnetic wave then, I think it becomes more complicated to talk about where it is, because, like, it's an electromagnetic field...now, instead of like...a particle.*

*Bryan: Yeah it's weird to think that it's like in more...places than one at a single time. Like, I don't know—*

In the dialogue, we see the students collectively negotiating the ontology of the photon. In this moment, the photon is described by Eric as a "blob of electromagnetic wave" which draws from both particle and wave characteristics. The pauses and hesitations in the conversation suggest in-the-moment construction of these ideas. Additionally, in lines 12-13 Bryan tentatively offers his thoughts, explicitly flagging "weirdness" of the idea that the photon could be un-localized. We model the above dialogue with the conceptual blending framework, and describe the group's conversation as a collective construction of a conceptual blend with classical particle and classical wave input spaces. As evidenced by Eric's first statement, particle characteristics include localization ("single point") and particle path ("flying through space"), while wave characteristics include creation of an interference pattern and interfering with itself. The blend space is the "blob of electromagnetic (EM) wave". This new entity inherits some of the localization property from the particle (the word "blob" suggests something contained in a finite space) as well as a non-localized wave property (it is "complicated to talk about where it is", line 10). A particle interacting with the screen would leave a dot, but a wave would create an interference pattern. The "blob of EM wave" inherits both of these properties—Tara explains that it "spreads out into the interference pattern...right before a dot appears" (lines 6-8).

The diagram in Figure III.6 illustrates this blend. The particle input space is on the left, the wave input space is on the right, and the "blob of EM wave" blend space forms the third vertex of the triangle. The horizontal lines on the diagram connect an element in one space to its counterpart in another space. The dashed lines represent projections of elements from the input spaces to the blend space [8]. The *composition* of localization and non-localization gives the "blob of EM wave" the property that it is somewhat localized, but it does not have an easily defined position. The elements "dot on the screen" and "interference pattern" form the blend space element described by Tara: the blob spreads out into an interference pattern and then appears as a dot on the screen.

As the conversation continues, we see the students elaborating on the ideas set out in the construction of the blend space, "blob of EM wave", which we model as an *elaboration*, or running of the blend process. Initiated by Eric's statement in line 11, the students are thinking about an

[8] These lines are features of the typical blend diagrams [65, 85, 97]. We do not use them for every blending analysis, but in this particular episode they are useful.

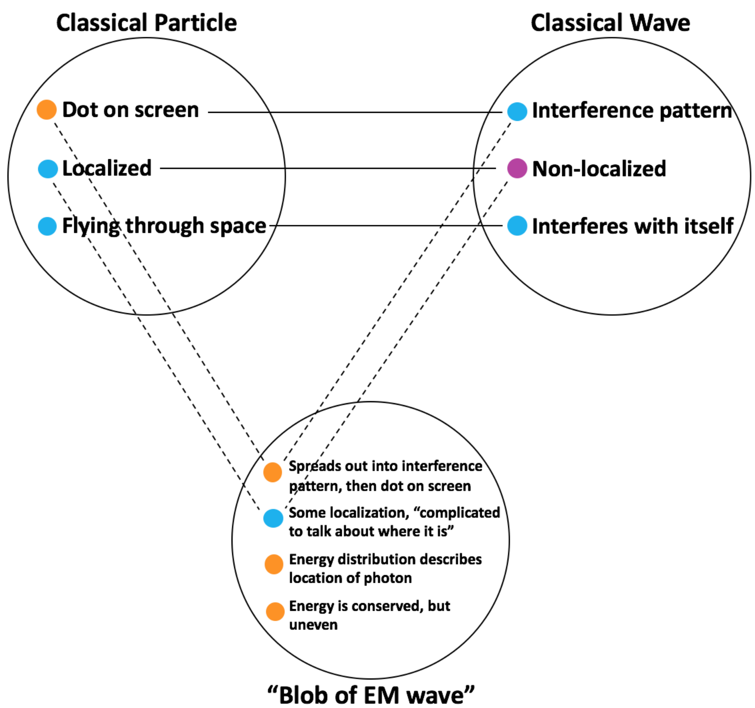



FIG. III.6. Conceptual blending diagram for Episode 1—a discussion between Eric (blue), Tara (orange), and Bryan (pink) about the behavior of a single photon in a double slit experiment. Classical particle and classical wave form the input spaces for the "Blob of EM wave" blend space.

electromagnetic field, and a question arises:

*Eric: So then we're saying that if the...if the amplitude of the wave is zero it's like because of destructive interference ...bec—if it's uh staying at zero, does that mean the photon isn't there, because there's no ...field there?*

*Tara: Right, 'cause it's interfering with itself, but can a photon cancel itself out?*

Prior to the construction of the blend space, this question of whether a photon can cancel itself out would have held no meaning for the group. However, in the blend space, the photon can simultaneously take on both wave and particle characteristics (e.g. localization and interference). The wave property of interference leads to this new question among the group; there is now a particlelike entity that exhibits interference behavior. Upon constructing the blend space, the students begin to try on new ideas. They are negotiating the structure of this blend space—*What are the rules? How does the photon behave? What are its fundamental properties?* They do this in a tentative manner, posing their ideas as questions (lines 24-17) as if to explicitly mark them as exploratory ideas that they are playing around with in order to make sense of the quantum phenomenon at hand. Eric responds to Tara's question about whether or not a photon could cancel itself out:

*Eric: Completely?...Like if you bounced a photon off of a wall, and then like halfway through the bounce it was like halfway over itself and it just canceled out to nothing—*

*Bryan: Is that possible?*

*Eric: But then you can't do that because of conservation of energy. Also, I'm not sure that photons bounce off walls.*

*Tara: Well, I mean if you're thinking of a photon as a wave that's sort of spread out, it's not like—it's like, it's spread out so if it doesn't exist at a certain point it doesn't mean it doesn't exist at other points.*

*Eric: Can you say that again?*

*Tara: Like, you can have points where there is no energy from the photon, but you can have points where there is a lot of energy from the photon so overall, energy is conserved, it's just uneven.*

The concepts of energy and energy conservation were not present in either input space initially—they emerge as a result of elaboration of the blob of EM wave blend. The students bring in their prior knowledge of energy conservation that has not been explicitly addressed in this prompt (this is an example of what F&T call *completion*). As they reason through their new description of a photon, the idea arises that a photon could bounce off of a wall and cancel itself out. The students reject this idea immediately, holding on to the idea that a particlelike entity cannot simply cancel itself out and disappear. After running into the insufficient description of a photon bouncing off of a wall, Tara presents a different explanation. Throughout the discussion, the students are negotiating collectively: *Does it make sense to describe a photon like this?*

The students negotiate the emergent meaning of energy conservation as it applies to the blob of EM wave. Through the running of the blend, students return to the input spaces in order to construct a reasonable description of energy and energy conservation in the blend space. It is this dynamic interplay between all elements of the network that characterizes the student reasoning about photons in this episode.

After Tara brings in explanations that use energy and energy conservation, Eric tries to clarify them, attempting to solidify some common understanding within the group about how the photon behaves:

*Eric: So, it would not be possible for a photon to completely cancel itself.*

*Tara: Right, it like at a point it would be possible for none of the energy from the photon to be there, but over the area that the photon is spread out on, it would still have the total energy*

*Eric: So then, a photon can either have all of its energy concentrated in one spot or spread out over a large area?*

*Tara: I didn't say that.*

*Eric: Oh, ok. [Chuckles]. That was just something I was throwing out there.*

*Tara: I don't know if you can have...um, I don't know if it's either-or, I don't think anything is either-or.*

In this last exchange, Tara maintains a tentative position about the nature of the photon. In this moment, she is unwilling to assign deterministic properties to the photon and ends the conversation by saying "I don't think anything is either-or". In addition to describing the indeterminacy of the photon, this statement could be a reference to the nature of knowledge itself. This last statement from Tara marks the end of the episode, because at this moment there is a long pause and then the group moves on to a different topic. The silence among the group members following Tara's statement can be taken as a signal of agreement that this type of incertitude is appropriate here. Because they end with this note of hesitation and do not have anything further to add to the sense-making process at this moment, we infer that in this local moment, the tentative stance is conceptually satisfactory for the group (i.e., the idea of uncertainty or indeterminacy is sufficient for the group in this moment). This is aligned with instructional goals where the course instructor explicitly argued for the value of tentative knowledge and claims in QM. An alternative interpretation could be that the conversation ended on this note because of an awkward social interaction (lines 36-39), or that the other group members used this tentative stance as an escape hatch [98] to get out of a socially uncomfortable or conceptually confusing situation and move on to the next problem. We take the tentative stance to be productive for them in this moment, whether it be conceptual or social productivity, or both.

We characterize Episode 1 as a conversation of dynamic ontological construction—the group collectively develops an understanding and negotiates meaning of the blob of EM wave ontology of the photon, which we label as a *blended ontology.* This structure is constructed on-the-fly by the group (i.e. the blob of EM wave was not a conception of the photon that the students brought with them into the reasoning episode). While "blob" and "EM wave" may be existing ontological structures (the word "blob" is likely brought in by the representation on the PhET simulation), the "blob of EM wave" is a new object for them to discuss and takes on new meaning during their conversation. The students elaborate on the blob of EM wave blend by questioning the possibility of a photon canceling itself out. Through this in-the-moment and messy reasoning, energy conservation takes on a new meaning for the students in the context of the single photon in a double slit experiment. The emergence of new ideas about energy and the ultimate rejection of the idea that the photon is able to cancel itself out is a dynamic process to which each group member contributes. While the episode certainly takes on the flavor of a blended ontology, we also see some hints of parallel ontological structure in the blend space element (coming from Tara's statement, lines 6-8) that the photon spreads out into an interference pattern and *then* forms a dot on the screen (the temporal order is the key thing here that suggests a parallel structure). This suggests not only that students can apply multiple ontologies in a given reasoning episode or construct new ontologies on-the-fly, but that they can use multiple types of dynamic ontologies (i.e. parallel and blended ontologies are not mutually exclusive). This flexible use and in-the-moment collective negotiation contributes to what we refer to as the messiness of student reasoning in this particular episode.

Also contributing to this messiness is the students' tentativeness around ascribing characteristics to the photon (namely, Tara's bid in line 38 that nothing is "either-or" in regards to the energy of the photon). In addition to the bid for uncertainty that concluded the episode, throughout the conversation, each of the three students flagged their contributions with hesitation and incertitude—posing their ideas as questions, and including the caveats "I'm not sure" or "I don't know". We take

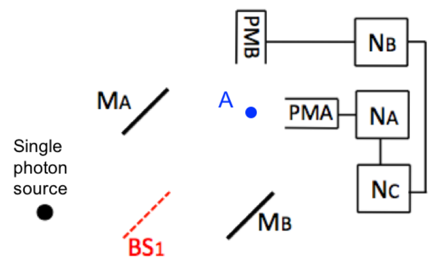



FIG. III.7. Schematic of a Mach-Zehnder interferometer with one beam splitter presented to students in Episode 2. Point A marks the point that Tara references in line 49 (this was not present in the diagram given to students). PMA, PMB are detectors, $N_A$, $N_B$, $N_C$ are counters, $M_A$, $M_B$ are mirrors, and $BS_1$ is a beam splitter.

these tentative stances to be productive for the students. They are engaging in patterns of scientific discourse with their peers—playing around with ideas to make sense of the quantum phenomenon at hand, pushing one another to articulate their ideas, and entertaining questions of interpretation of QM—activities that we would define as goals of this course. Additionally, these students are putting forth ideas they are unsure about, which is something we hope our physics students feel comfortable doing in our educational environments and is one way that novices and experts alike learn and make sense of physics concepts [50, 51].

Through modeling the group's conception of the photon as a collective blend of classical particle and classical wave input spaces, we are able to "see" the dynamics of the group's ontological negotiation. This conceptual blending analysis demonstrates the productivity of tentative and messy reasoning by highlighting the scientific discourse and reasoning practices the students are engaging in.

### 2. Episode 2: "Superimposed photon"

Episode 2 also involves Group A and occurred two weeks after Episode 1, during the first ten minutes of the third focus group session. In this episode, Eric, Tara, and Bryan are collectively negotiating the ontology of a single photon in the context of a Mach-Zehnder interferometer. Along with the schematic shown in Figure III.7, the students were presented with the questions: *1) What is going on when photons are sent through the experiment? 2) How would this experiment be different with classical particles or EM waves? 3) How do you think about the energy of the photon in this* *situation?* When this experiment is performed with a beam of light, half of the wave is reflected and the other half transmitted at the beam splitter (BS1). The reflected beam then follows the path to mirror A ($M_A$) and into detector A (PMA), while the transmitted beam goes to mirror B ($M_B$) and into detector B (PMB). When a single photon is sent through the interferometer, its state can be described as a superposition of the two paths. The photon will then be detected by *either* PMA or PMB. The students also had access to the IOP *Interferometer experiments with photons, particles and waves simulation* [99], which they set up on a laptop before they began to answer the questions. Looking at the schematic of the experiment, Eric begins the conversation:

*Eric: Umm...well when photons go through the beam splitter, and it splits the photon, and then, when it hits the detectors it goes into one of them randomly.*

*Bryan: Yeah, it's a 50-50 shot, which one it's detected by.*

*Tara: But, I–I think you have to be careful about your language there, because the photon itself I don't think is split, it has an equal probability of going through either of these paths, but the photon is still like...a photon [gestures a container/ball with her hands].*

*Eric: So it's not physically split, but like, it's in the ambiguous state of being—*

*Tara:—It's superimposed—*

*Eric:—on both paths. [nods]*

The students construct a collective conception of the photon as something that has equal probability of being detected by either of the two detectors, and does not physically split, but is "superimposed"—a term that at this point in the discussion refers to an "ambiguous state of being on both paths" (lines 46-48) and will later be referred to by the students as a "state of superposition". This characteristic of being "superimposed" arises in the conversation as a response to Tara's correction of Eric's language (lines 43-45) and assertion that the photon is not physically split. This is one of several times in this focus group session when the idea that the photon may physically split is rejected. The group as a whole is playing around with ideas, rejecting some elements that are not productive in this specific context. Elaborating on the superposition aspect, Eric and Tara seem to reach some shared understanding that the photon, after being "superimposed", decides which detector to go into:

*Tara: Right…And it's still superimposed after it crosses like this little point right here [draws on paper and identifies point A in Figure III.8]*

*Eric: Right*

*Tara: So once it goes this way it's still superimposed and somehow it decides one of these based on probability.*

*Eric: Yeah…Um, and there's no chance that they—that it would go into both…detectors at the same time because you're only shooting a single photon.*

*Tara: Right.*

*Eric: So it just chooses one and goes entirely into that one.*

In grappling with the Mach-Zehnder experimental results and with what it means for a photon to be in a state of superposition, Eric and Tara have anthropomorphized the photon by assigning it the ability to choose which detector it hits. This may suggest that the students are actually thinking of the photon as an entity with decision-making abilities. It could also be interpreted as language that the students do not take literally and that, as they negotiate in-the-moment, they are searching for the words with which they can talk about this new, "weird" phenomenon in a coherent physical way. In order to continue to make sense of this "superimposed photon", the students move on to the second question and compare the properties of the photon with those of a classical particle or electromagnetic wave. Referring to a classical particle, Tara begins the discussion:

*Tara: Like, if-if you think of this as like a, like a literal ball, it can either go like that, or like that [tracing out two paths on the paper]. And so can the photon, but not as physically as this would. Like this [particle] would exist at this path the whole time–*

*Eric: Yeah, you would be able to like see which path it was on as it traveled.*

*Tara: Right, but the photon's like, I'm gonna do one of these things…we'll find out. [shrugs]*

*Eric: [Laughs] Um, versus with EM waves, it would actually go down both paths.*

*Bryan: Yeah, it's actually split.*

*Eric: So like, it's the two extremes, like with the particle it would go down one path only and with the wave it would go down both paths, and with like the photon it neither—it does like something in between where it just super-positions itself along both paths and then goes into* *one of them.*

*Tara: Right, like with the EM wave you have half the wave going on one part and half the way–wave on the other.*

Here, scaffolded by the prompts, the conversation turns to explicitly identifying particle and wave characteristics between which the photon characteristics are situated. We model this conversation as a collective conceptual blend where the "superimposed photon" is the blend space, which draws from classical particle and classical wave inputs (Figure III.8). First, the group constructs an ontology of the photon and begins to negotiate the properties of the superimposed photon. Then they return to the input space elements in order to further make sense of the blend. Eric and Tara continue by identifying how the photon is different from either of these entities. They agree that the photon does not act like a wave "because it doesn't get detected by both detectors", and that it does not act like a particle because they "don't know where [the photon] is". Once the students have reached some shared understanding around the properties of the photon (elements of the blend space in the diagram) and how they do and do not draw from classical particle and classical wave properties (elements of the input spaces), new meaning arises for individual students in what we label as elaboration or running of the blend processes. Addressing the third prompt, the students begin talking about the energy of the photon and elaborate further on the superposition aspect of the photon:

*Bryan: [Looking at the paper] Um. How do you think about the energy? I don't really know. Um...I don't know. Like when I think of it, I think of it as it's like split, just because it's easier to think about that way. But, I know that's not...true.*

*[Eric and Tara laugh]*

*Eric: Yeah, it's weird to think about where the photon is when it's in this like superposition state of not being anywhere. But also being in both places, but not really, because it hasn't decided yet.*

*Bryan: Yeah.*

*Eric: It's like it goes back and changes history when it hits. But, I don't think that's a good way to think about it. [Laughs] I don't think that's accurate.*

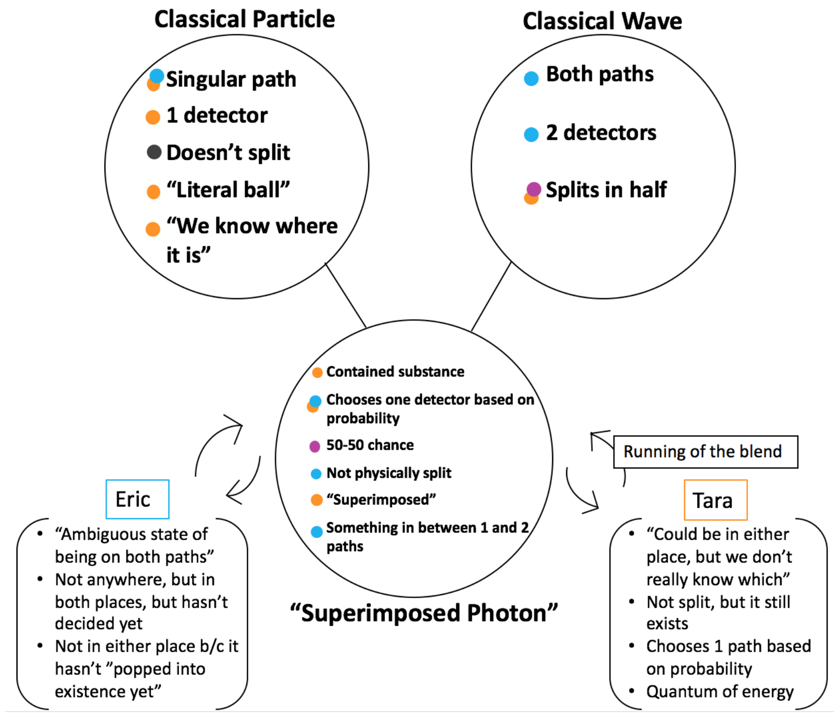



FIG. III.8. We model the collective conversation in Episode 2 between Eric (blue), Tara (orange), and Bryan (pink) as a conceptual blend. "Superimposed photon" is formed by input spaces of classical particle and classical wave. Eric and Tara run the shared blend to arrive at different interpretations of "superposition". The elements indicated with grey were not spoken by any one individual, but rather implied by statements from various group members.

*Tara: Well, I mean, I always think of the energy of the photon always being [gestures ball with her hands], like together. Like it can't really be split 'cause it's a quantum of energy.*

Through pauses, "ums", saying "I don't know" or labeling something as "weird", the three students explicitly flag their ideas with incertitude. Bryan (line 73) acknowledges that he knows the way he thinks about the photon is not the "right" way to think about it, making the distinction between his notion of the energy splitting in half versus the unarticulated "correct" way to think about the energy (an idea that the group is seeking to articulate and make sense of). Signaling the difference between these two ontological spaces, Bryan voices his thinking with the caveat that he does not think it is correct. Similarly, Eric brings in a new idea that the photon changes history when it hits one of the detectors (line 79). He then immediately distances himself from this idea with laughter, perhaps suggesting that a photon behaving this way would be "too weird". His subsequent statement, "I don't think that's accurate" shows his lack of confidence in and non-committal to the idea he just put forth, suggesting that Eric is making sense of the photon in-the-moment, and putting forth ideas he is unsure about. Tara also brings in a new idea when she invokes quantization. Following her description of the photon as a "quantum of energy" (line 82), Eric makes another bid for the photon physically splitting in half at the beam splitter. Similar to the exchange at the beginning of the episode (lines 40-45), the idea of splitting is rejected from the shared understanding of the photon. As the conversation continues, the students negotiate the meaning of the superposition state of the photon. In doing so, Eric and Tara draw different interpretations. Bryan takes much fewer turns of talk [9] and thus we cannot discern how he is thinking about the superposition element in running the blend. However, we do note that despite his minimal contributions to the conversation, Bryan *is* a part of the collective construction of the blended ontology of the superimposed photon. At the end of the ten-minute episode, Eric and Tara each articulate their ideas about what the "state of superposition" means:

83 *Eric: [Laughs]...Um, yeah I mean with the particle, obviously it's that same thing where*
84 *all the energy is traveling along one path and with the wave the energy is split between two*
85 *paths...So the ener–the energy like follows its position, but because we don't really know the*
86 *position of the photon, I don't really know where the energy is. I assume it's wherever the*
87 *photon is...So I guess the energy is also in, a sup–state of superposition.*
88 *Tara: I think that's a good way of thinking about it. Like it could be in either place, all of it*
89 *could be in either place, but we don't really know which, and it's not split, but it still exists.*
90 *Eric: And it's not actually in either of those places yet, because it hasn't been detected yet, so*
91 *it doesn't–it hasn't like popped into existence yet. If that's the correct way of thinking about*
92 *that...*

In this exchange, Eric elaborates on his original statement of the "ambiguous state of being on both paths" (lines 46-48) by describing the photon as "not being anywhere", but also "being in both places", while having "not decided yet". Despite the apparent agreement in their language towards one another, this contrasts Tara's descriptions of the photon as a "quantum of energy" that could be on one path or the other, we just "don't really know which". Tara is more confident in her interpretation while Eric approaches his ideas tentatively as if to suggest he is still trying

[9] In the conversation as a whole, but especially in the latter part when Eric and Tara are running the blend and negotiating the meaning of a superposition state.

them on and making sense of the quantum phenomenon at hand. We note that these roles are reversed from those in Episode 1 where Eric was attempting to be more concrete and Tara was hesitant to do so as she made a bid for nothing being "either-or". Two weeks later and in a different experimental, although conceptually similar, context Tara's statements are now more consistent with a deterministic interpretation (this language from Tara shows up repeatedly in Episode 2, leading us to believe she is, in this moment, utilizing an interpretation similar to what we would characterize as a deterministic hidden variable interpretation of QM), while Eric tentatively develops a notion of indeterminacy. We believe this longer time scale change in stances around indeterminacy suggests an underlying flexible nature of ontological structure, although we focus just on the local reasoning episodes for this analysis. Future work may explore this more, and follow the changes in ontological reasoning of one or more individuals within one group over the course of a semester.

In this episode we see evidence of blended ontologies. In order to explore the dynamic processes that help shape the negotiation of the ontology of the photon, we describe the collective conversation using the conceptual blending framework. At the beginning of the conversation, the group's conception of the photon begins to emerge when they decide that the photon is not physically split, goes into one of the two detectors with equal probability, and is "superimposed". These properties can be described as elements of a quantum photon blend space (Figure III.8). From this initial description the students explore the superposition element in order to refine their ideas about how the photon behaves. Eric and Tara agree that the photon "decides one of [the detectors] based on probability" to coordinate the idea that the photon is not able to physically split and the fact that it has equal probability of hitting either detector. This can be described as a running of the blend process, which contributes to the emergence of meaning for the quantum photon. Following the collective construction of the conception of the superimposed photon, the students identify classical particle and wave properties that the photon draws from. While a classical particle, or a "literal ball" (line 58) takes one path and does not physically split, an electromagnetic wave takes both paths simultaneously and is "actually split" (line 64). The students explicitly peg the classical particle and wave ontologies as the two extremes between which the photon lies; the photon exhibits some properties of each, but is itself a different kind of entity as it does not fully map onto either particle or wave. As shown in Figure III.8, in mapping the group's conversation onto the blending framework, classical particle and classical wave form the input spaces which merge to form the quantum photon blend space. We remind the reader here that our use of the terms *input space* and *blend spaces* do not refer to conceptual packets held in one person's mind as originally intended by F&T, but rather they are socially distributed conceptions constructed and utilized by the group to make sense of the phenomenon in question. The input and blend spaces in Figure III.8 reflect the shared meaning reached in the group.

The conceptual blending framework helps to elucidate the dynamic nature of the conversation: first the students come to shared understanding about the superimposed photon, then they articulate the particle and wave inputs in order to make sense of the blend space photon, and then elaboration of the constructed ontology brings in new ideas about energy, all scaffolded by the prompts. In the latter stage of the conversation, the students begin to use energy as a proxy with which to think about the photon. For example, now instead of saying that the photon doesn't split, Tara says that the *energy* doesn't split and thus describes the photon as a "quantum of energy". Through this running of the blend, Eric and Tara reach different interpretations of the state of superposition. Tara is more aligned with a hidden variable interpretation: the photon takes one path or the other, but we cannot know which one until it is detected. Although we ultimately want the students to reject the hidden variable interpretation, this may be an important stepping-stone for Tara in her learning of QM[10]. While she seems sure of her conception of superposition in this context, Tara still uses some unsure language and gestures throughout the conversation "I don't even know"; tone of voice; confused facial expressions; shrugging). In this episode, Tara's hidden variable interpretation of the photon in a Mach-Zehnder interferometer is elaborated upon and locally sustained. We make no claims about her cognition or interpretation beyond this situated moment, and we are not concerned with any of the individual students having "right" or "wrong" answers in this particular moment. In shifting away from the binary view of student learning, we

[10] We did not conduct a longitudinal study of individual students' reasoning over time, but we did observe that individual and group ideas, interpretations, and reasoning patterns changed within and across the hour-long focus group sessions. That is, a student who made use of the hidden variable interpretation in one week on a specific question may reject that interpretation the next week in a different context. We do not know what role this particular reasoning episode played in Tara's learning of QM, but we leave open the possibility that her embracing of the hidden variable interpretation in this local moment may not be detrimental to her learning, and in fact may give her helpful ideas to build off of in the future.

focus on the value of having students engage in messy and flexible discourse with their peers—they are able to consider difficult questions of interpretation (an expert-like activity), and play around with or negotiate ideas they are unsure of, which requires the ability to be reflective about one's own thinking and to externalize that for peers in a social setting. Contrary to Tara's apparent assuredness in her interpretation of superposition, Eric is less confident in his ontological stance, stating that the photon isn't anywhere because it doesn't exist yet, but that it could also be on both paths. We emphasize here the value and productivity of tentative knowledge structures as we note that this is one example (of many) in QM where certainty is not always appropriate. We distinguish students' tentativeness (being unsure about their ideas) from uncertainty in QM (the lack of ability to make predictions with certainty, inherent in quantum mechanical systems), noting that tentative reasoning may help students to develop a sophisticated understanding of inherent uncertainty. Additionally, we see the students flexibly playing with ideas they have put forth, and negotiating in-the-moment with one another in order to make sense of the quantum phenomenon at hand. These are skills we find important as they mirror activities of professional physicists, and demonstrate a depth of understanding and learning. The conceptual blending analysis of this episode provides a way to look beyond the checkbox of "they used the physics language, so they get it". That is, within the dynamic construction of a collective ontology, the quantum terminology of "superposition" takes on certain meaning for individual students. This episode provides evidence of collective dynamic construction and negotiation of ontologies and the conceptual blending framework allows us to see how different interpretations of shared meaning arise through running of the blend processes.

### 3. Episode 3: "Fuzzy ball of probability"

Episode 3 occurred in the first few minutes of the fifth focus group session in Group B (week 13 of the semester), with Zach, Fernando, and Jacob. The students were presented with an image of a copper wire that ends in space about half way across the page, along with the following question: *We have an electron in a copper wire. Can you draw the potential energy, and the wave function of the electron?* The second part of the prompt (that the students attend to later in the session, after this particular episode) brings a second copper wire near the first with a small insulating gap in between, and asks the students to again draw the potential energy and wave function of the electron. In this second part of the question we would expect tunneling, a quantum phenomenon where the electron can be found in the second wire despite not having enough energy to “get over” the potential barrier. In the first part of the prompt that comprises Episode 3, the potential is zero inside the wire and non-zero outside. The wave function of the electron is sinusoidal inside the wire, and exponentially decays outside of the wire (the “classically forbidden” region). After working independently for a brief time, the students compare the wave functions that they have drawn. All three students have indicated that the wave function extends outside of the wire in an exponential decay. Zach says, “the wave function extends on either side of the well—of the wire, into the classically forbidden zone, which has something to do with quantum tunneling.” To that, Fernando responds with a description of the electron:

*Fernando: Yeah I guess it’s like that, the fuzzy nature of, of electrons kinda like you don’t know. I think that—that’s always been kind of confusing, whether it goes out because it like, it’s like a fuzzy ball of probability that extends out–into that zone? Physically? Or...I don’t know that’s the way it seems to me.*

This language of an electron being “fuzzy” was never used in class, and had not come up in prior focus group discussions. Fernando may have been thinking about the electron in this way before (or a prior simulation may have cued the word “fuzzy” for him much like the “blob” in Episode 1), or he may have constructed this description in-the-moment. Either way, he proposes the idea of the “fuzzy ball of probability” in this moment as a tool for linking the wave function representation in front of him on the paper to a physical picture of what the electron is doing. Fernando’s initial statement about the fuzzy nature of electrons hints at some notion of probability or indeterminism when he says “kinda like you don’t know” (line 93-94). He then labels the electron as a fuzzy ball of probability and questions the physicality of this description. Is the electron something that physically extends outside of the wire, or is it something else? Fernando explicitly flags his idea with reluctance—he labels it as “confusing” (line 94), presents the fuzzy ball description as a question (line 95), and makes the reflective statement “I don’t know that’s the way it seems to me” (line 96).

Fernando throws out the "fuzzy ball of probability" as a catch-all phrase, and then the students begin to unpack what it might mean in the specific context of an electron in a wire:

*Jacob: Huh I guess I never really thought about...that. If it just extends out 'cause it's like a fuzzy ball of stuff, it's not really anywhere. [Laughs] But that's kind of—yeah, I never really thought about why it extends out.*

*Fernando: Yeah I feel like a lot of times we think about like only the math, and then like—I, I don't know, I guess physically thinking about a lot of this quantum stuff is not very intuitive, and kind of hard to do, but...*

*Jacob: I just kind of took it for granted 'cause I was like oh negative kinetic energy—pshh [puts hands up] don't tell me.*

*Fernando: [Laughs] Yeah, exactly. Doesn't really mean anything I guess, but—*

*Zach: I kinda imagine like a, an electron like going back and forth and like getting to the end and like–like going out, and then almost like magnetically like–like going outside a little bit and then shooting back in. [gesturing with his finger a particle moving back and forth]*

*Fernando: ...Yeah, I like the analogy in class where we had, where like the water was with the rubber [brings right hand to meet left]*

*Jacob: Yeah that's what I was just thinking. How it would like hit the rubber wall and like, can stretch out [gesturing stretching rubber wall] and then just shoot it back.*

Responding to Fernando's proposition for the electron as a fuzzy ball of probability, Jacob engages in metacognition as he recognizes that he has never thought about the physical meaning of the decay of the wave function. He attempts to take up Fernando's fuzzy ball description, but refers to a "fuzzy ball of stuff", which we take to be ontologically distinct [11] from a "fuzzy ball of probability". In lines 97-99, the ambiguity of pronouns in Jacob's statement is a signal of in-the-moment reasoning. We infer that Jacob means that the *wave function* extends out because the *electron* is a fuzzy ball of stuff. In lines 100-102, Fernando sets the tone for the conversation by revealing his expectations about the roles of math and thinking about physical interpretation in

[11] As was briefly mentioned in Section III A 3, Chi defines two categories to be ontologically distinct if they have no overlapping ontological attributes [68]. This is not how we use the term here. We take "fuzzy ball of stuff" to have different meaning from "fuzzy ball of probability", but do not consider it as a separate ontological category altogether. This is consistent with our description of ontological structure in which the boundaries between ontologies are not rigid, but rather flexible and can overlap.

QM. He says that thinking physically about quantum phenomena is difficult and not intuitive, yet the group continues attempting to do just that. For the remainder of the episode (and much of the entire hour-long session), they work collectively to make sense and construct a physical description of quantum tunneling. We note that in this focus group session, the students have not been explicitly prompted to engage in this interpretive discussion (although by this time in the semester, there are expectations established in the class that attending to interpretive aspects is part of learning QM). We take the word 'but' in lines 102 and 105 to represent an epistemic bid from Fernando that although thinking about the physical interpretation of QM is difficult, it is important and valuable for him and his peers to continue to engage in this discussion in the focus group setting.

The group puts forth different ideas for how to think physically about tunneling, beginning with Zach imagining the electron as a point-like particle (evidenced by his gesture) moving in and out of the wire "almost magnetically" (lines 106-108). The water and rubber analogy the students reference in lines 109-112 is an analogy for tunneling that was presented in class, where a rubber barrier is analogous to a potential barrier and the water wave is analogous to a wave function. As water sloshes up against a rubber wall, some of the wave will "leak" to the other side of the wall, going through the barrier and not over it (nor breaking through it). Although the analogy as presented in class was meant to draw on wave characteristics, the students are attending to the material aspects of the description in their conversation. This is evidenced by their particlelike language: it [the water] *hits* the rubber wall and then the rubber *shoots* it back. In this episode, the water and rubber analogy is brought up as a continuation of the description of the electron as a particle that moves out of and back into the wire magnetically.

Thus far in the conversation the students have been shopping around for ideas to try to link the mathematically derived wave function to a physical picture. In a metacognitive statement about his ontologies, Fernando highlights the difference between these spaces and then identifies what he sees as a conflict between them:

*Fernando: Yeah, that was pretty good. And then...I don't know, I think it's just that this, this concept still...escapes me a little bit. Yeah because I feel like the same thing [gestures to*

*Zach], I feel like, I, I always like when I look at this I see an electron kinda bouncing back and forth, but in class he keeps saying it's like distributed throughout this at all times. And then like at the nodes, it can never like bounce through that. [Shrugs.]*

*Zach: Oh yeah.*

*[Fernando Laughs]*

Fernando identifies that there is a difference between his intuition of a particlelike electron and the "right answer" from class that involves the electron being non-localized, and being represented as a wave function (which is sinusoidal inside the wire), such as the one drawn on his paper. In trying to reconcile these two ontologies, he encounters a conflict because the wave function representation suggests there are points (nodes) where the electron will never be found. With a shrug, Fernando marks this as something that needs to be reconciled. Zach agrees that this poses a problem, and returns to the fuzzy ball of probability idea:

*Zach: Yeah that's right...I like the fuzzy ball thing. It's like, in that um...what was it, the matter wave demo sim thing, where it like dims out when it splits [separates his fingers on the table]...*

*Fernando: Oh yeah when it hits a wall and it kind of dampens.*

*Zach: I kinda think about that. Yeah it's like not an electron, it's just like a dim bigger–electron...[gesturing a blob with his hands]*

*Fernando: Thing?*

*[Zach and Fernando laugh]*

*Jacob: It'd probably almost be useful to start thinking of electrons like we would think of photons or whatever, we would have to think about them like a particle in some situations but like a wave in others 'cause like right here [pointing to the wave function] we're thinking about it like a wave for sure, but when we're talking about it bouncing back and forth that's like a particle kinda deal to me.*

Here, Zach and Fernando have used the fuzzy ball to help reconcile their perceived contradiction between particlelike and wavelike characteristics of the electron. Zach references a simulation that he relates to the fuzzy ball description of the electron; from his gestures and the overall course context, we believe he is referencing the PhET Quantum Wave Interference simulation (the simulation used in Episode 1 to run a double slit experiment with a single photon [96]). In line 121, the "it" that dims out is ambiguous. We infer that Zach is suggesting that the electron is a fuzzy ball that acts similar to an entity in the sim which "dims out when it splits". In the class, the students have most often used this particular simulation to talk and think about photons, but the simulation gives the option of choosing a photon, electron, neutron, or Helium atom. This recall of the simulation helps Zach and Fernando bring meaning to the fuzzy ball. That is, the picture from the sim helps them link the wave characteristics that they know are part of the "right" answer with their intuitive physical picture of material entities. The simulation representation can be thought of as a material anchor [100] for the electron as a fuzzy ball of probability. Zach and Fernando share some common understanding around the blended entity, "fuzzy ball of probability", but Jacob on the other hand does not take up this language. Instead he advocates for holding the particle and wave ontologies in parallel: sometimes we should think about electrons as waves and other times we should think about them as particles.

In this episode, we see the group utilizing two different types of ontological structures—blended ontologies and parallel ontologies—both of which can be described as dynamic, although the nature of dynamics in these two cases are distinct. For the bulk of the conversation, the students are shopping around for ideas, trying out different analogies and descriptions. In doing so, they are attempting to reconcile two competing pictures: electron as a wavelike entity (invoked by mathematically derived representations on paper, and things the professor has said in class) and the more intuitive material or particlelike picture of the electron. Fernando begins the conversation by throwing out the phrase, "fuzzy ball of probability" as a possible way to link the wave function representation with a physical picture of the electron, explicitly acknowledging his hesitancy around this idea as he does so. Although attempting to link the particlelike and wavelike ontologies, in practice the students hold them in parallel to one another. This is evidenced by the word "but" in Fernando's statement, "when I look at this I see an electron kind of bouncing back and forth, *but* in class he keeps saying it's like distributed throughout this at all times [emphasis added]" (lines 115-117).

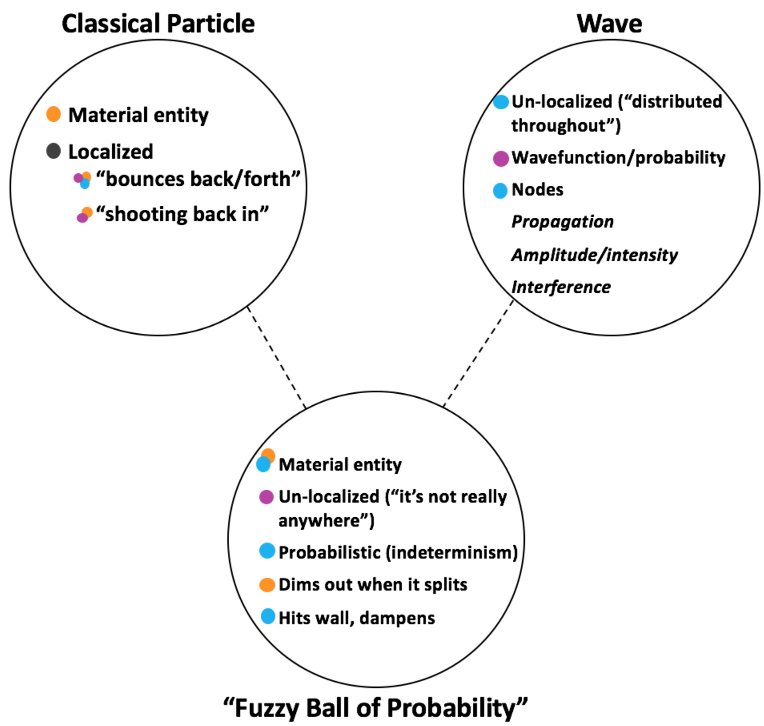



FIG. III.9. We model the conversation between Fernando (blue), Zach (orange), and Jacob (pink) in Episode 3 as a collective conceptual blend, where classical particle and (semi-classical or quantum) wave form the input spaces for the fuzzy ball of probability blend. Part of the wave input includes the material anchor [100] of a simulation that the students recall (elements indicated in italics). The elements indicated with a grey circle were not spoken by any one individual, but rather implied by statements from various group members.

The material or particlelike view of the electron includes thinking of it "bouncing back and forth", gestures of a small localized entity that has a definite location, and using language such as "it hit the wall" and "shooting back in". The wavelike ontology includes the mathematically derived wave function representation which has nodes and a notion of probability, and thinking of the electron as an unlocalized entity which is "distributed throughout". The students attempt to draw meaning from intuition as they try to connect the two parallel ontologies and form a cohesive description of the quantum tunneling phenomenon. The lack of reconciliation comes in line 117 when Fernando says "And then like at the nodes, it can never bounce through that." This marks a turn in the conversation where Fernando and Zach no longer find it constructive to hold the two ontologies in parallel. Using recall of a simulation representation as a material anchor, they turn to the fuzzy ball of probability and collectively negotiate meaning around this idea. The fuzzy ball is an entity which draws from both particle and wave characteristics. The name "ball" suggests a material property; it includes a notion of probability or indeterminism, a uniquely quantum characteristic that the students do not discuss in depth but is mentioned in the name of the "fuzzy ball of probability" and is buried in their wave function representations; the fuzzy ball is "not really anywhere", suggesting a wavelike property of unlocalization; it hits a wall and then dampens, drawing on both particlelike language ("hits") and wavelike characteristics of propagation and diffuseness; it dims out when it splits, eluding to wavelike interference; and it is like a "dim bigger electron thing" suggesting a material entity with a wavelike characteristic of intensity. Figure III.9 illustrates the fuzzy ball of probability as a conceptual blend, drawing from input spaces of particle and wave, and including elements (in italics) which come from or are informed by the recalled simulation.

While Fernando and Zach come to some common understanding of the fuzzy ball as a blend of two competing ontological spaces, Jacob does not take up this same language. The episode ends with Jacob making a bid for continuing to hold these ontologies in parallel by thinking about the electron as a wave sometimes, but a particle at other times. We note here that in this analysis there is no judgment as to whether the blended or parallel ontologies are "better". Rather, this episode provides evidence of *different types* of dynamic ontologies.

The conversation is full of hedging and tentativeness, signaling in-the-moment ontological negotiation. Even further, Fernando puts forth an idea that he questions and delivers with hesitation. We take this type of tentative reasoning to be inherently productive. As educators, we want our students to be able to put forth ideas they are unsure about and play with them in a flexible way to make meaning and develop understanding. Fernando does this here with the help of his peers, which we take to be valuable for the group because they continue to search for and make sense [8, 83] of a satisfactory physical description of the tunneling phenomenon they see represented in the form of a wave function drawn on their papers (i.e., the tentativeness helps sustain engagement in sense making processes) Fernando and Zach use the fuzzy ball idea to reconcile a contradiction they see between two ontologies, and in doing so they begin to develop a physical description of the wave function in this specific context, a goal that Fernando identified for the group at the beginning of the episode when he alluded to the value of thinking physically about quantum phenomena despite the unintuitive nature of doing so. For Jacob, the tentative reasoning and collective negotiation resulted in a different ontological structure which was useful for him in the moment. In this episode, we see two students (Fernando and Zach) shifting locally from a material ontology to something that can be defined as a hybrid between matter and wave ontologies (the fuzzy ball). We make no claims of stability beyond these observed moments. The blended ontology is locally sustained for Fernando and Zach, and the parallel ontologies are productive in the moment for Jacob. This episode provides examples of how ontologies can be dynamic in either construction (blended) or application (parallel).

### E. Synthesis and discussion

The three focus group episodes provide examples of students reasoning collectively about the nature of a quantum entity. In each example, we see nuanced and complex reasoning processes and flexible use of ontologies. In Episode 1 we see Eric, Tara, and Bryan collectively negotiating the ontology of a photon, which they come to describe as a "blob of EM wave", an ontological conception that draws from both classical particle and wave but that cannot be fully mapped onto either. We see the students construct emergent meaning around the energy of the photon, and we also see hints of parallel ontological structure within the blended ontology (lines 7-8), suggesting that the different types of dynamic ontologies may not be mutually exclusive. Episode 2 illustrates how two students can come to different interpretations of a superposition state from a shared understanding of the properties and behaviors of a photon. In Episode 3 we see evidence of different types of dynamic ontologies: parallel (switching back and forth between stable notions of particle and wave) and blended (constructing a new ontology on-the-fly which is a blend of particle and wave properties). All three episodes provide evidence of the dynamic nature and students' flexible use of ontologies, and demonstrate what it looks like for students to engage in messy and tentative reasoning when negotiating conceptual understanding of quantum phenomena. The large scale methods of selecting and analyzing the data are the same across all episodes, but the results are determined by which form of analysis the data lend themselves to. For example, in Episodes 1 and 2 we map the ontological reasoning using the conceptual blending framework, but in Episode

3 only part of the conversation can be mapped using the conceptual blending tool while the rest is best described with a parallel ontological structure.

Our analyses lead us to identify a preliminary model of four different approaches to understanding quantum entities (Figure III.10). The first is a purely classical ontology (e.g. electrons are classical particles). Chi's framework would say that if a student holds this (wrong) ontology, they must undergo radical conceptual change in order to reassign the electron to the correct category [68]. We would say that this ontology is bad in some circumstances (i.e. in a quantum context it is wrong to think of electrons only as classical particles). A dynamic ontologies framework [54] would suggest that students have the ability to reason across ontological boundaries and to use ontologies flexibly. So, a student holding a classical particle ontology of an electron in one moment, could be cued into activating different ontological resources and flexibly utilize a different ontology in another context or moment. The second type of quantum ontological structure is when one applies robust classical structures to quantum entities (unidirectional arrow). This could be the Bohr model of the atom (which is useful in many situations [101]), or a hidden variable interpretation of quantum mechanics (which may or may not be useful as a stepping stone to reason through); these can be destructive if they are the *only* models used. Similarly, the third type is a combination of classical and quantum structures, with a bidirectional arrow in between. That is, instead of simply applying the structure and properties of classical entities to a quantum entity, the student uses both classical and quantum properties flexibly. An example of this could be the analogical mapping of a rubber barrier and water wave to potential barrier and wave function in quantum tunneling. If students applied this analogy as a one-way mapping of one domain to another [102], we would classify it as the second type of approach. If it were applied as more of a blend analogy [103], blending the hybridity of quantum understanding with classical mechanics concepts, we would consider this to be the third type of approach. Another example could be particle-wave duality. The second and third approaches describe different applications of classical concepts that create tools for QM understanding. The fourth ontological approach is one that uses purely quantum descriptions of a quantum entity, identifying mathematical and physical entities which come together to describe QM. In Figure III.10 these four approaches to reasoning about

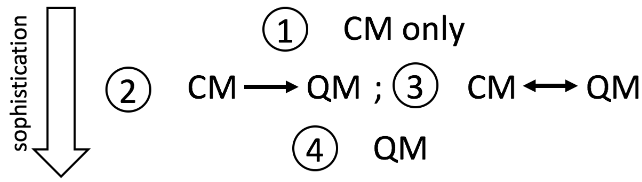



FIG. III.10. Preliminary model of four different ontological approaches to QM sense-making. CM=classical mechanics, QM=quantum mechanics

quantum entities are listed in increasing degree of sophistication from our perspective. We are not suggesting that a student learning quantum mechanics must go in order 1-4 (but maybe sometimes they do), or that types 2 and 3 are bad. In fact, most of our data live in these two middle types, and even expert physicists often make use of combined classical/quantum descriptions of quantum entities. We do not wish to make judgments about the relative utility or sophistication of types 2 and 3 (as shown in Figure III.10, we consider them to be equivalent in this sense). A deterministic hidden variable interpretation (such as we see reflected in Tara's statements in Episode 2) is ultimately not correct and not useful for physicists, but may be a stepping stone to understanding QM. In comparison, the Bohr atomic model, also an example of applying classical structures to quantum entities, is productive for students [101] and often used by expert physicists. We suspect that each of the four models could be applied productively or unproductively. This is a preliminary model based on the data and analysis presented in this paper as well as our experience teaching and learning QM. Future work will continue to investigate and refine these approaches.

### F. Limitations

A key limitation of this work is the small sample size. We do not seek to map the small group conversations between a total of six Modern Physics students onto a broader Modern Physics or QM student population (here at the University of Colorado, or to other universities or contexts). These six students were some of the top-performing students in the class, and self-selected into the focus group study. Nonetheless, the three episodes presented here demonstrate that this type of dynamic ontological reasoning is possible. The snippets of student reasoning we have selected and analyzed here are not representative of all students, but they *are* recognizable kinds of discourse. That is, not all students (or groups) go through these exact kinds of reasoning practices, but the methods we provide and the resulting refinement of a dynamic ontologies framework are particularly useful for thinking about the nature of student reasoning, and further, how to leverage these student practices and capabilities. While qualitative research of this kind may not be externally generalizable in the probabilistic sense of large scale quantitative work [104], we do believe the results are transferable and applicable to other individuals who matter in the enterprise of physics education.

Coming from a sociocultural perspective, we chose to focus on collective conversations, attending to both individual and collective reasoning. There are some limitations to this approach. First, analyzing group discourse makes it hard to know how individual students are reasoning. We often cannot tell if statements voiced by students are personally held ideas. Ultimately, knowing each student's personal relationship to the ideas put forth in the groups was not the purpose of this study, and thus this limitation did not present an issue for our analysis. It is, however, important to keep in mind when reading and interpreting these results. Furthermore, reasoning of individual students is explored in the next chapter. Secondly, there are social dynamics of the groups which intersect strongly with the collective reasoning. In this paper we have chosen to foreground the reasoning structures of the group, but note that this is certainly influenced by the social dynamics unique to each group of students. We unpack the social dynamics a bit further in Chapter IX.

As noted in Section III D, it is likely that in addition to the prompts and content areas, the simulations influence the type of ontological reasoning we see the students engaging in. In Episodes 1 and 2, the simulation is part of the task itself and so we cannot separate it from the prompt. In Episode 3, the students are not asked to use a simulation, but in discussing the phenomenon of tunneling they reference a prior simulation. We can think about (the recall of the) simulation in this case as playing the role of a material anchor [29] for the blended ontology. We are not able to account for the influence of the simulations on students' reasoning in the present analysis. However, we begin to address the impact of the framing of curricular materials more substantially in the next chapter.

### G. Conclusions

Using evidence from three episodes of student reasoning, we have outlined a framework of dynamic ontologies. This framework includes an underlying organizational structure which is flexible in nature, and in which reasoners can make flexible use of existing stable structures as well as develop and negotiate new blended structures. We argue that the messiness of student reasoning should be valued and that the tentative nature of students' contributions to collective discourse are valuable for learning QM. The opening quote in this section from Richard Feynman can have many interpretations. One is that there is a monolithic understanding of QM—you either get it or you don't. We choose an alternative interpretation which aligns with our argument for valuing the messiness of student reasoning: Everybody understands QM, incompletely, in their own messy ways. Lastly, we have demonstrated the utility of collective conceptual blending as an analytic tool to unpack the nuances of students' dynamic and tentative reasoning structures in QM, contributing to a research community that values complexities of student reasoning.

The theoretical contributions of this paper are not (yet) designed to focus on instructional implications, though the work here does suggest that instructors of, and curriculum developers for, Modern Physics and QM courses (and probably physics in general) should recognize and attend to students' flexible use of ontologies and allow students space to engage in messy reasoning. One specific instructional implication is a challenge to us; namely, how we should go about teaching "wave-particle duality". Does the way we teach this topic allow students the space to engage in messy and tentative reasoning and to construct their own nuanced ontological conceptions of photons and electrons? How do we encourage students to use blended or parallel ontologies with the words we use and the questions we ask? We speculate that when students hear (or read) the phrase "wave-particle duality", they get the message of a *parallel* ontology—the entity is sometimes a particle and sometimes a wave, with the dual natures kept separate from one another. While this ontological structure is useful in many circumstances, we do not want our students to be restricted to it. On the contrary, when experts use the phrase "wave-particle duality", we often have a *blended* ontology in mind—the entity is neither a particle nor a wave although it exhibits both particle and wave characteristics. That is, when we talk about "wave-particle duality" we are blending the dual natures together to form a new kind of thing entirely. It is worth investigating further the ways in which our wording and framing of this particular topic impacts students' ontological reasoning. The following chapter begins to investigate these questions about the impact of prompts (although not specific to the phrase"wave-particle duality"), and provides examples of how instructors can provide students this space to engage in messy reasoning.

# Chapter IV. A refined framework for investigating students' ontologies and their patterns of use

*This chapter comes from an article currently under review at Physical Review Physics Education Research: Hoehn, Gifford, and Finkelstein 2019 [74].*

Building on the prior study that explored the dynamics of students' reasoning in a focus group setting [61, 105], we engage in a triangulation study of students' ontological reasoning about photons and electrons in individual written work in a modern physics course context. Whereas the prior study focussed on collective oral student reasoning, we extend this work by now looking at *individual* student reasoning through their written work from a variety of artifacts (HW, exams, surveys) and across three topic areas. We ask the following research questions:

1) *Can we document evidence of dynamic ontologies in individual written work in the same way that we did for collective oral reasoning?*

2) *What are the patterns of use (frequencies, overlap, distribution) of ontologies in the individual written work?*

3) *How does the wording and framing of prompts impact students' ontological reasoning?*

To this end, this chapter makes the following four contributions: We present a full description of a refined framework to describe and distinguish between different kinds of dynamic ontologies. We then demonstrate the utility of the framework by presenting students' capacities for flexible use of ontologies in individual written work. We do this by exploring the patterns of use of ontologies, as a broad scale (class-wide) application of the framework, which provides evidence to show that students are capable of nuanced reasoning and flexible use of ontologies. Additionally, we investigate the impact of the wording of question prompts and find that students' responses are aligned with the ontologies or ontological structures that the prompts themselves lead to. That is, the way we ask questions matters. We provide examples that are indicative of the manner in which the flexible use of ontologies is dependent upon content domain, entity, question format, and wording or framing of the prompt. Through a few notable example questions, we illustrate the (messy) intersection between these factors.

In demonstrating that students are capable of varied ontology use, we do not make claims about wide scale prevalence or frequency across other course contexts. The evidence for flexible use of ontologies we present here is tied to the particular modern physics class in which our study takes place. The results are thus tied to the unique goals of the course (e.g., supporting students' ontological reasoning), but demonstrate students' capacities to engage in these flexible and varied uses of ontologies and ontological structures. The findings also demonstrate that how we ask questions influences student reasoning. This claim may seem obvious, yet it is still important to document experimentally with respect to ontology or ontological structure use. Furthermore, there are some current models of the nature of ontologies [69] that suggest that question framing should *not* matter because of the claimed robust nature of ontologies.

While we demonstrate that the subject of modern physics is particularly suited for examination of ontology use, we also note that student dynamic use of ontologies exists in other course contexts as well (e.g., ontologies for energy in any level of physics class). Prevalence of flexible use of ontology, and influences of content domain, course structure, and other contextual factors remain the subject of future work.

### A. Theoretical framing

In examining how students productively use multiple ontologies in learning quantum physics principles in a modern physics class, we dominantly draw from prior work on ontologies. Simultaneously, we ground this work in recent efforts that broaden the definition of productivity of reasoning, and with a theoretical commitment that foregrounds the situated nature of learning and reasoning.

In this paper, we build on prior work (Chapter III) and present a more complete framework (Figure III.4 below) to describe the different ways in which ontologies can be dynamic. This framework aligns with the view of ontologies as dynamic in nature, but also finds utility in the notion of a single stable ontology applied in a given moment or context. In this way, we move beyond the dichotomy between dynamic and fixed ontologies to include both stable and more flexible structures. Looking at the varied ways that students use ontologies gives us access to a wider range of student reasoning, thereby broadening the way we understand, and value, student reasoning. As in all of our work, we begin with a fundamental commitment to value the messiness of student reasoning. In the prior chapter, we highlighted two elements of messiness: 1) flexible use of ontologies and 2) tentativeness of reasoning. In this chapter, we focus on the former, although we do see some instances of tentative reasoning even in individual written responses on homeworks, exams, and surveys.

As noted above, we approach our research from a sociocultural perspective of learning [19]. That is, we consider both the objects and processes of learning to be social. As such, we value both individual ideas as well as collective negotiated meanings. In our prior work, we focused on collective reasoning and considered a blended ontology constructed within a group of students to be socially distributed [61, 78]. While the present study focuses on individual responses, we consider an individual's learning as a social process—individual cognitive processes are mediated by social tools (e.g., language) [28] as well as other people and their use of tools [29, 30]. We consider student responses to reflect both their own thinking and that of the context in which they are embedded [20]. To that end, part of the analysis presented below investigates the contexts in which students are engaged in sense making, looking particularly at the impact of our prompts on students' use of ontologies. The wording and framing of our questions is one piece of a broader educational environment in which our students are situated, and we see it as a vital part of education research to consider the ways in which our environments can contribute to and support students' learning.

## B. Framework

In Chapter III, we began to investigate the dynamics of students' reasoning and developed an initial framework to distinguish different types of dynamic ontologies. We use the term *ontological structure* to refer to the manner in which *ontologies* are applied or used. As a result of the analysis of collective student reasoning around canonical topic areas in QM, we moved beyond the dichotomy of fixed versus stable ontologies by drawing on notions of stable structures as well as flexible use of different ontological categories. The framework refines our understanding of how ontologies can be dynamic in nature by considering two dimensions that characterize students' reasoning using ontologies: construction (when are the ontologies developed?) and application (which ontologies are applied?). The data that form the foundation of this framework consisted of episodes of collective student reasoning around three canonical topics in modern physics: double slit experiment, Mach-Zehnder interferometer with a single photon, and quantum tunneling.

Upon development of the framework and application to episodes of students reasoning in groups, we wondered if we would see the same kinds of dynamics of ontological reasoning in *individual* and *written* work. We developed a coding scheme based on the framework and applied it to individual written and multiple-choice (MC) responses from the same seven students in the focus group study across the same three topic areas. Results of this pilot study triangulating between the collective oral reasoning and individual written work are presented in Ref. [105]. Through this process, we refined the framework itself to be applicable to both collective oral reasoning and individual written work. In the context of collective reasoning in focus groups, we described blended ontologies as being constructed in the moment, as we could see the negotiation of meaning unfolding (in a messy, and often productive, way) bit by bit in the video. However, when looking at individual written reasoning turned in on a homework, survey, or exam, we cannot comment on in the moment reasoning. In these instances, we do still identify and describe blended ontologies as a new category created by combining elements from multiple pre-existing ontologies. The blended ontologies are still dynamic, though in a different way than when we identify collectively constructed blends unfolding and being negotiated throughout a conversation. This difference comes from the difference between the theory of conceptual blending [85] as it was originally intended to describe the cognition of individuals and the adapted collective conceptual blending framework [78] which treats the blends as collectively constructed and socially distributed. Although the substance of our dynamic ontologies framework has not changed, we no longer label blended ontologies as being constructed in the moment. Additionally, in the refinement, we relabeled what was initially a "fixed" ontology as a "unitary" ontology.

| Construction \ Application | Single (Not dynamic) | Multiple (Dynamic) |
| --- | --- | --- |
| Stable (Not dynamic) | Unitary | Parallel |
| Novel (Dynamic) | Blended | [ Parallel Blends ] |

FIG. IV.1. Typology of ontological structures for a given individual reasoning episode. Construction refers to when the ontologies are developed. Application refers to which (or how many) ontologies are applied.

The first major contribution of the present chapter is to present a refined description of the framework along with a codebook that operationalizes it for a particular context; we then apply the codebook to a large-scale sample of student reasoning and demonstrate its utility by investigating the patterns of use of ontologies. The full version of the framework that describes ontological structure is shown in Figure IV.1. It includes four quadrants, three of which we observe in our data. The full coding scheme is described in the methods section below and provided in detail in Appendix A.

A *unitary* ontology is an application of a single stable ontology in a given local reasoning episode, where "stable" suggests the ontology is an already developed, perhaps robust, conception that the learner brings with them into the reasoning episode. For example, treating a photon solely as a classical particle in a given context would be a unitary ontology. In this case, a student arriving in a modern physics class is familiar with the notion of a classical particle, and thus the classical particle ontology is considered to be a stable structure. However, we note that the structures in the framework are defined at the level of an *individual reasoning episode*. Applying a unitary ontology in one context does not preclude using a different ontology, or different structure, in other instances or contexts. From our prior example of energy, a unitary ontology might be thinking of energy as a material (or quasi-material) substance. When attending to conservation of energy, a learner might treat energy as stuff that can be possessed by an object or transferred or exchanged between objects, e.g., "the particle *has* energy".

A *parallel* ontology consists of switching back and forth between two or more stable ontologies. For example, saying a photon travels as a classical wave and is detected on a screen as a classical particle is a parallel ontology because the two categorizations are held separate from one another—sometimes the photon is a particle and sometimes it is a wave. Again, in this example the classical particle and classical wave ontologies are considered to be stable because these are notions that a modern physics student would have brought with them into the reasoning episode. That is, they are already familiar with characteristics of classical particles and waves and they draw on these pre-existing ideas in order to construct the parallel ontology of the photon. From the energy examples previously, we also observe a parallel ontological structure in the student reasoning presented in Ref. [63]. In a conversation about leaves blowing in the street, the students switch back and forth between a substance ontology and stimulus to action ontology, often between turns of talk. First, a student questions whether "leaves in the street *have* energy" (substance ontology). In her next utterance, that same student says "wind is energy" (stimulus ontology, treating wind as the agent). Another student then says that the leaves are "*getting* wind energy" (back to substance). A third student says the leaves are "*pushed* by energy" (stimulus), and then the first student says again that "they *have* energy" (substance) [63, emphasis added]. A familiar occurrence in student collective reasoning, this conversation is an example in which the parallel ontological structure is socially distributed.

In the third quadrant of the framework, a *blended* ontology is a novel ontological category for the learner constructed in the moment, drawing from two or more stable (input) ontologies. From the combination of two or more input ontologies, new meaning emerges to describe a new kind of entity entirely. This structure is based on Fauconnier and Turner's theory of conceptual blending [85] applied to ontologies [61, 78]. A blended ontology of a photon might include describing the photon as being localized but that the position depends on a wave interference pattern. Contrary to the parallel ontology, this description combines the particle and wave characteristics, and in doing so new meaning of what a photon is emerges. The photon is neither a particle nor a wave, but something different that exhibits some characteristics of each. Further details of such blends are given in Ref. [61] and below in Section IV C 4 . From our prior example of energy, Dreyfus, Gupta, and Redish provide an example of a blended ontology [65]. Students in their study described the emission and absorption of energy by blending energy as a substance and energy as a location. The resulting blended ontology has the properties that “absorbing energy makes things go up” and “releasing energy makes things go down”.

The fourth quadrant of the framework contains a structure—“parallel blends”—that we do not see in our data, but would involve switching back and forth between multiple blended ontologies. We can imagine seeing this structure if we examined student reasoning over longer time scales, rather than constraining to individual reasoning episodes.

The framework centrally contains a description of the *structure* of ontologies (i.e., unitary, parallel, blended), but it also implicitly includes the ontologies themselves (e.g., in our context, particle, wave, etc.). When examining student reasoning, the ontological structure can only be determined after the ontologies themselves have been described. The three structures described in our framework apply across a range of ontologies (e.g., particle and wave ontologies for photons, or substance and location ontologies for energy), though of course, the specific ontologies that are used in analyzing student reasoning will depend upon the content area of focus. We do not provide an exhaustive examination of the particular ontologies (entities) that can be used in the framework, rather we present a framework that helps guide our understanding of ontological reasoning regardless of content area. Figure IV.1 illustrates the central focus of the framework, on ontological structure, noting this will always happen around specific ontologies being applied to given entities. In order to answer research questions around the kinds of dynamic ontologies used in individual written work, the patterns of use of ontologies, and the impact of prompts on students’ ontological reasoning, we present a concrete application of the framework to reasoning about photons and electrons in the context of three canonical topics in QM.

### C. Methodology

This study is one piece of a larger effort to understand students’ reasoning in QM. In our effort to present a more complete framework and apply it to students’ individual written work, we triangulate across multiple data sources. In applying the framework to new modalities of student reasoning (e.g., written work), we simultaneously sought convergence of findings from different sets of data and analyses, and used these results to refine the framework itself (the final version of which is presented in Figure IV.1 above). The students' focus group reasoning, individual written work, and the framework all mutually inform one another, resulting in a broader understanding of students' ontological reasoning about photons and electrons in the contexts of the double slit experiment, Mach-Zehnder interferometers, and quantum tunneling.

#### 1. Course context

This study takes place in the same Modern Physics for Engineers course as the prior study. One of the novel learning goals of the course is for students to develop sophisticated ontological reasoning skills. That is, we want students to be able to reason using different ontologies, to decide which ontology they should use when, and to metacognitively reflect on their ontological reasoning (something we call *meta-ontological competence*). The course strives to support students in this endeavor by asking questions directly about ontology (see below for many examples in our data set) and using materials such as tutorials designed to support students in reasoning flexibly across ontologies and reflecting on which ontology they are using and why (see Chapter V. This course seeks to encourage and support students in engaging in nuanced and complicated reasoning; we strive to value the messiness of student reasoning in the classroom through grading that emphasizes reasoning over final answers, attending to interpretation and asking questions that do not necessarily have correct answers, and holding students accountable for developing their own explanations and interpretations.

#### 2. Sampling

The data in this study come from one semester of the Modern Physics for Engineers course described above. There were 140 students enrolled in the course, 60% of whom were mechanical engineering majors. The next most common major was electrical engineering (13%), followed by a smattering of other engineering or science majors. Including the original seven students for whom we had already coded responses, we selected a sample of 28 students (20% of the total enrollment). Requirements for being included in the sample were that the student had completed the entire course and had completed at least 75% of the questions included in our data set. Of the students who fit these requirements, we selected a random sample. The sample is representative of the course overall by major, and includes twice the proportion of women than in the class overall (14% versus 7%). We oversampled for women [12] because the representation of women in this particular semester was lower than other semesters of the same class, and we are more concerned with being able to say something about what kinds of reasoning students are capable of and what kinds of patterns we might see in this particular pedagogical environment, rather than being representative of one particular semester's enrollment. We also slightly oversampled for students with A and B final grades, as grades were in part reflective of students' explanation of reasoning—Students with lower grades tended to have sparser answers which we more frequently would have to code as "can't tell", while students with higher grades tended to have more complete work from which we could infer information about ontologies, allowing us to draw more meaningful conclusions. Characteristics of the sample compared to the class overall are shown in Table IV.1. Percentages of majors do not sum to 100% because some students have multiple majors. University of Colorado is a predominantly white institution (68% of undergraduates are white [106]), as is the College of Engineering (61% of undergraduates are white [107]).

TABLE IV.1. Characteristics of the sample population compared to the whole class.

| | Full Class ($N = 140$) | Sample ($N = 28$) |
|---|---|---|
| Average final grade | 82% | 85% |
| % A's,B's | 72% | 79% |
| Average homework grade | 84% | 89% |
| % Female | 7% | 14% |
| % Mech. Engr. major | 61% | 61% |
| % Elec. Engr. major | 13% | 14% |
| % Other major | 34% | 36% |

[12] Gender information in these data comes from our perception of gender expression. We regret that we do not have gender data as reported by the students themselves, and thus our estimation of percent female may be slightly inaccurate.

### 3. Three topic areas

In this study we focus on homework, exam, and pre- and post-survey questions around the three topic areas that were the focus of the original focus group study [61]—double slit, Mach-Zehnder interferometer, and quantum tunneling. In this section, we briefly discuss the relevant physics of each topic area and how these topics are introduced to the students in this study.

Young's double slit experiment is a canonical example for introductory as well as upper division physics classes. As shown in Figure IV.2, a barrier with two small slits is a certain distance away from a screen. When one shines a beam of light on the two slits, an interference pattern of bright and dark spots appears on the screen. Considering the light coming from each of the two slits as sources of light in phase, the dark spots on the screen are the points at which total destructive interference occurs between the two waves (i.e., the path length difference is equal to an integer number of half wavelengths). Conversely, the bright spots on the screen are the places of complete constructive interference (i.e., the path length difference is equal to an integer number of wavelengths). Here we are applying mostly a classical wave ontology of light; the interference pattern is the ultimate indicator of wavelike behavior, and we are attending to mostly wavelike properties of the light (e.g., wavelength). Our description also includes describing light as a beam or ray, which is similar to, but distinct from, a wave ontology. When attending to two different light rays we are not focused so much on the wavelike properties, but rather thinking about a light ray as something that travels in a straight line from one location to another. After establishing how a beam of light behaves in the double slit experiment, we then "perform" the experiment with single photons. Figure IV.2 shows a screenshot of the Quantum Wave PhET simulation [96], where a single photon can be shot through the two slits. Each individual photon hits the screen in one (seemingly random) place and appears as a dot. However, after repeating this experiment many times with many individual photons, an interference pattern appears on the screen over time. That is, the location of detection of each individual photon was not in fact random, but related to the interference pattern that we see with the beam of light. In the lectures and homework, students in the class grapple with questions such as "Which slit did the photon go through?" and "Where was the photon just before

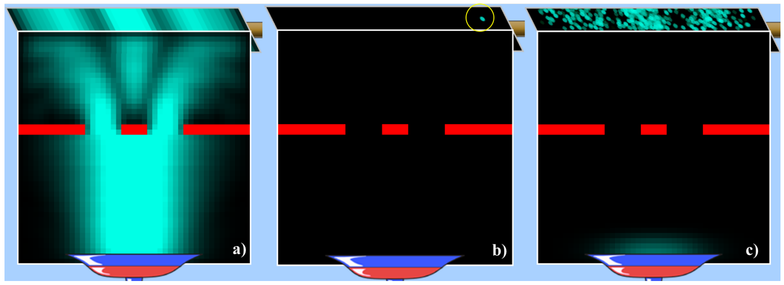



FIG. IV.2. Screenshots of the Quantum Wave PhET Simulation [96] showing the double slit experiment with: a) a beam of light, b) a single photon, and c) many photons over time.

it hit the screen?" In reasoning through this example, one might attend to particlelike (localized detection) or wavelike (interference pattern) characteristics of the photon. A beam of light is made up of *many* photons, so if light acts as a wave then an individual photon can be thought of as a little chunk of a wave, yet it still has non-wavelike properties such as localization on the screen. We can also conduct this experiment with individual electrons, and achieve results similar to those of individual photons. In the modern physics class, this is one example of how we introduce matter waves.

A Mach-Zehnder interferometer can be thought of as analogous to the double slit experiment. In the modern physics class, students investigate how single photons behave in this interferometer, first with only a single beam splitter (Experiment 1), then with two beam splitters (Experiment 2). As shown in Figure IV.3, there is one path from the source to the first beam splitter. There the light (or single photon) can be reflected and/or transmitted to mirrors A and B. Each mirror reflects the light (or single photon) toward the second beam splitter (in Experiment 2) at which point the light can be reflected and/or transmitted into detectors A and B. In Experiment 1, with only one beam splitter, there is no mechanism for interference of the two paths after the first beam splitter. Individual photons are either detected in detector A or B, with a 50-50 probability. If a photon is detected in detector A, we can infer that it was reflected at the beam splitter and reflected by mirror A. If it was detected in detector B, we can infer that it took the other path. Thus, with no interference, we can infer "which path information" based on where the photon is detected. Because the detector counts are split roughly 50-50 between the two detectors, we say that the photon is acting particlelike in Experiment 1 (i.e., a classical particle would have the same experimental results, and would take one path or the other). In Experiment 2, the second beam splitter provides the mechanism for interference between the two paths after the first beam splitter. Inserting a phase shifter in one of the paths creates an effective path length difference. Manipulating the path length difference changes the probability of detection at each of the detectors, resulting in an interference pattern at each detector. With the interference, there is no longer “which path information” (i.e., a photon detected at detector A could have been reflected at the first beam splitter and transmitted at the second, or vice versa). Because of the interference, we say the photon is acting like a wave in Experiment 2. However, each individual photon is only detected in one detector (a particlelike characteristic). Analogous to the double slit experiment where each photon is detected in a single spot on the screen but over time the interference pattern appears, in Experiment 2 each photon is localized in one detector, but over many counts the interference due to the difference in phase is apparent at the detectors. Following these two experiments, we introduce Wheeler’s delayed choice experiment [77] as experimentally conducted by Grangier, Roger, and Aspect [108] as Experiment 3. In this scenario, the setup begins in Experiment 1 with one beam splitter, and then after the photon has passed the first beam splitter, the second beam splitter is inserted into the experiment, to end up in Experiment 2. Students learn that the results of wavelike interference in Experiment 3 disprove a local hidden variable interpretation of QM because if the photon had a hidden variable it should have been acting like a particle and not been able to switch its behavior after passing through the first beam splitter. Students in our class grapple with the details of these experiments, and also with what the results tell us about the nature of photons. Is a photon particlelike in some instances and wavelike in others (parallel ontology)? Or is it something else entirely that draws on some particlelike and wavelike characteristics (blended ontology)?

The third topic area is quantum tunneling, a phenomenon in which an electron in one wire can appear in a second nearby wire despite not having enough energy to overcome the potential barrier (air gap) between the wires. We model the system of two wires as two one-dimensional finite square wells with a barrier in between them, as shown in Figure IV.4. The wave function of the electron is given by solutions to the Schrodinger equation. We touch on time dependence in the modern physics class, but primarily have the students work only with the *time independent*

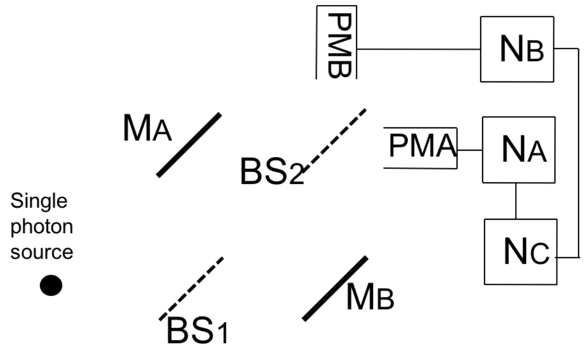



FIG. IV.3. Schematic of a Mach-Zehnder interferometer with a single photon source. $M_A$ and $M_B$ are mirrors, $BS_1$ and $BS_2$ are beam splitters, $PM_A$ and $PM_B$ are photomultiplier detectors, and $N_A$, $N_B$, and $N_C$ are counters for detectors A and B, and coincidences between the two.

Schrodinger equation. In our examples, the total energy of the electron is set to be greater than the potential energy of the electron in the wires (which is set to zero), and so the solutions are complex exponentials, or sinusoids. In the air gap in between the wires, the total energy of the electron is set to be less than the potential energy of the electron outside the wire (determined by the work function of the metal). Classically, the electron does not have enough energy to get out of the wire, but quantum mechanically, it is possible to measure the electron in the second wire. That is, the electron can tunnel through the barrier to reach the second wire. The solution to the Schrodinger equation in the air gap is a real exponential, suggesting that there is some probability of finding the electron in the gap, and that probability dominantly exponentially decreases as the electron moves farther into the gap. In this course, compared to the other topic areas, there are fewer opportunities for students to directly consider ontology in tunneling, but they do grapple with questions of interpretation which have the potential to elicit ontological reasoning: What does it mean physically for the electron that the wave function decays through the barrier? Where is the electron?

### 4. Codebook

Starting from an initial framework of dynamic ontologies which was developed through analysis of students' collective oral reasoning around the three topic areas discussed above [61], we developed

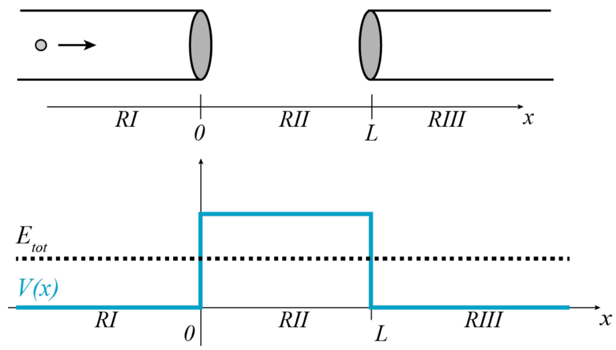



FIG. IV.4. System of two wires with an air gap in between that we model as a finite square well. The potential energy is given by $V(x)$. The total energy, $E_{tot}$, is greater than $V(x)$ inside the wires (Regions I and III) and less than $V(x)$ in between the wires (Region II) in the "classically forbidden region".

a codebook to code individual, written student responses for the nature of the ontologies they were using. Through an iterative process of creating, applying, and refining the codes, we refined both the codebook and the framework itself. In this refinement, the description of the ontological structures was expanded and relabeled; Figure IV.1 is the final result of this process. There are two main elements for which we coded each response: *ontology* and *ontological structure*. The ontology refers to how the *entity* (photon or electron) is being categorized. We infer ontology from the student's language—what words are they using to describe the entity [44, 63, 64] or what properties and characteristics are they assigning to the entity in a given moment or context. There are six *ontology* codes: "particle", "wave", "both particle/wave", "something else", "not particle", and "not wave". *Ontological structure* refers to the nature of the ontology (Figure IV.1)—is it an application of a single stable ontology (unitary), or switching back and forth between two or more separate ontologies (parallel), or a blending together of multiple ontologies to create a new kind of entity all together (blended)? Again, we note the fourth possible category (parallel blends) does not appear in our data set given our focus on analysis of individual episodes.

The ontology codes were a combination of *a priori* and emergent, and the ontological structure codes came directly from the framework for distinguishing different kinds of dynamic ontologies. For each aspect, there was also a "can't tell" code, used when we did not have evidence of a specific ontology or structure. Tables IV.2 and IV.3 provide examples for each code (see Appendix A for the complete codebook). The majority of questions in our data set are written responses, but there are also some MC questions. Although in a MC question we do not have access to a student's reasoning, we still code these responses for ontology and ontological structure. Each MC response is coded as *consistent with* a given ontology. In our data set, all of the MC responses are coded as having a unitary structure due to the way the answer choices were written. We could, however, imagine writing MC responses that were more nuanced and consistent with parallel or blended ontologies. Because we are primarily interested in the nuances of student reasoning, we do not focus on the MC questions, although we leave them in the data set for completeness and because they help us to answer the question of how the prompts might impact the ontologies we see students using.

TABLE IV.2. Codes for ontology—How is the entity being categorized?

| Ontology | Evidenced by | Example |
|---|---|---|
| Particle | Classical particle characteristics: localized entity (single location), bouncing, mass, etc. | "The photon ends up at one specific location." |
| Wave | Classical wave characteristics: interference, reflection/transmission, non-localized, etc. | "The single photon must travel through both slits so that it can interfere with itself to produce the interference pattern." |
| Both particle / wave | Both particle and wave language, "particle that interferes w/itself", "wave when travels, then detected in 1 spot", could be either parallel or blended. | "A single photon will be superimposed in both paths after it is split by the beam splitter and has an equal probability of ending up in detector 1 as it does in landing in detector 2." |
| Something else | May be more nuanced than the above options: e.g., a blend of classical particle characteristics and quantum properties ("quantum particle"), a particle but described by probability and not localized | "The position of the particle is anywhere throughout time. This is different [from a classical particle] because the position of a classical particle is not probability-based." |
| Not particle | Defined only by *not* having particle characteristics (with no mention of what it is) | "The electron is more probable in [one region] compared to [another region]. This is different from a classical particle which cannot overcome a hill without the required kinetic energy." |
| Not wave | Defined only by *not* having wave characteristics (with no mention of what it is) | "The wave is both reflected and transmitted at the same time, [but] a single photon will either be reflected or transmitted." |

### 5. Selecting questions and coding responses

In our analysis, we only include questions about photons and electrons. Questions are drawn from student homework, exams, and pre- and post-surveys. There are 24 individual questions

TABLE IV.3. Codes for ontological structure—What structure is used in the particular moment/context?

| Ontological structure | Evidenced by | Example |
|---|---|---|
| Unitary | Single ontology applied throughout response, e.g., only attends to particle characteristics | "[The photon] is like a particle because it only goes into one detector." |
| Parallel | Switching back and forth between two or more stable/robust ontologies; Often separated by time; e.g., "sometimes particle, sometimes wave", "travels as wave, then detected as particle" | "[Photons are] waves when [they] go through slits and particles when detected at [a] screen." |
| Blended | Combines elements from input ontologies and new meaning emerges to create a new category; Often referring to the same entity at the same moment in time; e.g., hybrid of particle and wave | "The photon goes through both slits and interferes with itself, ending in a spot that relates to the interference pattern." |

spanning the three topic areas—6 about double slit, 3 about tunneling, and 15 about Mach-Zehnder. We selected questions for which we had an opportunity to make inferences about ontologies and ontological structures in students' responses; in many cases, this means that the questions ask directly about ontology. While we attempted to select questions that span the various entities (photon, electron), ontologies (wave, particle, etc.), and ontological structures (unitary, parallel, blended), given the naturalist study in a real class we are not entirely able to disentangle these variables. As such, we present preliminary investigations into variation among how students use ontologies and ontological structures to reason about quantum entities.

Each question was coded for entity, ontology, and ontological structure; in some instances, during the analysis, we clustered questions together and coded the cluster overall to make stronger inferences about a student's ontology. For example, we clustered the following series of questions about the double slit experiment: *16) Briefly discuss which aspects of this experiment are consistent with photons acting like classical particles. 17) Briefly discuss which aspects of this experiment are consistent with photons behaving like electromagnetic waves. 18) Briefly discuss which aspects of this experiment are consistent with photons behaving like a particle and a wave at the same moment in time.* Questions were only clustered when they appeared next to each other on an assignment (homework or survey), and when the content of the questions fit together or built off of one another (i.e., asking about different elements of the same experiment or phenomenon). When considering the clustered questions, the 24 questions are condensed into 16 overall items—4 for double slit, 2 for tunneling, and 10 for Mach-Zehnder. In our analysis, we look at the individual questions separately as well as the clusters. Across the 28 students and 24 individual questions, there were 626 total responses, of which we coded 523 (the remainder were coded as "can't tell"). When considering the 16 clustered items, across the 28 students there were 418 total responses, of which we coded 349 (the remainder were coded as "can't tell"). That is, there are 523 total *coded* responses, and where possible we cluster associated questions, yielding 349 coded responses.

The first two authors independently coded subsets of the data, and then all three research team members came together to discuss any discrepancies. We repeated this iterative process, refining and elaborating on the codebook each time, until we had consensus on how to consistently apply each of the codes. On our final round of independent coding, for the ontology codes, we had 82% agreement with a Cohen's kappa of 0.76, and for the structure codes, we had 80% agreement with a Cohen's kappa of 0.71 (considered to be a good level of agreement [109]). After discussion, we had 100% agreement. The lead author completed the entirety of the coding, discussing with the team each time a difficult-to-code response arose.

### 6. Coding prompts

In order to investigate the extent to which the wording and framing of the question prompts impacts students' ontological reasoning, we coded each of the prompts as leading towards a specific ontology or ontological structure or not. There are several aspects that might contribute to how leading the prompts are: format of the question (MC versus written response), wording of the question, local framing (location of question among preceding questions in a series, and implied meanings in a question), or framing in the broader course context (how a topic was introduced, and messaging and culture of the course). We coded each prompt twice, once for whether it was leading towards a specific ontology, and then for whether it was leading towards a specific ontological structure. In coding for the "leading-ness" of prompts, we focus mostly on the wording of the questions, but also the local framing to some extent. We coded the prompts after the codebook (and thus our understanding of what constituted each ontology and structure code) was finalized, but before coding the entire sample of 28 students. This approach helps minimize the extent to which familiarity with the spread of student responses to each question could influence the way we coded the question itself. We coded the prompts based on the words used, as well as the instructor's intent in asking the question (the questions in our data set have been used in this particular course for many semesters, and were written primarily by the last author, building on years of course transformation efforts [43, 73]). For example, the question *"Which aspects of this experiment are consistent with photons acting like classical particles?"* is leading toward both a particle ontology and a unitary structure, while *"Does an electron lose energy when it tunnels?"* is not leading. A complete list of the questions and how they were coded is available in Appendix A.

#### 7. Analysis

There are two main approaches to analysis applied after completing all of the coding of student responses and prompts. First, we looked at the patterns of use of ontologies among the clustered questions. We looked at the distributions of ontologies and ontological structures overall and by entity, topic area, and question format. Where applicable, we performed chi-squared tests of homogeneity [110, 111] to determine if two distributions were statistically significantly different from one another. Next, we investigated the impact of the prompts. Looking at the separate individual questions, we compared the results of the distributions of codes (for ontology and ontological structure) for the two separate categories of leading and non-leading questions. We used a two-sample chi-squared test of proportions [110, 112] to determine if the percentage of a given code was statistically significantly different between the leading and non-leading questions.

### D. Results and discussion

#### 1. Patterns of use

In applying the framework to individual responses in our data set we are able to observe variation in students' ontologies and ontological structures. Beginning from a position of valuing the messiness of student reasoning, we demonstrate students' capacities to reason using varied ontologies and ontological structures. In presenting these results we also demonstrate the utility of the framework to capture differences in the dynamics of students' ontologies, in both collective and individual reasoning, oral and written reasoning, as well as looking at both separate individual questions and clustered items.

In this section, we present analysis of overall patterns of use of ontologies and ontological structure. We consider the coding of clusters rather than individual questions (where applicable) because this grouping gives us the most information about a student's ontology in a given moment or context. This analysis includes 349 coded responses. There are three results that all illustrate one main point—students are capable of drawing on multiple ontological resources, or using their ontological "toolkits". That is, we see students using a variety of ontologies and structures across the three topic areas for both photons and electrons. Much like in the collective oral reasoning in prior work [61], we see flexible use of ontologies in the individual written student work.

The first result is that we observe a variety of ontologies used for each entity. Figure IV.5 shows the distribution of ontologies for photons and electrons, including both written and MC questions. For photons, the primary ontology students used was "both particle/wave", and for electrons the most common ontologies were "wave" and "something else". For each entity, the ontologies are different from what one might expect with a classical perspective; both photon and electron might be considered more particlelike from a classical view. Here we observe different dominant ontological categories, and clearly varied use within each entity. We see that students' ontologies are not bound by entity (i.e., there is a distribution of multiple ontological categories for both photons and electrons). However, the patterns of use vary by entity. In our dataset, this variation is in part driven by the difference in content area as well as the format of the question. We return below to explore some of these differences.

The second result is that in individual written work, students use three of the ontological structures described by the framework: unitary, parallel, and blended. Attending to just the written questions (excluding MC), the frequencies of each ontological structure code are shown in Figure IV.6. These outcomes demonstrate the ability of students to use fixed ontological categories

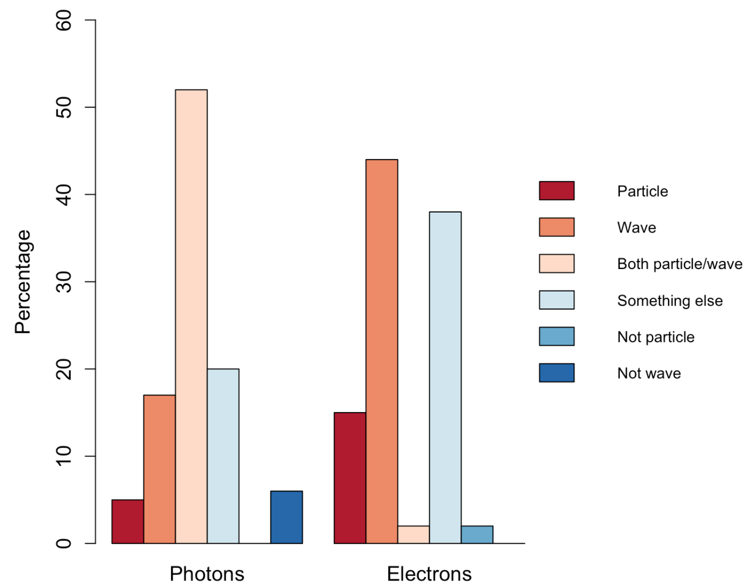



FIG. IV.5. Percentage of ontology codes, by entity

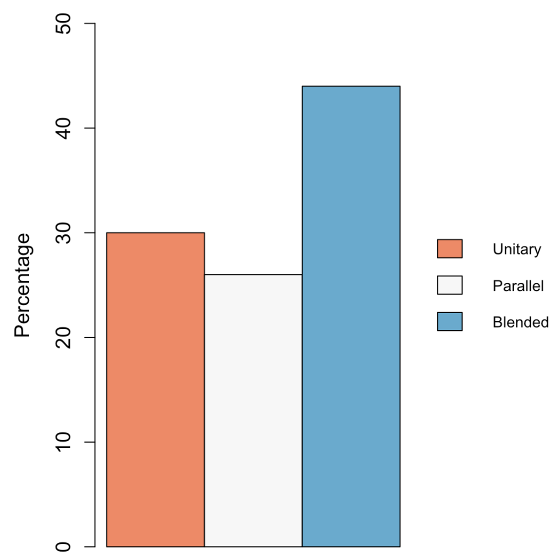



FIG. IV.6. Distribution of ontological structures among written responses (excluding MC).

(in this case 30% of the responses are unitary), to move rapidly between two or more ontological categories (26% are parallel), and to create new ontological structures (44% are blended) to reason about quantum entities.

Not only do students use multiple ontological structures, but we observe different patterns of use by entity. This is the third result which demonstrates students' capacities for drawing on multiple ontological resources. Examining the overall outcomes of Figure IV.6 above in more detail, we can disaggregate the distribution of ontological structures by entity and by topic area; this is presented in Table IV.4 below. In all cases (entity and topic area), we observe that students use multiple ontological structures. The distributions of ontological structure for photons and electrons are statistically significantly different from one another ($p = 0.003$ with Pearson's chi-squared test) [13]. Of the written questions, the most common structure for photons is blended, with an equal split between unitary and parallel. The codes for electrons are roughly split 50-50 between unitary and blended. One difference that jumps out in these results is that there are no tunneling, and thus no electron, responses coded as parallel. It may well be that entities and/or topic areas lead students to differential use of ontological structure. Perhaps it is tempting to make a conclusion about students not using parallel ontological structures when reasoning about electrons, but here, as noted in the Methods section, we have to keep in mind the strong connections between entity, topic area, and question format. Here, we have considered only the written-response questions—100% of responses to MC questions are coded as using a unitary ontological structure, due to the nature of the MC answer choices. In particular, had there been written-response questions to a double slit question with electrons it is possible we would see students using a parallel structure for electrons. This difference by question format foreshadows some of the results around impact of prompts presented in the next section. Interactions between content area, entity, and how the framing of questions impacts student use of ontologies and ontological structures is explored further below.

TABLE IV.4. Distribution of ontological structures by entity and topic area among the written-response questions (excluding MC).

| | Unitary | Parallel | Blended |
|---|---|---|---|
| Photons | 29% | 28% | 43% |
| Double slit | 33% | 15% | 53% |
| Mach-Zehnder | 27% | 32% | 41% |
| Electrons | | | |
| Tunneling | 46% | 0% | 54% |

The distributions of student responses coded for ontology and ontological structure illustrate that students in this particular pedagogical environment are capable of flexibly using different ontologies.

[13] This statistic comes from doing the analysis of just the written questions (excluding MC), but the distributions are also statistically significantly different from one another when looking at written and MC questions together ($p = 5\text{E-}8$).

In the pilot coding study [105] we confirmed that this variety also exists *within individual students' responses*, a crucial piece of information that tells us the variation is not only between students but also within responses from a given student. Here, we see this variation on a large (class-wide) scale. We see some differences in ontological structure use between photons and electrons, and note that there are likely strong intersections among entity, topic area, wording and framing of the prompt, and question format. To begin an investigation of the variation in ontologies documented here, and the intersection among some of these factors, we look at one particular element, the role of instructional prompts, and examine by content domain and entity.

### 2. Impact of prompt framing

To begin to examine contextual factors that influence when and how students use various ontologies and ontological structures, we examine the impacts of how the question prompts are framed. We compared the distributions for students' ontologies and ontological structures between prompts that were coded as *leading* versus *non-leading*. Some questions are leading toward both a specific ontology and ontological structure (e.g., designed to elicit a blended "both particle/wave" ontology), and some are leading toward a specific structure but not specific ontology (e.g., MC question where each answer is consistent with a unitary ontology but the question itself does not lead toward a specific answer). In our study, we find no questions that are leading toward a specific ontology but not ontological structure [14].

To examine the impacts of how specific question framing can influence student response, we primarily analyze individual questions rather than clusters. In considering this finer grain size, we can investigate the impact of the wording of individual prompts as well as how they are organized sequentially to form clusters. The results of how questions influence students' use of ontologies and ontological structures (for all entities, all topics, and all formats) are shown below. In examining the influence of prompts on student reasoning, we present overall data and examples of specific prompts for each of the two areas of focus: ontology use, and ontological structure use.

[14] We are not suggesting that it is not *possible* for a question to be leading to ontology but not ontological structure, just that we did not observe that in our context.

### *a. Leading to ontology*

As seen in Figure IV.7, there is strong indication that whether a question prompt is coded as leading or not is associated with students' use of ontology. That is, the way we ask questions informs the way students respond. For the non-leading questions, the ontology codes are distributed across most of the categories—primarily "wave", "both particle/wave", and "something else", with a few "particle" and "not wave". When we ask questions about ontology that do not guide or encourage students toward a specific ontology, we see that students use a variety of ontologies in their responses. (This result is consistent with evidence in the prior section that students are capable of using a variety of ontologies.) Turning to the questions that are coded as leading, there are prompts that are leading towards "particle" (2 prompts), "wave" (2 prompts), either "particle" or "wave" (1 prompt), "both particle/wave" (2 prompts), and "something else" (2 prompts). For those questions coded as leading towards a particle ontology, students primarily use a particle ontology in their responses. Similarly, in each of the other categories of questions that are ontologically-leading, we observe students primarily use the associated ontologies in their responses. For each of these groupings, we compared the percentage of responses coded with a given ontology between leading and non-leading questions; in each case, the fraction of responses using a given ontology for a leading question is statistically significantly different from that in the non-leading case at the $\alpha = 0.05$ level via a chi-squared two sample test of proportions. For the one prompt coded as leading toward either a "particle" or "wave" ontology, we compared the combined particle and wave percentages between the leading and non-leading questions.

To illustrate how these overall results appear for individual questions, we present an example of a cluster of three double slit questions. On a conceptual survey at the end of the semester students were asked the following three sequential questions: *16) Briefly discuss which aspects of this [double slit] experiment are consistent with photons acting like classical particles. 17) Briefly discuss which aspects of this [double slit] experiment are consistent with photons behaving like electromagnetic waves. 18) Briefly discuss which aspects of this experiment are consistent with photons behaving like a particle and a wave at the same moment in time.* Question 16 is coded as leading towards particle because it asks students to attend to particle characteristics. Likewise, question 17 is

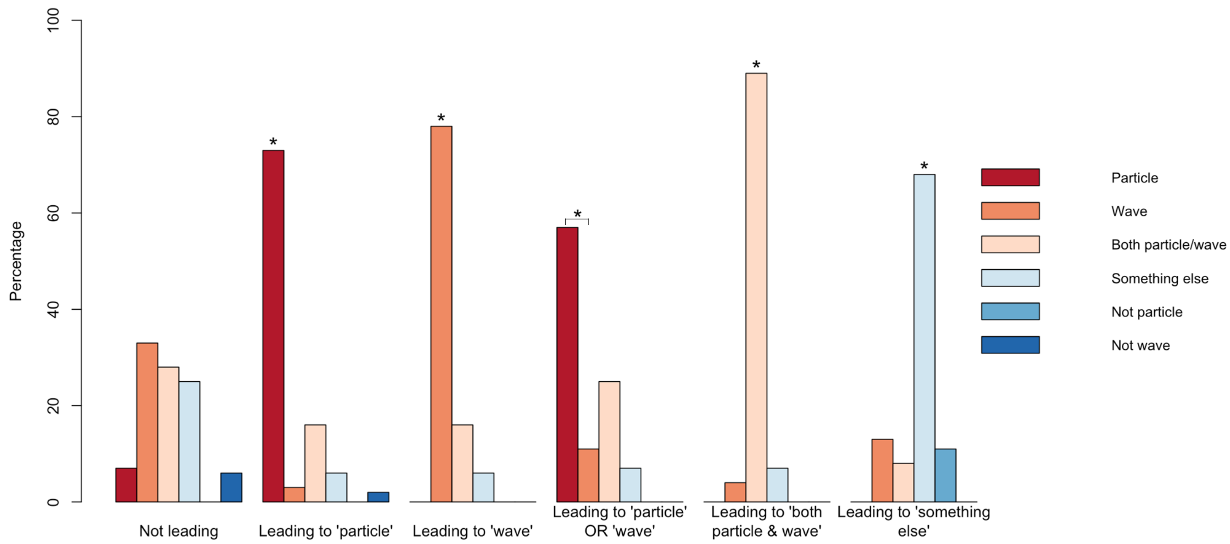



FIG. IV.7. Distributions of ontologies for leading and not leading questions. In each case, for prompts leading to a specific ontology, the percentage of codes for that ontology are statistically significantly different from the percentage of that ontology among non-leading questions (denoted by *).

leading towards wave. Question 18 is coded as leading towards “both particle/wave” because it asks students to consider the ways in which particle and wave aspects are present at the same time.

As shown in Figure IV.8, the results for student use of a given ontology align with the coding for the leading nature of the questions. Responses to question 16 are primarily “particle”, responses to question 17 are primarily “wave”, and responses to question 18 are primarily “both particle/wave”. It is interesting that there were no “not wave” codes for question 16 and no “not particle” codes for question 17. Compared to the non-leading questions within the double slit topic area, the percentage of “particle” codes for question 16, “wave” for question 17, and “both particle/wave” for question 18 are all statistically significantly different.

### *b. Leading to ontological structure*

Similar to the case of questions leading to given ontologies, the leading-ness of prompts to a given ontological structure appears to impact student use of ontological structures. See Figure IV.9. There are nine prompts coded as leading toward a unitary structure, one prompt coded as leading toward a blended structure, and three prompts coded as leading toward either parallel or blended structures. For questions that are leading toward a specific structure, the coded responses are more

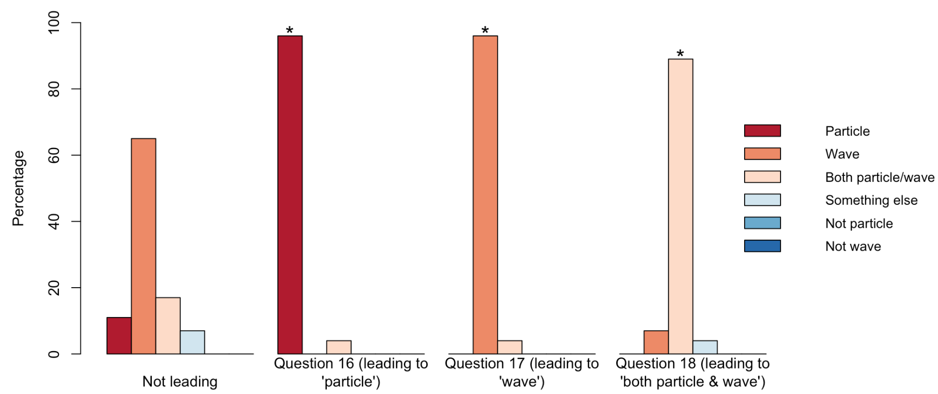



FIG. IV.8. Distribution of ontologies for one cluster of three double slit questions leading to particle, wave, and both particle/wave ontologies respectively.

skewed toward that structure. Again, we compared the percentage of the structure code in question between the leading and non-leading questions and they were all statistically significantly different (at the $\alpha = 0.05$ level via a chi-squared two sample test of proportions). For the prompts leading toward either "parallel" or "blended", we compared the combined percentages of "parallel" and "blended" codes between the leading and non-leading questions. Not only can the prompts lead students to specific ontologies, but they can also encourage use of specific structures. We see this same signal of the leading-ness of the prompts (for both ontology and structure) when looking at the clustered questions.

Among the questions that are coded as not leading towards a specific ontological structure, there is a distribution across the three structures, with unitary being the most common followed by blended and then parallel. We might expect that when not encouraged to use a specific ontological structure, students would apply a unitary ontology. (This is the most common and arguably easiest thing to do [69]. Experts do this all the time by attending to the most important or relevant characteristics of an entity in a given moment or context [54]). However, we code students as using a fair amount of blended (37%) and parallel (13%) ontological structure in non-leading questions. This result is suggestive, that in a course that values student development of more nuanced and complicated ontological structures, students use them, even when the questions are not leading to those structures.

Once again, to illustrate how these overall results appear for individual questions, we revisit the

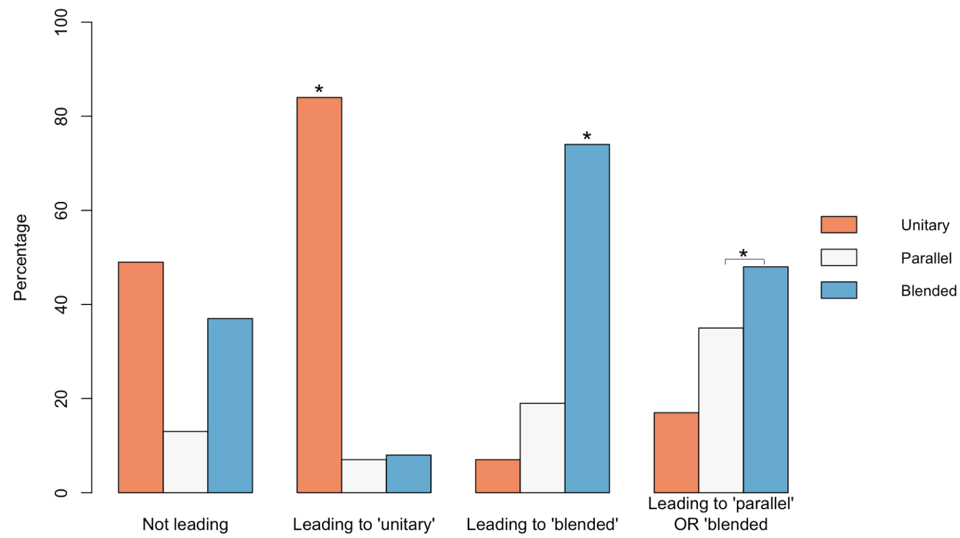



FIG. IV.9. Distributions of ontological structures for leading and not leading questions. In each case, for prompts leading to a specific ontological structure, the percentage of codes for that structure are statistically significantly different from the percentage among non-leading questions (denoted by *).

double slit questions from the post-survey. Questions 16 and 17 are coded as unitary because they ask how the photon behaves like a given object (particle or wave). Question 18 is coded as blended because it asks how particle and wave characteristics are present at the same time [15]. The student responses to questions 16 and 17 (leading toward unitary) are coded primarily as unitary while the responses to questions 18 (leading toward blended) are primarily coded as blended. Again, in each case the percentage of responses coded for an ontological structure in the leading questions is statistically significantly different from the percentage of that code among non-leading double slit questions (at the $\alpha = 0.05$ level via chi-squared two sample test of proportions). See Figure IV.10.

Among all questions and all topic areas, the "something else" ontology code comes up 25% of the time for the non-leading questions (Fig. IV.7). Most often when a response was coded as "something else", we labeled it as "quantum particle" or "quantum wave"; that is, the student's response described an entity that was either a classical particle or classical wave combined (in either a parallel or blended fashion) with other quantum properties like a wave function or probability. This is an

[15] Of note, the course context in which the students and this question are embedded in might also leave room for a parallel ontology. The instructor of the class (NDF) taught that one conclusion we can make from the delayed choice Mach-Zehnder experiment is that photons do not act like particles and waves at the same time. Thus, the broader framing of the class could encourage students to reject question 18 and use a parallel ontology to say photons behave like both waves and particles, but not at the same time. However, the message in class that the particle and wave characteristics are not simultaneous was given in the context of the Mach-Zehnder topic area, while this question is about double slit (which to experts is analogous to Mach-Zehnder, but students often view them as two distinct domains). While the contexts in which our questions are situated undoubtedly matter [20], here we attend to just the wording and local framing of the prompts to investigate one piece of this broader educational environment. Thus, question 18 is coded as leading toward a blended both particle/wave ontology.

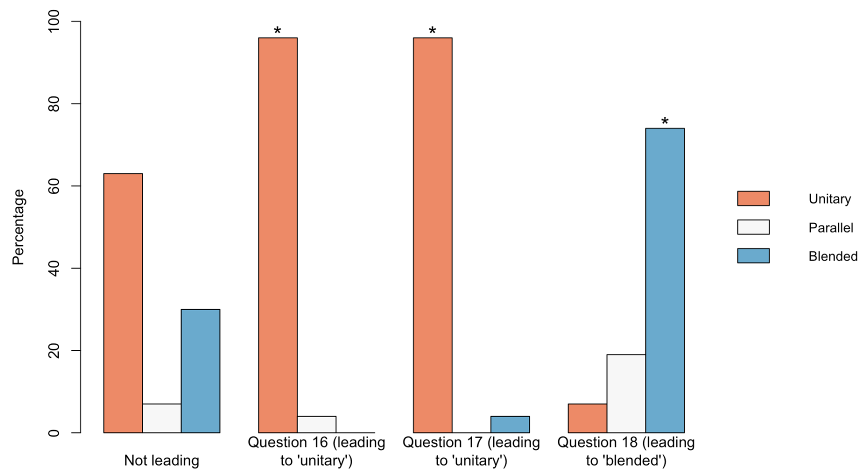



FIG. IV.10. Distribution of ontological structures for one cluster of three double slit questions leading to unitary and blended ontologies.

example of how the ontology and ontological structure aspects are inherently linked together to form a framework for identifying and describing students' ontological reasoning. Even when not encouraged or guided to do so, students make use of more complicated ontological structures. The pedagogical environment of this particular class works to support students in doing so, and this result is evidence of students' capabilities in that environment. The question of how much or what kinds of supports students need in order to make flexible use of ontologies in this way could be a topic for future study.

In our data set, blends arise more often than parallel ontologies. This leads us to question why—Is it a particular emphasis of the course, the specific topic areas we have chosen, the wording and framing of the prompts? Our current study is unable to answer these questions. What we can say, however, is that students make use of all three ontological structures even when not explicitly encouraged in the prompt to do so.

The results in this section suggest that the way students answer questions depends on how the question is worded or framed. One might also wonder if asking questions about different entities and within different content domains accounts for the signal of "leading-ness" we see in Figures IV.7 and IV.9. However, when we look at subsets of the data by topic area and also entity (photon versus electron), we still observe the "leading-ness" of the prompts. We present one example—the Mach-Zehnder content area—in the next section. There are *some* differences by entity and content area, and we suspect that content and context interact in complicated ways, but the overall point that we take from these results is that the way we ask questions matters. To begin to explore the interactions of the differences in students' ontology use by entity, content area, question format, and wording of the prompts, we review some specific examples.

### 3. Examining specific prompts and student approaches

In the prior sections, we found variation in the ontologies and ontological structures students use in individual written work, demonstrating students' capacities for flexible use of ontologies (in aggregate) both within and across content areas and entities. We also found variation in the use of ontologies and ontological structures associated with how questions are posed. This impact of question framing on students' ontology use across content domains and entities is notable. We now begin to explore the nuances of how these results differ by content area, entity, question format, and question framing, and how these factors interact with one another.

Overall, we see evidence of the "leading-ness" of prompts for the use of both ontologies and ontological structures (Figures IV.7 & IV.9). In the prior sections, we concretized the overall results by examining an example cluster of questions from the double slit content domain, illustrating what the role of prompts looks like at the level of an individual question. This same narrative is reinforced when we examine other topic areas. Here, we take a deeper look at the Mach-Zehnder topic area. In addition to presenting results that corroborate the overall findings—students' varied use of ontologies and ontological structures, and the leading-ness of prompts impacting student use of ontologies and ontological structures—this section begins to explore notable exceptions and variations from the broader patterns. Here, we first recapitulate prior results within the Mach-Zehnder topic area for both ontology and ontological structure. Then, we pull out two interesting examples and examine those specific prompts further.

All of the Mach-Zehnder questions analyzed here are about photons, and only one of the 15 is a MC question. The distribution of ontologies for all Mach-Zehnder questions are displayed in Figure IV.11. As before with the overall results and the double slit example, the coded responses to non-leading questions are distributed across several ontologies. That is, when the prompt does

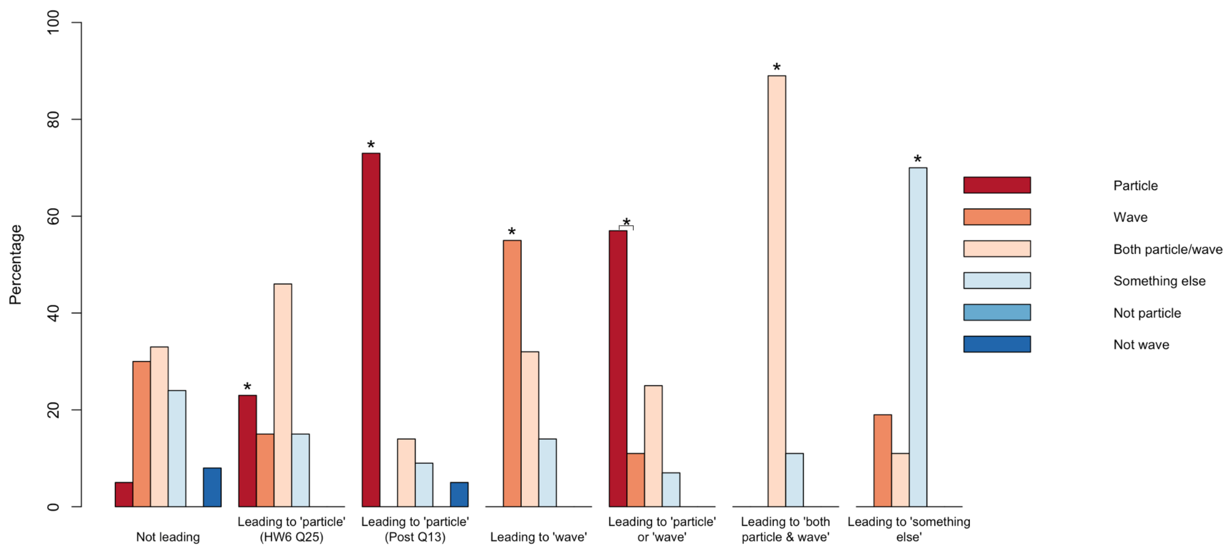



FIG. IV.11. Distribution of ontologies for leading and non-leading questions within the Mach-Zehnder content domain.

not lead students in a certain direction, students use a variety of ontologies. Of the 15 Mach-Zehnder questions, there are two coded as leading to "particle", one leading to "wave", one leading toward either "particle" or "wave", one leading toward "both particle/wave", and one leading toward "something else" (the remaining nine prompts are not leading). Again, we compared the percentage of codes for a given ontology between the leading and non-leading questions. For the prompt leading to either "particle" or "wave", we compared the combined percentages of "particle" and "wave" codes between the leading and non-leading questions. In each case, the differences were statistically significantly different (at the $\alpha = 0.05$ level via a chi-squared two sample test of proportions).

In this analysis, we did not combine the two questions that were leading toward "particle" because the results between the two are quite different. For question 13 on a conceptual post-survey (Post Q13), the results are primarily particle, as expected. For question 25 on the sixth homework (HW6 Q25), the ontology codes are primarily "both particle/wave" (46%), yet the percentage of "particle" codes is still higher (and statistically significantly different) for this question than for the non-leading questions. That is, we see a signal of the leading-ness toward a particle ontology when compared with the results from the non-leading questions, yet the modal ontology code is "both particle/wave". Further, the distributions of ontologies for HW6 Q25 and Post Q13 are

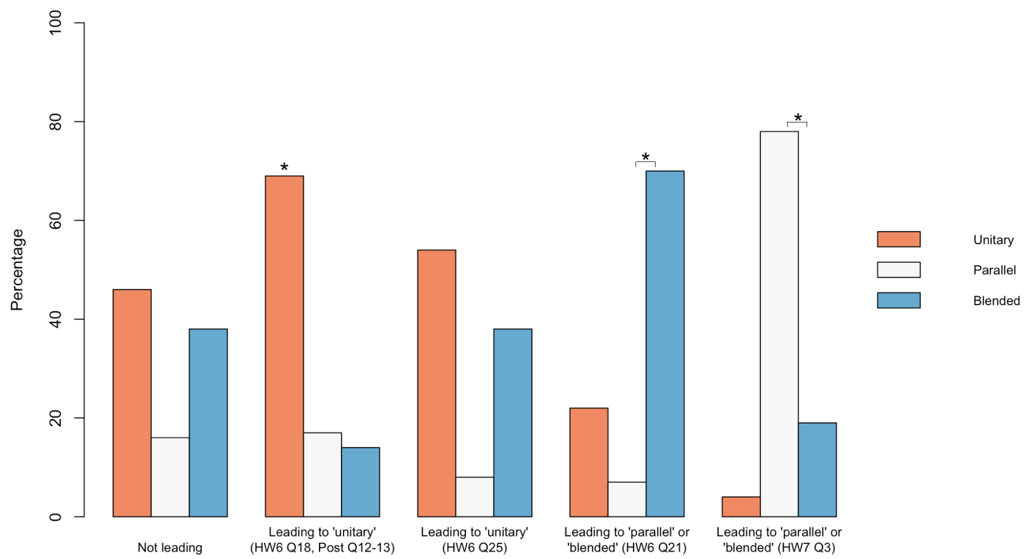



FIG. IV.12. Distributions of ontological structures for leading and non-leading questions within the Mach-Zehnder content domain area.

statistically significantly different from one another ($p = 3.1$E-13 with Pearson's chi-squared test). We wondered why the results for HW6 Q25, a question coded as leading toward "particle" were so different from Post Q13, also leading toward "particle". We investigate this result further in Section IV D 3a below.

We also observe a similar signal where the coding of prompts strongly aligns with the use of ontological structure. The distribution of ontological structures for the 15 Mach-Zehnder questions are shown in Figure IV.12. As before, with the overall results and the double slit example, the responses to non-leading questions are distributed across the three ontological structures. When the prompt does not lead students in a certain direction, students use a variety of ontologies in a variety of ways. There are five questions coded as leading to "unitary", and two leading to either "parallel" or "blended". Similar to the analysis of the ontology results, we present some of these questions separately in the analysis so as not to wash out differences in the results. In every case except one (HW6 Q25) the percentage of codes for the structure associated with the leading-ness of the prompt is statistically significantly higher for the leading questions.

The responses to the first three questions leading to "unitary" (HW6 Q18, Post Q12-13) are coded primarily as "unitary". The fourth question is HW6 Q25 that we mentioned above, and that we will explore further in Section IV D 3a below. The percentage of unitary codes for this question is higher than that among non-leading questions, but this difference is not statistically significant. The fifth question leading toward unitary is a MC question from the final exam (not shown in the graph), and 100% of the responses are coded as unitary. This is perhaps the strongest signal that the question framing can impact students' responses, although it is also arguably the least interesting since this difference derives from the format and not necessarily the wording of the prompt. The two questions leading toward either “parallel” or “blended” were from the sixth and seventh homework assignments (HW6 Q21 and HW7 Q3). Each of these two questions has a higher percentage of combined parallel and blended codes than those among non-leading questions (both statistically significant differences). However, as shown in Figure IV.12, the results for HW6 Q21 are primarily blended, while for HW7 Q3 they are primarily parallel. These distributions are statistically significantly different from one another ($p$ = 2.2E-16 with Pearson's chi-squared test). We wondered what about these two prompts, both coded as leading toward either “parallel” or “blended”, might result in such stark differences in students' use of ontological structure. We explore differences in these questions in Section IV D 3b below.

### *a. Leading prompt within broader framing*

HW6 Q25 (the prompt of which is shared below) is a notable example of a question for which the results are a bit unexpected. The question is coded as leading towards unitary particle, but as seen above in Figures IV.11 and IV.12 the most common ontology is “both particle/wave” and the most common structure is “unitary”. On the surface this is confusing because these two codes never overlap (in the codebook, “both particle/wave” is only ever coded as “parallel” or “blended”). We investigate these results further, but we must preface these results with the caveat that the sample size for this question is small (13 total coded responses) because many of the students' responses were coded as “can't tell”. Table IV.5 shows the full matrix of coded responses for this question. The most common ontology is “both particle/wave” (46% of codes), the majority of which were “blended”. The most common structure is “unitary” (53% of codes), which was almost evenly distributed amongst “particle”, “wave”, and “something else” (with a slight preference for “particle”). So, for the question that is leading toward unitary particle, the most common response was both particle/wave and blended (38%), while unitary particle was the second most common (23%).

TABLE IV.5. Matrix of ontology and ontological structure codes for HW6 Q25.

| | Unitary | Parallel | Blended | All structures |
|---|---|---|---|---|
| Particle | 23% | | | 23% |
| Wave | 15% | | | 15% |
| Both particle/wave | | 8% | 38% | 46% |
| Something else | 15% | | | 15% |
| Not particle | | | | 0% |
| Not wave | | | | 0% |
| All ontologies | 53% | 8% | 38% | |

In order to decipher these results, we look at the wording of the question and where it is situated within a larger assignment. The question is specifically about Aspect's delayed choice experiment [108]. In class, the students were introduced to the anti-correlation parameter ($\alpha$), which is a measure of the coincidences between counts from the two separate detectors. When the detectors are triggered simultaneously less frequently than they would at random, $\alpha$ is between 0 and 1, and we say photons are behaving (at least at detection) as classical particles (i.e., they are only detected in one detector at a time). If there are more coincidences than random, independent events ($\alpha > 1$), then the photons are acting as classical waves (i.e., detected simultaneously in both detectors). The results of the experiment showed that as the photon number became very small (approaching the single photon limit), $\alpha$ decreased, but was never exactly zero. The prompt in HW6 Q25 reads: *In your own words, explain what the anti-correlation parameter ($\alpha$) is, both in terms of its mathematical definition, and in terms of what it physically tells us, in the context of the single-photon experiments performed by Aspect. Why didn't Aspect measure $\alpha = 0$ if photons are supposed to be acting like particles?* In class, we explained that $\alpha < 1$ indicated a quantum, particlelike regime (i.e., individual photons are being detected in only one detector at a time) and discussed the real-world constraints of never having only a single photon in the apparatus at one time, and thus, $\alpha$ is never exactly equal to zero. This question was coded as leading toward unitary particle because of the stated ontology that "photons are supposed to be acting like particles", and a few responses (3 out of 13) were indeed coded as unitary particle. However, we will look more closely at the surrounding questions to consider this particular prompt in the context of a larger framing. As we mentioned above, the sample size for this question is small because of a high percentage of responses coded as "can't tell". This happened primarily when students described the anti-correlation parameter mathematically but did not relate it to any physical or ontological characteristics of the photon. Any statistical inferences we make from this question are limited by the sample size, yet considering this question as situated among other questions and in a specific course context still helps us to understand the possible impact of our prompts on students' use of ontologies and ontological structures.

HW6 Q25 was preceded by a series of about 10 questions about Mach-Zehnder experiments with one and two beam splitters. The question directly preceding this one (HW6 Q24) asked students to summarize and synthesize their findings of the prior questions in terms of how photons behave in the Mach-Zehnder experiments. The prompt, in part, reads: *In what sense do they behave like classical particles? In what sense do they behave like classical waves? Was there a case where the photons acted only like classical particles; or acted just like electromagnetic waves? Or must photons be something different to both electromagnetic waves and classical particles?* This question was coded as not leading to a specific ontology or structure. It prompts students to attend to particle and wave characteristics, but leaves room for "unitary" (only classical particle or only classical wave), "parallel" (switching between only particle and only wave), and "blended" structures (different from either particle or wave). "Not leading" is itself a category that can be further specified. There are questions that are considered "not leading" because: students are not asked directly about ontology, they ask about ontology but there is no specified ontological use, or as in the case of HW6 Q24, they ask students to consider *multiple* ontologies, none of which is preferred over another. The results of Q24 as compared to Q25, for both ontology and ontological structure, are given in Figure IV.13

The responses to Q24 were primarily "both particle/wave" and blended. When compared to all of the non-leading Mach-Zehnder questions (see Figure IV.11), the percentage of codes in Q25 are higher for "particle" and "both particle/wave", although for the "both particle/wave" ontology

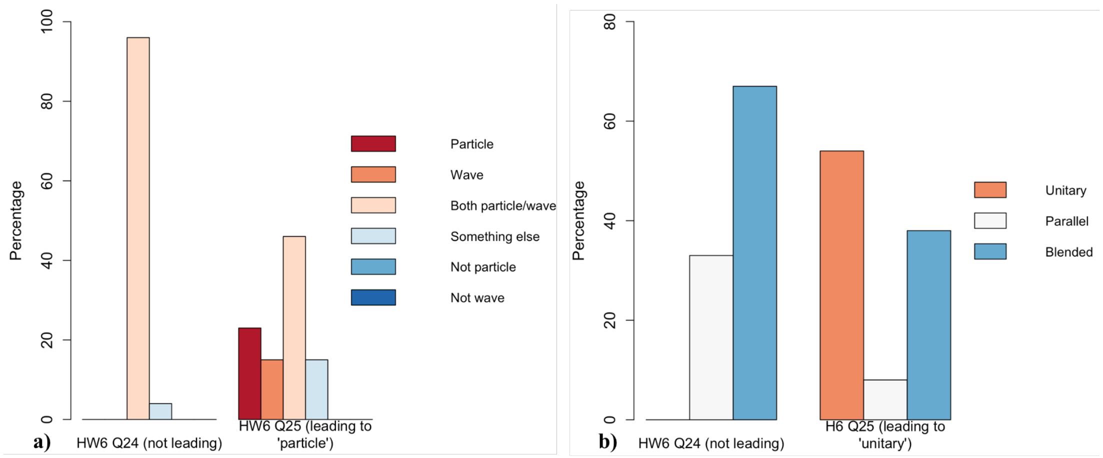



FIG. IV.13. Comparisons between HW6 Q24-25 for distributions of: a) ontologies and b) ontological structures.

codes this difference is not statistically significant. That is, we do see signal in Q25 for the "leadingness" of the prompt toward a particle ontology, yet this signal is much weaker than for other leading questions in the data set, evidenced by "both particle/wave" as the modal ontology despite the prompt leading toward "particle". The distribution of ontological structures for Q25 is not distinguishable from the distribution for non-leading questions. On its face, Q25 might encourage students to explain how a photon is like a particle, or perhaps how it is not like a particle (e.g., quantum particle). However, when considering Q25 as the culmination of a series of questions, and one that comes immediately after we have asked students to summarize what they know about the nature of photons in the context of Mach-Zehnder experiments, the question might instead be interpreted by students as: "Now that you have a solid and nuanced understanding of a photon's behavior in this context, explain why this paradox is not actually a paradox." Given this framing, it is perhaps unsurprising that the most common coded response was a blended particle/wave ontology. While we can write an individual prompt that is leading, we also have to consider where the prompt sits in the broader framing and context of an assignment, a topic area, or the class overall.

### *b. Leading to parallel versus blended*

As shown in Figure IV.12 and discussed above, HW6 Q21 and HW7 Q3 were both coded as leading toward either a parallel or blended structure, and yet they yielded notably different results for students' use of ontological structures. In the specific context of a Mach-Zehnder interferometer with two beam splitters, HW6 Q21 asks *"As the phase shifter is varied, how is the behavior of single photons similar to electromagnetic waves? How is their behavior different from electromagnetic waves?"* We coded this question as leading toward "something else" for ontology because it prompts students to attend to wavelike and un-wavelike characteristics, and students could either switch back and forth between these two ontological categories (parallel structure) or blend them together to form a new entity entirely (blended structure). HW7 Q3 was assigned a week later in the course and asks students to summarize what we can learn about the nature of photons from the single photon Mach-Zehnder experiments (Figure IV.3 and described in Section IV C 3 above): *"As discussed in class and in the readings, what do the two single-photon experiments performed by Aspect (Exp 1 & 2 from lecture) tell us about the nature of photons? How were these two experiments designed to demonstrate the particle and the wave nature of photons? When answering, don't concern yourself with technical details (such as how the photons were produced); focus instead on how the design of each experimental setup determined which type of photon behavior would be observed. How are the elements of Exp 1 & 2 combined in a delayed-choice experiment (Exp 3 from lecture), and what do delayed-choice experiments (along with the two Aspect experiments) tell us about the nature of photons?"* We coded this question as leading towards a "both particle/wave" ontology because it explicitly asks students about both the particle and wave nature of photons. Again, this question leaves room for different ontological structures, where particle and wave characteristics are kept separate from one another (parallel) or blended together (blended). The results for ontology codes for these two individual questions are shown in Figure IV.14. The ontological structure results are given above in the last two columns of Figure IV.12.

The modal ontology code for each question is as expected based on the leading-ness of the prompts—"something else" and "both particle/wave" for HW6 Q21 and HW7 Q3 respectively. The primary ontological structure code for HW6 Q21 is blended, while the primary code for HW7 Q3 is parallel. For HW6 Q21, the results suggest that students were most often talking about "wave" and "not-wave" characteristics blended together, not separating them out (in time or by

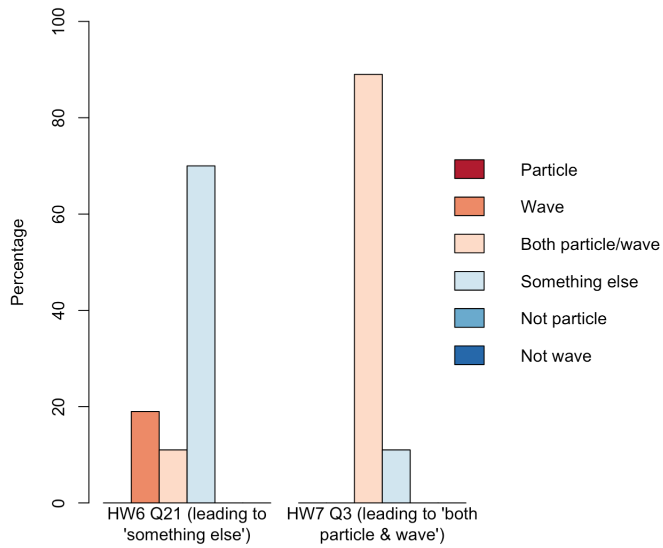



FIG. IV.14. Comparisons between HW6 Q21 and HW7 Q3 for distributions of ontologies.

context). We also might have expected some responses of "both particle/wave" if the students identified particle characteristics as the "not-wave" elements. The question does not explicitly guide students to a "both particle/wave" ontology and we see that reflected in the results. For HW7 Q3, the first half of the question encourages a parallel ontology because of the focus on the two separate experiments and "which type of photon behavior would be observed", suggesting separate particle and wave behaviors. The end of the question asks about how the delayed choice experiment combines the results of Experiments 1 and 2, thus possibly encouraging a blend of the particle and wave behaviors. Because the parallel structure dominates the results, it appears that the first part of the question may have set the stage for students' responses. Either students did not answer the second part of the question about the delayed choice experiment, or they answered it by keeping the particle and wave behaviors separate. This leads us to wonder how the results may have differed if the question had only asked about how the photon behaved in the delayed choice experiment (i.e., the second half of the question), or if the question has been separated into multiple questions. Another possible factor is the difficulty of the content. The delayed choice experiment is often difficult for students to grasp and HW7 Q3 requires the synthesis of many different conceptual elements. It seems plausible that a student who was struggling to understand the content might default toward a parallel rather than a blended ontology, especially if the beginning of the question encourages them to do so.

### E. Conclusions

We have refined and presented what we consider a full framework for examining students' dynamic ontologies and applied it to individual responses about photons and electrons across three topic areas. The first two major research questions of this study are: *1) Can we document evidence of dynamic ontologies in individual written work? 2) What are the patterns of use of ontologies in these contexts?* We initially developed the framework from data of students' collective oral reasoning, a setting that might naturally lend itself to nuanced and flexible reasoning. As such, it was not obvious that we would see the same kind of dynamics in individual work. Here we have extended prior work to investigate a representative sample of one modern physics class, where we do indeed see evidence of dynamic ontologies across these data. The results show a variety of ontologies used for each entity (photons and electrons), with some differences by format of the question. We also see three ontological structures (unitary, parallel, blended) used in the individual written work, again with differences by the question format, but also with some notable differences between entities—we observe all three structures for photons, but only unitary and blended ontological structures for electrons. In the distributions of ontologies and ontological structures we see strong interaction between entity, content domain, and question format. Overall, in an environment where students are encouraged to engage in nuanced reasoning, we see that the students are capable of using a variety of ontologies in a variety of ways.

To answer the third research question—*How does the wording and framing of prompts impact students' ontological reasoning?*—we coded the prompts for their "leading-ness" toward specific ontologies or ontological structures, and then compared the results between the leading and non-leading prompts. We see significant results, both overall and within each topic area, that the way we ask questions impacts the way students answer them. When a question is leading toward a "both particle/wave" ontology or a blended ontological structure, student responses are more likely to make use of a "both particle/wave" ontology or blended ontological structure, respectively. We see these results overall, in individual questions, and across topic area.

Of course, there are many factors that influence student responses and we have begun to examine the interrelated factors of course structure, framing of the question prompt, and sequencing of questions through two detailed examples. In one example, the leading-ness of a particular question, when considered individually, was quite different than when considered as situated amongst a series of questions. In another example, two questions that each left room for parallel or blended ontological structures saw different responses, possibly due to the beginning of one question setting the stage for a parallel rather than a blended ontology and overriding the remainder of the prompt and/or the possibility that parallel ontological structures are more accessible than blended structures for a difficult question requiring synthesis of many concepts.

Not only do we document variation in students' ontologies, but we take the position that such variation can be productive for student learning. Experts often use multiple ontologies for a given entity [54, 63, 71], with a fluency in determining when each ontology is most appropriate or useful and identifying the limitations of each ontology. Student use of different ontologies and ontological structures within and across contexts could be an indicator of such sophisticated reasoning, or *meta-ontological competence*. Our data suggest that students do make use of their ontological toolkits, and as researchers and instructors we ought to value and leverage this flexibility to support student learning.

As instructors, we should be aware that students are capable of flexible ontological reasoning (i.e., they are capable of making use of a variety of ontologies in a variety of ways within and across contexts). By documenting this flexible use, we work towards leveraging students' capabilities in support of student learning and development. Beyond recognizing students' capabilities for flexible use of ontologies, instructors should strive to support students' dynamic (and sometimes messy) reasoning. This can be done by using questions that ask specifically about ontology, asking questions that leave room for (or lead towards) a variety of ontologies and structures, or using resources like tutorials [75] that work to support students' meta-ontological competence.

# Chapter V. Creating tutorials to support ontological reasoning

As a curricular application of the above studies of students' flexible use of ontologies in a modern physics context, we participated in a collaborative effort between University of Colorado and University of Maryland (UMD) to write modern physics tutorials and associated instructor guides (given in Appendix B). The materials presented and discussed in this chapter are the result of collective work by Benjamin W. Dreyfus, Jessica R. Hoehn, Erin Ronayne Sohr, Noah D. Finkelstein, Ayush Gupta, Andrew Elby, and Kathleen Hinko. The examples of student reasoning are the result of data collected and analyzed by JRH. The primary goal of this suite of ten tutorials is to support students' *meta-ontological competence*—their ability to reason using different ontologies, to decide which ontology they should use when, and to metacognitively reflect on their ontological reasoning. Related to this primary goal, we also designed the tutorials with the goal of supporting students' content mastery, reflective ability, and group work and collaboration skills. For the most part, the tutorials are not meant to *introduce* new concepts, but to build on prior learning. That is, we expect students to be at least somewhat familiar with the topic(s) before engaging with the tutorial. Supporting meta-ontological competence, reflection, and group work—all goals of the tutorials—can help reinforce content mastery. The tutorials were designed for a sophomore-level modern physics environment. We designed and implemented them with mostly engineers majors, but have also seen them spark fruitful discussions among physics majors.

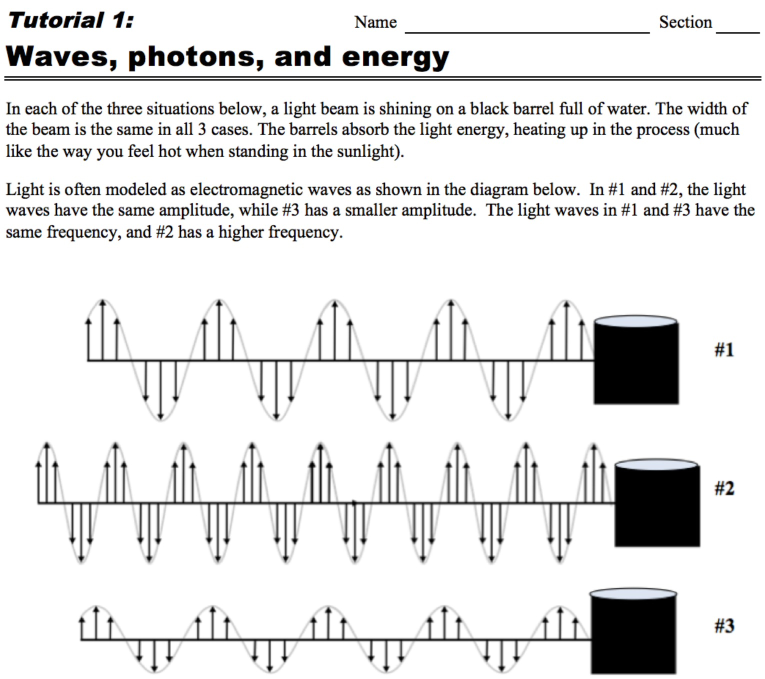
On the CU side of the collaboration, the tutorial design began with many of the prompts from the focus group study presented in Chapter III. In preparation for the focus groups each week, we assembled questions on topics related to the modern physics course that would lead students to conversations about ontologies of quantum entities (i.e., how does a photon behave in a double slit experiment?). Informed by how students in the focus groups engaged with the questions, we refined them and turned them into tutorial-style prompts. Drawing on our experiences as instructors, the results of the CU focus group study, as well as student interviews and focus groups at UMD, we engaged in a collaborative and iterative process of refining the tutorials.

Here, we summarize seven of the ten tutorials (for which JRH played a primary design role or became familiar with through implementation in our CU modern physics course) to externalize further some of the design principles, focusing primarily on the ways in which they embody the approach of supporting students' dynamic ontological reasoning. For four of the tutorials (1, 2, 3, and 6), we also present examples of student reasoning from focus groups to show how students engage with these materials and provide evidence that these prompts *can* elicit the kind of messy and dynamic ontological reasoning we are interested in supporting. The seven annotated tutorials, along with the corresponding instructor guides, are available in Appendix B. The full suite of tutorials and instructor guides (without annotations) are available for download on PhysPort [75].

**Tutorial 1: Waves, photons, and energy**

Tutorial 1 aims to guide students through an investigation of the nature of light and the nature of the models we use to describe and understand light. The beginning of the tutorial is pictured in Figure V.1. Considering light shining on barrels of water and heating them up, students are asked to compare and reconcile the classical electromagnetic wave and quantum photon models of light. Comparing two beams of light, represented as electromagnetic waves, that have the same amplitude but different frequencies, students should recognize that the two beams will supply the same amount of power (power goes as the square of the amplitude, and is independent of frequency) and thus the two barrels will heat up at the same rate. However, one beam of light has a higher frequency than the other, meaning that each individual photon has more energy. From this, students can conclude that the beam with the higher frequency must have fewer photons. They can either work this out conceptually or mathematically—the tutorial does not require students to write down symbolic expressions, but when we have asked these questions on homework assignments, we have asked which beam has more photons and *by what factor*, thus requiring students to work out the problem mathematically.

This tutorial supports students' meta-ontological competence because it provides a space for students to reason about an entity (light) in multiple ways. The questions explicitly prompt the use of different ontologies, and ask students to explore what conclusions they can reach when reasoning about light in these different ways (see Figure V.2 for examples of some of the questions). In Question 1, students are asked to use the ontology of light as an electromagnetic wave. Questions



**Tutorial 1:** Name ________________________ Section ____

**Waves, photons, and energy**

In each of the three situations below, a light beam is shining on a black barrel full of water. The width of the beam is the same in all 3 cases. The barrels absorb the light energy, heating up in the process (much like the way you feel hot when standing in the sunlight).

Light is often modeled as electromagnetic waves as shown in the diagram below. In #1 and #2, the light waves have the same amplitude, while #3 has a smaller amplitude. The light waves in #1 and #3 have the same frequency, and #2 has a higher frequency.



FIG. V.1. The beginning of Tutorial 1, where students consider wave and photon models of light.

2-9 prompt students to think of light as photons, with no explicit instruction around what a photon is. Instead, students are asked to negotiate this for themselves: What does a photon look like, how might photon characteristics be related to a wave representation of light (Question 2), how can we compare energies of individual photons (Question 4), how do individual photons collectively make a beam of light (Question 5), and how fast do photons travel (Question 6)? The open ended-ness of Question 2—How would you draw the three situations using the photon model?—gives students the flexibility to construct their own ontology of a photon (constrained by experimental observations, e.g., energy is proportional to frequency). Further, the tutorial introduces two models of light—Question 1 suggests not that light *is* a wave, but that we often *model* it as a wave. In Question 2 when students are asked to create a corresponding representation for the photon model, they are told "There are many good ways to do this, with no single right answer." This framing embodies a dynamic ontologies approach; students are encouraged to explore different ontologies for light, and to think about which ontology they are using and why.

The tutorial culminates in the question: Which model is correct? Many students often say

*Waves, photons, and energy*

1. Thinking of light as an electromagnetic wave, in which case(s) does the barrel heat up the fastest? Slowest? Explain your reasoning.

2. As you might have seen in your quantum or modern physics class, light can also be modeled as consisting of photons. How would you draw the same three situations using the photon model? (There are many good ways to do this, with no single right answer.) Try drawing this individually first, and then discuss with your group.

3. Thinking of light as photons, in which case(s) does the barrel heat up the fastest? Slowest? Explain your reasoning.

4. Now let's focus on some of the specifics of trying to depict the three cases in terms of photons. In which of the three cases does each individual photon carry the *most energy*? The second most energy? The least energy? Why?

*Waves, photons, and energy*

5. In which of the three cases are there the *most photons* per second reaching the barrel? The second most photons per second? The least? Why?

6. In which of the three cases are the photons *fastest*? Slowest? Explain your reasoning.

★ *Consult an instructor before you proceed.*

7. Are your answers to questions 4, 5, and 6 consistent with one another? Explain why or why not.

8. Do you still agree with the diagrams that you drew?

9. Let's revisit question 3. What does the photon model say about which barrel would heat up the fastest and the slowest?

FIG. V.2. The first nine questions of Tutorial 1 that explicitly prompt consideration of ontology.

that the quantum (photon) model is correct and the classical (wave) model is incorrect[16]. In our experiences running this question in focus groups and in class, this can lead to interesting discussions about the nature of science (i.e., How do we know if both of these models are valid?). Students should (and often do) come to the conclusion that the models must be constrained by experiment and that the classical (wave) model is indeed correct in this context. In reconciling the wave and photon model predictions about which barrel heats up the fastest, students will reach the conclusion that barrel 1 has more lower energy photons and barrel 2 has fewer higher energy photons (Question 5). This tutorial guides students to the conclusion that in this specific context, thinking of light as a wave or light as photons can both be useful and helps them develop a sense of when to use which model, thus encouraging and supporting flexible use of ontologies.

The associated instructor guide introduces and frames the tutorial by enumerating the instructional goals, listing what we expect students to know or be familiar with at the outset, and discussing common student responses we have seen. As mentioned above, many students' immediate instinct is to *not* try and reconcile the two models, citing the photon (quantum) model as the correct one. This response is perhaps not surprising in a modern physics class where a common reaction to learning quantum physics ideas for the first time is to throw classical physics out the window. The instructor guide explains that the language of "classical" and "quantum" is not used in the tutorial in order to avoid this perception that quantum model is new and therefore the only correct one. It subsequently walks through the reasoning we expect students to ultimately reach that reconciles

[16] The correct answer is that each model has correct elements, and in fact they must be reconciled in order to make conclusions about relative numbers of photons

the two models, but notes that it is ok if students have not yet concluded this on their own before the tutorial prompts them to consider if the models are or should be consistent with one another.

As an example of how this tutorial promotes sophisticated student reasoning, consider one focus group session where students grapple with the questions of which model is correct and can they be reconciled, leading to an explicit discussion about ontology. This example comes from the same group as in Episode 3 in Chapter III above. The four members (all pseudonyms) of the group are Fernando, Zach, Jacob, and Wei. Fernando starts to work out the problem mathematically to determine the relative number of photons between barrels 1 and 2 (same amplitude, but different frequencies). In doing so, he sets the power from one beam equal to another beam, and the group launches into a conversation around whether this assumption is valid:

*Jacob: I think classical just doesn't work*

*Wei: So they are maybe not equal to each other*

*Fernando: What's that?*

*Wei: They maybe not equal to each other*

*Fernando: Yeah, I guess that's a big assumption that I'm making. That classical theory says that they're the same power?..That's—*

*Jacob: —I think that classical [inaudible]—*

*Fernando: —that was my problem—*

*Zach: —I feel like they have to disagree—*

*[Wei and Jacob having inaudible side conversation]*

*Zach: —like be unequal, for there to like be a debate about it.*

*Jacob [to Wei]: feels like one of them. [Laughs]*

*Fernando: Yeah, I feel like that's one thing I'm confused about because at some points you can say that light acts like a wave, and some points you can say light acts like a particle, and sometimes you can say it acts as both. So this is a situation where we have to, I guess, really decipher whether if it is acting only like a quantum particle, then it's not going to be acting like a wave, or if it acts like both. You know what I—I mean, I feel like for every situation you have one of three options.*

*Wei: Yes, the quantum way is like—*

*Fernando: So you're saying it's acting like a particle, and not like a wave?*

*Wei: Yes, it's like particle.*

*Zach: Ok so, what makes you say it's acting like a particle?*

*Wei: Yeah particle, ok so, let's just assume there's only one particle, like in this situation. So if it only has one particle, the higher frequency means higher energy. So we don't need to talk about, like the classical theory. Because we only have one particle. It's not a wave. So it's not [inaudible].*

*Fernando: But, so you're saying it does act like a particle here but not like a wave?*

*Wei: Well the classical is like wave, because it's a sine, right? [gestures sine wave in the air with his pencil]. Sine k x plus...t...And for the quantum it's just assume it's a particle, because it, and also it has a frequency.*

*Fernando: So which one are you saying is right, here?*

*Jacob: Quantum.*

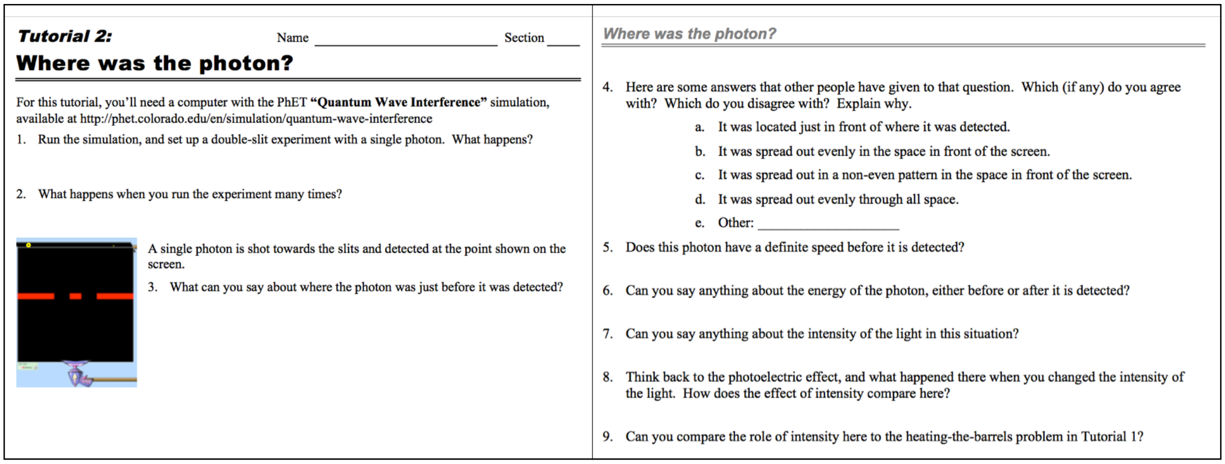
*Zach: Yeah, I feel like, I don't know why, but I wanna say I like the quantum theory just because it like seems more tangible that a photon and not a wave would come in and be able to like change...um...like it says it's heating up the barrel, so I feel like something like a particle would do that...just intuitively, instead of a wave. It would go in and [gestures hand churning in a circle] you know, kick up the velocity*

At line 145, Fernando has turned this epistemological question (Which model correctly describes nature in this instance? Or more implicitly, how do we know which model is correct?) into an ontological one—is this an instance where we should be thinking about light as a particle, as a wave, or as both? Fernando seems to think that it is appropriate here to treat light as both a particle and a wave, while Zach, Wei, and Jacob seem to think that in this instance "particle" is the most appropriate ontology. Zach provides reasoning behind his preference for a particle ontology, citing his intuition that a particle is better able to heat something up than a wave—the particle "comes in" and "kicks up the velocity". This episode of reasoning takes place about halfway through an hour-long focus group session. While the students have not yet settled on a correct answer (that the beam with higher frequency has less photons), we see their reasoning as being productive because they are making progress toward an understanding of the physics and are engaging in sophisticated discourse practices. They demonstrate meta-ontological competence by explicitly thinking about what ontologies they are using and why, and they hold each other accountable for articulating their ontological reasoning (e.g., Zach asks Wei to explain why he thinks light is acting like a particle in line 154).

**Tutorial 2: Where was the photon?**

In Tutorial 1, students are asked to think about the photon model of light with no instruction or guidance around what a photon *is* (i.e., no explicit exploration of the ontology of a photon)[17]. This second tutorial supports students in thinking about how photons behave (individually or collectively), by considering the results of a double slit experiment with a single photon, and then many photons one at a time. As stated in the instructor guide, one of the goals of this tutorial is to guide students in thinking about particle and wave characteristics of a photon. However, unlike Tutorial 1, this tutorial does not explicitly ask about "particle" and "wave" characteristics. Rather, it asks students to use the Quantum Wave PhET simulation [96] to explore the behavior of a single photon in a double slit experiment (Questions 1-2), and then asks about the photon's location (Questions 3-4), speed (Question 5), energy (Question 6), and intensity (Questions 7-9) (see Figure V.3). When students engage with these questions, they often bring up particle and wave language even though they are not directly prompted to do so. This is consistent with our findings in Chapter IV that the leading-ness of prompts impacts the ontologies students use, but even when not led to a certain ontology students can and do engage in flexible use of ontologies. Adding to this flexibility, many questions on this tutorial are about interpretation of QM and do not necessarily have correct answers (although some have decidedly incorrect answers, e.g., Question 4, which asks "What can you say about where the photon was just before it was detected?"). This creates space for students to practice reasoning about a photon with multiple ontologies, or to construct new ontologies (provided that in the implementation, the tutorial is framed as being about sense making and not about correct answers).

[17] As we saw in the example of student reasoning above, in Tutorial 1 students often treated a photon as synonymous with particle.



**Tutorial 2:** Name ____________________ Section ____

**Where was the photon?**

For this tutorial, you'll need a computer with the PhET **"Quantum Wave Interference"** simulation, available at http://phet.colorado.edu/en/simulation/quantum-wave-interference

1. Run the simulation, and set up a double-slit experiment with a single photon. What happens?

2. What happens when you run the experiment many times?

A single photon is shot towards the slits and detected at the point shown on the screen.

3. What can you say about where the photon was just before it was detected?

*Where was the photon?*

4. Here are some answers that other people have given to that question. Which (if any) do you agree with? Which do you disagree with? Explain why.
   a. It was located just in front of where it was detected.
   b. It was spread out evenly in the space in front of the screen.
   c. It was spread out in a non-even pattern in the space in front of the screen.
   d. It was spread out evenly through all space.
   e. Other: ________________
5. Does this photon have a definite speed before it is detected?
6. Can you say anything about the energy of the photon, either before or after it is detected?
7. Can you say anything about the intensity of the light in this situation?
8. Think back to the photoelectric effect, and what happened there when you changed the intensity of the light. How does the effect of intensity compare here?
9. Can you compare the role of intensity here to the heating-the-barrels problem in Tutorial 1?

FIG. V.3. Tutorial 2, which explores the ontology of a single photon in the context of a double slit experiment.

In addition to listing the instructional goals and framing this tutorial as one that has interpretive questions for which there are not necessarily correct answers, the instructor guide describes the features of the required simulation and how we expect students to engage with some of the questions. For example, although familiarity with the uncertainty principle is not required for this tutorial, students may draw on this in answering questions about what they can know about the position, speed or energy of the photon. If this tutorial is implemented before talking about the uncertainty principle (as is the case in our CU modern physics course), some of the prompts can be revisited later in the context of the uncertainty principle.

We have evidence that students engage in dynamic ontological reasoning when working on these questions. In particular, the students in Episode 1 ("Blob of EM Wave") in Chapter III above are talking about an earlier version Question 4, which asks for a single photon detected on the screen in the double slit experiment: "What is the most reasonable interpretation of where the photon was just before it was detected? a) It was located just in front of where it was detected, b) It was spread out evenly in the space in front of the screen, c) It was spread out in a non-even pattern in the space in front of the screen, d) It was spread out evenly through all space." Although this question does not directly ask "What kind of a thing is a photon?", Eric begins the conversation by talking about ontology, referring to the photon as a "big blob of electromagnetic stuff, instead of a single point". Throughout the conversation, the three students collectively negotiate the ontology of the photon. We see them being metacognitive about the ontologies they are using, evidenced by statements beginning with “When you think of the photon as...” (line 9), “It’s weird to think that it’s...” (line 12), and ”if you’re thinking of a photon as a wave...” (line 23). Asking interpretive questions that do not necessarily have correct answers gives students the space to engage in messy and dynamic reasoning.

As noted above in Chapter III, the PhET simulation used in this tutorial may play a role in how the students talk about the photon. We speculate that Eric uses the word “blob”, in part, because when the photon is fired in the simulation, it looks like a blob. Although in this tutorial we do not ask students directly about how the simulation effects their ontological conception, in our focus group study, students sometimes brought this up simultaneously. In Tutorial 10 (LEDs), we do ask students directly about the ontologies they see being depicted by a simulation.

#### Tutorial 3: Plane waves and wave packets

The primary goal of Tutorial 3 is to guide students in thinking about two different representations or models—plane waves and wave packets—that are often used to describe photons and electrons. The first five questions ask about plane waves and wave packets in an abstracted sense, not connected to any specific entity. After students think about and discuss definitions and properties of plane waves and wave packets and how they relate to the uncertainty principle (i.e., What is the difference between a plane wave and a wave packet, and for which one is position or momentum most well defined?), Questions 7-8 engage students in discussions about properties of photons and how they connect to the different representations (see Figure V.4). We have seen students engage in productive (and messy) sense making about how the frequency of a photon does or does not relate to its speed. For example, a line of reasoning might go as follows: Higher frequency means higher energy which means faster. Yet photons travel at the speed of light, so for photons speed must be independent of frequency. In contrast, students then consider this question in the context of an electron. Question 9 has students consider a wave ontology for an electron, by presenting two sine waves as representations of wave functions for two free electrons, and then asks which free electron has more kinetic energy. Here, students come to understand that higher frequency means higher kinetic energy (and thus speed) for an electron (which has mass), but not a photon. Overall, this tutorial guides students to reason about the *physical meaning* of the plane wave and wave

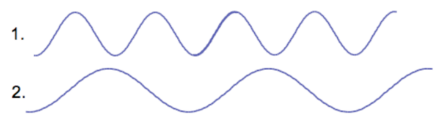
*Plane waves and wave packets*

6. What is(are) the difference(s) between a blue photon and a red photon?

7. How would you describe these differences in terms of (a) plane waves and (b) wave packets? Which description do you like better?

8. Imagine you have two photon generators, one for blue photons and one for red photons, and they are equidistant from a screen. At time *t*=0, a single blue photon and a single red photon are shot out by the generators. Which one will hit the screen first?

9. The wave-like graphs labeled #1 and #2 describe the wave functions for two **electrons**. (Note: We are talking about electrons here, not photons.) Which of the two free electrons have more kinetic energy? #1, #2, or do they both have the same energy?




10. In thinking about these questions, what kinds of assumptions were you making about what a photon is (or isn't), or what an electron is (or isn't)?

FIG. V.4. Questions 6-10 from Tutorial 3, which encourage students to connect various properties of photons to plane wave and wave packet representations.

packet representations. The message we expect students to take away is that it is useful to think about photons or electrons in different ways (different ontologies), and they might choose to use a specific representation depending on the property of interest (i.e., you might think about a photon as a plane wave when attending to the frequency). The tutorial culminates in a reflective question (Question 10) asking what assumptions they were making about what a photon or electron is or isn't, thus directly supporting the *metacognitive* aspect of meta-ontological competence.

In addition to framing the tutorial as above, the instructor guide states the expected prior knowledge (e.g., we expect students to be able to draw a plane wave and a wave packet, or know what they look like) and discusses possible student answers to some of the questions.

The three students from Group A (Episodes 1 and 2 in Chapter III)—Eric, Tara, and Bryan—engage in dynamic and tentative reasoning around these questions about plane waves and wave packets. After talking about the differences between plane waves and wave packets in an abstract sense (a plane wave has a well defined wavelength and extends over all space, while a wave packet is a superposition of many plane waves and thus does not have a well defined wavelength but is localized), the group considers what it means to represent a single photon as a plane wave or a wave packet.

*Bryan: I don't know, I guess I think of it more as a wave packet. Um, actually I don't really know.*

*Eric: I mean, do we know both the location and energy of a photon? Don't we? I think we do.*

*Tara: Do we? I think we know the energy of the photon, because that just has to do with the frequency. But the position we don't know until like you know, until it hits the screen. Like in the double slit experiment, the photon was everywhere in space until it hit the screen, we knew where it was. So, in that sense it's not really localized.*

*Eric: So the photon was like a plane wave until it hit the screen?*

*Tara: Maybe—I guess where I was going with that is that it behaves like a plane wave in terms of like we don't really know where it is, but once it starts to interact with something and we detect it, then it's more like a packet.*

*Eric: But then do we not really know its energy anymore? Because with a wave packet we don't really know the energy.*

*Tara: Hmm...I don't know.*

This snippet is characteristic of their entire hour-long focus group session around these questions—the students are grappling with questions of ontology, and "trying on" different ways of thinking about entities (in this case, a photon). They do not come to any conclusive answers, but they ask important questions that shows they are engaged in sense making around these sophisticated topics. For example, in line 182, Eric brings up the question, If a single photon is represented by a wave packet, does that mean we do not know its energy? They return to this same question a few minutes later:

*Eric: Something that I guess I don't fully understand with the wave packet is...so it's this sum of an infinite number of different waves, so what does that mean for its energy?? Because you've got all these different waves being put together, how do you determine its energy?*

*Tara: Would the energy be the sum of the energies of all the other waves? Or?*

*Eric and Tara: [simultaneously] Then we would have infinite energy!*

*[Eric laughing]*

*Tara: I don't know, well that's also like, how do you calculate the wavelength of a wave packet?*

The students continue to grapple with this tension between knowing that a single photon can be described by a frequency (and thus an energy) and thinking about what it means to represent a single photon as a wave packet. They continue by considering the differences between a red and blue photon:

*Bryan: Well first off, the wavelength is different between a blue and red photon, obviously. The frequency is different.*

*Tara: And that's related to energy, because the smaller the wavelength, the more energy it has.*

*Eric: So, the blue photon would have a shorter wavelength and it would travel faster—*

*Tara: —So it would have a higher kinetic energy—*

*Eric:—When we're looking at it as a plane wave.*

*Tara: And then, in terms of the plane waves and the wave packets...I guess, would a blue photon...how would you describe that in terms of a wave packet?*

*Bryan: No idea.*

*Eric: I don't know. [Laughs]*

*Bryan: Is a blue photon wave packet like a bunch of blue light superimposed, or is it different colors?*

*Eric: See that's what I don't understand about the wave packet is you have to have the infinite number of frequencies, that wouldn't all be the same blue frequency, so like...I guess the blue would be the first harmonic and then you have harmonics above that to—no. Actually, I don't think that's how this works. [Laughs]*

*Tara: When you make a wave packet does it go...I don't know where I was going with that.*

*Bryan: Because if you have infinite amount of...wavelengths or whatever, wouldn't that create white light? Don't all the colors make white?*

We see the students reasoning in a tentative manner, putting forth ideas they are unsure about (e.g., lines 207-209), phrasing their ideas as questions, and saying "I don't know" while continuing to try and make sense of the question at hand. As discussed above in Chapter III, we consider this tentativeness—one element of "messiness"—to be valuable for student learning, in that it allows students to sustain engagement in collective sense making processes and to engage in scientific or expert-like discourse practices. This group's negotiation of the tension around representing a photon in different ways is an example of the way in which these curricular materials can lead to messy and nuanced reasoning.

**Tutorial 4: Particle in a box**

The bulk of the design and research work around Tutorial 4 was conducted by our collaborators at UMD. At CU, we have asked parts of these questions in focus groups and in class, but never the full tutorial. This tutorial guides students to examine the nature of a quantum particle in an infinite square well potential, a cornerstone topic in QM that is typically taught with an emphasis on calculation. Through exploring the particle in a box system, this tutorial asks students to consider when it is useful to think about the quantum entity as particlelike, wavelike, or neither. It does so by presenting a standing wave on a string and a classical particle in an actual box as explicit comparisons to the quantum particle in a box. First, the students consider solutions to the infinite square well potential (Section A, pictured in part in Figure V.5). They are asked to sketch the wave function (but not necessarily write down the equation, although we expect many students to do so in order to draw the graph or check that their graph makes sense). Next, the students are asked to explain the physical meaning of the wave function. This question does not have one correct answer, but is designed to help students consider what physical principles might apply to the quantum particle. The following questions scaffold this consideration—why can the particle not have zero energy, and what position, energy, and speed might you measure in this system?

Sections B and C (pictured, in part, in Figure V.6) then walk students through the two classical analogs before returning to the quantum system. The structure of this tutorial can be thought of as encouraging a parallel or blended ontological structure between the inputs of classical wave (standing wave on a string) and classical particle (classical particle in a box). By beginning with, and then returning to, the quantum particle after the classical systems, we expect students to draw comparisons among the systems. The last section directly asks them to do so (Questions 18 and

**Tutorial 4:** Name ______________________ Section ____

## Particle in a box

**A. The quantum particle in a box**

Consider the "particle in a box": a quantum system where one particle (for example, let's say an electron) is confined between positions 0 and $L$. This can be in 1, 2, or 3 dimensions, but for now, let's focus on the 1-dimensional system.

1. Let's say the system is in the ground state ($n = 1$). Sketch a graph of the wave function.

2. How would you explain the physical meaning of the wave function?

3. Why isn't the ground state $n = 0$? That is, why isn't it possible for the particle to have zero energy?

4. If you were to measure the position of the particle at some point in time, what position(s) would you expect to measure? Why? If you repeated this experiment, would you get the same measurement every time?

FIG. V.5. The first section of Tutorial 4 asks students to consider properties of a "quantum particle in a box."

*Particle in a box*

**B. Standing wave on a string**

Now let's put aside the PIAB for now, and think about some systems in classical physics that might have some similar properties. First, let's think about a standing wave on a string. (Just a regular old string, not anything from quantum mechanics.)

9. Does it make sense to talk about the energy of this system? What physical properties does it correspond to?

10. Can we define a "speed" for the wave? If so, what would that mean? If not, why not?

11. What is the relationship between the energy (in #9) and the speed (in #10)?

*Particle in a box*

**C. Classical particle in a box**

Now let's consider a classical particle, moving in 1 dimension inside an actual box. (Again, this isn't the same as the quantum "particle in a box.")

12. What would the "ground state" look like for the classical particle? (I.e., what is the least possible energy that the particle could have, and what would the particle be doing in that case?)

13. Suppose the particle has some energy, and is bouncing back and forth. If you were to measure the position of the particle at some point in time, what position(s) would you expect to measure? Why? If you repeated this experiment, would you get the same measurement every time?

14. Graph the probability of measuring the particle at each position, as a function of position. How does this compare to the graph you drew in #1?

FIG. V.6. Sections B and C of Tutorial 4 ask students two consider two classical analogs to the quantum particle in a box.

19): In what ways is the quantum particle more like a classical particle, classical wave, both, or neither? This question is leading in that it asks students to choose the ontology that *most* fits the quantum particle, but it also leaves room for nuanced and messy reasoning by providing the "both" and "neither" options. This type of question is the embodiment of the dynamic ontologies approach, encouraging students to construct or negotiate meaning of an ontology of the quantum particle that draws on both classical particle and wave characteristics.

The instructor guide introduces and frames the tutorial and notes that it does not need to be implemented in class immediately after covering the particle in a box system. We have seen students find utility in working through these questions even after moving on to more complicated systems in class. Additionally, the instructor guide walks through each question (or group of questions) stating our answers to the questions along with typical student responses, noting that although there are multiple ways to approach some questions, we expect students' answers to be consistent with one another.

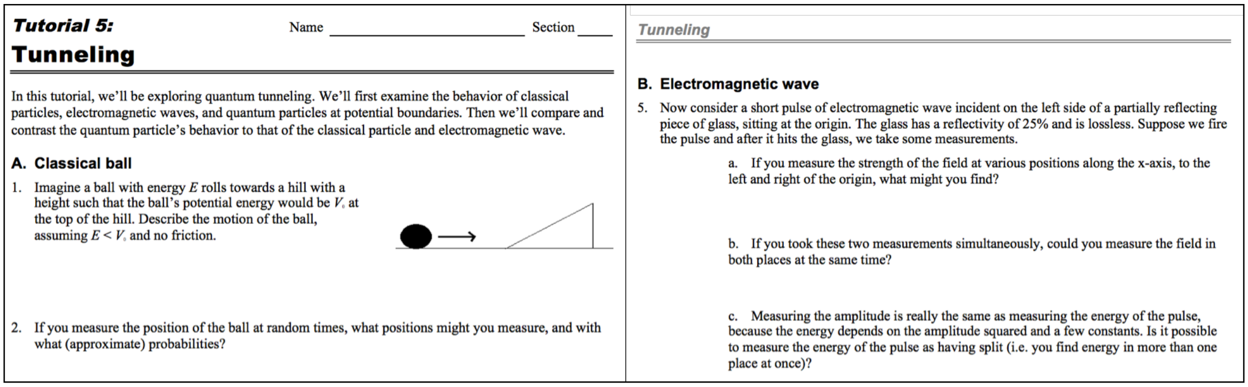
**Tutorial 5: Tunneling**

The design of this tutorial was the result of Danny Rehn's work [113] and was subsequently refined by the UMD side of the collaboration. We have implemented it in the Modern Physics for Engineers course at CU. As stated in the instructor guide, a goal of this tutorial is to have students consider a blended ontology for a quantum particle as a way to make sense of tunneling. The inputs of the blend, classical particle and (classical) electromagnetic wave, are explored in sections A and B (see Figure V.7), by imagining a classical ball rolling towards (and maybe up) a hill, as well as considering an electromagnetic wave incident on a partially reflecting piece of glass. We expect students to recognize things like: a classical ball can get over the hill only if it has enough energy ($E > V_0$), it is measured in one place, it loses kinetic energy as it rolls up the hill but the total energy stays constant (assuming no friction), and an electromagnetic wave is split into two parts and we can measure energy from the pulse on both sides of the barrier. The tutorial then asks them to think about how these properties or behaviors do or do not apply to the quantum particle in the tunneling situation (see Figure V.8). Similar to the particle in a box tutorial, this tutorial culminates in questions that directly ask students how a quantum particle compares to either of the inputs (classical particle or EM wave). The goal is that students will come to see (at least in this context) the quantum particle as an entity that has both particle and wave characteristics, but is a different kind of thing entirely. This tutorial is a lead in to the following one, where students will use the potential barrier to model a real-world situation.

The instructor guide introduces the goals of the tutorial, as above, and provides clarifications or suggested follow-up questions for each question along with expected student answers.

**Tutorial 6: Electron in a wire**

As a continuation of the previous tutorial, Tutorial 6 guides students in applying the finite



**Tutorial 5:** Name ____________ Section ____

**Tunneling**

In this tutorial, we'll be exploring quantum tunneling. We'll first examine the behavior of classical particles, electromagnetic waves, and quantum particles at potential boundaries. Then we'll compare and contrast the quantum particle's behavior to that of the classical particle and electromagnetic wave.

**A. Classical ball**

1. Imagine a ball with energy $E$ rolls towards a hill with a height such that the ball's potential energy would be $V_o$ at the top of the hill. Describe the motion of the ball, assuming $E < V_o$ and no friction.

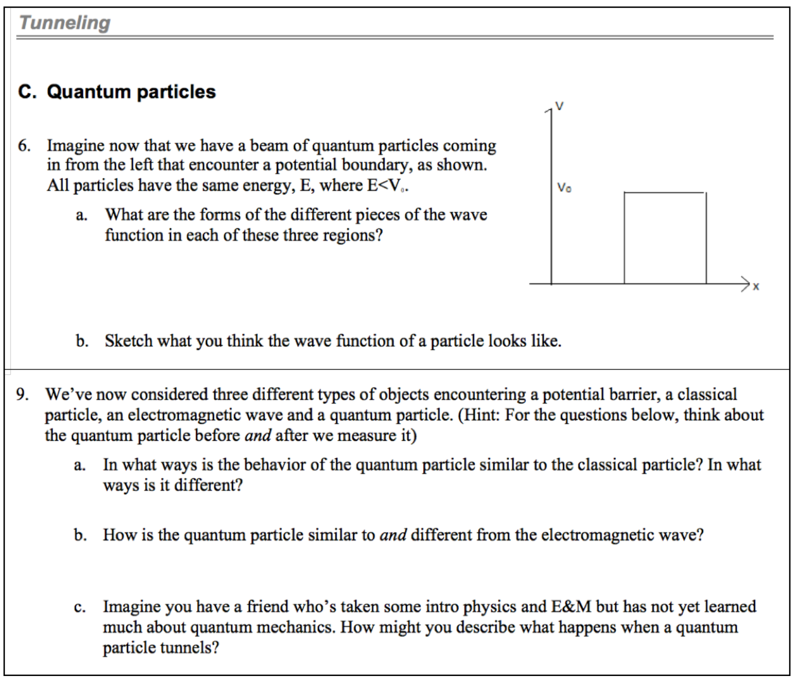
2. If you measure the position of the ball at random times, what positions might you measure, and with what (approximate) probabilities?

*Tunneling*

**B. Electromagnetic wave**

5. Now consider a short pulse of electromagnetic wave incident on the left side of a partially reflecting piece of glass, sitting at the origin. The glass has a reflectivity of 25% and is lossless. Suppose we fire the pulse and after it hits the glass, we take some measurements.
   a. If you measure the strength of the field at various positions along the x-axis, to the left and right of the origin, what might you find?
   b. If you took these two measurements simultaneously, could you measure the field in both places at the same time?
   c. Measuring the amplitude is really the same as measuring the energy of the pulse, because the energy depends on the amplitude squared and a few constants. Is it possible to measure the energy of the pulse as having split (i.e. you find energy in more than one place at once)?

FIG. V.7. Sections A and B of Tutorial 5 have students consider classical particles and electromagnetic waves as inputs to draw on when thinking about quantum tunneling.



*Tunneling*

**C. Quantum particles**

6. Imagine now that we have a beam of quantum particles coming in from the left that encounter a potential boundary, as shown. All particles have the same energy, E, where $E<V_o$.
   a. What are the forms of the different pieces of the wave function in each of these three regions?
   b. Sketch what you think the wave function of a particle looks like.



9. We've now considered three different types of objects encountering a potential barrier, a classical particle, an electromagnetic wave and a quantum particle. (Hint: For the questions below, think about the quantum particle before *and* after we measure it)
   a. In what ways is the behavior of the quantum particle similar to the classical particle? In what ways is it different?
   b. How is the quantum particle similar to *and* different from the electromagnetic wave?
   c. Imagine you have a friend who's taken some intro physics and E&M but has not yet learned much about quantum mechanics. How might you describe what happens when a quantum particle tunnels?

FIG. V.8. Section C of Tutorial 5 asks students to compare quantum particles tunneling to classical particles and electromagnetic waves.

potential barrier model to a real situation of an electron in a wire. We ask students to consider the conditions for tunneling and what we know about the potential, kinetic, and total energy in the system. In addition to these content mastery goals, this tutorial encourages students to consider what it physically means for an electron to tunnel. This starts in Question 7 (see Figure V.9), where we ask students to qualitatively describe what the wave function tells us about the electron. That is, we ask the students to connect the graphical representation of the wave function to physical

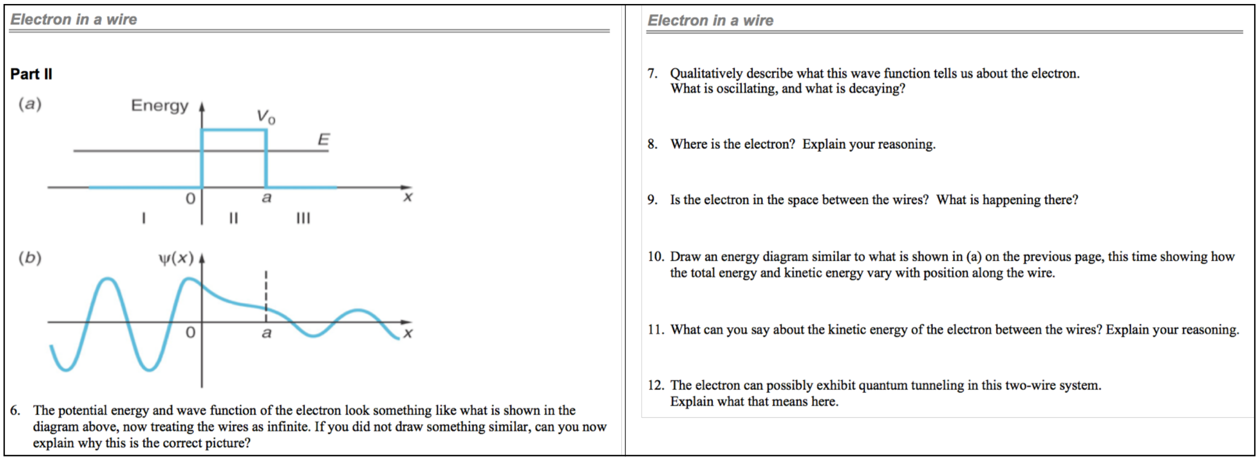



FIG. V.9. Example questions from Tutorial 6 that have students consider what it physically means for an electron to tunnel.

properties or characteristics of the electron. Next, the tutorial asks students where the electron is and if it exists in the space between the wires. This leads students towards recognizing that the phenomenon of tunneling suggests that there is a possibility of measuring the electron in the barrier (if only our measurement didn't change the system...), but leaves room for different interpretations (i.e., the wave function *is* the electron versus the wave function *carries information* about the electron). Finally, Question 12 asks what it means that the electron can possibly exhibit tunneling, supporting students in considering the ontological question, "What kind of a thing is an electron and how does it behave?"

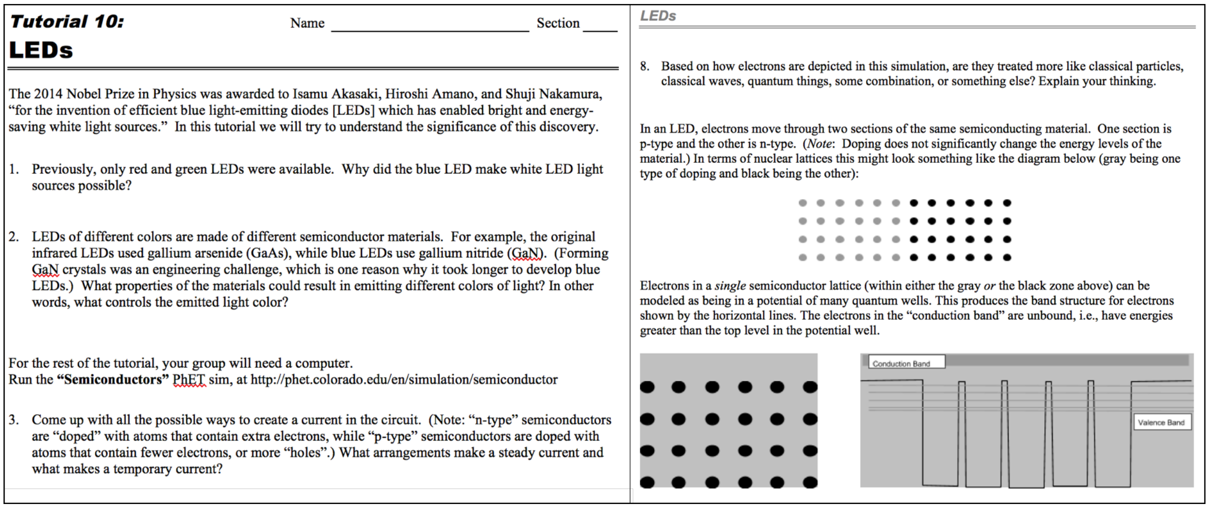
The instructor guide provides logistical guidelines such as giving the students part two (the beginning of which is shown in Figure V.9) separately and after they have completed part one. Additionally, it points out ambiguous or open-ended questions like Question 8 ("Where is the electron?") where the instructor has the choice of encouraging reasoning in a certain direction or letting the students consider multiple ways of thinking about the issue.

We have seen that these open-ended prompts lead to messy and productive conversations among students. Episode 3 ("Fuzzy Ball of Probability") from Chapter III provides an example of this. After drawing the wave function for the one wire situation, Fernando, Jacob, and Zach discussed the electron as a "fuzzy ball of probability", using both parallel and blended ontological structures to make sense of the quantum tunneling phenomenon. In this conversation, the students demonstrate meta-ontological competence by talking directly about the ontology of the electron and questioning which models are best for this physical system (including identifying tensions between how *they* picture the electron and how the professor talks about it in class). These things are evidence of students thinking metacognitively about which ontologies they are using when, and why.

**Tutorial 10: LEDs**

The primary goal of Tutorial 10 (shown in Figure V.10), in addition to understanding what an LED is and how it works, is for students to use and coordinate multiple models for electrons, as represented in a simulation. This is a slightly different "meta" aspect of meta-ontological competence—considering what ontology is depicted in a simulation or representation, and how that is impacting how *you* are thinking about the entity. Question 8 asks students to do this by asking if, in the simulation, electrons are treated more like classical particles, classical waves, quantum things, a combination, or something else. Students are also asked to explain their thinking. We hope that in the conversations surrounding this question, students will consider the ways in which they find the depiction in the simulation useful or not useful, and what the limitations are. The fact that we are asking what ontology the simulation uses suggests to the students that there are different possible ways of thinking about the nature of an electron, hence encouraging dynamic and flexible use of ontologies. The reflective part of this question helps students to be aware of how they are thinking about the electron (and why), and to begin to consider the benefits and limitations of different ontological conceptions. Building on this, and also supporting students engagement in mathematical sense making [41], Question 9b asks students to attend to the properties of an electron ("features of electron motion") and connect them to features of the representation. Students may recognize, for example, that the direction of motion (direction of current flow) is shown directly in the simulation but must be inferred from a potential energy diagram.

This tutorial has a heavy emphasis on understanding the simulation(s) and the mechanism of how an LED works. After exploring this application of QM, the students are asked to conclude by thinking about when they used particle or wave models for the electron and what the benefits and disadvantages of each were. Further, they are asked if they have a preference between these two models (ontologies). The framing of this question is slightly leading toward a unitary ontological



**Tutorial 10:** Name ______________________ Section ____

**LEDs**

The 2014 Nobel Prize in Physics was awarded to Isamu Akasaki, Hiroshi Amano, and Shuji Nakamura, "for the invention of efficient blue light-emitting diodes [LEDs] which has enabled bright and energy-saving white light sources." In this tutorial we will try to understand the significance of this discovery.

1. Previously, only red and green LEDs were available. Why did the blue LED make white LED light sources possible?

2. LEDs of different colors are made of different semiconductor materials. For example, the original infrared LEDs used gallium arsenide (GaAs), while blue LEDs use gallium nitride (GaN). (Forming GaN crystals was an engineering challenge, which is one reason why it took longer to develop blue LEDs.) What properties of the materials could result in emitting different colors of light? In other words, what controls the emitted light color?

For the rest of the tutorial, your group will need a computer.
Run the **"Semiconductors"** PhET sim, at http://phet.colorado.edu/en/simulation/semiconductor

3. Come up with all the possible ways to create a current in the circuit. (Note: "n-type" semiconductors are "doped" with atoms that contain extra electrons, while "p-type" semiconductors are doped with atoms that contain fewer electrons, or more "holes".) What arrangements make a steady current and what makes a temporary current?

*LEDs*

8. Based on how electrons are depicted in this simulation, are they treated more like classical particles, classical waves, quantum things, some combination, or something else? Explain your thinking.

In an LED, electrons move through two sections of the same semiconducting material. One section is p-type and the other is n-type. (*Note*: Doping does not significantly change the energy levels of the material.) In terms of nuclear lattices this might look something like the diagram below (gray being one type of doping and black being the other):

Electrons in a *single* semiconductor lattice (within either the gray *or* the black zone above) can be modeled as being in a potential of many quantum wells. This produces the band structure for electrons shown by the horizontal lines. The electrons in the "conduction band" are unbound, i.e., have energies greater than the top level in the potential well.



FIG. V.10. Example questions from Tutorial 10 on LEDs.

structure because it asks students if they had a preference for *either* particle or wave, but may also leave room for a parallel or blended structure of both particle and wave.

Compared to prior tutorials, the physics here is slightly more complicated. Thus, in addition to listing the instructional goals and pre-requisites, the instructor guide provides answers for each question (where applicable) or ideas for students to consider that we have found valuable for students' learning.

### Summary

This suite of modern physics tutorials [75] is a curricular application of our studies of the dynamics of students' ontological reasoning, and an enactment of the implications of leading students to use different ontologies in different ways through the wording and framing of prompts. By using materials that include questions explicitly about ontology, questions that require students to reflect on how they are thinking about entities, and interpretive questions which do not necessarily have correct answers, we can encourage students to engage in flexible ontological reasoning and be metacognitive about their own and others' ontologies. That is, we can support students' meta-ontological competence. Implementing such curricular materials is one way to value the messiness of student reasoning.

# PART THREE: EPISTEMOLOGIES

Over the last decade or more, the physics education community has grown to acknowledge the importance of not only attending to students' content mastery, but also their views about the nature of knowing and learning physics [49, 114, 115]. These attitudes and beliefs have been shown to be associated with measures of conceptual learning [116, 117], student performance in courses [118], and persistence in the major [119]. Within thinking broadly about students' attitudes, a major thread in physics education research has focused on students' *epistemological* views about learning physics [114, 120]—their views about what it means to know and learn physics. These beliefs have been assessed with a number of surveys, including the Maryland Physics Expectations Survey (MPEX) [121] and the Colorado Learning Attitudes about Science Survey (CLASS) [116]. A student's response to each item is scored as being "favorable" or "unfavorable" (or "neutral"), and the student receives a score for each cluster (subset) of items into which the overall survey is divided (see [116] for details). The "favorable" response is determined by the expert consensus for a given item. It is widely cited that in typical physics classes, with both traditional and reformed pedagogy, students' views decline (i.e., become less "favorable", or less "expertlike") or, at best, remain the same, after a semester of physics instruction [115, 116, 121]. Exceptions include courses with an explicit focus on fostering epistemological development (e.g., [122–124]) or modeling instruction (e.g., [117]). See Ref. [115] for a review of these and other approaches that see positive CLASS gains. Further, it has been reported both in the US [125] and the UK [126] that the students who go on to major in physics (or pursue graduate studies in physics) are the students who started out with more expert-like views and higher interest in physics; these studies can be taken as evidence that our physics classes do not help students develop epistemological expertise, but rather they select the students who come in with high levels of epistemological sophistication (as measured by the CLASS). While surveys like the CLASS and MPEX break down the construct of expert-like beliefs about physics into multiple dimensions or clusters, they treat the construct of "physics" itself as unitary. In this section, we explore the possibility that students display variability in their attitudes and beliefs about physics depending on the specific subfield within physics.

Beyond physics education, there is a multi and interdisciplinary research area (comprising education researchers of all disciplines, philosophers, and psychologists) that focuses on questions of epistemology. Greene, Sandoval, and Bråten refer to this research field as "epistemic cognition" (others use the terms "personal epistemology", "epistemological resources", "nature of science", or "public understanding of science"), and they provide an overview of the varied and sometimes contradictory terminology used by different researchers within the field [127]. Researchers frequently use the adjectives *epistemic* and *epistemological*, but often intend different meanings. In philosophy, *epistemic* (deriving from the Greek word *episteme*) means "of or relating to knowledge" [128, p. 92]. Thus, an *epistemic belief* would be a belief related to knowledge. *Epistemology* (deriving from both *episteme* and *logos*) is the "theory of the epistemic" [128, p. 92], or an account of knowledge. The *epistemology of physics* thus refers to "warrants and means by which experts in [physics] justify claims as knowledge" [127, p. 2]. An *epistemological belief* then, according to philosophers, is a "belief about epistemology" [128, p. 92], or a belief about the theory of knowledge. In the realm of philosophy, *epistemological* beliefs are more formal and held by people like philosophers, while students and non-philosophers are more likely to have *epistemic* beliefs [129]. Kitchener describes the difference between epistemic and epistemological as analogous to the difference between"folk physics" and "proper physics":

> *"Folk (naive) physics might be said to be the set of beliefs, possibly innate, of ordinary folk about space, time, the movement of bodies, etc. Many of these beliefs of common senses physics are, according to modern scientific physics, false. Scientific physics, on the other hand, has a true or truer account of the movement of bodies in space and time as well as a better conception of what the field of physics is all about, about how one proceeds to "do" physics, etc.* [128, p.86]

Along with many education researchers, we depart from the philosopher's approach to epistemology and use the terms *epistemological* and *epistemic* loosely, often treating them as synonyms. We value the naive, intuitive, and implicit theories of everyday thinkers, who we believe still have "theories" even if they are not formal or explicit. In line with Hammer and Elby, we consider "naive epistemologies [to be] made up from fine-grained, context-sensitive resources" [14, p.54]—resources that can be cued by instruction and curricular materials. Distinct from, but intertwined with, epistemologies, we also consider students' expectations—their views about what counts as knowing and learning for the purpose of doing well in a particular course [114]. In this section, we primarily attend to students' epistemologies while noting that many of these constructs—epistemologies, expectations, attitudes, beliefs—are all intertwined and likely interact in complicated ways [130].

The studies in this section contribute to two ongoing debates among researchers studying students' epistemologies and expectations. One debate addresses the extent to which students display general, domain-independent epistemological views about the nature of knowledge and knowing, or display different beliefs about different domains. While some (e.g., [131]) consider "knowledge" to be made up of increasingly useful models that humans use to describe an "objectively real" world, others (e.g., [17, 132]) argue that we can only describe knowledge in terms of interactions between individuals and specific contexts [127]. Greene, Sandoval, and Bråten state that the latter view has "led some scholars to argue that research into epistemic cognition must be situated in context, and that it is a mistake to characterize particular epistemic stances or beliefs as adaptive or sophisticated in all contexts. Rather, the knowledge status of a particular proposition or procedure, and its "sophistication," depend upon the context in which they are being evaluated [83, 131, 133]." [127, p. 4-5]. Empirical work directly comparing students' views about different domains, e.g., chemistry versus psychology knowledge (see [134] for review) has led to a growing consensus that at least some aspects of students' epistemological beliefs are domain-specific. Here, "domain" typically means an academic discipline [127, 135], but researchers have left open the possibility that "domain" may correspond to something else. Our work adds to this line of research by demonstrating that students can exhibit different epistemological views about the classical physics encountered in first-year courses and the quantum physics encountered in third semester and later courses. In other words, the "domains" of domain-specific epistemological beliefs may be finer-grained than most previous literature has suggested.

A second debate to which we contribute concerns the nature of epistemological expertise or sophistication. In the survey- and interview-based literature on students' epistemologies of physics and science, researchers have scored as more sophisticated the ideas that learning consists of active construction of ideas as opposed to absorption or memorization, that scientific knowledge is a product of human invention rather than read directly from nature, that knowledge is tentative rather than certain, that physics consists of interconnected concepts as opposed to just largely disconnected facts and equations, and that problem solving should involve conceptual and sometimes even intuitive ideas rather than just formulaic use of equations [120, 121, 136]. More recently, however, many researchers have argued that epistemological sophistication consists not of blanket assertions such as "knowledge is tentative", but rather, of contextualized judgments, e.g., explanations for why falling objects accelerate are much more certain than explanations for how black holes dissipate [83, 137, 138]. If we conceptualize epistemological expertise as holding certain views at the expense of other views, then differences in students' attitudes towards classical and quantum physics might be interpreted as showing that students have less sophisticated epistemological beliefs about quantum physics than about classical physics. However, the notion that epistemological sophistication consists of the ability and propensity to make contextualized, nuanced judgments about knowledge and knowing opens up an alternative interpretation: students are judging quantum physics to be less intuitive and less tangibly connected to everyday experience, and quantum problem solving to rely more on mathematics and less on conceptual or intuitive reasoning, than is the case in introductory classical mechanics. Perhaps the recognition of these differences is itself a component of epistemological sophistication.

This section consists of three chapters. Chapter VI presents our initial study of students' domain-specific epistemologies. We administer a bifurcated CLASS survey which asks students to respond to each item twice, once asking about classical physics and once asking about quantum physics. We observe that on some items, students respond differently to the classical and quantum versions. That is, we provide evidence of the existence of students' "epistemological splits" between classical and quantum physics. Supplementing the quantitative data and analysis with qualitative coding of interviews, we clarify that at least some of the split results on the surveys correspond to students' conscious domain-specific views and are not an artifact of the bifurcated survey itself. Through these results, we challenge the tendency of treating physics as a monolithic discipline, and also begin to question the epistemological sophistication in holding domain-specific views.

Chapter VII is an extension of the initial study presented in Chapter VI, which expands the dataset of survey responses and presents the next step of qualitative analysis for understanding what it means for students to reflect split epistemic stances. In this extended study, we present survey results from 15 modern physics and QM classes across 8 semesters at 4 universities. The results show that students' epistemological splits (on some items) are robust across institutional and instructional contexts. We engage in a qualitative coding of new focus group interviews to identify and categorize elements of student reasoning that we might attend to when trying to further understand students' split epistemologies. Through the interaction among the distinct themes identified, we can being to understand *why* some students hold views of knowing and learning quantum physics that differ from those in classical physics. Further, we argue that these split epistemic views can be sophisticated and productive for student learning and epistemological development.

While in Chapter VI and Chapter VII we demonstrate that students' split epistemologies exist across instructional contexts, there are many indications that the specifics of instructional contexts and practices will impact the development of students' domain-specific epistemic views. Chapter VIII is a preliminary investigation into the impact of instructors' personal epistemic views toward QM and the emphasis on specific aspects of epistemology in their teaching of QM. Exploratory analyses suggest that an instructor's personal epistemological views can impact students' views—with differing instructor views we see corresponding differences among the class distributions. However, we note that the instructor's personal stance is one of many factors that interact to impact or guide students' epistemological development. We also explore differences in instructors' stated emphases on a given epistemological topic—we see different student responses in classes where the instructor reported explicitly addressing the relation of QM to the real world compared to classes where the instructor stated avoiding those topics. Further, we explore instructors' *stated* versus *enacted* emphases on epistemology and consider the impact on students' epistemic views.

# Chapter VI. Documenting the existence of sub-domain-specific epistemologies

*This chapter is largely from a publication currently under review at European Journal of Physics on which JRH is a co-first author: Dreyfus and Hoehn et al. [139].*

In this chapter, we present a study in which we gave students modified versions of selected items from the Colorado Learning Attitudes about Science Survey (CLASS) [116], distinguishing between "classical physics" and "quantum physics." We observed differences between views about classical and quantum physics—which we refer to as "split epistemologies"—in individual students' responses. This study is exploratory in nature and demonstrates the existence of this phenomena rather than the breadth or mechanisms of the outcomes. The split CLASS results challenge various assumptions that we, as physics education researchers and instructors, might make when measuring or attending to students' epistemologies of physics. We provide initial implications for research and for instruction, and discuss further research directions that could address some of the questions raised here.

Most studies specifically designed to look for context dependence in students' epistemologies of physics have probed variability across activities and contextual features of problem presentation, not variability across subfields of physics. For instance, in a high school physics course, Muis and Gierus [140] administered the same physics epistemology survey [141] twice, a few weeks apart. They found gender-linked differences in students' epistemologies of physics based on whether students had been primed by conceptual versus "standard" procedural physics problems. In one condition, students took a conceptual physics test immediately before completing the epistemological survey. In the second condition, students instead took a more traditional procedurally-oriented physics test. All students participated in both conditions, in counterbalanced order. "Results revealed that girls espoused more constructivist beliefs about physics for conceptual knowledge than for procedural knowledge, whereas the opposite was found for boys" [140].

Other studies have found other aspects of context dependence (though not gender-linked) within individual students' physics epistemologies, such as variation in views about the role of theory in data interpretation when probed in the abstract versus in a specific experimental context [142], and differences in a student's views about the role of intuitive, everyday knowledge when addressing physics problems in the classroom during group work versus outside the classroom during an interview [143]. Although context dependence in students' epistemologies of physics is fairly well-established, the contexts studied have primarily been different types of physics knowledge or practices, or in and outside of formal classroom environments, and not context dependence across subfields of physics.

Most published survey-based studies of student epistemologies and expectations report pre-post results in introductory physics courses, making it impossible to break down the results in terms of specific physics topics. A few longitudinal studies, however, may bear on subfield dependence of physics epistemologies. For instance, Gire et al. [144] administered the CLASS to physics majors in all four years, and found no significant change in favorable responses during the first three years, but a shift towards expert-like views in year four. The courses in which the CLASS was given included quantum mechanics, but the analysis bins students by year rather than by course and does not address the issue of subfields. Bates et al. [126] did a similar "pseudolongitudinal" study, giving the CLASS to cohorts from high school to physics faculty. Again, the analysis bins undergraduate students by year rather than by course. They observed a statistically significant decrease in favorable responses for undergraduates in year three, and explain this result as anomalous. We wonder whether this is the year when most students were taking quantum mechanics, though there is not sufficient information in the paper to draw any conclusions about this.

McCaskey [145] also looked at the impact of the modern physics context. He found that one reformed introductory physics course demonstrated epistemological gains on the MPEX, while a similar reformed introductory course which included modern physics topics showed losses. There were other differences between the courses, so McCaskey could not conclude that modern physics was the relevant variable. However, when the professor of the latter course scaled back the modern physics coverage the following year, the course showed MPEX gains. McCaskey hypothesizes "that modern physics ideas are often hard to reconcile with experience. They can seem like disconnected pieces, especially when compared to previous parts of the course. Students may feel like modern physics ideas are wacky and require acceptance, and not reconciliation or sense-making" [145].

Finally, Mason and Singh [146] gave the AAPS (Attitudes and Approaches to Problem Solving) survey to physics graduate students, asking them to complete the survey twice: once when thinking about problem-solving in their own graduate physics courses, and once when thinking about problem-solving in introductory physics. The responses about introductory courses were more expert-like (favorable), and the authors attributed this both to graduate students not yet being "experts" in graduate-level physics and to the traditional instructional approaches used in graduate courses. Again, we wonder if graduate quantum classes may have contributed to the divergent results seen in this study.

Crucially, all of these studies interpret survey results in the traditional manner of associating higher sophistication with responses that are favorable in introductory classical physics. This is an interpretation we will challenge below.

Additionally, researchers have recently been looking at students' attitudes and beliefs around the nature of experimental physics, situated in laboratory rather than lecture or theory classes [147, 148]. This is yet another version of attending to context dependence of students' epistemic views—students who display "expert-like" views in an introductory lecture class may *not* display such expertise in a laboratory setting, and students' epistemologies around theory and experiment may develop at different rates. Considering students' views toward both of these domains (theory and experiment) might give insight into their broader understanding of the nature of science.

As noted above and discussed in detail below, our study uses a bifurcated survey, a survey in which each item from a previously-existing survey is turned into two items. Our methodology is inspired and informed by several past PER studies involving bifurcated surveys. McCaskey [145] gave students the Force Concept Inventory (FCI) [149], and asked them to give both the answer "you really believe" and the answer "you think scientists would give." The former prompt was later changed to "that makes the most intuitive sense to you." Splits were observed at the end of the course, suggesting that students had not reconciled their physics knowledge with their intuition. However, the splits were smaller in a reformed course promoting this kind of reconciliation. Gray et al. [150] similarly bifurcated the CLASS, asking students to select both the choice "that best expresses your feeling" and "the choice that you think a physicist would give." They found that students correctly identified the "expert" views but did not personally agree with those views. This result was used to demonstrate both the validity of the CLASS, by showing that students were willing to express their own views instead of the "right answer," and the failure of some courses to help students see expert-like beliefs as useful. This approach is also used in the context of experimental physics—the Colorado Learning Attitudes about Science Survey for Experimental Physics (E-CLASS) gives two prompts for each item, one that asks *"What do YOU think when doing experiments for class?"* and another that asks *"What would an experimental physicist say about their research?"* [147, 148]

McCaskey's study bifurcated a content survey (the FCI) along epistemological lines, while Gray et al.'s study along with the E-CLASS bifurcated an epistemological survey (the CLASS or E-CLASS) along epistemological lines. Our study occupies a third category, bifurcating an epistemological survey along content lines (classical versus quantum). The most similar previous study is Mason and Singh's study of graduate student views of intro versus graduate physics classes [146], discussed above. We expand on that work in two ways. First, problem solving is one dimension that we examine, but we look at others as well. Second, while the graduate physics courses in Mason and Singh's study probably included quantum mechanics, our survey asks specifically about quantum versus classical physics.

This investigation arises out of a larger study of student thinking in quantum mechanics. Within the larger study, we observed anecdotally that students' attitudes about quantum mechanics differ in some ways from their attitudes towards physics in general. We designed and administered a bifurcated survey to address the following research questions.

> *1) Are there differences in students' self-reported attitudes toward quantum and classical physics?*
>
> *2) Are the pre/post shifts in students' attitudes before and after a modern physics course different for quantum and classical physics?*

Investigating the existence of differences between students' views toward classical and quantum physics requires quantitative analysis of survey responses. To understand these quantitative results, we also use qualitative data to address a third research question:

> *3) Do students in interviews spontaneously express epistemological views related to differences between classical and quantum physics?*

A "yes" answer to this third question would enable us to interpret our bifurcated survey results as reflecting students' consciously held views, not just their tacit views that crystallize into survey responses shaped largely by the bifurcated structure of the survey itself. We explore student quotes from interviews to answer the question, and then in Chapter VII, further analysis of qualitative interview data will examine more deeply the specific reasons students provide for their selected answers that represent splits (or similarities) between quantum and classical physics. The following two subsections describe the quantitative methods used to address research questions (1) and (2), and the qualitative methods used to address research question (3).

## A. Quantitative methods: Do classical-quantum epistemological splits exist?

### 1. Data collection

To investigate whether students report different attitudes towards classical and quantum physics, we selected Likert-scale items from the CLASS, taken from multiple categories [116], and bifurcated them into classical and quantum versions. For example, the CLASS item "Knowledge in physics consists of many disconnected topics" was modified to "Knowledge in classical physics..." and "Knowledge in quantum physics..." One block of questions was introduced by the instruction, "For these questions, please think about your previous physics courses on motion, electrical phenomena, etc., which we are calling "classical" physics in the items below," and the other was introduced by, "For these questions, please think about past/upcoming courses on modern physics and/or quantum physics."

The CLASS assesses attitudes on a number of dimensions, and we were specifically looking at *epistemological* attitudes, so we did not include items such as "I enjoy solving physics problems." Using our instructional and researcher intuitions, not a systematic procedure, we further narrowed the survey and chose six CLASS items (shown in Table VI.1) based on two factors: alignment with the epistemological agenda of the modern physics course at CU (the same course that was described in Part Two), and our guesses about the likelihood that students would "split" their responses. So, for instance, we anticipated that in comparison to classical physics, students may perceive quantum physics to be less tangibly connected to real-world experiences, less conceptually intuitive, and more needful of a "shut up and calculate" approach to problem solving. Some of these items (e.g., item 41, see Table VI.1) were also chosen because of prior work [151] suggesting that students' responses to these items may be influenced by the quantum context. Our approach to selecting specific CLASS items is appropriate for this study because our goal was to see if students displayed different epistemological views about classical and quantum physics *at all* (which could be established by splits on a single item), not to characterize the range and substance of the differences or to develop a new survey. Because we cared only about construct validity (e.g., do students understand the question in the same way we do, and do their answers reflect their epistemologies and expectations rather than other factors), not psychometric properties, we did not re-validate the bifurcated items. We assumed that: (i) students' comments during interviews would provide insight into how they interpreted "classical" and "quantum", and (ii) given the validation studies on the original CLASS items, students would still know what the items mean when the word "physics" is replaced by "classical physics" or "quantum physics." Students' comments during interviews, as discussed in Section IV, did indeed suggest that they interpreted "quantum" and "classical" in the way we intended.

The bifurcated survey questions were administered in four different courses spanning two universities and four semesters: Physics 2130 (Modern Physics for Engineers) at the CU in Semesters 1 and 3 of the study, Physics 420 (Modern Physics) at the University of Maryland (UMD) in Semester 2, and Physics 3220 (Quantum 1) at CU in Semester 4. As described in Part Two of this thesis, CU Physics 2130 is the third semester of the introductory physics sequence, enrolling primarily engineering students. It focuses on introductory quantum mechanics and emphasizes applications and interpretations of quantum mechanics [52]. The Semester 1 CU modern physics course was taught by Instructor A. In Semester 3, this course was co-taught by Instructors A and B (though

TABLE VI.1. Six bifurcated CLASS items used in this study.

| CLASS Item # | Item |
|---|---|
| 6 | Knowledge in [classical/quantum] physics consists of many disconnected topics. |
| 23 | In doing a [classical/quantum] physics problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem. |
| 28 | Learning [classical/quantum] physics changes my ideas about how the world works. |
| 35 | The subject of [classical/quantum] physics has little relation to what I experience in the real world. |
| 40 | If I get stuck on a [classical/quantum] physics problem, there is no chance I'll figure it out on my own. |
| 41 | It is possible for physicists to carefully perform the same measurement in a [classical/quantum] physics experiment and get two very different results that are both correct. |

using most of the original course materials), resulting in shifted emphases—less on applications and on interpretations—compared to Semester 1. UMD Physics 420 is an upper-level modern physics elective for engineering students that spends several weeks on special relativity and the rest of the semester on quantum mechanics. This particular semester was a traditional lecture-style course. CU Physics 3220 is the first semester of upper division quantum mechanics for physics majors. In Semester 4, this course was taught by Instructor C with a spins-first pedagogical approach and included clicker questions in lecture as well as tutorials in an optional one-credit weekly tutorial section.

Students answered the bifurcated CLASS questions as part of a larger survey that also included some epistemological questions specific only to quantum mechanics [27]. For the Modern Physics courses in Semesters 1-3, the survey also contained some quantum mechanics conceptual questions taken from the Quantum Mechanics Conceptual Survey (QMCS) [29] and elsewhere.

### 2. Data analysis

Rather than grouping items into clusters, we looked for classical-quantum splits on each bifurcated pair of items. Scoring a bifurcated *cluster* of items would require making decisions about which response is "favorable" and having that decision apply to both the classical and quantum version of the item. In this study, we wanted to leave open the possibility of challenging precisely that assumption.

The survey items are on a 5-point Likert scale from Strongly Disagree to Strongly Agree, but as is common in PER studies using Likert-scale data [116, 121], we collapsed Strongly Agree and Agree into a single category, and likewise for Strongly Disagree and Disagree. This permits us to see only strong splits (e.g., between "agree" and "neutral," or between "agree" and "disagree"), not weak splits (e.g., between "strongly agree" and "agree"). We avoided treating the Likert scale—even the collapsed 3-point version—as interval data, where the points on the Likert scale are treated as uniformly spaced, making it possible to calculate an average score for each given item). We did this both for general psychometric reasons [152] and for the reasons discussed by Adams et al. [116]: validation interviews for the CLASS showed that students had a variety of reasons for choosing Neutral, so we cannot assume that Neutral represents a point halfway between Agree and Disagree. Instead, we treated the Likert scale as ordinal data, where the options are discrete categories rather than points on a continuum.

Because ordinal data cannot be averaged, we could not use traditional statistical tests such as t-tests to determine whether the classical/quantum splits were statistically significant. Instead, we used the Bhapkar test [153–155], a generalization of the McNemar test [156] to more than two categories. The McNemar test compares two dichotomous distributions to determine whether they are significantly different. It has been used in the PER literature [157–159] to compare student performance on individual concept inventory items under different conditions (e.g., pre and post, or two versions of the same question), where the two categories were "correct" and "incorrect." However, our data set has three irreducible categories (Agree, Neutral, and Disagree), so we used the more general Bhapkar test. Notably, the Bhapkar test (like the McNemar test) requires knowing not only how many students gave each response (Agree, Neutral, and Disagree) under each condition (classical and quantum), but also requires filling a $3 \times 3$ matrix with each permutation of responses for a given student (e.g., how many students selected Agree for classical and Neutral for quantum, and so forth for the other 8 permutations). Thus, the Bhapkar test "knows" how many students actually split their responses, and not only the population totals. The Bhapkar test is similar to the

Stuart-Maxwell test [160, 161], and is asymptotically equivalent to it at large N, but is considered more powerful at smaller N [153, 162].

To determine significant classical-quantum splits, we performed the Bhapkar test on each bifurcated CLASS item for each administration of the survey. Students who left either the classical or quantum question blank were excluded from the analysis for that item. The $n \times n$ Bhapkar test (where $n = 3$ in our case) yields a chi-squared statistic with $n - 1$ degrees of freedom, which can then be converted into a p-value to determine significance.

One aspect of this methodology initially troubled us. The Bhapkar test treats Agree, Neutral, and Disagree the same way it would treat Vanilla, Chocolate, and Strawberry—as unordered categories. For the purposes of this study, however, it makes sense to think of Neutral as between Agree and Disagree. To determine whether this made a difference, we tried collapsing the six off-diagonal elements of the $3 \times 3$ matrix into two: "$classical > quantum$" (which includes classical-Agree/quantum-Neutral, classical-Agree/quantum-Disagree, and classical-Neutral/quantum-Disagree), and "$quantum > classical$" (which includes the other three permutations where the classical and quantum responses differ). We ran a standard $2 \times 2$ McNemar test on this reduced matrix, which is possible because the Bhapkar and McNemar tests ignore the diagonal elements, and found no difference in terms of which items showed statistically significant splits between these results and the Bhapkar results. Therefore, we report the results from the Bhapkar test, which is established in the literature, and not from our own ad hoc test.

We treated each semester of survey data as a separate data set, because the four groups had different student populations, resulting in different starting points on the pre-semester survey, and because the courses differed in content and instructional approach (one upper division QM course versus three modern physics courses; three reform-oriented courses (although reformed to varying degrees), and one traditionally taught class). By treating the groups separately, we observed significant splits that may have been washed out inappropriately had the groups been combined. It may have made sense to combine data from Semesters 1 and 3 because these came from the same course at CU, with similar student populations. However, the Semester 3 course was co-taught, resulting in slightly different instructional emphases on applications and interpretation. The fact that we obtained similar results from the two semesters, as shown below in Section III, suggests that these splits are robust across different instructional contexts. That is, they are not necessarily the result of one instructor teaching the course in a specific way.

We used the Holm-Bonferroni correction method [163] to account for the fact that we ran many comparisons (Bhapkar tests) and hence were likely to obtain one or more "statistically significant" results ($p < .05$) that are actually just fluctuations. For each bifurcated item, we performed four tests: comparing classical and quantum for both the pre- and post-survey administrations, and comparing pre and post for both the classical and quantum versions of the item. Because, for now, our main goal is to explore the existence of classical-quantum splits, not to explore how the targeted courses influenced students' views, we only report on the classical-quantum comparisons here; pre-post shifts are left for future work. Thus, keeping each semester's data separate and conducting our analysis item-by-item, we corrected each p-value resulting from the Bhapkar test for four comparisons. The p-values from Semester 4 are not corrected since there was only a post survey that semester and thus we only performed one test on each item.

### B. Qualitative methods: Do students spontaneously express splits?

#### 1. Data collection

As one of the multiple qualitative data streams collected for the larger project, we individually interviewed seven students recruited from the CU modern physics course in Semester 1, both mid-semester and near the end of the semester. These were the seven students in the focus group study described in Chapter III. For this chapter, we analyzed the end-of-semester interviews. The purpose of these videotaped interviews was to check in with the students individually on their experiences both in the Modern Physics course and in the focus groups in which they had previously participated about the nature of quantum entities and related interpretational issues. The thirty-minute semi-structured individual interviews often involved unprompted meta-level conversations about learning in QM, which is why we chose to code them for this study. The interview protocol is provided in Appendix C. Crucially, the interviews did not explicitly address the CLASS items.

### 2. Data analysis

We transcribed the interviews, and coded them for *spontaneous epistemological comparisons* between classical and quantum physics. We note that from their previous experiences with the research team, the interviewees likely knew we were interested in their epistemological views about learning quantum physics. Still, the interviewer did not generally ask students directly to compare their learning in classical versus quantum physics.

Starting from the beginning of the interview, the coder identified "epistemological statements"—ones that pertain to how ideas in quantum physics connect with everyday experiences and/or intuitions, or to how learning or problem-solving in quantum mechanics should be approached, and other issues in the space spanned by the six bifurcated CLASS items. Upon identifying an epistemological statement, we then coded for "comparison"—whether the student positions the ideas, concepts, results, or equations encountered when learning quantum in contrast to those encountered previously in physics. (Students' previous courses were entirely classical.)

Having identified an epistemological comparison, the coder then reviewed the previous few turns of conversation to determine if the comparison was spontaneous. The coder classified the preceding prompts as prompting or not prompting a reflection on the comparison. So, for instance, if an interviewee responds to a prompt such as "How is learning QM different from previous courses?" by making an epistemological comparison, the comparison is not spontaneous. By contrast, suppose the interviewer asks, "What do you see as the role of math in QM?", and the interviewee responds by stating that math is more important in QM than in classical physics because QM is counterintuitive. We code that as a spontaneous epistemological comparison because the interviewer did not ask for a comparison. Thus, "spontaneous" doesn't mean "out of the blue." It just means that the student, uncoerced, can articulate an epistemological comparison between classical and quantum physics, indicating that our split survey results do not correspond entirely to tacit beliefs or to artifacts of the bifurcated survey design.

Instead of using a second coder to increase the accountability of our coding-based analysis, we include in the Results section an example of a spontaneous epistemological comparison from each interviewee who (we claim) offered one.

## C. Results

### 1. Bifurcated survey results: Classical-quantum splits

Table VI.2 summarizes the survey results: we found statistically significant splits between the classical and quantum versions of most of the questions, in both pre and post administrations. Note that Table VI.2 lists a few results as undefined or blank. For item 28, Semester 2, pre, no students chose Disagree for classical and Neutral/Agree for quantum, and no students chose Disagree for quantum and Neutral/Agree for classical. For item 35, Semester 3, post, every student responded Disagree for the classical version. In these instances, running the Bhapkar test would involve dividing by zero. We can understand this to mean that there was not a classical/quantum split on this item. For item 41, the quantum version of the item did not appear in this form on the post survey in Semester 1.

More detailed results, with the percentage of students who "split" in each possible permutation, are given in Appendix D. A condensed version of this information is displayed graphically in Figure VI.1. There are separate graphs for each administration of the survey. Each bar graph shows "no split" ($Q = C$, the percentage of students who gave the same responses for classical and quantum, on the 3-point Likert scale), "quantum $<$ classical" (students who chose classical-Agree/quantum-Neutral, classical-Agree/quantum-Disagree, and classical-Neutral/quantum-Disagree), and "quantum $>$ classical" (students who chose the other three permutations where the classical and quantum responses differ). We discuss some of the specific results below in Section VI D.

TABLE VI.2: Classical-quantum splits on the bifurcated CLASS questions. The p-values are determined with the Bhapkar test and corrected using the Holm-Bonferroni method. Items with significant splits at the $\alpha = 0.05$ level are indicated in boldface.

| CLASS Item # | Item | Survey | $N$ | p-value |
|---|---|---|---|---|
| 6 | Knowledge in [classical/quantum] physics consists of many disconnected topics. | **Semester 1 pre** | 118 | **1.8E-04** |
| | | Semester 1 post | 114 | 0.58 |
| | | Semester 2 pre | 21 | 0.17 |
| | | Semester 2 post | 22 | 0.22 |

| CLASS Item # | Item | Survey | $N$ | p-value |
|---|---|---|---|---|
| | | **Semester 3 pre** | 43 | **4.0E-05** |
| | | Semester 3 post | 51 | 0.43 |
| | | Semester 4 post | 40 | 0.45 |
| 23 | In doing a [classical/quantum] physics problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem. | **Semester 1 pre** | 118 | **<5.0E-05** |
| | | **Semester 1 post** | 113 | **6.0E-04** |
| | | **Semester 2 pre** | 21 | **1.5E-03** |
| | | **Semester 2 post** | 22 | **0.03** |
| | | **Semester 3 pre** | 43 | **1.8E-03** |
| | | **Semester 3 post** | 51 | **2.9E-03** |
| | | **Semester 4 post** | 40 | **2.9E-03** |
| 28 | Learning [classical/quantum] physics changes my ideas about how the world works. | Semester 1 pre | 118 | 0.98 |
| | | **Semester 1 post** | 108 | **<5.0E-05** |
| | | Semester 2 pre | 21 | Undefined |
| | | Semester 2 post | 21 | 0.98 |
| | | **Semester 3 pre** | 43 | **0.03** |
| | | **Semester 3 post** | 49 | **3.6E-04** |
| | | **Semester 4 post** | 40 | **0.01** |
| 35 | The subject of [classical/quantum] physics has little relation to what I experience in the real world. | **Semester 1 pre** | 117 | **<5.0E-05** |
| | | **Semester 1 post** | 108 | **6.0E-04** |
| | | **Semester 2 pre** | 21 | **0.03** |
| | | **Semester 2 post** | 22 | **4.0E-03** |
| | | Semester 3 pre | 43 | 1 |
| | | Semester 3 post | 51 | Undefined |
| | | **Semester 4 post** | 40 | **8.93E-09** |
| 40 | If I get stuck on a [classical/quantum] physics problem, there is no chance I'll figure it out on my own. | **Semester 1 pre** | 118 | **<5.0E-05** |
| | | **Semester 1 post** | 104 | **<5.0E-05** |
| | | **Semester 2 pre** | 21 | **2.1E-03** |
| | | **Semester 2 post** | 22 | **<5.0E-05** |
| | | **Semester 3 pre** | 43 | **4.0E-05** |
| | | **Semester 3 post** | 51 | **1.5E-04** |
| | | **Semester 4 post** | 40 | **0.04** |
| 41 | It is possible for physicists to carefully perform the same measurement in a [classical/quantum] physics experiment and get two very different results that are both correct. | **Semester 1 pre** | 118 | **<5.0E-05** |
| | | Semester 1 post | N/A | N/A |
| | | **Semester 2 pre** | 21 | **0.01** |
| | | **Semester 2 post** | 22 | **4.0E-05** |
| | | **Semester 3 pre** | 43 | **4.0E-05** |
| | | **Semester 3 post** | 51 | **4.0E-05** |
| | | **Semester 4 post** | 40 | **7.1E-18** |

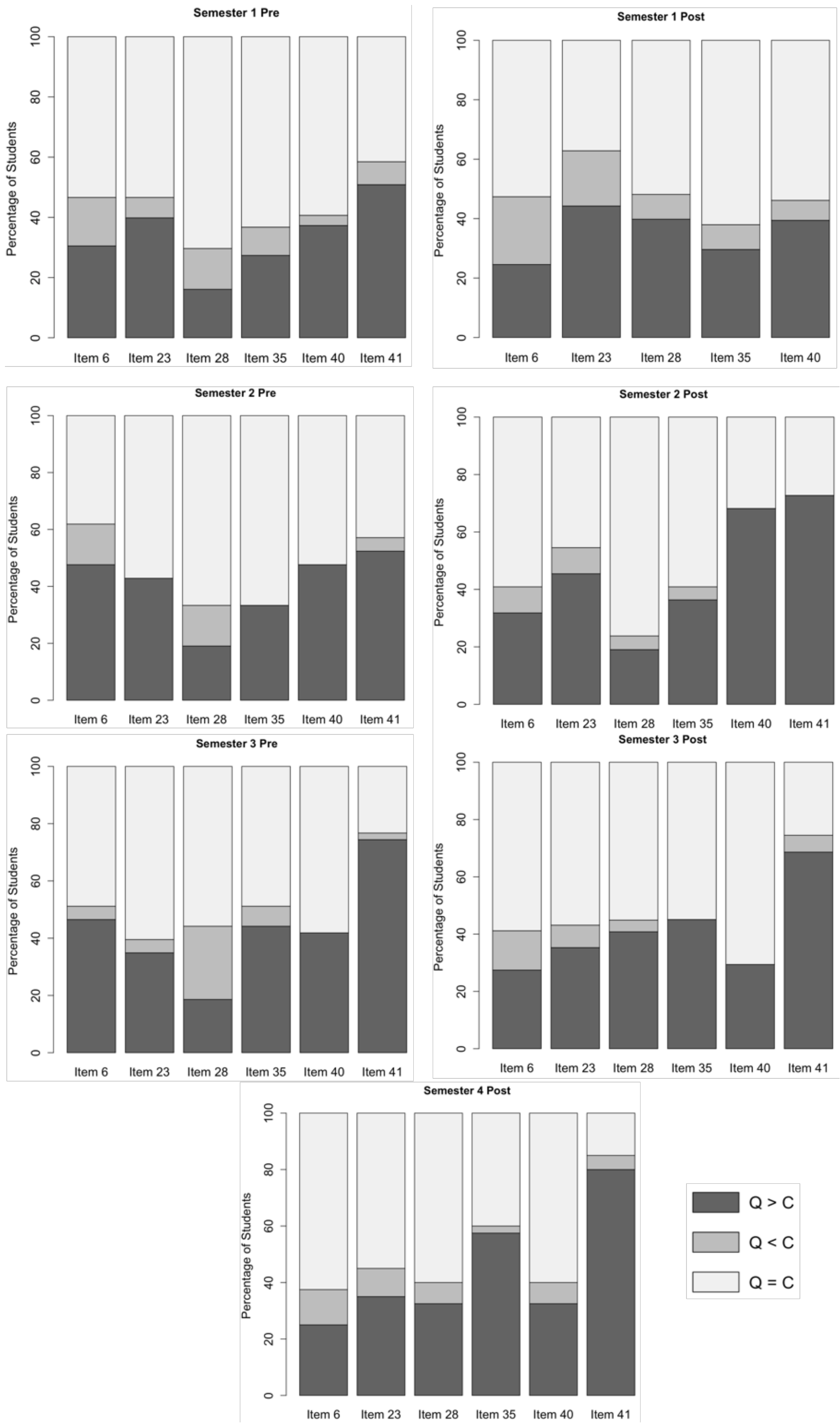



FIG. VI.1. Classical-quantum splits on the six bifurcated CLASS items for the seven survey administrations. Each bar shows the percentages of students who did not split their response ($Q = C$), split their responses with less agreement for quantum than classical ($Q < C$, i.e., quantum-disagree/classical-agree, quantum-disagree/classical-neutral, quantum-neutral/classical-agree), and split their responses with more agreement for quantum than classical ($Q > C$).

### 2. Spontaneous epistemological comparisons

Of the seven interviewed students, six expressed a spontaneous epistemological comparison, suggesting that the classical-quantum splits found in the survey results correspond to something psychologically robust for students. We present one spontaneous epistemological comparison from each of those six students, all pseudonymous.

***Wei:*** Wei initiated a segment of conversation by stating, “Physics 1 and 2 is just really straightforward and you...can do the exams and homeworks really easy. But for quantum, after you understand what’s going on and you see the questions, but you still really confused about it. So it’s really changed the way to think about physics.” The interviewer followed up by asking “So it’s like changing the way you think about physics but then that sometimes doesn’t match with doing the homeworks or exams, not like with your previous experience, right?” Wei responds, “After you know the formulas about the physics like on physics 1 and 2...you just plug them into the questions. And then for quantum you must understand what he’s saying and what he’s asking and you can know the formulas but you sometimes cannot understand what he’s saying at you.” We see Wei’s first statement as spontaneously comparing classical physics courses (Physics 1 and 2) with the quantum course he’s currently taking. When prompted for clarification, Wei implies that the comparison has an epistemological component: in Physics 1 and 2, knowing the formulas is sufficient for solving problems, whereas in quantum mechanics, that’s not the case, because you can “know the formulas” but still not “understand what he’s saying at you” or “what he’s asking.”

***Jacob:*** Responding to a prompt about the role of math in the QM class, Jacob says, “Some of the math that we would’ve gotten into would be so complex I feel like it would just take up a ton of time to try and understand and maybe make us more confused than we were earlier and just be really hard. But I don’t know, I feel like I’m typically a math kid. I like the math parts of that stuff more, like in physics 2 and 1 I totally liked the math more. I thought it was easy and fun...” Jacob here compares the mathematics he encountered in classical physics, which was “easy and fun,” to the math he encountered in the QM class, which was “complex” and “really hard.” Crucially, from an epistemological perspective, Jacob says that investing the “ton of time” needed to understand some of the QM math would have risked making "us more confused"—a hindrance rather than an aid to understanding.

***Tara:*** Responding to an open-ended prompt about her experience in the Modern Physics course, Tara spontaneously compared approaches to learning quantum versus classical physics: "Learning how to learn in quantum physics was kind of a challenge...With classical mechanics, you sort of have an intuition...but with quantum mechanics nothing really makes sense. Like there's situations in which the potential energy is greater than the total energy...Quantum physics doesn't really follow the same trajectory as those courses do." Tara here implies that the trajectory of her earlier, classical courses connected to intuition, whereas "with quantum mechanics nothing makes sense"—leading to a different course trajectory.

***Fernando:*** When asked about how physics and mathematics relate in QM, Fernando replied, "When it all comes down to it, if I don't understand what's going on conceptually, that's a little bit more difficult. Especially with this quantum stuff, I guess the conceptual work you need that mathematics behind it." We interpret "especially with this quantum stuff" to indicate a comparison with non-quantum stuff. Fernando is asserting that, in QM, the "conceptual work"—the intellectual work needed to make sense of the concepts—relies more ("especially") strongly on the underlying "mathematics behind it." This sentiment coheres with thoughts he expressed elsewhere in the interview.

***Eric:*** Asked about the role of conceptual versus mathematical reasoning in QM, Eric responds by drawing a distinction with classical physics courses: "I'd say the conceptual is less reliable now than it used to be, because in previous physics courses, the concepts were always intuitive...In quantum, it's more complicated, and less intuitive, so I'm less likely to trust my instincts on the concepts and more likely to trust the math."

***Zach:*** This student's one spontaneous epistemological comparison comes very close to the border between "comparison" and "not a comparison." The interviewer asked, "What did you think about the focus group, now that it's kinda over. Um, yeah, what's your experience?" Zach responded, "I think it's cool. Like I definitely think like quantum mechanics is weird. I definitely think there's like merit in finding out how it's thought about and learned, just because it's so strange, like the ideas are so like really brand new. I have like, I had nothing in my head to attach it to before coming in. So, I mean, like yeah it's definitely like weird to like get your head going and wrap around some of the ideas..." Zach here does not explicitly compare the quantum weirdness he repeatedly mentions with any aspect of classical physics. We see evidence of an implicit comparison, though, when Zach calls QM "really brand new," with "nothing in my head to attach it to" before the course begins. These phrases connote a discontinuity with his previous experiences, including possibly prior physics courses. If we make the inference that the "strange," "weird" nature of quantum ideas constitutes part of the discontinuity, then Zach is saying that QM is new and weird compared to classical physics. We acknowledge, though, that this is a higher-level inference than we made in attributing a spontaneous epistemological comparison to the five previous students in this section.

For readers who do not agree with this inference, we note that Zach, responding to an open-ended prompt, spontaneously starts talking about the weirdness of quantum ideas. So, even if it is too much of a stretch to infer that he views quantum ideas as weirder than classical ideas, it is clear that quantum weirdness is salient to him.

In summary, of the seven interviewed students, five expressed clear spontaneous epistemological comparisons and one expressed spontaneous epistemological views about QM that involved an implicit comparison with classical physics.

### 3. Bifurcated survey results: Pre-post shifts

In addition to looking at the splits between the classical and quantum responses within a pre- or post-administration of the survey, we also tested to see if there were pre to post shifts for either the classical or quantum versions of the items. The focus of this chapter is on the classical-quantum splits we presented above, but we also include some of the pre-post results as an emergent finding that warrants further investigation beyond the scope of this chapter. In the Semester 1 CU modern physics class, we observed statistically significant pre-post shifts for CLASS items 28 and 40, and the most notable of these shifts occurred for the *classical* version of the questions. That is, for item numbers 28 and 40, the Semester 1 students' beliefs about classical physics seem to have changed after a semester of QM instruction. The results of the pre-post analysis for this particular class are displayed in Table VI.3.

TABLE VI.3. Pre-post shifts on the bifurcated CLASS questions for the Semester 1 CU modern physics class. The p-values are determined with the Bhapkar test and corrected using the Holm-Bonferroni method. Items with significant splits at the $\alpha = 0.05$ level are indicated in boldface. Item 41 was not included on the post survey in this particular semester and is thus excluded from this table.

| CLASS Item # | Item | Version | $N$ | p-value |
|---|---|---|---|---|
| 6 | Knowledge in [classical/quantum] physics consists of many disconnected topics. | Classical | 86 | 0.56 |
| | | Quantum | 85 | 0.09 |
| 23 | In doing a [classical/quantum] physics problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem. | Classical | 86 | 0.31 |
| | | Quantum | 85 | 0.31 |
| 28 | Learning [classical/quantum] physics changes my ideas about how the world works. | **Classical** | 86 | **0.02** |
| | | **Quantum** | 82 | **0.03** |
| 35 | The subject of [classical/quantum] physics has little relation to what I experience in the real world. | Classical | 85 | 0.51 |
| | | Quantum | 82 | 0.47 |
| 40 | If I get stuck on a [classical/quantum] physics problem, there is no chance I'll figure it out on my own. | **Classical** | 86 | **0.03** |
| | | Quantum | 81 | 0.56 |

Figures VI.2 and VI.3 display these results graphically. For item 28, *Learning [classical/quantum] physics changes my ideas about how the world works*, for both versions both pre- and post-instruction, the modal response was agree. That is, students overall see both classical and quantum physics as changing their ideas about how the world works. The responses to the classical item from pre- to post-survey shifted away from agree. That is, after a semester of QM instruction, the students were less likely to agree that classical physics changed their ideas about how the world works. On the quantum version of the item, the shift went in the opposite direction—after QM instruction, the students were more likely to agree that quantum physics changed their ideas about how the world works. For item 40, *If I get stuck on a [classical/quantum] physics problem, there is no chance I'll figure it out on my own*, the modal response for the classical version of the item both pre- and post-instruction was disagree. Overall, students have confidence in their problem solving

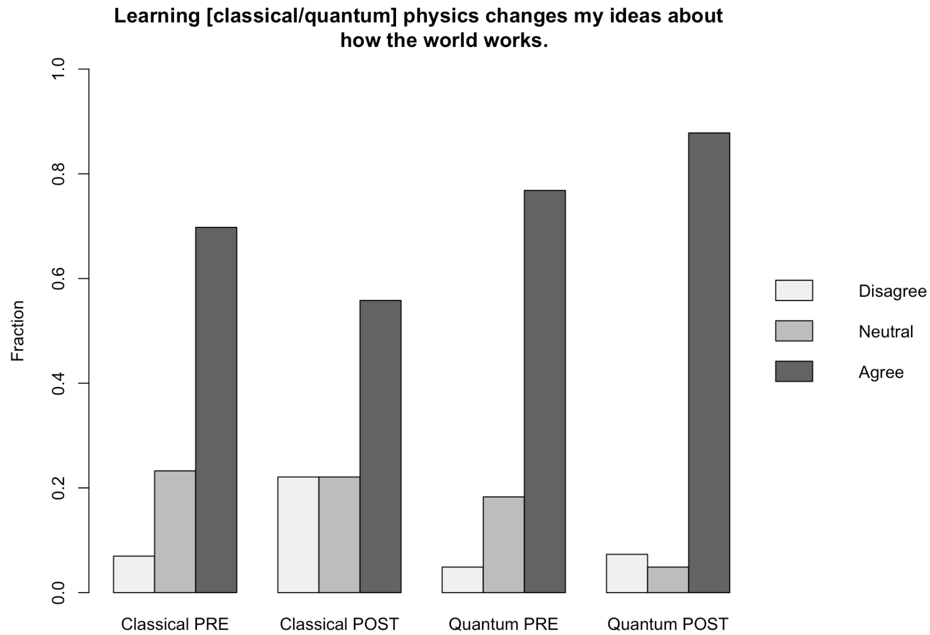



FIG. VI.2. Pre-post shifts on CLASS item 28 in the Semester 1 CU modern physics course. For both the classical and quantum versions of the item, the pre-post shift is statistically significantly different (with the Bhapkar test, at the $\alpha = 0.05$ level).

ability when they get stuck on a classical physics problem. There was a statistically significant pre-post shift towards agree on the classical version of this item. That is, after instruction on quantum physics, students were *more* likely to report confidence around solving classical physics problems. We did not observe a pre-post shift on the quantum version of the item—here, the modal response was again disagree, but students were less likely to disagree with the quantum item compared with the classical item (52% on the post-survey for quantum compared to 88% on the post-survey for classical). These results indicate that in this one semester of a modern physics class, students shifted toward being *less* likely to see classical physics as changing their ideas about how the world works and having *more* confidence in their problem solving ability in classical physics.

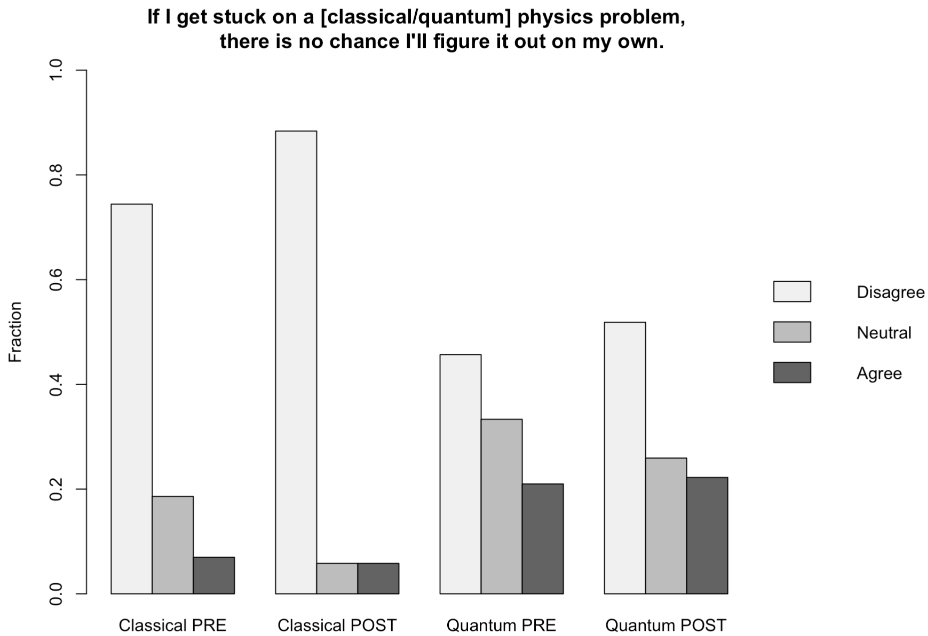



FIG. VI.3. Pre-post shifts on CLASS item 40 in the Semester 1 CU modern physics course. On the classical version of the item, the pre-post shift is statistically significantly different (with the Bhapkar test, at the $\alpha = 0.05$ level). The shift on the quantum version of the item is not statistically significant.

## D. Discussion

### 1. Existence of split epistemologies

Before summarizing what this study shows, we want to reiterate our disclaimers about what it does not show. We cannot make claims about patterns in the differences between students' epistemological views about classical and quantum physics, because validated clusters of survey items would be needed to support such claims. Nor can we draw generalizations about students' reasoning in quantum mechanics or even students in the CU modern physics course from the interview analyses, because N is small and because the spontaneous epistemological comparisons emerged in the context of an ongoing relationship between the research team and the interviewees.

The "split" results indicate that most students do perceive epistemological differences to exist between classical and quantum physics, and the interview analyses suggest that at least some of these epistemological "splits" correspond to students' held views. So, when we talk about students' epistemologies of physics, we need to be careful about treating "physics" as monolithic; these results show that a significant number of individual students display different approaches to knowledge in physics depending on the specific subdiscipline within physics.

An emergent finding worth examining further is the observation of significant pre-post shifts in a given semester on select questions. We observed statistically significant pre-post shifts at CU for both versions of item 28. For both pre and post, and both classical and quantum, the majority of students agreed with the statement. However, nearly 25% of the students who responded to the classical question on both pre and post surveys (N=86) shifted away from Agree. That is, prior to learning QM these students professed belief that learning classical mechanics changed their ideas about how the world worked, but after the semester of Modern Physics instruction, they responded with Disagree or Neutral. The opposite shift occurred for the corresponding quantum physics statement—Of the students who responded to the quantum statement on both pre and post surveys (N=82), 22% of them shifted towards Agree (mostly from Neutral). This latter result is what we might expect to happen after a semester of QM instruction, but the shift in *classical* epistemologies is intriguing.

Tara, a student in the CU Modern Physics course, shifted from Neutral to Disagree for the classical statement (suggesting a decreased belief that classical physics changes her ideas about how the world works) and from disagree to strongly agree for the quantum statement (suggesting an increased belief that quantum physics changes her ideas about how the world works). The quotes from Tara presented in Section VI C 2 help us understand what this pre-post shift might mean.

Tara articulates a difference between classical and quantum physics—she does not have the same intuition and everyday experiences with quantum physics that she does for classical physics, and thus she reports having to find a new way to make sense of the new concepts she has learned. Tara's explanation could be related to the change in her beliefs about learning classical and quantum physics as observed in her responses to the statement "Learning physics changes my ideas about how the world works". We find it interesting that Tara talks explicitly about the differences in learning classical and quantum physics without being prompted to do so, and we posit that this could be a result of the explicit emphasis on epistemology within the course. The fact that instruction in the course explicitly addresses things like epistemology in and interpretation of quantum mechanics may help the students be more aware of the difference between classical and quantum physics and the nature of knowing in each. We did not observe a comparable shift on this item in the other three courses in the dataset, which did not share this emphasis.

Additionally, on the post survey the students could be responding to the classical statement relative to the quantum statement. That is, we could imagine a student stating, "Now that I know what quantum physics is, classical physics doesn't change my ideas about the world all that much in comparison." At the culmination of the course, classical physics is no longer a new subject and thus might no longer be viewed by some students as a subject that changes their ideas. In another student interview, Fernando—also enrolled in the CU Modern Physics course—responds to the question, "Has this course changed the way you think about physics?" He answers that the course has definitely changed his view of physics. He elaborates:

*Fernando: Umm, I think really going into, so I did not take physics in high school. I was actually, I came in as a biology major. So I had no real prior physics experience, so I went into physics 2 which was electricity and magnetism. Completely new concepts, but at least they kind of sort of made sense, you know like understanding how electrons work. And now going into this view that you know things can be waves...even looking at electrons, like electrons can be waves and things like that. It totally changes how I look at even electricity and magnetism, which is super awesome and super crazy.*

Although Fernando was not one of the students who contributed to the overall shift toward disagree for "Learning classical physics changes my ideas about how the world works" (in fact, he shifted from Agree to Strongly Agree for classical and from Neutral to Strongly Agree for quantum), his interview response may provide an example of a student thinking of the two versions of this statement comparatively. On the survey Fernando agreed that learning both classical and quantum physics changed his ideas about how the world works, but in his interview he spoke of the differences between learning classical and quantum physics. Learning classical physics, he said, involved learning "completely new concepts, but at least they kind of sort of made sense", whereas learning quantum mechanics involved learning totally new concepts, changing the way he thought about even things he had learned in classical physics. Another statement that resulted in interesting shifts was "If I get stuck on a [classical/quantum] physics problem, there is no chance I'll figure it out on my own". The CU Semester 1 data show a statistically significant shift from Neutral to Disagree for the classical statement. While the majority of students disagreed with the statement on both pre and post, 15% of the students shifted from Neutral to Disagree. This result suggests that learning QM has given these students more confidence with regard to solving classical physics problems. There was no shift for the quantum statement, and the majority of responses were Disagree, followed by Neutral and then Agree. There were no statistically significant shifts in the UMD data, although we see an interesting result. All (pre) or nearly all (post) of the students disagreed with the classical statement—these students feel confident in their problem-solving abilities when it comes to classical physics. For the quantum statement, initially 50% of students disagreed followed by Neutral and then Agree but on the post survey the opposite trend appears: Agree had the most responses (39%), followed by Neutral and then Disagree. Again, the small N (18) and lack of statistical significance makes this hard to interpret.

Overall, it is no surprise that the students' problem-solving confidence is higher for classical physics than it is for quantum. They have had more experience with classical physics (both in class and in the "real world"), and the level of mathematics needed for QM is much higher. Yet the finding that students' responses to some classical items shifted over a semester of QM instruction is interesting and merits further investigation.

#### 2. What counts as epistemological sophistication?

These results highlight an important interpretational issue about how to interpret the "split" results. Usually, CLASS responses and other epistemological survey responses are scored as favorable, unfavorable, or neutral. Applying this scoring procedure to the bifurcated survey results, one could conclude that students hold less favorable views about quantum physics than they do about classical physics. Specifically, from the post-semester results in Table 1 and Figure 2, we would conclude—to the extent that responses to individual survey items allow conclusions—that students who have completed a quantum physics class hold less sophisticated views about the utility of sense-making in quantum physics than they hold about the utility of sense-making in classical physics

(CLASS item 23, item 40). One would also conclude that students hold less sophisticated views about how quantum physics relates to the real world than they hold about how classical physics relates to the real world (CLASS item 35). In addition, one would conclude that students are more sophisticated about how quantum physics necessitates a rethinking of how the world works than they are about how classical physics necessitates a rethinking of how the world works (CLASS item 28).

We question this interpretation of our split results for both theoretical and empirical reasons. The theoretical argument is that epistemological sophistication centers around nuanced contextualized judgments, and the empirical support comes from experts displaying epistemological differences between classical and quantum physics. As discussed in previous literature on students' epistemologies of science [83, 138, 164], the default scoring scheme for epistemological surveys, counting some responses as more favorable than others, is not merely a matter of convenience or habit ("that's what everyone does!"). Such scoring schemes encode an assumption about the nature of epistemological expertise, that it involves holding certain favorable views at the expense of other, less favorable views. So, for instance, the coding and scoring schemes used in interview- and survey-based epistemology research encode the assumption that scientific knowledge should be viewed consistently as tentative rather than certain, coherent rather than piecemeal, constructed by people rather than "read off" of nature or transmitted from authority, and so on [83].

An emerging consensus, however, holds that epistemological sophistication consists not of adherence to "blanket generalizations" [83], but to the ability and propensity to make contextualized judgments about knowledge-generating processes, coupled to criteria for monitoring and evaluating those processes and their results [131]. For instance, it makes epistemological sense to treat "the Earth is round" as certain knowledge, and telephone numbers as transmitted from authority to recipients. It makes epistemological sense to treat the learning of complex concepts like Newton's 2nd law as more "constructivist" than learning the value of the universal gravitational constant.

Applying this argument to classical versus quantum physics, we could argue that students are sophisticated to display epistemological splits on our bifurcated survey—to treat classical and quantum physics as epistemologically different in certain ways. It is reasonable to view quantum concepts as less intuitive than classical concepts and therefore to rely more on mathematics and less on intuitive reasoning when solving problems. Along the same lines, it is reasonable to view quantum mechanics as requiring a deeper rethinking of how the world works than classical mechanics does; at least some physics experts would agree with these "split" views. Of course, epistemological expertise does not demand these splits; depending on one's interpretation of quantum mechanics, experience with quantum concepts, and so on, an expert could decide that classical and quantum mechanics are more epistemologically similar than different. Thus, we would expect some experts not to split. We argue that there are sophisticated reasons for splitting and for not splitting, and hence the "default" interpretation of epistemological splits should *not* be that students possess greater epistemological sophistication about one subfield of physics than another.

This theoretical argument makes empirical predictions about the views of epistemologically sophisticated physicists: different physicists will display different epistemologies of quantum physics, likely leading to differences in their tendencies to express classical-quantum epistemological splits.

Accounts produced by historians and philosophers of quantum mechanics align with these predictions. Max Jammer [46] and others have documented the lively debates among Bohr, Einstein, Heisenberg, and other early developers of QM about how to interpret quantum mechanics. The interpretational dilemmas and disagreements have not all been resolved (see, e.g., any issue of Foundations of Physics). Preliminary results from another thread of the larger study also align with those predictions [165]. We briefly review relevant aspects of their results. We administered the bifurcated survey to two instructors—one from the CU modern physics course and the other from the CU upper division quantum 1 course. On the quantum versions of the items, the two instructors' responses differed from one another on four of the six items. Furthermore, instructor 1 expressed classical-quantum splits on 3 of 6 items (two of these being "weak" splits between strongly disagree and disagree), and instructor 2 split on 3 of the items (one of these being a "weak" split). Obviously, we cannot generalize from these small-N results, but they suggest the likely value of a larger, partly interview-based study of experts' classical-quantum splits (and non-splits). In Chapter VIII, we will further discuss this preliminary investigation (and others) into expert's responses to the bifurcated CLASS items.

We also see that students' reported attitudes towards classical physics can change after quantum physics instruction. This challenges a formulation of "pre" and "post" that implies that the "post" result is the end of the story, since we see that students' attitudes toward classical physics (after completing introductory physics) could still shift further. This result again challenges the standard interpretation of CLASS (and other) survey results. By the standard interpretation, taking quantum physics led to students becoming less epistemologically sophisticated about classical physics, because students became much less inclined to see "learning classical physics [as changing] my ideas about how the world works." However, in Section VI D 1 above, Tara and Fernando give us reason to interpret this shift as a gain, not a loss, in epistemological sophistication. Those students affirmed that learning classical physics necessitates a rethinking of how the world works, but a rethinking that now seems minor as compared to what quantum physics demands, since in classical physics they could rely at least to some extent on basic intuitions about reality.

### E. Initial implications for instruction

As we attend to how students think in domain-specific ways about the nature of learning and knowing, we ought to consider implications for not only how we measure impacts of our classes but also how we teach. The consistency of our "split" findings across multiple instructional contexts, for most of the bifurcated survey items, leads us to offer one thought about implications of this work for instruction. Although experts may disagree about what constitutes the "right" degree of classical-quantum splitting with respect to the value of conceptual reasoning (versus the need to "shut up and calculate"), we worry that quantum physics students are in danger of sliding too far away from conceptual reasoning. Students may internalize messages (intended or not) that quantum mechanics is a place to rely solely on mathematical calculation [34, 166, 167] and that sense-making will not avail them here, which may impede the activation of the productive epistemological resources that they bring to classical physics. As instructors, we should not assume that students' "plug and chug" tendencies are entirely a "natural" epistemological response to quantum weirdness; we need to monitor ourselves and our students to avoid amplifying that tendency. In Chapter VIII, we present students' reactions to one instructor's attempt at doing just that by continually reiterating to the class that "quantum is weird, but not that weird".

### F. Future directions

There are many avenues of future work deriving from this exploratory study: extending the dataset of students' responses to investigate the prevalence of splits across different contexts, further analysis of students' pre-post shifts on the bifurcated CLASS items, conducting a large scale survey and interview study of experts' epistemological splits (or lack thereof), or formal validation of the bifurcated CLASS items. The following chapter investigates the first item, extending the quantitative dataset of this study as well as analyzing new focus group interviews to identify some common reasons that students express epistemological splits between classical and quantum physics.

# Chapter VII. Advancing our understanding of students' sub-domain-specific epistemologies

In Chapter VI, we demonstrated the *existence* of splits in students' attitudes toward classical and quantum physics, and began to challenge the assumption that a "favorable" response to a given item in the context of introductory classical physics (or "physics" as a monolithic whole) should also be the "favorable" response in the context of quantum physics. We call for attending to domain-specificity when it comes to thinking about, measuring, and attending to students' epistemologies. This chapter is an extension of that work, expanding the quantitative data set of student responses to the bifurcated CLASS survey and including a new qualitative study of students' reasons and justifications for their responses to those bifurcated items. As this chapter is a continuation of our initial work (presented in Chapter VI), the quantitative data presented here *include* those from the above chapter, though with responses from several hundred additional students. Through this extension, we: a) document the *prevalence* of students' epistemological splits across several institutional and instructional contexts, b) identify and categorize elements of student reasoning to attend to that can help us understand why students might split their epistemic stances toward classical and quantum physics, and c) argue that epistemological sophistication does not hinge on the absence or presence of splits (i.e., epistemological splits *can* be productive for students). In general, we consider students' ideas or reasoning to be productive if they support progress toward understanding the physics, sustain engagement in sense making processes, provide opportunities for students to engage in expert-like practices, or support cohesive social interactions. Specifically in the context of epistemology, we focus on the expert-like practices aspect of productivity. Building on the theoretical argument that epistemological sophistication includes being able to make contextualized judgements [83, 131], we present examples of students articulating these contextualized judgements. Further, in students' justification for reporting epistemological splits we identify other elements or practices—like metacognition and valuing sense making—considered to be productive for students' learning and development. Parallel to Chapter VI, within each section below (Methods, Results, Discussion) we present two subsections, for the quantitative and qualitative parts of the study.

### A. Methodology

In this study of domain-specific epistemologies, we use a mixed methodological approach. Quantitative analyses of survey responses help us determine the presence and prevalence of students' epistemological splits between classical and quantum physics, and qualitative coding of focus group interviews help us identify and categorize reasons that students might report different epistemic stances to the two domains.

#### 1. Quantitative: Bifurcated survey study

The survey methodology for this continued study is the same as that presented in the prior chapter. We bifurcated six CLASS [116] items into pairs of items asking about classical and quantum physics (see Table VI.1) and included them on a survey along with other questions asking about attitudes towards quantum physics specifically. This survey has now been administered in 15 classes across 8 semesters (with 11 instructors) to a total of 1,089 students—typically the survey is given both pre-and post-instruction, but in some instances we only administered the post survey. In the initial exploratory study (Chapter VI) we analyzed and discussed classical-quantum splits as well as pre-post shifts, although the bulk of the discussion focused on the former. While in the initial work we discovered an interesting result in one of the CU modern physics courses—that students' views toward *classical* physics changed after instruction on *quantum* physics—here, we hone in on only the classical-quantum splits, and leave further pre-post analysis for future work. The courses in the expanded dataset presented in this chapter span four universities—CU, University of Maryland (UMD), California State University Fullerton (CSUF), and California Polytechnic Pomona (CPP)—and include both lower- and upper-division, and both modern physics and quantum mechanics courses. We have pre- and post-data from ten of the fifteen courses, and only post-data from the remaining five. Table VII.1 lists the courses in our dataset. There are two courses at CPP taught by different instructors and with different pedagogical approaches—a required position-first upper-division QM class, and a spins-first upper division QM elective. One thing that slightly complicates our analysis is that some students took both of these courses concurrently. There were

40 students enrolled in the traditional position-first course, and 17 enrolled in the elective, 10 of whom were co-enrolled. Each student only took the survey once. Since our primary goal with the extended quantitative analyses is to investigate the prevalence of epistemological splits, we do not double count students. Thus, we split these 47 students up into two groups: those who only took the traditional upper-division course (N=30), and those who took the elective course (including students who took both, N=17). If we were to use these results to make conclusions about the impact of position-first versus spins-first QM classes on students' domain-specific epistemologies, this grouping would be problematic. However, for now we focus on investigating the *prevalence* of split responses and thus find that this grouping is sufficient. In doing so, we still acknowledge that there may be instructional or pedagogical factors that influence students' epistemic splits, which is why we avoid aggregating the two groups together entirely. If there are significant differences in students responses in the position- versus spin-first courses, combining the two groups may wash out splits we might otherwise see. In Table VII.1, we provide an Instructor ID so that we can compare the existence of splits between different courses taught by the same instructor, or different instructors teaching the same course. This serves to demonstrate that epistemological splits are not the result of a few instructors teaching in particular ways, and will be helpful in Chapter VIII when we conduct preliminary investigations into the impact of instructors on students' development of (domain-specific) epistemic stances.

The four schools in our dataset are all large, four-year, public, research universities. CU and UMD are predominantly white institutions [106, 168], and CSUF and CPP are Hispanic serving institutions [169, 170].

As in the prior chapter, we collapsed the student responses to a three-point Likert scale (disagree, neutral, agree) and performed the Bhapkar test [153, 154] to determine if two distributions (classical versus quantum of the same item) were statistically significantly different from one another. For courses that we have pre-post data for, we corrected the p-values resulting from the Bhapkar test for four comparisons using the Holm-Bonferroni correction method [163]. For the post-only datasets, no corrections were needed since for each item we only performed one test (classical versus quantum).

TABLE VII.1. Courses in which we administered our bifurcated CLASS survey. Course ID is defined by *university–course #–semester #*. (Semester #'s and instructor ID's are a continuation from those defined in Chapter VI.) Topic is either modern physics (MP) or quantum mechanics (QM). All MP courses are lower-division with the exception of UMD 420-2 which is an upper-level modern physics elective. All QM courses are upper-division. Courses for which we only have post-data are denoted by *.

| Course ID | Topic | Student pop. | Pedagogical approach | $N$ | Instructor |
|---|---|---|---|---|---|
| CU 2130-1 | MP | Engineering | Reformed, emphasis on epistemology | 120 | A |
| UMD 420-2 | MP | Engineering | Traditional | 27 | D |
| CU 2130-2 | MP | Engineering | Traditional | 87 | E |
| CU 2170-2 | MP | Physics | Traditional | 102 | F |
| CU 2130-3 | MP | Engineering | Reformed, co-taught | 59 | A & B |
| CU 3220-4* | QM | Physics | Reformed, spins-first | 41 | C |
| CPP 401-4* | QM | Physics | Reformed | 30 | G |
| CPP 499* | QM | Mixed | Reformed, spins-first | 17 | H |
| CSUF * | QM | Physics | Reformed, spins-first | 17 | I |
| CU 2130-6* | MP | Engineering | Traditional | 68 | J |
| CU 2130-7 | MP | Engineering | Reformed, emphasis on epistemology | 121 | A |
| CU 2170-7 | MP | Physics | Traditional | 92 | F |
| CU 3220-7 | QM | Physics | Reformed, spins-first | 46 | C |
| CU 2130-8 | MP | Engineering | Reformed | 114 | A |
| CU 2170-8 | MP | Physics | Traditional | 148 | K |

### 2. Qualitative: Focus group study

In order to investigate the reasons that students might exhibit different epistemic stances toward classical and quantum physics, we conducted a two-semester focus group study with upper division QM students. The participants were from the CU 3220-4 and CU 3220-7 courses listed above in Table VII.1—the same class (upper division QM for physics majors) taught by the same instructor (with a spins-first approach) three semesters apart. In addition to the required lectures and other course components, the instructor offered an optional tutorial section once a week (some students took it as a one-credit course, others attended but did not receive course credit). In Semester 4, we conducted a focus group study during this tutorial section the last week of the semester. Thirteen students (four women and nine men[18]) voluntarily participated in the hour-long study, in exchange for food and a chance to reflect with their peers on their learning in QM as well as review content

[18] In this particular semester we did not collect demographic information, so the gender information we report is our perception of gender expression. We include this information because it may be important for interpreting students' interactions and comments within the focus group setting (see Chapter IX).

for the final exam. The instructor was not present during this hour, and we had three groups of students (two groups of four and one group of five) simultaneously engaged in conversation in one room. The students had completed the bifurcated CLASS survey the week prior to the focus group study. Each group was audio and video recorded, and JRH walked around dropping in briefly on each group to listen and ask follow up questions. The protocol (given in detail in Appendix E) consisted of three parts. Part one presented three of the bifurcated CLASS items (items 6, 23, and 41) with the classical items grouped together followed by the quantum versions. Students were asked to first answer the questions individually. Then, they were asked to discuss with their group members, answering the questions, *"What did you answer, and WHY? Can you come to consensus?"* The latter question was meant to encourage students to articulate their reasoning and to include every member of the group in the conversation, not to suggest that they *needed* to come to agreement on the answers. Most groups did eventually come to consensus on each item, but in the instances where group members had differing views from one another, we found that these prompts led to extensive discussion (regardless of whether consensus was ultimately reached). Part two of the protocol was a QM content section, including a tutorial on interpretation of operators that they had completed in the optional tutorial section earlier in the semester. The goal of having the students collectively work through this tutorial was geared toward another project on mathematical sense making and will not be a focus of analysis in this study. Part three consisted of more open-ended questions about the students' experiences learning QM (e.g., *What was your experience like this semester? What were the challenges for you in this course?*).

In Semester 7, we conducted a second round of this focus group study, with a few methodological changes. This time, we recruited students from the overall class and the study did not take place during the normal tutorial section time. Students were compensated monetarily, and also received food and course extra credit for their participation. We again had thirteen participants (one woman, one gender fluid person, and eleven men[19]). One might be concerned about self-selection affects in each of these rounds of focus groups since the types of students who voluntarily participate in an extra weekly tutorial section or sign up for a research study might be the students who are more

[19] We received responses to a demographic survey for nine of the thirteen participants. For those who did not provide their gender, we report our perception of their gender expression. We do so because this information may be important when interpreting students' interactions with one another

inclined to get good grades or be reflective about their learning. While the focus group participants were likely not representative of the overall class enrollment, this is not a methodological concern for this study because the goal is to determine the types of reasons students *might* give for similarities or differences between their epistemic views of classical and quantum physics, not the prevalence of this reasoning. In this second round of focus groups, Part one of the protocol (available in full in Appendix E) was the open-ended questions about students experiences learning QM. We changed this order to see if students would have unprompted discussions around epistemology (compared to in Semester 4 when the students' "open-ended" conversations were in a sense primed by the bifurcated CLASS items). Although it did not explicitly prompt students to compare their views toward classical and quantum physics, this section included a few questions that were more directly related to epistemology, like *"Has this course changed the way you think about physics?"* and *"What role does intuition play in quantum mechanics?"* Part two then included five of the six bifurcated CLASS items (excluding item 41) along with the same prompts as in Semester 4 encouraging students to discuss the items with one another and see if they could reach consensus. Part three included content questions from the QMCA [32] which the students had recently taken as a post-assessment in class. In Semester 7, there were two researchers (JRH and a post-baccalaureate researcher) facilitating the focus groups. In looking through the video data collected in the first round (semester 4) of focus groups, we noticed that the students' conversations were less generative and candid when the facilitator came over to listen or ask a follow up question. For this reason, in the second round, we did not interrupt the groups (when possible), letting the students themselves direct the conversations (as guided by the hard copy prompts given to each group).

We analyzed the audiovisual data from the six focus groups across the two semesters through emergent, or generative, coding [171]. Focusing on the two sections of the protocol bearing directly on epistemology (parts one and three in Semester 4, and parts one and two in Semester 7), we identified instances of epistemological comments. An epistemological comment (or epistemic comment) is any comment a student makes about or relating to the nature of knowing or learning. We further tagged these epistemological comments as being "split", or domain-specific, if the students make a comparison between classical and quantum physics. This method of analysis is in line with that used in Chapter VI to identify "spontaneous epistemological comparisons", except here we are not concerned with the spontaneity of students' comments. We are, however, focused on the domain-specificity as the goal of this analysis is to identify things to attend to in order to understand why students might hold or exhibit different epistemic stances between classical and quantum physics. Treating individual student comments as the unit of analysis, we watched the videos several times and grouped these epistemological comments thematically (i.e., how were students comparing classical and quantum physics, and what justifications or explanations were they giving?). In identifying instances of students' epistemological comparisons, we do not include discussions around CLASS item 41 (included in the protocol for Semester 4, but not Semester 7). As mentioned previously, this item emphasizes QM concepts in addition to epistemology, which differs from the other five items. Further, in Semester 4 when we included this item in the focus group prompts, it was the only one for which most groups reached unanimous consensus with little to no discussion. For these reasons, we exclude item 41 when identifying and categorizing student reasoning around the epistemological splits. Through multiple pass-throughs of the videos and accompanying transcripts and discussions among the research team, we iteratively refined the list of themes with which we categorized the episodes of student discussion. Next, we condensed the themes into broader categories. The themes are fine-grained and distinct reasoning elements, although often intertwined with one another, while the categories are more general and encompass multiple related themes. The themes and categories that resulted from our coding process are listed in Table VII.3 in the Results section below. We coded each epistemological comment as falling under one or more themes. A small percentage ($< 5\%$) of the students' domain-specific epistemological comments did not fit into any of the themes.

## B. Results

### 1. Quantitative: Prevalence of epistemological splits

A summary of the survey results for all 15 courses is presented in Table VII.2. For each classical-quantum pair, we include the number of survey administrations that had a split result, out of the 25 total administrations (10 pre-post and 5 post-only). Item 41 was only given on 23 administrations (9 pre-surveys, and 14 post-surveys). Complete results, including p-values resulting from the Bhapkar test are presented in Appendix F.

TABLE VII.2. Summary of bifurcated CLASS results for the full dataset. We report the total number of times we observed splits for a given item, and how many of these occurred on pre- and post-surveys. There were 25 total survey administrations (23 for item 41)—10 pre-surveys (9 for item 41) and 15 post-surveys (14 for item 41). To count as a "split", the survey responses for the classical and quantum versions of the same item had to be statistically significant with the Bhapkar test at the $\alpha = 0.05$ level.

| CLASS Item # | Item | Total splits observed (out of 25) | Splits on pre-surveys (out of 10) | Splits on post-surveys (out of 15) |
|---|---|---|---|---|
| 6 | Knowledge in [classical/quantum] physics consists of many disconnected topics. | 13 | 6 | 7 |
| 23 | In doing a [classical/quantum] physics problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem. | 23 | 10 | 13 |
| 28 | Learning [classical/quantum] physics changes my ideas about how the world works. | 11 | 3 | 8 |
| 35 | The subject of [classical/quantum] physics has little relation to what I experience in the real world. | 25 | 10 | 15 |
| 40 | If I get stuck on a [classical/quantum] physics problem, there is no chance I'll figure it out on my own. | 23 | 9 | 14 |
| 41 | It is possible for physicists to carefully perform the same measurement in a [classical/quantum] physics experiment and get two very different results that are both correct. | 23 | 9 | 14 |

We observed splits on every administration of items 35 and 41 (25 and 23 administrations respectively). Figures VII.1 and VII.2 illustrate what these splits look like for two example courses: CU 2130-3 (Modern Physics for Engineers) and CU 3220-7 (QM1). On item 35, students generally see classical physics as related to their real world experiences, but are less likely to report this real world connection for quantum physics (Figure VII.1). The students in CU 2130-3 were more likely to see QM as related to their real world experiences than the students in the CU 3220-7 course. Although every course in our dataset had splits on this item, we often see differences

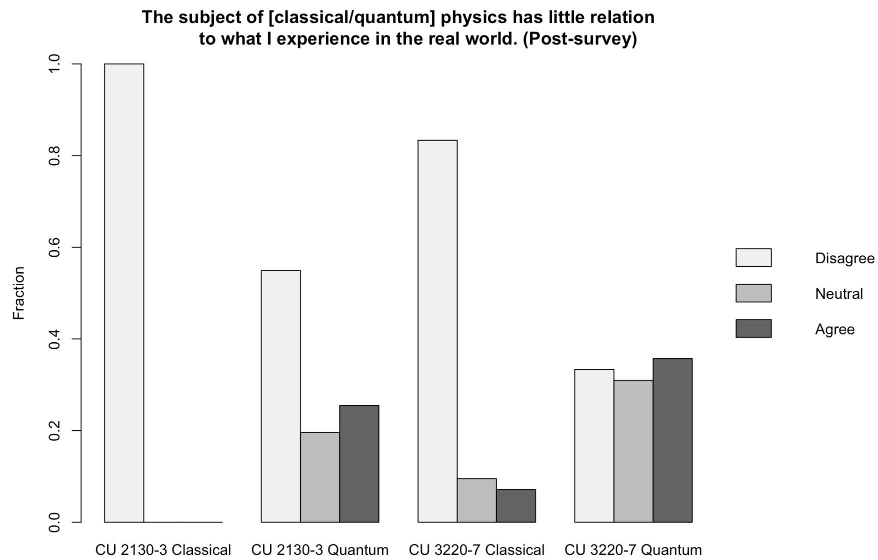



FIG. VII.1. Examples of post-instruction classical-quantum splits on CLASS item 35 for two courses: CU 2130-3 (Modern Physics for Engineers) and CU 3220-7 (QM1).

between courses in the quantum responses. We begin to explore some of these differences and their connection to the individual instructors in Chapter VIII. On item 41, students typically disagree with the classical version and agree with the quantum version. These responses reflect an understanding of the intrinsic probabilistic nature of QM. It is interesting to note that even the pre-surveys had splits on item 41, which we consider to be an item that reflects QM content in addition to epistemological views. For the two classes show in Figure VII.2, the pre-survey splits look similar to the post-survey, with disagree the primary answer for classical and agree the most common answer for quantum. As we discuss in the qualitative results section below, students are often familiar with quantum physics concepts coming in to a modern physics or quantum mechanics class. Additionally, in most courses the pre-survey was given on the first homework assignment one or two weeks into the term. At this point, it is likely that the students had already received some epistemological messaging from the instructor. It is for this reason, in part, that we focus our discussion on the post-administrations of the survey.

Items 23 and 40 had split results on most (23 out of 25) survey administrations. We again present examples of what these splits look like for two courses (see Figures VII.3 and VII.4). Students overall disagree with item 23, saying that they would *not* trust their calculation rather

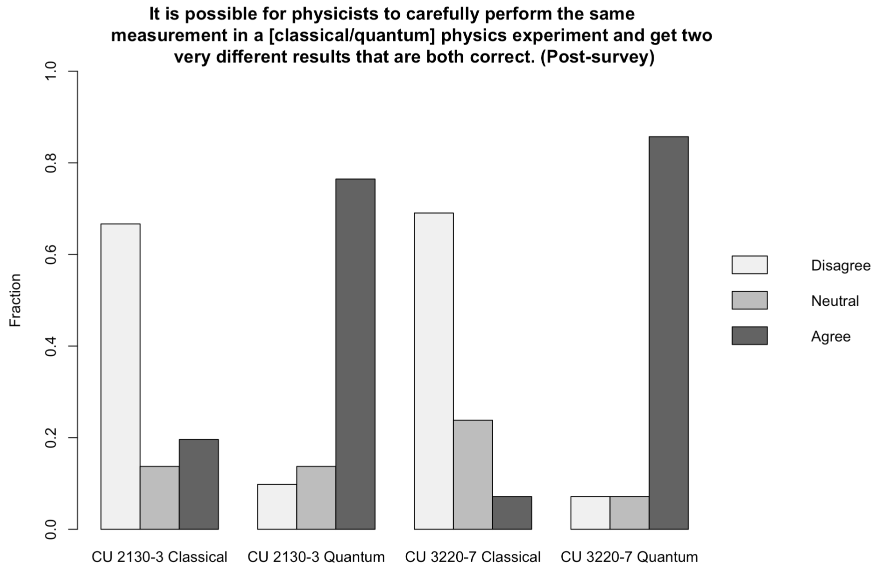



FIG. VII.2. Examples of post-instruction classical-quantum splits on CLASS item 41 for two courses: CU 2130-3 (Modern Physics for Engineers) and CU 3220-7 (QM1).

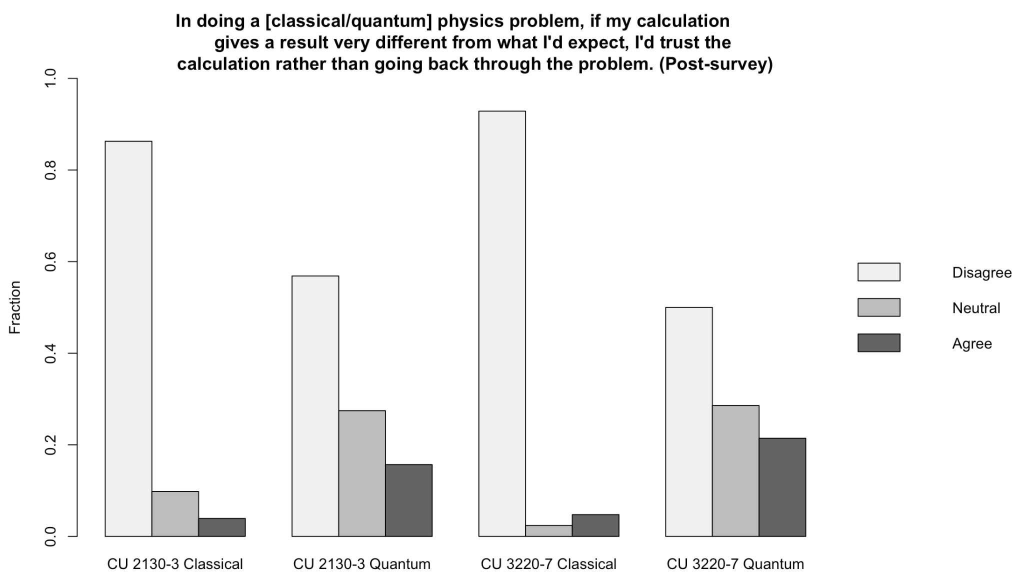



FIG. VII.3. Examples of post-instruction classical-quantum splits on CLASS item 23 for two courses: CU 2130-3 (Modern Physics for Engineers) and CU 3220-7 (QM1).

than going back through the problem, in either classical or quantum physics. However, students are less likely to disagree with this item in quantum physics. That is, they are more likely to trust their calculation in quantum physics as compared to classical physics (Fig. VII.3). We see similar results on item 40—students overall disagree that they have no chance of figuring a classical or quantum problem on their own, but they are less likely to do so for quantum physics (Fig. VII.4).

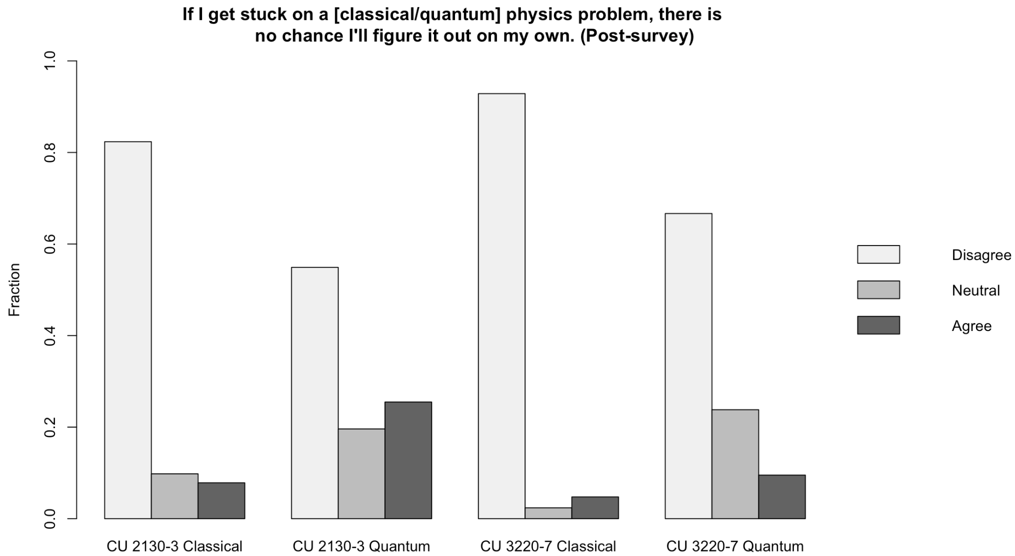



FIG. VII.4. Examples of post-instruction classical-quantum splits on CLASS item 40 for two courses: CU 2130-3 (Modern Physics for Engineers) and CU 3220-7 (QM1).

Items 6 and 28 only saw splits on about half of the survey administrations. When splits did occur on these items, students were more likely to view quantum physics as consisting of disconnected topics (item 6) and more likely to say that quantum physics changes their views of how the world works (item 28), as compared to classical physics.

### 2. Qualitative: Students' reasons for epistemic splits

The coding analysis described above resulted in twelve themes encompassed by five categories; these are presented in Table VII.3, along with an example student quote for each theme and the number of students (out of 26) for which each theme was present.

TABLE VII.3: Resulting from analysis of focus group data, twelve themes encompassed by five categories of reasons that students cite for epistemological splits between classical and quantum physics. Note: the "Intuition" and "Math" categories have one overlapping theme. For each category and theme, the number of students (out of 26 students in the focus groups) for which the category/theme was present is reported in parenthesis.

| Category | Theme | Quote |
|---|---|---|
| Intuition (18) | **1:** Students have two distinct "intuitions", one for classical and one for quantum (12) | "We just learned a different intuition...The axiom of our [classical] intuition comes from our experiences. But now we have a different axiom of which to pull our intuition from."–Ryan |
| | **2:** Students recognize that they can develop intuitions for quantum but might not be there yet (8) | "It's very different to try to be intuitive about this class I think. But there are some intuitions...you can gain."–Jane |
| | **3:** Students struggle and/or find new ways to visualize quantum systems (11) | "We all really struggled with adjusting to not being able to visualize exactly what we were doing, which was a big change from most of our other physics classes."–Stephanie |
| Math (17) | *(Duplicate from above)* **3:** Students struggle and/or find new ways to visualize quantum systems (11) | *(Duplicate from above)* "We all really struggled with adjusting to not being able to visualize exactly what we were doing, which was a big change from most of our other physics classes."–Stephanie |
| | **4:** Math in quantum plays a different role than in classical (7) | "I had a problem transitioning to the new math style 'cause...it was like learning a new language, especially with something that wasn't real."–Daniel |

| Category | Theme | Quote |
|---|---|---|
| | **5:** Learning quantum requires learning new mathematical representations of physics (6) | "Like you're using totally different like kind of operators and things and...the introduction of kets and like these new notations for like, weird vector quantities. And just weird things that are written completely different."–Shane |
| | **6:** Students tend to trust the math more in quantum (7) | "The math tells us that there's no hidden variables...We...have to trust the math in saying that this is actually the nature of quantum mechanics."–Jane |
| Social, culture, experience (20) | **7:** Students find quantum less tangible to real-world experiences (13) | "Classical physics is right in my eyes. Quantum physics is kind of nonsense with respect of what I see in the world." |
| | **8:** Students have differing levels of relation to the real world (10) | "This is the class where physics...became directly applicable to the research I love."–Luke |
| | **9:** Quantum has more reputation than other classes (9) | "We're kind of walking into [QM] expecting our brains to go on a trip."–Jon |
| Quantum weirdness (18) | **10:** Students see quantum as weird (9) | "I still am a little creeped out by that whole wave function collapse deal. I don't really see how they...interpreted...that [it] just happens."–Sean |
| | **11:** Students see quantum as cool (10) | "It's actually kind of a really beautiful theory even though everyone thinks of it as this counterintuitive thing."–Rafael |
| Coherence (4) | **12:** Students see differing levels of connection between the topics of quantum (4) | "For quantum, I felt like most of it was all linked by a thread of probabilities or positions...we were always looking at the same type of thing...you're always trying to measure the same things."–Shane |

The five categories are: intuition, math, social/culture/experience, quantum weirdness, and coherence. The themes within each category are described below.

### *Intuition*

The intuition category comprises three themes: *(1) Students have two distinct "intuitions," one for classical and one for quantum, (2) Students recognize that they can develop intuitions for quantum but might not be there yet, (3) Students struggle and/or find new ways to visualize quantum systems.* Theme 1 is directly connected to an epistemological comparison—students recognize that "intuition", an important resource for learning physics, means something different to them in quantum as compared to classical physics. Beyond that, theme 2 reflects a recognition that their quantum intuition is a "work in progress." That is, students see the possibility of developing an intuition for QM even though at first it seems as if quantum physics concepts are unintuitive because they do not relate as directly to real-world experiences as do ideas in classical physics. Themes 1 and 2 often occurred together in student reasoning. Diana reflects both of these sentiments, stating "I was nervous at the beginning 'cause I knew I didn't have any intuition for this. I thought you just can't have intuition for quantum, and so, but now I feel a lot better because [the instructor] really did build in some intuition into all the equations. 'Cause I really did think I just have to accept whatever we learn about it and have no way of...yeah." In her statement, Diana implies that "quantum intuition" is different from "classical intuition" although she does not use the latter phrase. The fact that she was "nervous" at the beginning of the course because she did not have an intuition for the QM content suggests that, to Diana, intuition is an important part of learning physics (and perhaps it served her well in prior non-quantum classes). In line with theme 2, Diana also recognizes that she can in fact develop intuition for QM, citing that the instructor built "intuition into all the equations." The last sentence in Diana's comment suggests that she now believes that she does *not* have to accept knowledge as given.

Theme 3 references a particular kind of intuition—that related to being able to have a visual representation of the physical system in question. This theme falls into two categories simultaneously, "Intuition" and "Math", because the way students talked about visualizing physical systems (in QM) was often connected to both math and intuition. For example, James says, "Before I always had to think of things physically in my mind. Like imagine a picture of something. You can't do that [in quantum mechanics], you have to rely on just the math to solve things." Here, James is citing that in QM he does not have the intuition that comes from being able to picture the system, and for him this results in a certain stance toward math—you have to rely on the math because QM is un-intuitive or inaccessible. Not *every* comment coded as theme 3 straddled the two categories, but some of the instances of students talking about visualizing quantum systems also involved a discussion of the role of math, which is why we include this theme in both the "intuition" and "math" categories.

***Math***

There are four themes in the math category. In addition to theme 3 about visualizing quantum systems described above, they are: *(4) Math in quantum plays a different role than in classical, (5) Learning quantum requires learning new mathematical representations of physics, (6) Students tend to trust the math more in quantum.* Theme 4 is a general stance that math in quantum physics is different from math in classical physics—within this theme, students often describe math as being more important in their quantum class as compared to other courses. In one group of students, James and Shane discuss this role of math:

*James: Especially in our class that's so math, well not math intensive, but math heavy*

*Shane: Math reliant, I feel like is the best way to put it—*

*James: —yeah, yeah—*

*Shane: —Because I feel like the math itself isn't hard—*

*James: —No. But it's—*

*Shane: —except for maybe some of the derivations, but like—using the math is relatively easy, it mostly just like you know a new type of dot product, like pretty much most of the time, and then like adding time dependencies—and like, I mean obviously you have to know some complex variable stuff, but even that is just like the basic easy i stuff...so like, I think that like the math is less intensive, but like it's soooooo crucial to like understanding the concepts.*

*James: Yeah especially if you're like a physical, or a visual learner,*

*Shane: —yeah, yeah—*

*James: —and if you're just reading the book you're not going to learn too much.*

One of the main points of this snippet of conversation is not that the math required for QM is harder than in classical physics, but that it plays a more substantial role in conceptual understanding.

In addition to math possibly playing a different role, theme 5 zooms in on the ways in which QM requires different mathematical tools. The same group as in the above conversation brings this up as well when Shane says, "Like you're using totally different like kind of operators and things and...the introduction of kets and like these new notations for like, weird vector quantities. And just weird things that are written completely different." To this, Daniel responds, "Still not sure what a Hilbert space is..." Much like with theme 2 where students recognized their intuition for QM was different but that they were working to build it up, they often discuss how they are becoming more comfortable with the new mathematical tools and are working towards being able to use math as a tool for sense making just as they can in classical physics. As we saw with theme 3 straddling the intuition and math categories, students comments about math often connected to intuition. This is particularly evident with theme 6, which addresses the need to trust the math, or rely on the math more in QM, and is directly related to CLASS item 23. Shane states that in a situation where his intuitive answer disagrees with his calculation, he is more likely to trust the math because "my expectation could be far from what the actual thing should be. Especially in the beginning of the semester when you're not used to the quantum way of thinking, if I get a weird answer that's really different from what I expect, I'm more inclined to say...I just don't understand this." Later in the focus group, he states "I trust the math more than my conceptual knowledge," specifically referring to learning QM.

***Social, culture, experience***

The social/culture/experience category includes three themes: *(7) Students find quantum less tangible to real-world experiences, (8) Students have differing levels of relation to the real world, (9) Quantum has more reputation than other classes.* These themes are connected together in one category because students in our focus groups often talked about their real-world experiences as driven by cultural or professional practices in their lives as physics students. Theme 7 is directly related to CLASS item 35, and one thing that comes up when students (and experts) discuss this item is that people might mean different things by "real world experiences". For example, one group of students negotiates the tension between quantum physics underlying all physical phenomena, yet all of their day-to-day experiences are macroscopic.

*Diana: ...I was thinking about...how the Sun wouldn't even be able to work if it wasn't for quantum tunneling, because you wouldn't get fusion...I was like, I experience the Sun every day, so [laughs].*

*...*

*Stephanie: ...I guess you're thinking about like details, and I was thinking more, big picture. Because I was thinking most of the things we interact with are too big to be like quantum.*

*...*

*Fernando: ...I guess one of the things I think about is like light and photons. Like that's all quantum.*

*Diana: Yeah, and we experience that every day too. Even if we're not aware that that's what we're experiencing.*

As demonstrated in the above conversation, in Chapter VI and above in the quantitative results, students *overall* tend to say that quantum physics is less tangible than classical physics, but in our focus group study, the ways that they describe this often differed from one another depending on their personal experiences (theme 8). For example, a student involved in physics research might have different "real-world experiences" than a student not involved in research, and they might cite these as being strongly related to the QM they are learning in class. Luke states that "This is the class where physics...became directly applicable to the research I love." Aaron on the other hand voiced the opposite perspective, saying that QM is "a lot of really cool concepts and I really want to apply and—and figure out how they work and how they would change my perception, but as it is right now, it's kinda very abstract. I just—I don't apply it anywhere in my life right now...Anyway, until I'm like deep into physics I don't think this is gonna change my view of the world." Theme 9 reflects the role quantum physics plays in the physics curriculum as well as in popular culture. It has been documented that, more so than other sub-domains of physics, quantum physics holds social and cultural importance for physics students [11]. In our focus group study, students often alluded to cultural perceptions of quantum physics that gave them certain expectations coming in to the course, as well as the importance of QM as a cornerstone class in the physics curriculum. In one group, when discussing the question “Has this course changed the way you think about physics?”, Claire began:

*Claire: Um...I guess, I mean quantum was this thing that I kind of read up on the concepts of well before ever coming in to study physics. ’Cause I actually have a prior career as a...in culinary arts...so uh I, but yeah I—would read up on these concepts. So I...I mean it taught...More of this was teaching me more of the math, and how they apply it.*

Claire also wrote in a follow-up survey, “I was worried that I would be completely lost after the first month or so, but I’m glad that wasn’t the case. QM1 is my favorite class of all the classes I’ve taken so far.” The group continued around in the circle, each answering the question about QM changing the way they thought about physics.

*Ryan: [to Claire] Yeah, I’d have to agree with you there. The fact that—I mean we’re all physics students, and we kind of have an idea of Schrodinger’s cat and an entangled particle and, you know those—the big ones. So, we’re kind of walking into it, expecting our brains to go on a trip. Um so I wouldn’t really say that it changed the way I think about physics, but now I feel like I actually understand when people are talking about quantum, and like when I’m reading poster boards of people’s experiments I can be like “Oh I know that notation, oh I can see what you’re doing here. Oh yes, these all make sense.” So, I felt much more at home with reading the literature, but as far as changing how I thought about physics...probably not.*

*Jon: ... Um yes, in fact, to be the odd one out. My dream job is philosopher of physics... Actually. Um, so this class mattered a lot to me. Um. And it mattered because I—I don’t believe in just believing something. So yeah, we learned the concepts of quantum mechanics. We learned that Heisenberg’s uncertainty principle is that—that’s just the way it is. That was modern physics. It was them just telling us “that’s just the way it is.” And um, I don’t like just accepting that. So, uh, yeah in terms of QM—I’m not gonna say I understand it, but I can explain why it’s weird. And that matters a lot to my philosophical understanding of physics. Because it’s consistent. It’s weird, but it’s consistent with everything else. It always felt like it was some kind of choice—right, classical or quantum. But now it—no, it’s consistent. It’s* *all over consistent. And that's important to my overall physical understanding.*

In this conversation, the students describe having been familiar with quantum physics concepts before taking the class (e.g., Schrodinger's cat), that they had expectations that the course would be difficult (Claire wrote about thinking she "would be completely lost"), and they describe the QM1 course as a cornerstone course in the physics curriculum. Further, after taking the course Claire states that it was not as difficult as she thought and that it ended up being one of her favorite classes, Ryan feels more comfortable engaging with QM content in scientific (or other) communities, and Jon is satisfied that he can now explain *why* quantum is weird.

#### ***Quantum weirdness***

In the above conversation, Ryan and Jon alluded to the idea of "quantum weirdness". In our data, students talk about this idea in two different ways, and thus the "Quantum weirdness" category has two themes: *(10) Students see quantum as weird, (11) Students see quantum as cool.* The two themes differ in their emotional tone—theme 10 generally involved frustration or confusion at the unintuitive or "weird" concepts or phenomena, while theme 11 included more of an appreciation or awe for these things. For example, Sean reflects the "quantum as weird" theme when he says, "I still am a little creeped out by that whole wave function collapse deal. I don't really see how they...interpreted...that [it] just happens." Rafael reflects the "quantum as cool" sentiment when he says, "It's actually kind of a really beautiful theory even though everyone thinks of it as this counterintuitive thing." Although these two sentiments are different from one another, they are not mutually exclusive. Often, students expressed both ideas within or across comments.

#### ***Coherence***

The final category, "Coherence", has one theme—*(12) Students see differing levels of connection between the topics of quantum*—which stems directly from CLASS item 6, asking students if they view quantum physics as consisting of many disconnected topics. Some students, like Luke, view QM as being a single self-contained topic: "Quantum physics is its own topic. It can't be disconnected within itself." Other students view quantum physics as consisting of several topics that are all linked together. Shane describes the idea of probability as the thing that connects all of the topics: "For quantum, I felt like most of it was all linked by a thread of probabilities or positions...we were always looking at the same type of thing...you're always trying to measure the same things." On the contrary, James views QM as a series of disconnected topics *because* of probability: "I think there's slightly more disconnect in [the topics of] quantum. Just cause...you have to include probability in the physics now. And in classical, it all kinda stems from the same equations."

## C. Discussion

### 1. Prevalence of epistemic splits

In our initial exploratory study presented in Chapter VI, we demonstrated the *existence* of students' epistemological splits when it comes to some aspects of learning classical and quantum physics. The quantitative analysis in this chapter shows not only that these splits exist, but that they are prevalent across multiple institutions, instructional contexts, course levels, and student populations. On four of the six bifurcated CLASS items, we observed significant classical-quantum splits in almost every administration of the survey. Of the two items that had splits on only half of the administrations (items 2 and 28), we do not see any clear patterns in terms of which courses had splits. There are no obvious associations between level of course, university, or instructor and splits on these two particular items.

These quantitative analyses demonstrate that students reporting epistemological splits between classical and quantum physics is a prevalent phenomenon that we should attend to. Triangulating with the results of the qualitative coding and identification of themes in students' reasoning helps us to see a bigger picture of what these splits mean for students' learning and understanding of the nature of science.

### 2. Why students might report epistemic splits

The results of coding students' epistemological comments in focus groups gives us themes to attend to when trying to understand students' split epistemic stances. As illustrated above, we find that the themes identified in the coding often overlap and interact with one another. It is in these interactions that we can begin to understand why students might hold different views between classical and quantum physics. Many of the themes identified have bearing on the bifurcated CLASS items and the resulting splits we see in students' responses.

In the quantitative results (Table VII.2, Appendix F, and Figures VII.1, VII.2, VII.3, and VII.4), we found that students were more likely to trust their calculation in quantum physics (item 23), and they were less likely to see quantum physics as related to the real world (item 35) or as changing their views of how the world works (item 28). In the focus group conversations, some of the QM1 students reported a tendency to rely on math more in quantum physics because they find QM less tangible and intuitive or less related to the real world. For these students, the unintuitive nature of QM results in math playing a more authoritative role, as something that cannot be checked or predicted by intuition in the same way that it can in classical physics. Sometimes this experience of QM as unintuitive is related to students not being able to visualize a quantum system. These interactions between the math, intuition, and social/cultural/experience categories provide us some insight into student thinking around domain-specific epistemologies that we see reflected in the aggregate survey results.

In interactions between the social/cultural/experience and quantum weirdness categories, we see that for some students, the role that QM plays in popular culture as well as within the physics major informs how students view "quantum weirdness." Further, the relations that students perceive between QM and "the real world" can be dictated by their experiences as a physics student outside of a formal classroom environment. These connections have important implications for the development of students' physics identities. That is, students' identities as physicists or physics students and their epistemic views about QM mutually inform one another. This idea of "quantum weirdness" is an underlying theme that may have bearing on any of the six bifurcated CLASS items.

The themes and categories presented above in the Results section resulted from the analysis of conversations among 26 physics majors at the end of their first semester of an upper division QM class. While the views that students' shared cover a wide range, they are not exhaustive nor predictive of a broader population of physics students. The goal of the qualitative analysis was to look for things that students might say when talking about epistemic differences between quantum and classical physics. The themes in Table VII.3 provide some things to attend to when trying to understand students' epistemological splits, and the interactions among the themes begin to provide insight into why students may hold different views about learning and knowing in quantum physics as compared to classical physics.

### 3. Evidence of epistemological sophistication

As discussed in Chapter VI, a default interpretation of the split CLASS results might be that they reflect a lack of expertise around QM (i.e., that splits are undesirable and need to be changed). We warn against this assumption and argue that while there are certainly instances when, or contexts in which, the split epistemologies might manifest in a way that is detrimental to students' learning, they can also reflect epistemological sophistication and play a valuable role in students' learning of QM. In line with the emerging consensus in epistemology research that epistemological sophistication consists of the ability to make contextualized judgements about what counts as knowledge and how knowledge is generated [83, 131], we provide examples of student quotes that reflect the ability to articulate epistemological differences between the classical and quantum contexts within physics. We argue that these quotes reflect epistemological sophistication, and that they can be productive stances for students' learning of physics. In Chapter III, we defined an idea as productive if it "supports, initiates, or sustains progress" [8, p. 3] and a conversation as productive if it reflected discourse practices of professional physicists (e.g., flexibly reasoning across ontological categories, or tentatively trying out ideas). In thinking about productivity of students' ideas, practices, epistemic stances, etc. we must define productivity *to what end*. A given student contribution to a group conversation might be productive because it helps the group move towards a correct answer or a deeper understanding of the phenomenon or topic, or it might be productive because it begins to describe science as a messy and nonlinear process, or because it navigates an awkward social exchange and keeps the conversation flowing. In this section, we focus on elements of students' reasoning that are *epistemologically* productive, such as: distinguishing contextual differences in knowing and learning classical and quantum physics, engaging in metacognition, and valuing sense making and intuition as an important part of learning physics.

In describing the "social, culture, experience" theme above, we quoted one group of students (Diana, Stephanie, and Fernando) who articulated the differences between how classical and quantum physics were each related to their real world experiences. They drew a distinction between the macroscopic experiences of classical physics ("most of the things we interact with are too big to be like quantum") and the underlying, microscopic, quantum mechanical nature of their experiences ("we experience [light and photons] every day...Even if we're not aware that that's what we're experiencing"). Further, we see this group metacognitively reflecting on how they are thinking about the role of physics in their everyday experiences when Stephanie says, *"I guess you're thinking about like details, and I was thinking more, big picture."*

Jane and George also articulate the differences in how classical and quantum physics are related to the real world:

*Jane: I think like everything that's happening around us is all made possible by quantum mechanics and classical mechanics, it's just that like classical mechanics is more tangible. Ya know, like I can see gravity's effects on my pen dropping, or something but like...but quantum mechanics, the sun works through quantum tunneling, you know?*

*George: So you could always make an—like if I'm looking at the math and you're gonna say that has nothing to do with the experience that I have day to day. But if you're looking at the big picture of why the math exists, I can say that's the foundation of my experience day to day. So, like kinda depends on how you look at it. I think what [the instructor], like, two months said there was some quote that somebody, I don't remember who it was, but he was like, "Isn't it weird to think that everything that exists is built out of things that don't exist."*

Much like Diana, Stephanie, and Fernando above, Jane and George draw a distinction between the macroscopic relation of classical physics and the microscopic relation of quantum physics to their experiences, stating that "classical mechanics is more tangible". Not only are the students saying that QM is less tangible than classical physics (theme 7), but they make a contextual judgement about the knowledge generation in the two sub-domains—classical physics is based on observations and macroscopic things that we directly experience (e.g., the effect of gravity on Jane's pen), while QM consists of math that is not directly related to, but forms the foundation for, day-to-day experiences.

Another group made contextual judgements about the role of intuition in classical and quantum physics:

*Adi: Intuition is important but most of the time...the intuition—your intuition from classical mechanics sucks. You try to apply intuition [from] classical mechanics into quantum mechanics, it sucks.*

*Jean: Um, I don't think that it necessarily means the intuition you build up from classical mechanics. I use my intuition of what I think a probabilistic state will be. I use my intuition for what the mathematics should look like there's always that guess and check thing you're supposed to be doing, which is called sense making, that like I think my intuition plays a huge part. I know where I've gone wrong when solving a problem. Like I know when it went wrong, I just have to find where it went wrong.*

*Aaron: I think intuition played a major role in just like the building of actually understanding quantum. Because again for me, I'm more conceptual than mathematical, but in things like homeworks and things like the exams if I looked at an answer, and it seemed wrong, the intuition wasn't, "Oh, let's check the math immediately, what did I do wrong?" My intuition would say, well I probably misunderstood this aspect. It wasn't—I didn't think back to, "Oh, of course I missed, ya know, an r squared somewhere in there." It was—it was between more, ya know, what concept didn't I understand. So, I think the intuition was really important because it helped me fact check through my actual calculations.*

*Jean: Helped you sense-make. Yeah.*

*Luke: I can't think of a specific example but it was critical. I can't do physics if I don't have* *an intuition on the physics of it.*

*Aaron: Yeah, exactly.*

*Luke: Actually maybe learning physics is the process of gaining the intuition.*

In this conversation, the students all recognize the importance of intuition in learning physics. We see it as beneficial for students' learning if they value being able to use and gain intuition; this productive stance is reflected in Luke's final statement, *"learning physics is the process of gaining the intuition"*. Adi begins the conversation by recognizing that the intuition they have for classical mechanics does not apply so well in quantum mechanics, and Jean follows up by clarifying that intuition in QM looks different than it did in classical mechanics (theme 1). She also comments on the value of sense making, which she describes as "that guess and check thing". Jean and Aaron both identify intuition as playing an important role in sense making—Jean has intuition for what the math should look like, while Aaron uses his intuition to determine which concepts he didn't fully understand. We see this conversation as representing epistemological sophistication because the students recognize and value the intertwining roles of intuition and sense making in learning physics, and they are able to articulate differences between intuition in classical and quantum physics.

When discussing his answers to item 23 (*In doing a [classical/quantum] physics problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem*), Shane metacognitively reflects on his thinking when solving classical and quantum physics problems:

*Shane: I disagreed with classical cause like unless it's like just a really hard problem and I'm like, "Okay, I've got an answer, it's wrong, I give up, whatever." But if I get an answer wrong I'm like, "Okay, I wanna know why this is wrong because it should be right," and then I'll pour it back and say, "Oh, either this should be a negative, or this should be a positive." Or, "Oh, wow, I have a misunderstanding of this topic." While in quantum- and like- and like I wanna correct that cause it's classical physics and I feel like I should know it. But like quantum, if I run into something that's like pretty different from what I expect, I guess on the context it could be a little- like, depending on what the problem is and what the subject* *is, but most of the time I may be like, "Oh, I just don't fully understand this topic yet and I just need to read more," or something, ya know? I don't, I—I trust the math more than like my conceptual knowledge, I guess.*

Shane thinks about his own thinking and externalizes that to discuss the differences he sees in what it means to learn or solve problems in classical and quantum physics. To be fair, we are explicitly asking the students to engage in this metacognition by asking them to answer and compare the classical and quantum versions of the bifurcated CLASS items. However, this explicit prompting does not take away from the sophistication or productivity of this reasoning, but rather illustrates that we can support students in developing these skills with the questions that we ask and the environments we construct.

Beyond the productive elements we have highlighted here in the student reasoning, we might also define students' epistemological splits as sophisticated if experts themselves reported these splits. We have evidence from a *small* sample of instructors that when experts respond to the bifurcated items, they also report splits on some items between classical and quantum physics. We present preliminary investigations into these expert responses in the next chapter.

### D. Conclusions

In this chapter, we have presented an extension of our initial study into students' split CLASS results (Chapter VI). While the initial study demonstrated the existence of such epistemological splits, this extended study has demonstrated that the split results are prevalent across multiple institutional and instructional contexts. Students' epistemologies are closely linked with their conceptual understanding of physics, persistence in the major, identity as a physicist or scientist, and so the prevalence of domain-specific epistemic stances suggest that we should not treat "physics" as monolithic when attending to students' epistemologies (or at least be aware of when we are doing so). In coding focus group interviews, we have identified some elements of student reasoning that we can attend to when trying to understand students' epistemological splits. The interactions between the themes identified in student reasoning begin to tell a bigger story about *why* students might hold different epistemic stances toward classical and quantum physics. We argue that these splits *can* reflect epistemological sophistication.

# Chapter VIII. Preliminary investigations of instructor impact on students' sub-domain-specific epistemologies

*Some of this chapter (VIII A) appears in print in the 2017 PERC Proceedings: Hoehn and Finkelstein 2017 [165].*

We have presented evidence that students sometimes report different epistemic beliefs or stances toward classical and quantum physics. These split epistemologies occur in multiple instructional contexts, but we also expect the specific instructional factors to impact the development of students' domain-specific stances. It is widely reported in K-12 education, and increasingly in higher education, that individual teachers' implementation of curricula and their teaching practices in general impact student learning (e.g., [172–175]). Turpen and Finkelstein documented the ways in which individual instructors' implementation of *Peer Instruction* [176] impacted students' opportunities for learning and resulted in different degrees of emphasis on sense making within the classroom [173]. Individual instructors' pedagogical practices impact student learning in part because they send certain epistemological messages (implicitly or explicitly) or support certain epistemic views of doing, learning, and knowing science. For example, Linder identifies manifestations of "metaphysical realism"—the idea that science is a description of an objective reality independent from human minds—in physics teaching and argues that these epistemic messages: support rote-learning, portray conceptual understanding as the ability to solve "typical tutorial problems", value rapid instruction that can cover lots of material, and discourage coherent understanding of physics [177]. He thus calls on physics teachers to consider the epistemological implications of their teaching.

Prior work in Modern Physics courses has shown that an instructor's interpretive stance, and how they attend to interpretation of quantum mechanics (QM) in the classroom, matters for student learning [43, 52]. Students in a class with faculty who held (and taught) from local-realist perspectives of quantum mechanics were more likely to take realist perspectives (e.g. the electron has a definite but unknown position in an atom). Students in a class taught from a quantum-matter view were far more likely to hold such views, and students in an "agnostic" perspective were likely to make their own interpretations, usually in line with a classical-realist perspective. Along the same vein, we now argue that instructor *epistemological* stance toward QM also matters. This chapter includes preliminary investigations into the impacts of individual instructors on the development of students' *domain-specific epistemologies*. In line with Ref. [177], we call for QM teachers to consider their own domain-specific epistemic views and the epistemological implications of their teaching.

In Chapter VI, we mentioned that in preliminary investigations of instructors' personal responses, on some items the instructors reported splits and answered differently from one another. Understanding instructors' responses to the bifurcated CLASS items is useful for two reasons: 1) expert responses can help us to define and interpret epistemological sophistication in students' split epistemologies, and 2) instructors' personal responses along with the emphasis on epistemology in their teaching can help us understand the impacts of instructors on students' epistemological development. We focus on the second point in this chapter, and we present preliminary investigations into the impact of instructors' personal epistemic stances, and their stated emphases on epistemology when teaching, on students' domain-specific epistemologies, where the domains are different sub-fields of physics.

In these exploratory investigations, we focus on one particular CLASS item that concerns a topic central to our goals of physics teaching and examine the nature of the difference in students' beliefs about the relation of classical and quantum physics to the real world (item 35: *The subject of [classical/quantum] physics has little relation to what I experience in the real world*). In Chapter VII, we observed splits on this item on every survey administration. The results from this item across multiple instructional contexts provide motivation for future studies. We argue that instructor epistemological stance can contribute to students' epistemological development, although it can be difficult to disentangle the various factors such as instructor's personal view, stated or enacted emphasis on a specific epistemic topic, or pedagogical approach. This chapter is more tentative and exploratory than others in this thesis, but the initial results provide motivation for future studies and help support the argument that it is important to be aware of your own domain-specific epistemologies and attend to them when teaching.

In this chapter, we explore survey data from five courses (four modern physics and one QM) in the dataset described in Chapter VII (Table VII.1)—CU 2130-1, CU 3220-4, CU 2130-6, CU 2130-8, and CU 2170-8. We first present an investigation that compares CU 2130-1 (Modern Physics for Engineers) and CU 3220-4 (QM1), which suggests that an instructor's personal epistemological stance is one factor that impacts students' responses to the bifurcated CLASS items. Next, we explore differences in students' and instructors' responses between CU 2130-8 (Modern Physics for Engineers) and CU 2170-8 (Modern Physics for Physics Majors) that suggest in addition to an instructor's stance, the intentional emphasis (or avoidance) of certain epistemic topics in class impacts students' responses. Finally, we explore the differences in responses between CU 2130-8 and CU 2130-6 (both Modern Physics for Engineers classes) to suggest that instructors' enacted emphases in class, which can differ from their stated emphases, impact students' responses. Again, these investigations are all exploratory and serve to bring up interesting questions and motivate future studies.

### A. Do instructors' epistemic stances impact students' epistemic stances?

We first present a brief study that compares CU 2130-1 and CU 3220-4. As a reminder, CU 2130-1 is Modern Physics for Engineers taught by instructor A with a reformed curriculum emphasizing applications, interpretation, and the connection of QM to the real world [43, 73]. CU 3220-4 is an upper division QM1 class taught by instructor C with a spins-first pedagogical approach (using McIntyre's textbook [178]). This QM1 course included clicker questions in lecture and had an optional one-credit co-seminar associated with it that approximately 25% of the class regularly attended, and the content of the course had a focus on analytic problem-solving and preparation for continued advanced level study of QM. The instructors of these courses are both award winning instructors and have published on student learning in QM.

Students participated in this study as part of their course participation. There were 84 students from modern physics (68% of class) who were typically 2nd year engineering majors, and 40 students from QM1 (93% of class), typically junior physics or engineering physics majors. The modern physics course was approximately 12% female, QM1 was 37% female, and both courses enrolled predominantly white students.

In addition to the student bifurcated CLASS survey responses, we requested that the instructors of the courses also take the survey. They responded to the survey as they would personally answer the questions, and also reported for each quantum physics item the extent to which that particular topic was covered in their class. A free response section was also provided for the faculty to explain their reasoning.

We focus our analysis on CLASS item 35: *The subject of [classical/quantum] physics has little relation to what I experience in the real world.* Although we ran both pre- and post-surveys in the modern physics class, we focus here only on the post administration because we are interested in looking at the classical/quantum splits at the end of the modern physics and QM1 classes, and not at pre-post shifts. We matched the data, and only included students in the item-level analysis who responded to both the classical and quantum version of a given item. The survey questions are given on a 5-point Likert scale from Strongly Disagree to Strongly Agree, but for analysis we collapse the results to a 3-point scale (Disagree, Neutral, Agree). We treat these data as ordinal, with three discrete categories rather than ordered points on a continuum, since we cannot assume the Neutral response to lie on a linear spectrum halfway between Disagree and Agree [116]. In testing whether or not the classical/quantum splits were statistically significant, we use the Bhapkar test [153, 154] which uses information about how many students actually split their responses to determine if the overall distributions are different. We also test whether or not the distributions of responses on the quantum item are different between the two classes using a Mann-Whitney test.

In both classes, there is a highly statistically significant split between students' classical and quantum responses ($p < 0.0001$ in both cases). The majority of students disagreed with the statement for classical physics (Table VIII.1); that is, they think classical physics *is* related to their experiences in the real world. In both classes, far fewer students disagreed with the statement for quantum physics than they did for classical physics. Overall, students are less likely to report that quantum physics is related to their real-world experiences than classical physics. Although both classes had significant splits between classical and quantum physics, the quantum responses for the two courses look different from one another. Whereas the majority of modern physics students disagree with the quantum item, the distribution of responses for the QM1 students is bimodal. Collectively, the modern physics students think quantum physics *is* related to their real-world experience, but less strongly so than for classical physics. The QM1 students on the other hand are just as likely to disagree with the quantum statement as they are to agree. The two class distributions for the quantum version of the question are significantly different at the $\alpha = 0.05$ level ($p = 0.028$).

Interviews with the QM1 students help to explain what this bimodal distribution or epistemological split means for the students. One group of four students was responding to item 35, discussing what they answered for both classical and quantum versions, and why. Everyone responded Disagree for classical, but the group was split between Disagree and Agree for the quantum version (Fernando and Diana responded Disagree, while Lucy and Stephanie responded Agree). Their conversation surrounding this question, presented above in Chapter VII, included the following exchanges:

> ***Diana:*** *...I was thinking about...how the Sun wouldn't even be able to work if it wasn't for quantum tunneling, because you wouldn't get fusion...I was like, I experience the Sun every day, so [laughs].*

TABLE VIII.1. Percentages of students who gave each possible combination of responses (Disagree, Neutral, Agree) on both versions of the item, *The subject of [classical/quantum] physics has little relation to what I experience in the real world.* Instructor responses are indicated in boldface.

| **Modern Physics** | | | Quantum | | |
|---|---|---|---|---|---|
| $N = 84$ | | D | N | A | Total |
| | D | 51 | 17.9 | 13.1 | 82 |
| Classical | N | 2.4 | 4.8 | 1.2 | 8.4 |
| | A | 2.4 | 2.4 | 4.8 | 9.6 |
| | Total | 55.8 | 25.1 | 19.1 | |
| **Quantum 1** | | | Quantum | | |
| $N = 40$ | | D | N | A | Total |
| | D | 40 | 12.5 | 42.5 | 95 |
| Classical | N | 0 | 0 | 2.5 | 2.5 |
| | A | 2.5 | 0 | 0 | 2.5 |
| | Total | 42.5 | 12.5 | 45 | |

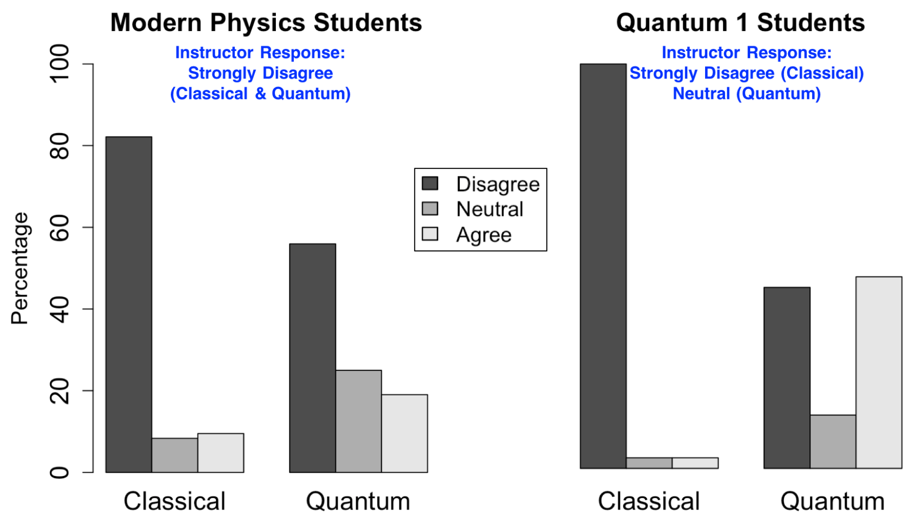



FIG. VIII.1. Post-survey responses from Modern Physics (CU 2130-1) and Quantum 1 (CU 3220-4) students to the bifurcated item: *The subject of [classical/quantum] physics has little relation to what I experience in the real world.* Instructor responses are indicated at the top of each plot.

> ...
>
> ***Stephanie:*** *...I guess you're thinking about like details, and I was thinking more, big picture. Because I was thinking most of the things we interact with are too big to be like quantum.*
>
> ...
>
> ***Fernando:*** *...I guess one of the things I think about is like light and photons. Like that's all quantum.*
>
> ***Diana:*** *Yeah, and we experience that every day too. Even if we're not aware that that's what we're experiencing.*

Following this conversation, the group came to a consensus that as a collective group they would choose Neutral.

It is not surprising to see different results for the quantum item in the two classes, given that they are different student populations (engineering versus physics majors) and different course contexts (lower level modern physics versus upper level QM). However, the difference in the distributions that we do see is consistent with how the respective instructors answered the question themselves. The modern physics instructor answered Strongly Disagree, and throughout the course emphasized applications of quantum physics to students' worlds (quantum basis of color, the periodic table, and applications such as the laser, MRI, and nuclear energy/weapons). Furthermore, the course engages students in debating about the nature of how and when QM shapes their worlds. The QM1 instructor answered Neutral, with the following accompanying explanation: "*Everything I experience arises in part from QM (light, material properties, everything) but nothing I experience is quantum mechanical, I have no direct experiences that are non-classical.*" The QM1 course engages in some discussion about the relevance of QM and its connections to other phenomena and branches of physics, but this was not a large portion of the class.

In both the modern physics and QM1 classes, students almost unanimously agreed that classical physics is related to their real-world experiences, but were less likely to say the same for quantum physics. As discussed above in VII C 3, we consider the QM1 students' conversation about their differing views about classical and quantum physics to be indicative of epistemological sophistication. Fernando and Diana explain that they see quantum physics as related to their experiences because QM underlies things like nuclear fusion, light, and electronics (even though they might not be aware they are experiencing these things). Lucy and Stephanie explain that they see quantum physics as *not* being related to their experiences because everything they interact with is macroscopic (Lucy says she experiences turning on a TV every day, but not building one). This conversation is closely aligned with the QM1 instructor justification for answering Neutral. Ultimately, the selection of Neutral is a proxy for the instructor's own preference for a bimodal response (in some ways agree and in some ways disagree), which parallels both the course response overall and the focus group discussions. We consider being able to articulate these differences and engaging in this type of epistemological discussion to be sophisticated, and we aim for our physics students to be able to do these things.

We have seen that experts do not agree on the response to the quantum version of this question, and that the item is subject to different interpretations: real-world experience *arises from* versus *is* QM. We note that we have not validated these bifurcated questions from classical test theory, and as a result do not seek to use them to issue a survey to review all students' views about real-world connections of quantum and classical physics. Rather, we use these results to demonstrate that students can and do engage in sophisticated (and messy) reasoning about the connection of physics to the real world, and that they do have domain-specific perspectives. Such perspectives reflect the debates between the two course instructors and those in the broader community.

The distributions of responses to the quantum question from the modern physics and QM1 classes are significantly different from one another (see Figure VIII.1). Roughly 43% of QM1 students answered Disagree for classical physics and Agree for quantum physics, compared to only 13% of the modern physics students. This difference is consistent with the difference in instructor responses. At present, we cannot argue for causality, but we do argue that instructor stance and extent of emphasis in their teaching are contextual factors contributing to students' epistemologies. We do not wish to suggest that one instructor stance is better or worse than the other, but we argue that the correlation between instructor stance and student responses implies that we should be aware of our own domain-specific epistemologies and attend to them when teaching. Furthermore, we note that whether or not we teach issues of interpretation or epistemological commitment, students will engage in such meaning-making [43]. One critique of this work could be that if professors' stances matter for students' epistemological development, then we should be looking for impact over time. We have pre and post data for the modern physics class and we do see pre to post shifts for some quantum items. However, existence of such shifts, or lack thereof, does not necessarily help us evaluate the impact of instructor stance because often we see regress on CLASS items over the course of a physics class [115], and thus the lack of a shift *could* be the result of instructor stance. Additionally, the pre survey was administered one week into the semester, at which point the modern physics instructor stance may have already been made clear to the students.

In this first exploratory investigation, we find that students' responses to the bifurcated CLASS item 35 reflect the corresponding instructor response, and we see this in both the aggregate quantitative results and in students' conversations. We take these findings to indicate that the instructor's personal stance is *one factor* that can impact students' epistemological development. However, there are many factors likely contributing to these results (such as student population, level of the course, content of the course, emphasis on epistemology in the class, pedagogical factors, etc.) that warrant further investigation.

### B. Does emphasis on epistemology in class impact students' epistemic stances?

The above study led us to wonder if the differences we were seeing in students views about the relation of QM to the real world was due to the content of the class—modern physics (essentially introduction to QM) versus more in depth QM. Thus, we now continue the above investigation looking only at CLASS item 35, but focus just on modern physics classes. In the above study, we looked at the CU 2130-1 course taught by Instructor A. As seen in Table VII.1 (Chapter VII), Instructor A taught this class four different times in our dataset. For the present investigation, we look at CU 2130-8, the same class as that described above but seven semesters later. For item 35 in particular, the student results are statistically indistinguishable between the Semester 1 and Semester 8 instances of the course. Further, Instructor A's personal stance and stated emphasis on the topic of relation between QM and real world remained the same between the two semesters.

We eliminate the modern physics versus QM confounding factor, by comparing two modern physics classes—CU 2130-8 and CU 2170-8—noting that the former primarily enrolls engineering majors while the latter primarily enrolls physics majors. The differences in how these two modern physics courses are taught is largely instructor-independent, but historically the course for physics majors has a higher degree of emphasis on mathematical formalism. Additionally, the CU 2130-8 class (taught by Instructor A) did not include special relativity, while the CU 2170-8 class (taught by Instructor K) spent the first two weeks of the semester on special relativity and the remainder of the semester on QM topics.

We employ the same methods as in the above study, and look specifically at CLASS item 35. We observe classical-quantum splits on this item in both classes. Unsurprisingly by now, students overall were more likely to say that classical physics was related to their real world experiences compared to quantum physics (see Figure VIII.2). While both classes saw significant classical-quantum splits on this item, and in the same direction (more likely to disagree for classical than quantum physics), the distribution of responses to the quantum version of the item is different between the two classes. As seen in Figure VIII.2, around 60% of students in the 2130 class answered disagree (i.e., they see QM as related to their real world experiences) with roughly 20%

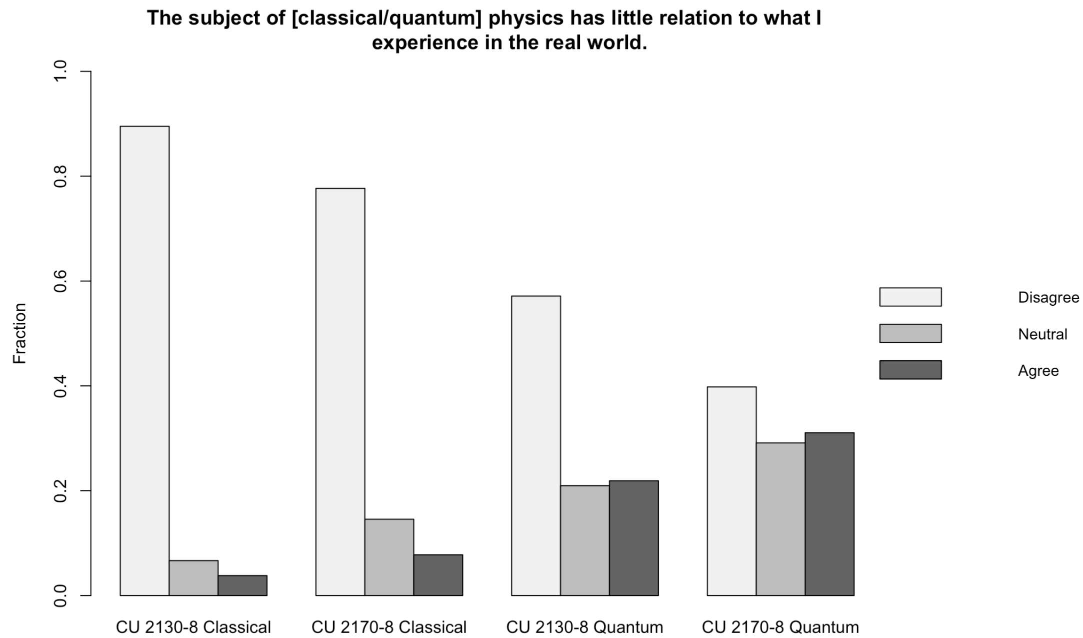



FIG. VIII.2. Post-survey responses from Modern Physics for Engineers (CU 2130-8) and Modern Physics for Physics Majors (CU 2170-8) students to the bifurcated item: *The subject of [classical/quantum] physics has little relation to what I experience in the real world.*

of students answering neutral and 20% answering agree. In the 2170 class, the responses were more evenly distributed with less than 40% of students disagreeing with the item and around 30% answering neutral and 30% agree. The difference between these two distributions is statistically significant ($p = 0.02$ with the Mann-Whitney test).

We turn to the instructors' responses to the bifurcated survey to try and explain the differences we see in the students' responses. Both instructors strongly disagreed with the quantum version of the item—they both see QM as being related to their real world experiences—but they reported different emphases on this topic in their classes. On a scale of 0 to 5, Instructor A (from the 2130-8 class) reported an emphasis of 4 ("many class sessions/homeworks focused on this topic") and Instructor K (from the 2170-8 class) reported an emphasis of 0 ("not at all, I explicitly avoided it"). The difference between the reported emphases of Instructors A and K is consistent with the differences we see in the student responses. In a class where the relation between QM and the real world was an explicit focus, the students were more likely to consider QM to be related to their real world experiences, and in a class where the instructor explicitly avoided the topic (despite personally viewing QM as related to the real world), the student responses were more evenly distributed.

Even when comparing two modern physics classes (rather than a modern physics and upper division QM class), Instructor A's students tend to find QM more related to the real world. In addition to the instructor's personal stance, we posit that the instructor's stated emphasis on this particular epistemic topic in class has an impact on the students' responses. Beyond these two factors, the student populations are different (in terms of their majors—primarily engineering versus primarily physics)[20], and the instructors' *stated* emphasis on epistemology might not tell the whole story. That is, an instructor's stated emphasis may vary in some ways from the emphasis enacted in their teaching. We turn now to one more investigation to address these remaining questions.

### C. How might an instructor's enacted emphasis on epistemology differ from their stated emphasis and does this impact students' epistemic stances?

We now compare the CU 2130-8 and CU 2130-6 modern physics classes, two instantiations of the same course taught by Instructors A and J respectively, and explore the instructors' stated and enacted emphases. Both instructors disagreed with the quantum version of item 35 (A strongly disagreed and J disagreed, but we collapse these and consider just the 3 point scale), and reported an emphasis of 4 out of 5 ("many class sessions/homeworks focused on this topic"). The results of the students' responses to this item are given in Figure VIII.3. Like the other courses we have considered in this section, the CU 2130-6 students largely disagreed with the classical item and were less likely to disagree on the quantum item. This is a statistically significant split. Again, the distribution of student responses to the quantum version of the item differs between the two modern physics classes. Students in the course taught by Instructor A were more likely to find QM related to their real world than students in Instructor J's class. This difference is statistically significant with the Mann-Whitney test ($p = 0.01$). In fact, the distributions for the quantum item in the 2170-8 and 2130-6 classes (taught by Instructors K and J) are statistically indistinguishable

[20] Arguably, these students are not that different from one another. Given that the modern physics classes are a third semester course, students in either version of the class have likely taken the same amount of college physics (Physics 1 and 2).

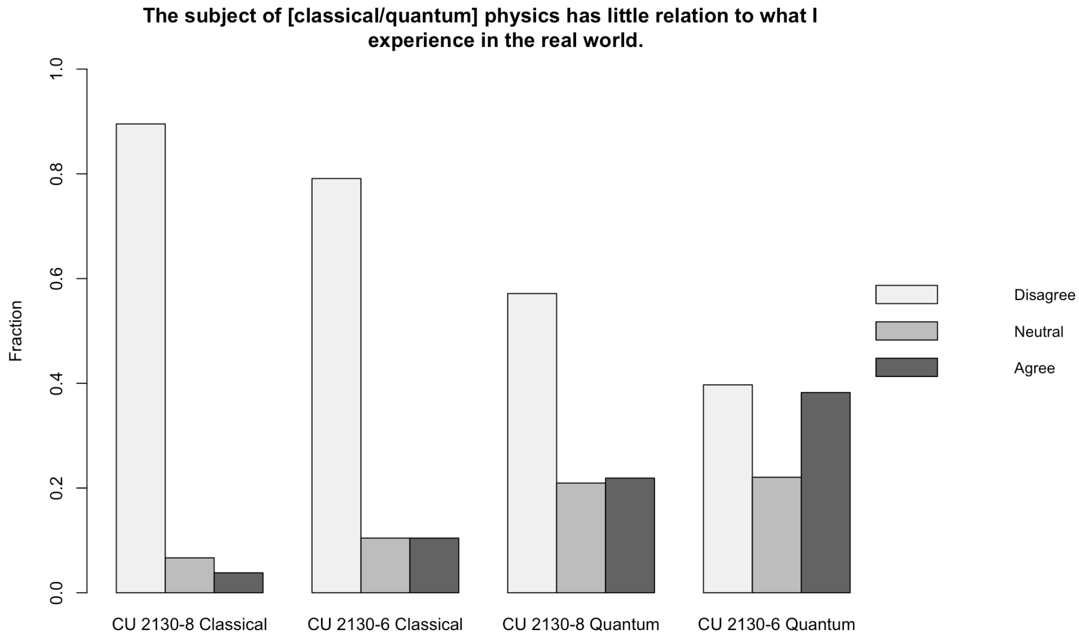



FIG. VIII.3. Post-survey responses from two Modern Physics for Engineers classes (CU 2130-8 and CU 2130-6) to the bifurcated item: *The subject of [classical/quantum] physics has little relation to what I experience in the real world.*

from one another.

Above, we connected the differences between students responses on this item to the different reported emphases by Instructors A and K. Yet here, Instructors A and J report the same personal stance and emphasis in class, and we still find that Instructor A's students are more likely to see QM as related to their real world experiences. We thus investigate the instructors enacted emphases by examining course materials.

We have described the 2130-8 class taught by Instructor A in depth elsewhere in this thesis (see Part One). A result of several years of course transformation [43, 73], this course emphasizes applications of QM. This is evident in: the homework assignments which situate problems in the context of real physical situations (e.g., introducing quantum tunneling by considering an electron in a wire and having students determine at which scale they would need to worry about quantum tunneling) and sometimes include estimation problems, the exams which test understanding of QM concepts through applications such as lasers, discharge lamps, or a scanning tunneling microscope, and the lecture slides and verbal messages from the instructor which often explicitly state that the quantum physics principles they are learning are related to everything they experience (i.e., color, chemistry, shocking your finger on a door).

In teaching the 2130-6 class, Instructor J used some of the same materials as in Instructor A's class. While some homework questions were connected to "real world" situations (i.e., a problem about the number of photons hitting a solar panel, a version of which was used in both classes), the homework assignments in Instructor J's class tended to emphasize mathematical proofs and derivations more so than in Instructor A's class (e.g., starting with Maxwell's equations, show that both E and B satisfy the one dimensional wave equation[21]). The lecture slides included similar applications and connections to the "real world" as in Instructor A's class (e.g., scanning tunneling microscope). However, we hypothesize that the ways in which these materials were framed in lecture may have differed between the two classes. A more in depth study would be needed to further examine the differences between the courses and the associated differences in split epistemological results.

A cursory look at the course materials suggests that both classes did indeed attend to applications of QM, but that Instructor A's class prompted students more often to consider these connections while Instructor J's course (at least in the homework assignments) placed more emphasis on mathematical calculations or derivations. We cannot make any strong conclusions from this exploratory investigation, but it is clear that despite the same instructor stance and reported emphasis, the enacted emphasis on the relation of QM to the real world is not identical between the two classes. This warrants further investigation.

### D. Motivation for further studies: Students cite instructor practices as impacting their own epistemic stances

In addition to the quantitative results that show differences in student responses associated with instructor views, and stated and enacted emphasis on epistemology, another approach to investigating whether and how instructors impact students' domain-specific epistemologies would be to ask the students themselves. We did not explicitly ask students about their perception of the impact their instructor had on their views, but many students in our focus group study presented

[21] For comparison, Instructor A's class also included a homework problem about the wave equation and solutions to the wave equation. It was followed up with a question asking students why there are cold spots in a microwave.

in Chapter VII often brought this up the idea of instructor impact spontaneously. In discussing the bifurcated CLASS questions or their experiences learning QM in general, several groups talks about things that their instructor did or said that helped them come to the conclusions they were making about the epistemological differences (or similarities) between classical and quantum physics. For example, QM1 students from the CU 3220-4 and CU 3220-7 classes spoke about how Instructor C helped them see QM as weird, but understandable:

*Fernando: Yeah, I think it's like just something that [the instructor] says a lot of the time, that quantum is weird but not that weird. And so I think that's gotten me a little more comfortable with it. At least the idea that you can have good intuition—that it's possible to have good intuition about it. And so a lot of the time I try think about, you know, what is—what is possible in this situation.*

*Diana: I think it also weirdly helps to know that quantum is weird. Because had I done a problem at the beginning and I got something completely different than what I expect, then I'd be like, "Oh, I just don't understand quantum, this must be right." So later in the semester, if I get something different from what I expected, that's still possible, obviously, but it's like part of me is like you know, maybe there is some validity to why I'm, ya know, questioning this problem.*

The instructor's repeated statement that "quantum is weird but not that weird" helped Fernando to recognize that he *can* have an intuition for QM. The message was also helpful for Diana who alludes to not only having intuition for QM, but also confidence in her problem solving—she speaks about this stance as being comforting in an otherwise confusing context. Ryan and Jon, in a different group (and a different semester), also talk about this particular message from the instructor:

*Ryan: I'm not gonna say I feel like I understand it, but I can explain why it's weird. And that matters a lot to my philosophical understanding. Because it's consistent. It's weird, but it's consistent with everything else. It always felt like it was like some kind of choice, right? Classical or quantum, but now it, no, it's consistent. It's all-over consistent, and that's important to my overall physical [Ind.].*

*Jon: It's very much like [the instructor] said at the beginning of the semester. "It's weird,* *but it's not that weird."*

The instructor's message that quantum was weird but not that weird was also helpful to Ryan because it helped him see physics as consistent, and it was important to him that he could understand and explain *why* QM was weird (i.e., articulate the ways in which quantum physics is less intuitive or less tangible than classical physics).

These example quotes suggest that we should attend to and investigate further the impact of instructors on students' epistemological development. Not only do we see differences in aggregate student responses associated with instructor-dependent factors, but the students themselves (in one particular class) spontaneously cite the instructor's epistemological messaging as impacting their own epistemic stances. One instructional implication we drew from our first study of the existence of students' epistemological splits (Chapter VI) was that as instructors we should avoid amplifying the "plug and chug" tendency as a response to "quantum weirdness". Instructor C demonstrates, with their repeated message to students that "quantum is weird but not that weird", one approach—not only do they avoid amplifying the message that QM is a place to rely solely on mathematics, but they counteract it by sending the message to students that they can gain an intuition for and make sense of QM.

# PART FOUR: SOCIAL DYNAMICS

In this dissertation, we have an overarching theoretical commitment to a sociocultural perspective of learning. In the prior two sections, we focused on ontological and epistemological aspects of students' learning, while recognizing the situated nature of learning within each study. In part one—studies of students' ontological reasoning around quantum entities—we investigated collective discourse, modifying a methodological tool [85] historically used for thinking about an *individual's* thinking to one that describes *collective* conceptual blends (Chapter III). Additionally, investigations into the impact of instructional prompts on students' ontological reasoning (Chapter IV) consider some of the ways that students' learning is situated in the instructional and curricular environments that we construct. In part two—studies of students' domain-specific epistemologies—we began to investigate how the instructor's personal epistemological stance may impact students' development of split epistemologies between classical and quantum physics (Chapter VIII), again considering how the educational environments we construct impact various aspects of students' learning. Now, we take a closer look at the social dynamics of one group and explicitly foreground the social nature of learning to identify how things like positioning and role-taking within the group are connected to epistemology and sense making. In doing so, we examine not only the social environment in which physics problem solving takes place, but how students' epistemological beliefs about learning physics and more specifically about the nature of collaboration in learning physics, interact with the social dynamics to impact students' learning.

With the increase in use of interactive engagement techniques in physics education, we are more frequently asking our students to work in groups. This group work and interactive engagement has been shown time and again to be beneficial for student learning as measured by performance on conceptual or problem solving assessments [179–182]. However, other research suggests that we should carefully evaluate the impact of group work along other dimensions of student learning as well as investigate factors that contribute to the construction and productivity of group work environments (e.g., [183–185]). For example, while Lorenzo, Crouch, and Mazur report that use of interactive engagement reduces the gender gap on the FCI [149, 186], Pollock, Finkelstein, and Kost report that interactive engagement alone is not sufficient to reduce the gender gap and that we should investigate additional factors such as the roles both instructors and students play in constructing norms around collaboration [183]. In biology education research, Tanner outlines the benefits of collaborative or cooperative learning [187], but also calls for instructors to design learning environments that attend to both individual students and interactions among students through teaching strategies that cultivate equitable classroom environments [188]. Likewise, Eddy et al. call on instructors to structure their classroom activities in a way that promotes equity by understanding the differential participation and barriers to participation in peer discussions for some students [189]. In a study of introductory biology students working in self-selected groups, they found that students' preferred roles in group work were correlated with their social identities (e.g., race, nationality, gender); the preference of roles is taken as a proxy for the actual roles that students play in group work. Eddy et al. identified three barriers to participation in peer discussions: exclusion from discussion by group members, anxiety around participating in discussion, and low student perceptions of the value of group work. They argue that in order to promote equitable classroom environments, instructors must first understand the barriers that lead to differential participation in group work—they call on instructors to not only focus on implementing interactive engagement and group work, but to attend to the dynamics among students. In this section, we respond to that call by investigating the social dynamics of one group of students in a collaborative problem solving setting and ask how the interactions between the social dynamics, epistemology, and sense making result in the social positioning of group members.

Just as our community has documented that physics is more than just content mastery, so too is group work more than just assigning roles to individual group members. In K-12 mathematics education, Barron illustrates through case study analyses how both cognitive and social factors play a role in defining a mathematics problem solving environment among sixth-grade students [185, 190]. "Less successful" groups had "relational issues", which included "competitive interactions, differential efforts to collaborate, and self-focused problem-solving trajectories" [185, pg. 348] that impeded the group's ability to engage in productive problem solving. Barron suggests that one way the cognitive and social factors work together to construct a joint problem solving environment is through the "development and maintenance of a between-person state of engagement" [185, pg. 349], which she describes as the awareness group members have for one another ranging from "complete lack of joint attention" to "continual coordinated participation." Along a similar vein, in this chapter we attend to the ways in which social factors are intertwined with sense making and epistemology in the construction and dynamic evolution of a joint problem solving space.

Pawlak, Irving, and Caballero identified four modes of collaboration by attending to three dimensions—social, discursive, and disciplinary—of introductory physics students' interactions [191]. Along the social dimension, they characterized the overall tenor of students' collaboration as *consonant* or *dissonant* [181]. The discursive dimension identifies the interaction patterns among students: consensual (one student makes substantial contributions), responsive (multiple students make substantive contributions), elaborative (the substantive contributions from multiple students build off of one another) [192], and argumentation (involving evidence, a subsequent claim, and justification of how the evidence supports the claim) [193]. Regarding disciplinary content, they characterized students' conversations as specific or abstract. Treating the social, discursive, and disciplinary dimensions independently, Pawlak, Irving, and Caballero identified four distinct modes of collaboration—debate, informing, co-construction of an answer, and building understanding towards an answer—each being a unique combination of codes along the three dimensions [191]. The simultaneous attention to multiple dimensions is a new approach to understanding collaboration in physics education. Similar to Pawlak et al.'s study, we attend to multiple dimensions of students' collaboration, but our analysis that we present in this section differs because we consider the *interactions* between those dimensions and attend to finer-grained details of the social dynamics in a collaborative setting.

Sohr, Gupta, and Elby also attend to the interactions between multiple dimensions of students' group work [194]. By investigating the intertwining of conceptual, epistemological, and socioemotional dynamics, they illustrate the multifaceted ways in which conflict can arise in collaborative settings, and identify one way students may resolve those conflicts. In particular, they describe an "escape hatch" as a series of discourse moves that serve to relieve tension in the group by ending or shifting the conversation *before* a conceptual resolution has been reached. "Escape hatches" can be productive in that they can help to establish more equitable group norms, or open up space for the group to engage collaboratively in conceptual discussion. This work provides an example of how interactional dynamics in a group can be the result of entanglement of many different factors. We build on this work by investigating the interactions between sense making, epistemology, and social dynamics in group work, although not necessarily in the presence of conflict or tension.

Researchers and instructors in the physics education community call for utilizing and valuing group work in our classes, and conduct studies of the various impacts of, and factors contributing to, collaborative environments. However, as a community we have thought less about how the *students* think about group work, and how that interacts with their learning. One way researchers examine students' perceptions or expectations of learning physics in a collaborative environment is through the lens of epistemological framing (e.g., [42, 195–199]). Generally, in anthropology and linguistics, framing refers to an individual or group's sense of what is going on in a given situation, including expectations about what could and should happen, what should be attended to, and how one should act [200–202]. *Epistemological framing* specifically refers to a sense of what is taking place with respect to knowledge (e.g., Is this a situation for sense making, or for rote manipulation of formulas?); these frames can be considered for individuals or groups and can vary moment to moment [195, 196]. There are other aspects of framing as well, such as *social framing* which refers to the sense of what to expect of people and their interactions in a social setting. As the nature of knowledge has particular importance in school settings, the physics education community primarily attends to *epistemological framing*, although it is noted that in collaborative settings different aspects of framing can interact with one another [196]. Scherr and Hammer describe a resource-based account of epistemological framing in which a frame is a "locally coherent pattern of activations" [196, p. 151] of epistemological resources [14, 203]. As epistemological resources may exist within an individual's mind *or* be distributed across a group of people, this resource-based account links two disparate approaches to framing—one with respect to individual reasoning and another with respect to social dynamics across groups [204]. In a study of collaborative tutorial-style activities in an introductory physics class, Scherr and Hammer illustrate that verbal and nonverbal actions together provide evidence of students' epistemological framing and insight into the dynamics of this framing [196]. Irving, Martinuk, and Sayre also explore the dynamics of epistemological framing by looking at the transitions or shifts between frames [197]. They identified two axes along which discussions could be categorized—expansive versus narrow, and serious versus silly—and observed that the majority of frame shifts were initiated by the teaching assistant facilitating the collaborative learning situation.

The notion of epistemological framing informs the analysis we present in this section in which we investigate a case study of one group of four students. Through the lens of epistemological framing, we might consider the members of this group to have shared understanding around what kind of activity they are engaged in, as evidenced by both their verbal and nonverbal actions[22]. Yet despite this alignment, we see one student in the group positioned as less knowledgeable. To understand this positioning, we identify an epistemological aspect of group work, and we associate students' behaviors with expectations as to how knowledge will be generated in the group—we refer to this construct as *epistemic stances toward group work*, or *epistemology of group work*. Our analysis and discussion are distinct from epistemological framing because while framing can include group work as one of many factors, we focus on a finer grained analysis of just the epistemic stances toward group work and how these interact with other aspects of the social learning environment. We consider these epistemic stances toward group work to be interactions between epistemological and social aspects of framing [196]. Part four of this thesis contains one chapter, in which we investigate the interactions between epistemology, sense making, and social dynamics in group work environments. We present a case study analysis in which we identify *epistemic stances toward group work* and argue that the misalignment of these stances among group members is one factor that contributes to the social positioning of individuals within the group.

[22] Considering a global view of their overall behaviors, we characterize the students in the group as sharing epistemological frames, while noting that individual and group's frames can and do vary moment to moment

# Chapter IX. Investigating how misalignment of epistemic stances toward group work can contribute to social positioning of students

The physics education community values collaboration as an important part of learning physics; we often ask students to work together in groups (e.g., on tutorials, during class discussions, and on laboratory or classroom-based projects), yet we have not studied as much how students think about group work and how that interacts with their learning. In this chapter, we focus our analysis on one group of four students in order to understand the intersection between epistemology, sense making, and social dynamics in group work. While watching video of this particular group, we noticed something puzzling: one student (Penny) who was continually making moves to open up the group for collective sense making was simultaneously positioned as less knowledgeable in the group. This positioning is evidenced by Penny frequently getting cut off or interrupted, explained to in a didactic manner, and the group taking up Penny's contributions as questions. While Penny (a woman) conveys a tone of tentativeness and inclusion in her contributions to the group, two other (masculine performing) students convey a tone of authority and assuredness. We first chose this particular group to analyze because we noticed gendered interactions among the students and marginalization of a female student who was trying to lead and contribute to the group's collective sense making. The interactions we observed among this group of students are in line with the documented gendered speech patterns and social discourse norms in science and engineering that favor masculine ways of engaging in conversation [205–207]. Motivated to understand the positioning of members within this group and the potential marginalization of a student who steers the group's sense making, we zoom in on epistemological aspects of these social interactions. Although we do not directly address the gendered interactions in the present analysis, focusing on the intertwining of epistemology, social dynamics, and sense making may provide one plausible mechanism for the gendered interactions and resulting social positioning. We attend to this epistemological aspect as a first step, and then in future work will dive more deeply into understanding the roles that gender, power dynamics, and ideology play in creating and driving interactions in a group problem-solving setting.

We characterize this group as being cohesive along some dimensions and not cohesive along others. The three most vocal group members are all on the same page about what counts as making sense of the QM problems at hand, and they engage in productive sense making. We view this epistemological alignment in contrast to other groups of students in our data set (that are not included in the present analysis) in which we observe an epistemological tension because one student wants to talk about the problems conceptually, while other students want to plug and chug through the mathematics and get answers to the problems. We view the sense making that Penny's group engages in as productive because they demonstrate making progress toward content mastery[23], engage in metacognition, find different ways to check their answers, and seek coherence among multiple representations. While the group members seem to be on the same page when it comes to epistemology of learning physics (*what counts as sense making in this context?*), we observe different orientations toward what it means to collaboratively generate knowledge. We identify two distinct aspects of epistemology: epistemic stances about learning physics, and epistemic stances toward group work. In our analysis, we focus on the latter and argue that the group members enact different stances toward group work, and that this misalignment is one factor which contributes to the overall social positioning of members within the group.

### A. Epistemic stances toward group work

Group work and collaboration can be one aspect of an epistemology of physics. For experts, and hopefully physics students, what it means to know, learn, and do physics includes group work or collaboration. However, the form that group work takes and how it functions to generate knowledge may be different for different people. We might consider the form group work takes as the ontology of group work, and can imagine different people to have different ontologies for group work. (We might even consider how people utilize ontologies for group work in a flexible manner that varies moment to moment or across social, conceptual, or epistemological contexts). Views about how the group work functions to generate knowledge are then considered to comprise the epistemology of group work, or epistemic stances toward group work. One stance might be that group work entails generating and making sense of ideas collectively, where individuals might contribute tentative ideas

[23] Making progress toward content mastery includes getting the right answer, but also understanding the concepts along the way well enough to be metacognitive, ask follow-up questions, check their answers, etc.

for the group to collectively negotiate and the sense making process is characterized by individuals thinking out loud and building off of one another's ideas. Another stance might be that group work means that individuals will come to understand the ideas at hand and then explain them to others, a mutual process where if you understand something or know the answer, you explain it to the group. We will identify both of these epistemic stances toward group work below in the case study analysis of one group of four students.

### B. Methodology

As part of an investigation of the ways students engage in mathematical sense making in QM, we conducted a focus group study with modern physics and QM students in which we gave them QM problems to work on in a group for one hour. While watching the video of one group and looking for elements of mathematical sense making, we noticed that one student was being positioned as less knowledgeable despite doing most of the work to guide the group's sense making. This motivated us to focus on this one particular group and conduct a case study analysis of the intersections between epistemology, sense making, and social dynamics. The group consists of four students—Penny, Morgan, Cam, and Sarah—from a modern physics class. We did not have relationships with these students prior to the one-time focus group session that took place in the last week of the semester. We recruited the students by sending an email to the class which framed the focus group study as an opportunity to work through QM problems with their peers before the final exam, but was not directly connected to their grade in the course. The professor, however, offered extra credit to students who participated. We organized the groups based on scheduling constraints, and the students were monetarily compensated for their time. Penny (she) identifies as a white female, and is a sophomore majoring in chemistry and physics. Morgan (they) identifies racially as white and responded to the gender question on our demographic survey with a question mark. They are a junior computer science major. We use "they" pronouns for Morgan[24], and note that their performance is more masculine, which is important information for interpreting the social

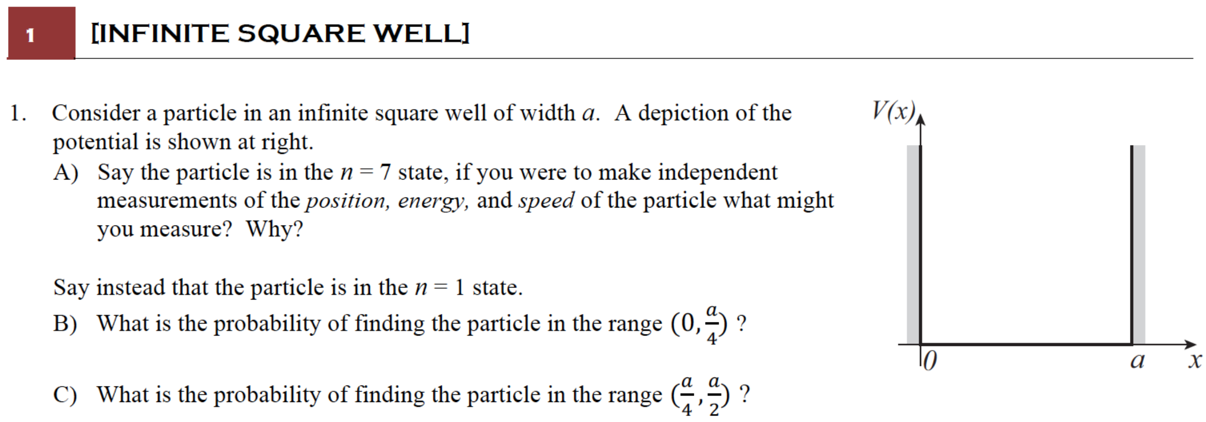
[24] We regret that we did not ask the students which pronouns they prefer, and thus use "they" for Morgan given the "?" response to the gender field. In the focus group and interview studies we have conducted since this one, we have asked students for their preferred pronouns.

dynamics we observe among the group. Cam (he) identifies as a white male, and is a sophomore physics major. Sarah (she) identifies as a white female and is a sophomore majoring in astrophysics and creative writing.

The focus group took place in a small room with the students seated around a small rectangular table—Morgan and Sarah sat on one side of the table, Cam on the other side, and Penny at the head of the table. We framed the focus group session as a chance for the students to work on a few QM problems together. We asked them to talk to each other and say what they were thinking out loud as much as they could, and we noted that we were more interested in their sense making rather than them arriving at a correct answer. One researcher was present in the corner of the room, and chimed in every once in a while with follow up questions for the group. We prepared four different problems (each with several sub-parts) for the students to work on, all in line with QM content they had covered in their course. The three episodes we present here include the students working on two different problems. The first problem (see Figure IX.1) is about an infinite square well. Part A asks the students to determine which values they might measure for position, energy, and speed for a particle in the $n = 7$ state of the infinite square well. Parts B and C ask for the probabilities of finding the particle in the first and second quarters of the well. The second problem (see Figure IX.2) is about a double square well, and has the students consider the cases when the well separation is zero (i.e., one wider well), is comparable to the width of the wells, and is very large compared to the width of the wells.

We transcribed the whole video, and watched it several times from different lenses attending to: the mathematical sense making students were engaging in (our original intention in conducting the focus groups), the interplay between social dynamics and sense making, the perceived gendered interactions, the ways in which certain students get positioned as more or less knowledgeable, the inferred epistemic aspects of students' sense making, and the inferred epistemic stances toward group work. On individual episodes within the hour-long focus group, we conducted a discourse analysis in order to infer epistemic stances toward group work and explore the ways in which those interacted with the social dynamics and sense making to result in the positioning of students as more or less knowledgeable. The analysis focuses primarily on Penny, Cam, and Morgan—the fourth student, Sarah, is engaged and listening to the conversation but makes very few of her own verbal contributions. The social dynamics involving Sarah (including her silence) are worthy of a study in their own right, but they remain outside of the scope of the present chapter. In our analysis, we pay attention not only to the words students are using, but their body language, physical positioning, gestures, tone of voice, pitch and pace of speech, who is talking when, and who has control over the sense making the group engages in. In the transcriptions, ellipses (...) represent pauses longer than those natural in speech; gestures or nonverbal actions are indicated in [square brackets]; square brackets sometimes also contain information added to the transcript by the researchers for clarity; em dashes (—) indicate interruptions in conversation or people talking over one another.

In the analysis, we have attempted to remain as objective and unbiased as possible, but note that we personally view group work in a way that is aligned with how we perceive Penny's actions in the group (i.e., collective consensus building, valuing externalization of confusion, and thinking through ideas in the moment with other people). We do not pass value judgement on which stance toward group work is "right" or "better", but note that our personal stances are generally aligned with collective consensus building (although these stances are certainly context dependent) and that there are many other stances held by people within the physics and physics education communities. Over the course of two years, discussions among seven physics education researchers led to preliminary versions of the analysis and arguments of this chapter. The analysis we present here is the result of discussions and consensus of three of those researchers, a team that includes two men and one woman. As a woman in physics, JRH (the lead author of this work) has privileged access to interpretation of the social cues and norms we see play out in the conversations among feminine and masculine performing physics students. This perspective thus shapes how we perceive and interpret the interactions among this particular group of students. Further, after reaching consensus among the research team, we validated and refined our analysis through additional discussions with physics education researchers external to the project.


1 [INFINITE SQUARE WELL]

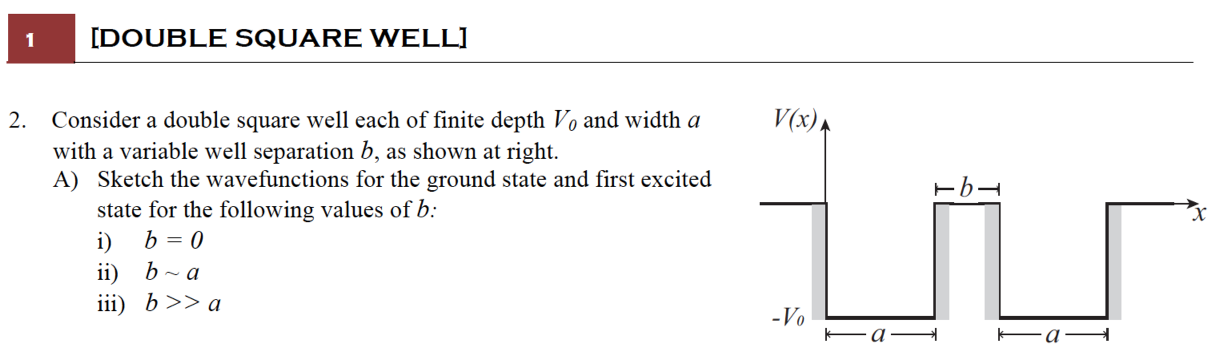
1. Consider a particle in an infinite square well of width $a$. A depiction of the potential is shown at right.
   A) Say the particle is in the $n = 7$ state, if you were to make independent measurements of the *position, energy,* and *speed* of the particle what might you measure? Why?

   Say instead that the particle is in the $n = 1$ state.
   B) What is the probability of finding the particle in the range $(0, \frac{a}{4})$ ?

   C) What is the probability of finding the particle in the range $(\frac{a}{4}, \frac{a}{2})$ ?



FIG. IX.1. Problem 1 from the focus group study asks about an infinite square well. In Episode 1, the students are discussing part A) and in Episode 2, they are determining how to check if their answers to parts B) and C) are correct.


1 [DOUBLE SQUARE WELL]

2. Consider a double square well each of finite depth $V_0$ and width $a$ with a variable well separation $b$, as shown at right.
   A) Sketch the wavefunctions for the ground state and first excited state for the following values of $b$:
   i) $b = 0$
   ii) $b \sim a$
   iii) $b >> a$



FIG. IX.2. Problem 2 from the focus group study asks about a double square well. In Episode 3, the students are discussing part A.ii) when $b \sim a$.

### C. Episode 1

Episode 1 takes place at the very beginning of the hour-long session. The students begin with question 1a (described above and shown in Figure IX.1). They are each looking down at their individual paper, reading the prompt. The conversation begins when Penny reads the question out loud and begins to share her thoughts:

*Penny: "If you were to make independent measurements of the position, energy, and speed, what might you measure?" Uhmmm. So since the, like at every, every time you measured you'd just like get a different position. Right?*

*Morgan: Yeah*

*Sarah: Yeah*

*Cam: Mhmm*

*Penny: Those would just, like according to the probability density.*

*Morgan: But they'd conform to whatever density it is, I'm not quite sure.*

*Penny: Like, whatever the, like the psi squared thing...? [writing on her paper]*

*Morgan: Yeah*

*Cam: Yeah*

*Sarah: Yeah*

*Cam: That would represent the distribution.*

*Penny: And it would have to be zero at the edges so it would be like that. [drawing on her paper]*

*Morgan: And since it's the seventh excited state would there be... —*

*Cam: —Yeah, because we're infinite—*

*Morgan: —seven peaks I think?*

*Cam: Yeah*

*Sarah: Yeah*

*Morgan: It would be like [draws wavy sin peaks in the air]*

From the beginning few seconds of the episode, we infer that all four students have the expectation that they are there to work together. This is evidenced by their posture, who responds to the questions, and the tentative nature of their questions. All four students are leaning in with their elbows on the table, looking down at their own papers but as the conversation begins they also look up at each other and each other's papers (see Figure IX.3). When Penny puts forth her idea in lines 329-330 that you would measure a different position every time, and follows up with a question ("right?"), the three other group members respond simultaneously ("yeah", "Mhmm", lines 331-333). Penny looks at Cam when she says "right?", but he is looking down at his paper, as are Morgan and Sarah. The fact that all three students respond despite not being addressed directly or not looking at the person who asked the question suggests an expectation that any idea put forth is for the group to make sense of and respond to. Additionally, to begin the hour-long


FIG. IX.3. Four modern physics students—Cam, Penny, Sarah, Morgan (starting on the left, going clockwise)—working collaboratively on QM problems.

problem solving session, the students put forth ideas with tentativeness, phrasing their ideas as questions (Penny lines 330, 336) and explicitly saying they're "not quite sure" (Morgan line 335) or qualifying an idea with "I think" (Morgan line 344). In the first few seconds of the episode, we see Penny and Morgan using this tentativeness, setting the tone for a collaborative sense making environment. As the hour progresses, however, we see this dynamic change as Penny continues to be tentative while Morgan and Cam adopt different ways of engaging in the conversation. Cam continues the conversation by putting forth his own tentative idea:

*Cam: Well, your n would be seven times pi I believe, right? Inside the k equals...*

*Morgan: Right.*

*Penny: How does n change the number of like... —*

*Cam: —Uhmm because...*

*Morgan: So, I think—*

*Penny: —waggle things...*

*Morgan: —I think it's...right. That's the, that's what's inside the sine function.*

*Penny: Oh, like the sin(kx)*

*Cam: So it's 7 pi over a.*

*Morgan: That's what I thought, but I think what that... I think it means that if you draw that there's gonna be seven peaks —*

*Cam: —And really—*

*Morgan: —and nine nodes—*

*Cam: —if you actually did the probability it'll actually like bounce up like that many times.—*

*Morgan: —Right.*

*Penny: Ohhh yeah—*

*Morgan: —'Cause in the second state it goes up once and then down—*

*Penny: —It has that many nodes. Or does it have seven minus one or something?*

*Cam: I don't think we have a minus one here...*

*Morgan: I think there's a, I think it's a... Well because there's one peak, I think that what you've drawn [to Penny] is the, is the—*

*Penny: —Yeah—*

*Morgan: —ground state—*

*Cam: —n = 1—*

*Morgan: —n = 1. So that has one peak. So I think it would make sense for it to equal the number of peaks.*

*Penny: And like—*

*Morgan: —Oh, it's n +1 uhh, zero points. That's what it is.*

As the students try to figure out how many peaks the n=7 wave function has, Cam and Morgan begin to talk over Penny (and one another). Penny attempts to ask a question and join in the sense making, but gets interrupted at lines 350 and 363. At line 365, she continues her thought that began with "Ohhh yeah" (line 363), talking at the same time as Morgan and looking directly at Cam. She asks if the number of nodes is "seven minus one or something", and Cam and Morgan both begin to answer her question. They do so tentatively, qualifying their explanations with "I think" (Morgan line 367) or "I don't think" (Cam line 366). This exchange represents a turning point in the overall dynamic, where Penny begins to assume the role of "question asker" and Cam and Morgan begin to assume roles as "explainers". These positioning and role-taking tendencies that we see developing in the first episode continue to evolve over the course of the hour. Here, Cam and Morgan explain to Penny as they try to figure out how the number of nodes depends on the state. This culminates when Morgan has an “aha moment” and states in an excited and authoritative manner that “it’s n+1...zero points. That’s what it is” (line 375). We note that in this instance we see Morgan’s excitement around figuring out the answer (the “aha moment”) coincide with their assumption of the role of explaining to Penny. Next, Penny steers the conversation in a different direction by bringing up a question about the graphs they are drawing on their paper (sketching the wave function on top of the potential energy graph given with the prompt):

*Penny: Well, also it doesn’t really make sense to draw this on this diagram right? Because, or wait, like what is the height of this?*
*Morgan: Well that... the height represents—*
*Penny: —Isn’t that just like the number of—*
*Cam: —The probability—*
*Penny: —Yeah—*
*Morgan: —It represents the probability—*
*Sarah: —Yeah—*
*Morgan: —of finding it there.*
*Cam: Right, so that’s where you’re most likely to find it in the center there.*
*Penny: So it just doesn’t belong... Like, what is V(x)? Isn’t that potential?*
*Morgan: Right. So that’s why, that’s why it’s sorta —*
*Penny: —So you wouldn’t really draw this on this graph—*
*Morgan: —not so great to draw there—*
*Penny: —You’d just draw it like this—*
*Morgan: — because the axes aren’t the same.—*
*Penny: —and this [y axis] would be like, probability—*
*Morgan: —Exactly—*
*Penny: —and then this [x axis] would still be position.*
*Morgan: And so that, drawing the probability curve makes sense, but yeah you’re right on that it doesn’t represent potential so it’s, it’s not the most meaningful curve there.*

Again, Penny is the one asking the questions (lines 376 and 386) that Morgan and Cam take up by explaining to her. At line 378, Morgan immediately takes up the question about drawing multiple graphs on the same axes; whereas in lines 364-367 Cam and Morgan's explanations were tentative, here Morgan answers in a more didactic manner. The tentative qualifiers ("I think") are no longer present and both Morgan and Cam contribute ideas with more conviction: "Well...the height represents" (line 378), "It represents the probability" (line 382), "Right, so that's where..." (line 385), "Right. So that's why..." (line 387). Amidst these explanations, Penny tries to chime in and join the sense making but gets cut off (lines 379, 388, 390, 392).

In this episode, Penny has control over the conversation in that she is the one asking questions that prompt the group to engage in sense making (considering what the wave function looks like, and then what it means to draw the wave function on the same axes as the potential energy). One might interpret Penny's questions to indicate that she is confused about the content (or more so than the other students, or at least more vocal about the confusion than the other students). We take a different interpretation, in this episode and in the hour-long session as a whole, that Penny often frames ideas in the form of questions or bids[25], and not that she is more confused about the content. This is evident in that her questions are often an idea followed by "right?" (lines 330 and 376). Penny does not have agency in the conversation in the sense that she is continually interrupted, and explained to in a didactic manner. With the confluence of these interruptions and explanations we begin to see Penny as being positioned as less knowledgeable within the group despite the fact that her ideas or contributions are the ones guiding the flow of the conversation.

We focus on Penny and Morgan in this episode and identify them as enacting two different epistemic stances toward group work. Penny's tentativeness and orientation toward inclusion are aligned with an epistemic stance that the way knowledge will be generated in the group is through collective consensus building or sense making. By adding "right?" to the end of her ideas, Penny invites others to contribute or weigh in on her ideas, consistent with a notion of collaboration that involves throwing out ideas for everyone to collectively grapple with and build on. Additionally, Penny throws out ideas and thinks through them as she is talking. For example, in line 363 Penny begins a thought with "Ohhhhh yeah" and finishes it in line 365 ("It has that many nodes") after

[25] The tendency of women to frame ideas as questions is well documented in feminist discourse [208]. This, among other finer grained analyses of these interactions, will be the subject of a forthcoming paper.

Morgan inserts a statement. In line 376, Penny brings up the idea of graphing multiple things on the same axes and interrupts herself to ask, “Because, or wait, like what is the height of this?” She continues by beginning to answer her own question, “Isn’t that just like the number of ” (line 379). This happens again in lines 386-394, where Penny’s five contributions are all a continuous thought interspersed with Morgan’s contributions. Penny’s thinking through ideas as she says them is characterized by slower speech. Overall, this slower pace of speech and asking questions where her pitch goes up at the end convey a tone of tentativeness and inclusion. We associate these actions and this tone with a view of group work as a collective consensus building process.

Morgan’s actions on the other hand are aligned with a stance toward group work that it should involve individuals working to understand the ideas in order to explain to other group members. This is evidenced by their immediate responses to or taking up of Penny’s questions with didactic explanations. In lines 351-352, both Cam and Morgan interrupt Penny before she finishes asking her question. Morgan explains, tentatively still at this point in the episode, how n is related to the number of peaks or nodes in the wave function. In line 378, Morgan immediately takes up Penny’s question about the graph, and through line 396 explains to her why it does not make sense to draw the wave function and potential on the same axes. In this exchange, Morgan’s explanations come across as certain and authoritative, in particular in lines 378 (“Well...the height represents”) and 387 (“Right. So that’s why...”). Additionally, in line 395 they validate Penny’s original idea that superimposing the two graphs does not make sense. This validation recognizes Penny’s contribution, while also placing Morgan in a position of authority. Morgan speaks at a quick pace, and their posture is directed toward Penny especially at the end of this episode when they point at Penny’s paper while explaining. Morgan’s fast speech, posture, immediate responses to questions, and didactic explanations convey a tone of authority and assuredness (primarily in the second half of episode 1). We see these actions and this tone as an enactment of a stance that group work means one person with the desired knowledge will explain to others.

In Episode 1, we see the interactions between epistemology, sense making, and social dynamics begin to result in the positioning of Penny as less knowledgeable. Penny’s slower speech means that Cam and Morgan have time to interrupt her, and her questions and tone of inclusion help to position Cam and Morgan as the explainers. In particular, we see the interactions of these factors in the exchange between lines 386-394. Penny asks and answers her question all at once ("what is V(x)? Isn't that potential?"), and continues her thought through line 394. However, her slower speech and tentative framing of questions, in addition to Morgan's faster speech and tone of authority, result in Penny being interrupted, explained to, and positioned as less knowledgeable.

In summary, Episode 1 sets the stage for Penny, Sarah, Morgan, and Cam to engage in group work around QM problems—we infer that they all have the expectation that they will work together, yet as the episode unfolds we see each individual engaging in group work in different ways. Penny begins to take on a role of the question asker, while Cam and Morgan start assuming roles as explainers. Sarah is engaged and listening, and every once in a while, affirms others' statements. We infer two different epistemic stances toward group work: Penny reflects orientation toward collective consensus building, and Morgan is aligned with an orientation of understanding in order to explain to others. In this first episode, we see these epistemic stances interact with the social dynamics of the group in a way that begins to position Penny as less knowledgeable despite the fact that her contributions are the ones driving the sense making that the group engages in. These roles and positioning continue to evolve and begin to solidify as the group continues to engage in the collective problem solving.

### D. Episode 2

Following Episode 1, the students continue to discuss question 1a, but when they get stuck wondering if the speed of the particle in the well is constant and if this violates the uncertainty principle, they decide to move on to part b which asks them to find the probability of finding the particle in the first quarter of the well if it is in the ground state (n=1). The students immediately begin writing down an integral. They recall the ground state wave function, and integrate the square of the wave function from 0 to a/4. They do the same thing for part c which asks for the probability of finding the particle in the second quarter of the well (this time, integrating from a/4 to a/2). Less than seventeen minutes in to the problem-solving session, the group has finished the two integrals resulting in symbolic expressions for the answers to question 1 parts b and c. Episode

2 begins when they are looking at their resulting answers and trying to figure out how to check if they are right. Penny begins the conversation by making a bid to check their answers against their intuition:

398 *Penny: Yeah, I don't really know, I dunno how to have a good intuition about—*

399 *Cam: —Well one thing that does make sense—*

400 *Penny: —Well wait the probability should be less than one—*

401 *Morgan: —Uhh, those, those sum to one.. that plus that plus that again plus that again.*

402 *Cam: Well the originally... yeah—*

403 *Morgan: —'Cause if it's symmetric around a over two*

404 *Cam: Yeah*

405 *Morgan: Then these two should add to get one half—*

406 *Cam: —Yeah!—*

407 *Morgan: —And they do.*

408 *Penny: Aah. Wait... a over...*

409 *Cam: Aaaah. Yeah, they do!*

410 *Penny: Wait, isn't—*

411 *Cam: —That makes sense.*

Although Penny's statement about intuition in line 398 is not phrased as a question, functionally it serves as a question to the group, and is taken up as such. Penny herself answers it in line 399, when she says "Well wait the probability should be less than one". For the first few lines of this episode, Morgan was hunched over their paper doing math—they wrote down an expression that was the sum of their answers to parts b and c (twice), and found that they summed to one. They finish this math right as Penny suggests the probability should be less than one (line 399) and look up and immediately begin explaining (line 400), pointing to the graphs the students had previously drawn on a big piece of paper in the center of the table. Here we see Morgan again taking up the role of the explainer, speaking quickly and with certainty. They explain the symmetry of the wave function, with Cam chiming in affirmatively. Penny attempts to join in and make sense of what they are saying (out loud, in the moment), but gets cut off by Cam in lines 408 and 410 as he "gets it" and expresses excitement over understanding Morgan's explanation. Penny finally gets in a question and asks Morgan to explain again:

412 *Penny: Can you explain that again?*

413 *Morgan: So like, if we have... essentially this is our function right here.*

414 *Penny: Mhmm.*

415 *Morgan: And this divides it, a over two—*

416 *Penny: —So we went—*

417 *Morgan: —is the halfway point—*

418 *Cam: —Yeeahhh—*

419 *Morgan: —so we basically, if you add these two together—*

420 *Cam: —You should be at the halfway point—*

421 *Morgan: —Then we should have this area—*

422 *Cam: —Which is one half [gestures at Penny with a quick bouncing finger pointing motion]—*

423 *Morgan: —Which should be one half—*

424 *Cam: —Which happens—*

425 *Penny: —Ohh—*

426 *Cam: —'cause these cancel*

427 *Penny: Ohhhhh! [smiles and silently claps]*

428 *Morgan: —the 4 pi's over two cancel*

429 *Penny: Oh that's so!—*

430 *Cam: —And another way—*

431 *Penny: —And this is n = 1 so like, that is symmetric.—*

432 *Cam: —And it makes more se—*

433 *Morgan: —Yeah—*

434 *Penny: —So we know that it's symme—*

435 *Morgan: And so, yeah*

Penny asks Morgan, "Can you explain that again?" (line 411), reinforcing her position as the question asker and Morgan's position as the explainer. Morgan immediately takes up the explanation and the following conversation is characterized by Cam and Morgan talking over one another while Penny tries to join in, but gets cut off (lines 415, 428). In this section (412-427), Morgan and Cam are explaining to Penny (in this case, she explicitly asked them to). Yet Penny still tries to join in and make sense of the argument along with them (e.g., “So we went...” in line 415). From the very beginning of the episode, Cam has been trying to contribute an idea about how they can know their answers make sense (“Well one thing that does make sense...” line 398), but he keeps getting talked over (lines 429, 431). He finally contributes his thought as they finish up the conversation:

*Cam: You should be able to say too that it makes... B) makes sense because, B) being the first part of it, should be less—*

*Morgan: —Should be less than the second part—*

*Cam: —than the second part*

*Penny: Mhmm...*

*Cam: So if we're taking the difference—*

*Penny: —Ohhh that makes sense!—*

*Morgan: —And that makes sense because...ya know, otherwise it'd be weird for it to sum to one.*

*Cam: Okay...*

*Penny: Good job team—*

*Cam: —alright. Well, at least it makes sense, or seems to make sense...Except I still don't understand what that first one is asking for. [laughs]*

Cam puts forth the idea that their answer makes sense because the integral for part b (first quarter of the well) should be less than that of part c (second quarter of the well) given the shape of the wave function, but Morgan interrupts to finish his sentence for him (line 437). When Morgan completes the explanation of the symmetry argument, Penny signals that she gets it by saying “Ohhh that makes sense!” (line 441), and concludes the episode by “saying good job team” (line 445).

The majority of this episode consists of Morgan and Cam explaining to Penny. Penny has assumed the role of question prompter (line 397) or question asker (line 411), while Morgan and

Cam have taken up the roles of "knowers" or explainers. These two different roles inform one another; for example, in line 411 Penny asks "Can you explain that again?" which positions Morgan and then Cam (who take up the question) as explainers. We see Penny being positioned as less knowledgeable in this episode. Right off the bat, she prompts the question about connecting to intuition (line 397), and as she puts forth an answer gets interrupted by Cam (line 398) and then Morgan (line 400). Then when Morgan begins to explain the symmetry argument, Cam understands it first and his affirmative interjections ("Aaaah. Yeah, they do!" line 408 and "That makes sense" line 410) prevent Penny from being able to join the collective sense making process. She continues to be interrupted throughout the episode when she attempts to contribute to the sense making (lines 415, 428, 433). Building on the tendency we noticed in Episode 1 for Cam and Morgan to explain to Penny, Episode 2 is dominated by these didactic explanations (lines 400-406 and 412-427). Additionally, in line 421, as Cam joins Morgan in explaining to Penny that their answers from parts b and c should sum to one half, he gestures to Penny with a quick, bouncing finger-pointing motion. One might interpret this gesture as a manifestation of Cam's excitement for figuring out that their answer makes sense, or as a reflection of his position as knower and transmitter of knowledge to Penny. We think both of these interpretations (and likely others) can coincide. Regardless of the intent or emotion behind the finger-pointing gesture, it functions in the group as a symbol of the social position of Cam as a knower and explainer and Penny as a questioner or one who gets explained to. Despite the ways in which we see Penny being positioned as less knowledgeable, she expresses excitement at the end of the episode (lines 441, 445). That is, she does not seem to notice or be bothered by the way this is being interrupted or explained to[26].

We identify the students' actions in this episode to be again aligned with two different epistemic stances toward group work: Penny as aligned with a stance of collective consensus building and Morgan and Cam (and possibly Sarah) aligned with a stance of individual understanding and explaining to others. Perhaps most indicative of these stances is in the different ways that Penny and Morgan react to not being sure about something. As Episode 2 begins, Morgan is hunched over their paper working out some math, and once they add up their symbolic expressions and find

[26] This is not necessarily evidence that these things aren't happening, as we know that these kinds of interactions happen all the time in the broader physics culture and that members of underrepresented groups don't notice that they are being marginalized or positioned in a certain way, or they notice and deal with it.

that they sum to one they look up and begin explaining to the other group members. This is one example of how when Morgan is not sure about something, they retreat into individual problem solving mode and when they figure it out they re-engage with group to explain their solution or idea to the other members of the group. We see this happen multiple times throughout the hour-long session; this tendency is aligned with an individualistic stance toward group work with the goal of understanding for yourself so that you can explain to others. When Penny is confused or unsure about something, she vocalizes her confusion, contributing ideas tentatively and thinking through them out loud in the moment. One example of this occurs at the beginning of Episode 2 when Penny makes a bid for considering intuition (line 397) and then continues to put forth an answer to her own question (line 399). We see this tendency as being aligned with a stance that considers group work a process of collective sense making where individuals put forth ideas they are unsure about and others grapple with and build on them so that the group collectively makes sense of the idea in the moment.

In addition to Penny's tentativeness, she tries to join in on collective sense making (lines 407, 415, 430, 433), and praises the group's accomplishment ("Good job team", line 445). Through these actions, Penny conveys a tone of tentativeness, inclusion, and excitement to be a part of a team. Cam and Morgan on the other hand, through fast speech and immediately jumping in and talking over people (lines 400-406, 414-428, 433-440), didactic or authoritative explanations (lines 400, 412-427), and pointing at Penny's paper or the common paper in the middle of the table in order to explain (400, 425), convey a tone of authority and assuredness, positioning them as knowers or explainers. We identified these two different epistemic stances toward group work in the first episode and see them again in the second episode and throughout the hour-long session. We note, however, that each individual student does not always act in alignment with these stances or roles that we have identified. In line 411, Penny explicitly asks for an explanation from Morgan, which could be evidence of a stance that group work involves one individual explaining the answer to another individual. In this brief moment, we might infer this stance for Penny. However, a few lines later, Penny tries to join in on the explanation, which we take as a bid for collective consensus building. This moment to moment variation in inferred epistemic stances toward group work is in line with the idea that epistemologies can be context dependent or vary moment-to-moment. Likewise, there are instances where we see Cam and Morgan engaged in practices that suggest an orientation to collective sense making (e.g., tentativeness in the first half of Episode 1). We do not take these as contradictions to the overall interpretation, but instances of context dependent and fluid epistemic stances. Looking at the hour-long session as a whole, we associate Penny's engagement with a stance of collective consensus building, and Cam and Morgan's actions with more individualistic stances focused on one individual with knowledge explaining to another.

We have largely left Sarah out of our analysis, because she does not often contribute verbally to the group, which makes it more difficult for us to make inferences about her involvement in the group. There are two possible interpretations we might take from Sarah's silence. One is that her silence is indicative of an epistemic stance similar to that enacted by Cam and Morgan—that is, group work is a process in which one individual with knowledge explains to the other individuals. If Sarah took this stance in this particular setting, and felt that she did not have knowledge relevant to their problem solving, then she might be inclined to not make many verbal contributions, and would assume more of a role of a listener or consumer of the explanations provided by other group members. An alternative interpretation is that Sarah takes a stance more aligned with Penny—one of collective consensus building—but that she does not see herself as part of the collective group and thus she does not contribute her ideas or vocalize her confusions. Both of these interpretations (and likely many others) are plausible. For now, we continue to focus our analysis primarily on Penny, Morgan, and Cam for whom we have more social, epistemological, and sense making cues to go off of.

In Episode 2, the interactions between the epistemic stances toward group work, social dynamics, and sense making result in Penny being positioned as a question asker with Cam and Morgan being positioned as knowers and explainers. Penny's slower speech and throwing out tentative ideas to think through on the fly and Morgan and Cam's faster and quick-to-explain speech result in Penny being interrupted and explained to (e.g., lines 399, 407). In this particular episode, these dynamics lead to Penny not contributing as much (in terms of turns of talk) as in other episodes. When Penny asks for an explanation (line 411), Morgan and Cam take it up with fast-paced and authoritative explanations that make it hard to contribute when Penny attempts to join in on the collective sense making (lines 407, 415, 430, 433). Despite this social positioning, it is Penny's contributions that again guide the group in the sense making they engage in—she prompts them to consider intuition in line 397 and symmetry in line 430. We note that before this episode began, Morgan had already begun to check that their answers summed to one half, indicating that they were already considering symmetry and possibly intuition (although we suspect "intuition" might mean something different to Morgan than it does to Penny). However, if Penny had not directly asked about these aspects, we do not know if the group would have engaged in conversation around them in the way that they did in response to Penny's prompting.

### E. Episode 3

Episode 3 takes place 46 minutes in to the hour-long session, when the students are working on question 2a (see Fig. IX.2 above), sketching the ground state and first excited state wave functions for a double square well. They spent a small amount of time on part i, considering the situation in which b=0 (a single finite square well of width 2a). In this episode, they are working on part ii where $b \sim a$. The group has agreed that the wave function must be an exponential decay towards infinity on either end and they have mentioned a sine solution within each well, but they are unsure what the solution inside the barrier (the middle region) should be. They have talked about an exponential decay coming from either well to meet in the middle, and this episode begins as they seek mathematical (formulaic) justification (Morgan in particular has been pushing for this). They are trying to remember where the sine and exponential solutions come from in the first place, and Morgan begins to write down the Schrodinger equation on the large piece of paper in the center of the table:

449 *Morgan: So if we have a Schrodinger equation which looks essentially like negative—*

450 *Penny: —hbar squared—*

451 *Morgan: —a positive constant...*

452 *Cam: Yeah, okay—*

453 *Morgan: —times the second derivative of this [wave function]...uh...equals...*

*Cam: The E minus U of x—*

*Morgan: E minus U of x—*

*Cam: —Times…Yuuuup—*

*Morgan: —times that [wave function] then when you rearrange this you get… double prime of x equals…so if E minus U of x is negative, is less than zero, right this [$E - U(x)$ term] is negative this [$-K\psi''$] is negative, so we get…essentially the solutions to that differential equation. [circling $\psi'' = +k\psi$ on the paper]*

From the beginning of the episode, Morgan assumes the role of "math doer" as they have control of the large piece of paper and thus the conversation. The other three group members are leaning in and all watching as Morgan writes down equations and explains out loud what they are doing. While Morgan explains what they are doing, Cam chimes in with affirmations (lines 451, 455) that suggest he understands what Morgan is doing and also starts to help out with the explanation (line 453). Penny continues the conversation by asking a question:

*Penny: How does, uhm, the I guess like location along x determine whether E is like E minus U is less than zero or greater than zero.*

*Morgan: E is constant, but U of x is a function of x.*

*Cam: Mhmm*

*Penny: Ohhh, yeah yeah yeah—*

*Cam: —Yes. And so yeah, you could look at the potential—*

*Morgan: —So as x changes this is gonna change. If E-U(x)—*

*Penny: Is gonna change from being zero to being V of—*

*Cam: —V naught*

*Penny & Cam: Yeah*

*Morgan: If it's greater than zero though we get this one with a negative sign there, that's a little hard to read. Uhm, and so the solutions of this are roughly…This is e to the x or e to the -x.*

*Penny: Yeah, that's usually…*

*Morgan: And this is*

*Penny: What does that say? Psi...?*

*Morgan: Sorry. This this? Yeah, I wrote that badly. The double derivative of—*

*Penny: —Equals negative psi? Okay.*

*Morgan: Yeah. This leads to e to the i x which is the same as cos(x) + sin(x)—*

*Penny: —Isn't it also just sine?—*

*Morgan: —With some, ya know, there's constants here.*

*Cam: Right.*

*Penny: Just sin(x)?*

*Morgan: Hmm?*

*Penny: Doesn't sin(x) satisfy this?*

*Cam: Yeah. But he's writing out the general form and then A or B would be zero —*

*Penny: —Ohhhh. Yeah.—*

*Sarah: — Yeah—*

*Cam: —depending on—*

*Morgan: Yeah*

*Cam: —basically you'd model it to fit the well. And you'd decide cosine or sine. Usually, in most cases, we'd use sine. But if it did need to start at one instead of zero then you would use cosine.*

In line 460, Penny prompts the group to connect the equation Morgan has written down (particularly the E-U(x)) term to the graph, a move that we consider to be part of mathematical sense making. Morgan takes up this prompt and explains to Penny that the total energy is constant but the potential energy is a function of x, and Cam chimes in to refer to the graph (line 465). Penny then starts to join in the collective sense making (line 467), and Cam jumps in to finish her thought or help her out, saying "V naught" and pointing at the graph on her paper (line 468). Morgan returns to the differential equations and continues to work out the solutions, with Penny asking for clarification about what Morgan has written (lines 470-476). When Morgan says that the solution in the region where E-U(x) is positive is cos(x)+sin(x) (with some constants), Penny vocalizes a question—"Isn't it also just sine?" (line 478). Once the group hears this question, Cam jumps in to explain to Penny what Morgan is doing (line 484). He continues the explanation after both Penny and Sarah have indicated that they understand ("Ohhh yeah", line 485 and "Yeah", line 486). Midway through Cam's explanation in lines 489-490, Penny looks down as if she has stopped listening to the explanation. Morgan returns to the math on the paper and continues:

*Morgan: Right...Okay...So it does have to be the decaying one in the middle. It can't be a sine wave. Well that's nice.*

*Penny: Wait, why?*

*Morgan: Because here we have the... we're in this [circling* $\psi'' = +k\psi$ *on the paper] condition, because E minus the potential, the potential is high up here and we presumably don't have the energy.*

*Penny: Yeaahhh. I mean... I dunno what E is. What determines... So I guess the thing that determines E is n.*

*Morgan: Right.*

*Penny: Through like [laughing] a relationship that we talked about already that we don't remember*

*Cam: What you could do is you could actually do your double derivative here with your uh you know your n value and your k stuff and you could actually solve for the energies and stuff if you needed.*

*Morgan: Right...But like. That, that. This observation. So like when U of x is high, above what we think the energy of our electron is...*

*Penny: Then —*

*Morgan: —Then—*

*Penny: —then it must be—*

*Morgan: —we know that...uh psi of x has to take on a form that's a negative exponential. Or a positive exponential, but the positive exponential can't happen because of regularity conditions.*

*Penny: And then that's, that is intuitively right.*

*Morgan: Yeah. There's a lot of constants here and stuff that I left out 'cause I don't actually know how to do ODE's—*

*Penny: —That if the energy is, if the potential energy is higher than the energy of the thing than the thing is not like as likely to be there.*

*Morgan: Right, well, yeah it drops off exponentially as you get further away from the uh low regions.*

In lines 492-493, Morgan says, “It can’t be a sine wave. Well that’s nice” with certainty as if to say, “Great, we have the answer!” Penny begins to ask a question about the total energy but then interrupts herself and answers her own question, in a tentative way (“I guess”, line 498). Earlier in the focus group session, the group recalled that the total energy (E) was determined by the value of n, but they could not remember the formula. Here, Cam jumps in to explain how they could get that formula by solving the differential equation, although they do not actually pursue this. To conclude the episode, Penny prompts the group to check their answer against their intuition (line 512). Morgan takes up this “intuition” idea by considering mathematical formalisms, talking about the constants they left out because they “don’t actually know how to do ODE’s” (line 513-514). Penny’s response indicates that she meant something different by “intuition”—“if the potential energy is higher than the energy of the thing than the thing is not as likely to be there” (lines 515-516). Morgan acknowledges Penny’s idea, but modifies it (“well, yeah it drops off exponentially” line 517), ending the episode in the didactic manner in which they started.

In this episode, Penny again assumes the role of the question asker. As is characteristic of her contributions throughout the hour, she is particularly interested in connecting to intuition, and it seems as though she does not want to let her questions go until this intuitive aspect is satisfied. Morgan in this episode takes on the role of the “math doer” and a teacher or explainer, controlling the shared piece of large paper and driving the mathematical argument that the group constructs in order to answer their question of what the wave function should look like in the barrier in between the two wells. Cam plays a slightly different role in this episode in that his contributions are primarily validating, repeating, or adding on to explanations. While he does not “own” or create the explanations, the act of validating what other people (usually Morgan) have said places him in a position of authority in the group. Sarah is quiet as usual, but is paying attention and leaning in to watch Morgan’s orchestration of the algebra that leads them through the sense making process.

We see Morgan has having most of the control over the conversation in this episode, yet Penny still opens up the group for sense making and guides the conversation with her questions. She prompts the group to connect their algebraic equation to the graph (line 460) and to check their result against their intuition (line 512). As the question asker and prompter of these moments of sense making, we simultaneously see Penny being positioned as less knowledgeable within the group. Much like in the prior two episodes, Penny is interrupted (lines 472, 507, 509) and explained to (lines 462-468, 470-479, 484-491, 495-497, 502-504) throughout the episode.

We again identify two different epistemic stances toward group work. Penny's tentative questions and ideas and thinking out loud (lines 460-461, 498) and attempts to join in on the sense making process as Cam and Morgan explain to her (lines 467, 472, 507-509) are aligned with a view of group work as a collective sense making process. Morgan's actions and contributions are consistent with a more individualistic stance of understanding and explaining to others, evidenced by their didactic explanations taking control of the large piece of paper and sometimes pointing at Penny's paper (lines 505-506). Cam's actions are also aligned with this more individualistic stance toward group work, but they vary from Morgan's actions because Cam assumes more of a role of validator and explainer, as he continually validates, affirms, or repeats ideas (lines 455, 463, 465, 468, 480) and explains what Morgan is going or adds to their explanations (lines 484, 489-491). Notably, in lines 502-504 when explaining how the group could figure out how exactly E depends on n, he uses "you" instead of "we". We interpret the "you" to not be directed at any one individual group member, but rather a replacement for "one". Had Cam used "we" instead, that would have reflected a different stance toward group work, perhaps suggesting he saw himself as part of a collective team, or that the knowledge could (and would) be generated in the group through a collective process involving everyone. The fact that he used "you" signals to us that he is not enacting an epistemic view of group work as a collective consensus building process.

Episode 3 occurs almost at the end of the hour-long problem solving session. By this point, the tendencies and role-taking that we noted in Episode 1 have solidified more as the students have learned or become comfortable in these particular roles. Throughout the hour, Penny has been interrupted, explained to, and (in our view) positioned as less knowledgeable. Yet more than 45 minutes into the session, she continues to make space in the group for sense making, and is still attempting to contribute to a collective sense making rather than just listening to Morgan and Cam's explanations (e.g., lines 467, 507). The interactions between the different epistemic stances toward group work we infer, the social dynamics, and the sense making processes result in the social positioning that we see develop in the group—Penny as less knowledgeable and a question asker, Morgan and Cam as knowers or explainers with positions of authority, and Sarah as a quiet listener who is not directly acknowledged by the other group members.

## F. Discussion

We were motivated to conduct this case study analysis because we noticed a puzzling phenomenon in this particular group—the student who was steering most of the group's sense making (Penny) was simultaneously positioned as less knowledgeable within the group. Investigating the intersections between epistemology, sense making, and social dynamics gives us some insight into how this social positioning can happen in collaborative problem solving situations. The work that the students are doing is primarily guided by the prompts we gave them and by Penny's questions and bids for sense making. Throughout the hour, Penny largely assumes the role of the question asker or question prompter, doing most of the metacognitive work that moves the group's sense making along. Her interactions in the group are characterized by slower, tentative speech, and thinking through ideas out loud. Morgan and Cam assume roles as knowers and explainers, in many instances conveying a tone of authority and assuredness through their pace of speech, diction, posture, and gestures. From Penny's actions, we infer an epistemic stance toward group work that collaboration means collectively grappling with, and building on, ideas to make sense of the topic at hand. We infer a different epistemic stance toward group work for Morgan and Cam, one that focuses on individual understanding in order to explain to other members of the group. The misalignment of these two different stances along with the roles the students assume and the ways in which they engage in discourse, result in the social positioning of Penny as less knowledgeable and Cam and Morgan as occupying positions of authority. Penny's slower speech along with Morgan and Cam's quicker speech and inclination to jump in and explain, allow Penny to be interrupted.

Likewise, Penny's questions and orientation to inclusion along with Morgan and Cam's inclination to explain, result in Penny being explained to in a didactic manner. We recognize that there are many factors at play in this social problem-solving setting, including gender, power dynamics, and cultural norms and discourse patterns, and we identify the misalignment of epistemic stances toward group work as *one* factor that contributes to the social positioning within the group.

This chapter presents a case study analysis of one group, but we have examples from other groups in our data set where we have seen this positioning happening, although it unfolds in different ways in different groups. We focus on one case study here to propose this construct that we refer to as *epistemic stances toward group work*, and document how it can contribute to social dynamics and social positioning of students in collaborative problem solving environments. We see the epistemic stances toward group work as being distinct from, but reflexively entangled with, role-taking in the group. The sense making that this particular group engages in is beneficial for Penny, Morgan, and Cam in terms of making progress toward content mastery (we do not have evidence to say the same for Sarah). However, there are other dimensions along which we might consider the group's interactions to be productive or unproductive. Considering who has access to certain roles or positions, we might characterize this group as unproductive, or at least not as beneficial as we might like our educational environments to be. While Penny engages in scientific practices that we see as productive (metacognition, checking ideas against intuition, seeking coherence, etc.), she probably does not benefit from *always* being the questioner. That is, Penny, and the group as a whole, would likely benefit from her sometimes assuming the role of the explainer; in this hour-long problem solving session, Penny does not have the opportunity to take up this position. Likewise, we see Morgan and Cam engaged in sophisticated sense making processes and clearly demonstrating some level of content mastery, but we posit that these students could also benefit from engaging in practices like questioning and checking ideas against intuition.

### G. Conclusions, implications, and future work

We frequently ask students to work together in our physics classes, and while there are some instances where we attend to the group work by assigning roles or constructing groups based on gender or performance levels [209, 210], it is not a leading discourse in the physics education community to consider how to support students in engaging in group work productively and equitably. Through this exploratory case study analysis, we echo the calls of other researchers [183–185, 188, 189, 191, 194] to not only implement group work in our physics classes, but to attend to the dynamics among students, investigate factors that contribute to collaborative environments, and construct group work settings that promote equity. Motivated by the observation that one student (Penny) who continually steered the group's sense making and was simultaneously positioned as less knowledgeable, we investigated the interactions between social dynamics, epistemology, and sense making. We inferred students' epistemic stances toward group work—their stances about what it means to generate knowledge in a group—and argued that the misalignment of these stances between individual students was one factor that contributed to the social positioning of Penny as less knowledgeable.

Understanding how multiple factors interact to privilege or exclude contributions from students in a group is a first step in learning how to cultivate, and support students in cultivating, equitable group work. There are several avenues of possible future work that have emerged from this initial study. A forthcoming paper will include a complementary analysis of this same group that takes an even finer-grained look at how the local interactional dynamics reproduce, and derive from, broader patterns of discourse and cultural practices [211]. In particular, we will address the gendered interactions between Penny, Cam, and Morgan that we observed in this case study. Additionally, it would be interesting to investigate further Sarah's role in this group in order to understand the possible barriers to her participation [189]. We could also conduct additional case study analyses with different groups of students—as mentioned above, we see similar social positioning happen in other groups in our data set, but the ways in which it unfolds differs by group.

In the case study we presented in this chapter, we see a group of students engaged in conceptually productive yet *inequitable* group work. We urge researchers and instructors to pay attention to the social interactions among students and access to participation for individual students when designing, facilitating, or evaluating collaborative environments. Not only should we attend to and support students' content mastery, their attitudes and beliefs toward learning physics, but also the ways they engage in group work, given that social interactions and collaboration with other people is paramount to the doing and learning of science. We should focus on supporting students in engaging in equitable group work and recognize the ways that epistemology, social dynamics, and sense making can interact, and should leverage these interactions in order to support students' learning, identities, and sense of belonging.

# PART FIVE: CONCLUSIONS

## Chapter X. Summary and concluding remarks

In this dissertation, we have investigated ontological, epistemological, and social aspects of students' reasoning in QM, valuing the messy nature of learning along the way and thereby shifting away from a dichotomous "get it or don't get it" view of learning. In regards to ontologies, we investigated the dynamics of students' reasoning around the nature of quantum entities and support this dynamic reasoning through the development of research based curricular materials. In regards to epistemology, we explored the ways in which students' views about the nature of learning quantum physics can differ from those of classical physics, and began to investigate the impacts of individual instructors on students' epistemological development. Lastly, we investigated how social dynamics, epistemologies, and sense making can interact to result in social positioning of students in a collaborative problem solving environment. The main points and conclusions from each section are enumerated below.

**Ontologies.** We developed a framework to describe and distinguish between different ontological structures—unitary (application of a single stable ontology), parallel (switching between multiple stable ontologies), and blended (creation of novel ontology by drawing on multiple stable ontologies yet blending them to create a new entity entirely). We observed these ontological structures and students' flexible use of ontologies in individual, collective, oral, and written reasoning. The demonstration of this flexible use of ontologies is novel for the PER community which has previously recognized the dynamic nature of ontologies, yet not elaborated on the different forms those dynamics can take. We attended to the situated nature of learning by examining the ways in which our prompts impact students' ontological reasoning; we find that the way we ask questions impacts the ontologies students will use. This finding is important because it has not yet been demonstrated in this way, and from it we suggest that as instructors we should recognize and attend to the ways in which students can engage in flexible use of ontologies. Additionally, we identified two elements of messiness in students' ontological reasoning—tentativeness and flexible use of ontologies—and argue that these can be productive for student learning. We presented an example of how we can work to support this messiness in curricular materials, through a suite of ten modern physics tutorials designed, in part, with the goal of supporting students' *meta-ontological competence*—the ability to think about how they are thinking about an entity and to determine when it is best to use a given ontology (e.g., when it is best to think of a photon as a particle and when it is best to think of it as a wave, or both, or some other type of entity entirely). Future work to be considered in the efforts to understand and support students' ontological reasoning includes further applying and studying the use of these (and other) curricular tools to demonstrate and understand how such tools, and educational environments more broadly, can support the development of students' meta-ontological competence. Additional studies of the relation between students' meta-ontological competence and their conceptual mastery, problem solving skills, reflective practices, and engagement in group work would also be of interest.

**Epistemologies.** We conducted a study of students' domain-specific epistemologies, where the different domains corresponded to classical and quantum physics. On some bifurcated CLASS items (one question asking about classical physics and the other about quantum physics), students reported "epistemological splits" between the two sub-domains. Students were more likely to consider quantum physics to be less tangible or connected to the real world, and to perceive problem solving in QM to rely more heavily on the math. We observed these epistemological splits across multiple institutional and instructional contexts (surveying over 1,000 students). The existence and prevalence of these splits suggests that we should be careful about treating "physics" as a single domain when attending to students' views about the nature of knowing and learning physics. Further, we challenged a default interpretation that split results mean that students are less expertlike in quantum as compared to classical physics. Through analysis of focus group interviews, we identified some of the reasons that students might report epistemological splits, and argued that these stances *can* reflect epistemological sophistication. Thinking about student reasoning as situated within a broader instructional context, we began to investigate the impacts that individual instructor's stances and emphases on epistemology in their teaching might have on the development of students' domain-specific epistemologies. This exploratory investigation raises questions worthy of further study, namely how instructors' personal stances and pedagogical practices impact students' epistemological development. Systematic studies will be needed to isolate these factors, to the extent that they can be separated from other contextual factors. Other future work in this area could include formal validation interviews with the bifurcated CLASS items, including a larger sample of expert responses to determine to what extent experts report splits and if there is an expert consensus on any of the paired items. Lastly, we can use this current work to consider how to construct instruments that recognize the variation among epistemology, and to build educational environments that promote productive epistemologies for students.

**Social dynamics.** We observed that in a group of modern physics students engaged in collaborative problem solving, one student who continuously steered the group's sense making through her questions and prompts to connect to intuition was simultaneously positioned in the group as less knowledgeable. We conducted a case study analysis of this group and identified *epistemic stances toward group work*—the inferred stances about how knowledge should be generated in the group—as one factor that contributes to the social positioning of the students. The analysis describes the ways in which epistemology, sense making, and social dynamics are intertwined and how they function together in a group that is cohesive and productive along some dimensions yet inequitable along others. The construct of epistemic stances toward group work is a novel contribution to the PER field, and the attention to the fine-grained social dynamics and the intertwining of multiple elements of students' collective reasoning represents a new approach to the study of group work. There are many avenues of future work in considering the role that social dynamics play in students' collective reasoning, including further exploration of the nature of epistemic stances toward group work in order to identify the variation among students and the prevalence of such stances. Additionally, we will continue to explore the ways in which these stances can interact with other factors such as epistemology of physics, sense making, curricular or pedagogical framing, or cultural norms and practices to influence the collaboration among students. Ultimately, such understanding could be used to develop more productive and inclusive educational practices.

We investigate and value the messy nature of learning by: identifying and describing the dynamics of students' ontological reasoning, attending to domain-specificity of epistemologies and identifying the sophisticated elements of students' epistemic stances, and investigating the interactional dynamics of students' collaborative work. Through focusing on the kinds of reasoning that students are capable of, we value their creativity, identity, and engagement in our educational environments, in service of supporting and cultivating their learning of physics.

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# Appendices

## A Chapter IV: Coding scheme for student responses and instructional prompts

CODEBOOK: The following three tables present the complete codebook used to code students' responses to written response and multiple-choice questions about photons and electrons, in the context of three introductory quantum physics topics (double slit experiment, Mach-Zehnder interferometer, and quantum tunneling). The codes for ontological structure come directly from the framework for identifying and describing the dynamics of ontologies, presented in the paper. The specific codes for entity and ontology depend on the specific context and topic area—this codebook (along with the corresponding results presented in the paper) is one example of a concrete application of the framework to a specific context.

**ENTITY—What is being categorized?**

The entity (thing that is being categorized or reasoned about) is usually determined by the question. In instances when the student's language does not match that of the question, both the student's language and wording of the question should be used holistically to determine what the student is reasoning about and how. For multiple choice questions, the entity is only determined by the language of the question.

| **Entity** | **Evidenced by** | **Notes** |
|---|---|---|
| Photon(s) | "Photon" or "photons" | Distinguish between single and plural; Please make a note if the question and answer or different parts of the answer use different pluralities. |
| Electron(s) | "Electron" or "electrons" | Distinguish between single and plural; Please make a note if the question and answer or different parts of the answer use different pluralities. |
| Other | Anything that does not fit into the above two codes | If coding "other", label the specific entity in the notes. |

**ONTOLOGY—How is the entity being categorized?**

The ontology is inferred from the student's language–properties/characteristics being assigned to the entity suggest a certain ontological categorization. For multiple choice questions, a given answer is marked as consistent with a given ontology.

| **Ontology** | **Evidenced by** | **Notes** |
|---|---|---|
| Particle | Classical particle characteristics: localized entity (single location), bouncing, mass, etc. | This code is specifically a *classical* particle. |
| Wave | Classical wave characteristics: interference, reflection/transmission, non-localized, etc. | This code is specifically a *classical* wave. |
| Both particle / wave | Both particle and wave language, "particle that interferes w/itself", "wave when travels, then detected in 1 spot", could be either parallel or blended. | Could be blended or parallel. |
| Something else | More nuanced than the above options: e.g., a blend of classical particle characteristics and quantum properties ("quantum particle"), a particle but not localized and described by probability | Try to assign brief label and describe in notes. Usually blends, but sometimes unitary. If the ontology is "something else", and the structure seems like "can't tell", code as "something else/unitary" |
| Not particle | Defined only by *not* having particle characteristics (with no mention of what it is) | A blend of "not particle" and "wave" would be coded as "something else/blended" (likewise for a parallel structure). |
| Not wave | Defined only by *not* having wave characteristics (with no mention of what it is) | Switching between "not wave" and "particle" would be coded as "something else/parallel" (likewise for a blended structure). |

| **ONTOLOGICAL STRUCTURE—What structure is used in the reasoning episode?** | | |
|---|---|---|
| There are two factors to consider when determining ontological structure: *construction* (when are the ontologies applied) and *application* (which ontologies are applied). Coding for ontological structure will often happen simultaneously with coding for ontology—the two mutually inform one another, and both are determined by the student's language (how are they treating a given entity?) | | |
| **Ontological structure** | **Evidenced by** | **Notes** |
| Unitary | Single ontology applied throughout response, e.g., only attends to particle characteristics | If a specific *ontology* is defined in a response, but the structure is unclear, code as unitary (this means there is no evidence to suggest parallel or blend, and the scale is too small for parallel blends) |
| Parallel | Switching back and forth between two or more stable/robust ontologies; Often separated by time, e.g., ”sometimes particle, sometimes wave”, ”travels as wave, then detected as particle” | An ontology is considered stable or robust in a given reasoning episode if it was a conception that the learner brought with them into the reasoning episode (in contrast to thinking it through and developing the categorization in the moment). *Classical wave* and *classical particle* are canonical ontologies that we expect modern physics students to be familiar with and draw on when reasoning about quantum entities. |
| Blended | Combines elements from input ontologies and new meaning emerges to create a new category; Often referring to the same entity at the same moment in time, e.g., hybrid of particle and wave | Attending to both particle and wave characteristics at the moment of detection would be a blended ontology, whereas attending to wave properties while the entity travels and particle properties when detected would be a parallel ontology. The key element of a blended ontology is that new meaning emerges—a blended particle/wave ontology for a photon considers the photon to share some characteristics with particles and with waves, but it is a different entity entirely. |

CODING OF PROMPTS: The following table lists each of the questions in our data set and how we coded them in terms of being leading to either a specific ontology or a specific ontological structure. The three topic areas are: double slit experiment (DS), Mach-Zehnder interferometer (MZ), and quantum tunneling (T). The question ID reflects what kind of assignment the question appeared on: homework (HW), post-survey (Post), midterm exam (Exam), or final exam (Final). Note: we only included questions from *post*-surveys in our data set, although we also asked some of them on *pre*-surveys. Also, there are two different post-surveys from which we drew questions for this study, labeled as "Post1" and "Post2". Charles Baily wrote and administered the second post-survey ("Post2").

| **PROMPTS** | | | | | | |
|---|---|---|---|---|---|---|
| **Topic** | **Question ID** | **Question** | **Context notes** | **Entity** | **Leading to ontology** | **Leading to ontological structure** |
| DS | HW5 Q6-7 | Which slit did this photon go through? Why? | Combined MC with written explanation. Screenshot of the Quantum Wave PhET sim with one photon on the detection screen circled. Answer choices: A) left, B) right, C) both, D) neither, E) either left/right, we cannot know which one | Photon | — | — |
| DS | Exam2 Q13 | Which pattern would you expect to observe when electrons pass through the double slit? | MC image options: one (A) or two (B) bright spots, interference pattern (C), none of the above | Electrons | — | Unitary |
| DS | Post2 Q16 | Briefly discuss which aspects of this experiment are consistent with photons acting like classical particles. | Image and description of double slit experiment with a low intensity light source where photons are detected one at a time and a fringe pattern becomes visible after many photons. | Photons | Particle | Unitary |

| Topic | Question ID | Question | Context notes | Entity | Leading to ontology | Leading to ontological structure |
|---|---|---|---|---|---|---|
| DS | Post2 Q17 | Briefly discuss which aspects of this experiment are consistent with photons behaving like electromagnetic waves | Same as Q16 | Photons | Wave | Unitary |
| DS | Post2 Q18 | Briefly discuss which aspects of this experiment are consistent with photons behaving like a particle and a wave at the same moment in time. | Same as Q16 | Photons | Both particle / wave | Blended |
| DS | Post1 Q7 | Which pattern would you expect to observe when electrons pass through the double slit? | Same as Exam2 Q13. MC image options: one (A) or two (B) bright spots, interference pattern (C), none of the above | Electrons | — | Unitary |
| T | HW9 Q8 | Does an electron lose energy when it tunnels? Explain. | | Electron | — | — |

| Topic | Question ID | Question | Context notes | Entity | Leading to ontology | Leading to ontological structure |
|---|---|---|---|---|---|---|
| T | HW9LA Q5 | How does the amplitude of the electron's wave function compare in each of the three regions? [Hint] Explain what physical meaning we can make from the shape of the wave function in Region II | Tutorial assigned as the long answer part of the HW. The same system of two wires modeled as finite square wells as illustrated in Fig. 4 of the paper. Region II is the air gap between the wires, where the form of the wave function is an exponential decay (if $E_{tot} < V$) | Electron | — | — |
| T | HW9LA Q7 | Using the solution to #6, what conclusions can you make about the possible position of the particle? How is this different than a classical particle in the same situation? Can you offer an explanation of why classical objects (people) don't exhibit the same property, called tunneling? | The prior question (Q6) asked students to draw the wave function. | Electron | Something else | Parallel or blended |
| MZ | HW6 Q15 | Play with the simulation for a few minutes, getting to understand the controls and displays. Note down 5 things you have found out. | IOP simulation of single photon experiments | Photon | — | — |

| Topic | Question ID | Question | Context notes | Entity | Leading to ontology | Leading to ontological structure |
|---|---|---|---|---|---|---|
| MZ | HW6 Q16-17 | 16) Compare the experimental results for classical particles and for electromagnetic waves and list the ways in which they are similar or different. Explain any differences between them in terms of how each behaves when passing through the experiment. 17) How do the experimental results for single photons compare with the results for classical particles and for electromagnetic waves? How does the behavior of a single photon passing through the experiment compare with classical particles and with electromagnetic waves? | Single beam splitter. Use response to Q16 as context to help code the ontology of photon in Q17. | Photons | — | — |
| MZ | HW6 Q18 | In terms of this experimental setup alone, is a single photon more like a classical particle or more like an electromagnetic wave? Briefly explain your answer. | Single beam splitter. | Photon | Particle or wave | Unitary |

| Topic | Question ID | Question | Context notes | Entity | Leading to ontology | Leading to ontological structure |
|---|---|---|---|---|---|---|
| MZ | HW6 Q19-20 | 19) Compare the experimental results for classical particles and for electromagnetic waves, and list the ways in which they are similar or different. Explain any differences between the two in terms of how each behaves when encountering the second beam-splitter. 20 ) How do the experimental results for single photons compare with the results for classical particles and for electromagnetic waves? How does the behavior of a single photon at the second beam-splitter compare with classical particles and with electromagnetic waves? | Two beam splitters and no phase shifter. Use response to Q19 as context to help code the ontology of photon in Q20. | Photons | — | — |
| MZ | HW6 Q21 | As the phase shifter is varied, how is the behavior of single photons similar to electromagnetic waves? How is their behavior different from electromagnetic waves? | Two beam splitters with phase shifter. | Photons | Something else | Parallel or blended |

| Topic | Question ID | Question | Context notes | Entity | Leading to ontology | Leading to ontological structure |
|---|---|---|---|---|---|---|
| MZ | HW6 Q23 | Explain how the phase shifter is able to affect the behavior of a single photon at the second beam-splitter. | Two beam splitters with phase shifter. | Photon | — | — |
| MZ | HW6 Q24 | Summarize your findings from questions 16-23 by characterizing how photons behave in these experiments: In what sense do they behave like classical particles? In what sense do they behave like classical waves? Was there a case where the photons acted only like classical particles; or acted just like electromagnetic waves? Or must photons be something different to both electromagnetic waves and classical particles? | | Photons | — | — |

| Topic | Question ID | Question | Context notes | Entity | Leading to ontology | Leading to ontological structure |
|---|---|---|---|---|---|---|
| MZ | HW6 Q25 | In your own words, explain what the anti-correlation parameter (alpha) is, both in terms of its mathematical definition, and in terms of what it physically tells us, in the context of the single-photon experiments performed by Aspect. Why didn't Aspect measure alpha=0 if photons are supposed to be acting like particles? | | Photons | Particle | Unitary |

| Topic | Question ID | Question | Context notes | Entity | Leading to ontology | Leading to ontological structure |
|---|---|---|---|---|---|---|
| MZ | HW7 Q3 | As discussed in class and in the readings, what do the two single-photon experiments performed by Aspect (Exp 1 & 2 from lecture) tell us about the nature of photons? How were these two experiments designed to demonstrate the particle and the wave nature of photons? When answering, don't concern yourself with technical details (such as how the photons were produced); focus instead on how the design of each experimental setup determined which type of photon behavior would be observed. How are the elements of Exp 1 & 2 combined in a delayed-choice experiment (Exp 3 from lecture), and what do delayed-choice experiments (along with the two Aspect experiments) tell us about the nature of photons? | | Photons | Both particle/wave | Parallel or blended |

| Topic | Question ID | Question | Context notes | Entity | Leading to ontology | Leading to ontological structure |
|---|---|---|---|---|---|---|
| MZ | Post2 Q6-7 | 6) What happens when an electromagnetic wave encounters a beam splitter? 7) What happens when a single photon encounters a beam splitter? | Use the response to Q6 as context to help code the ontology of photon in Q7. These questions were prefaced with: *Depending on the experiment, we can describe light in terms of electromagnetic waves, or in terms of photons. In modern optics experiments, two devices that are typically used are beamsplitters and detectors. For the following questions, don't worry about the details of how each device actually functions. Just explain the basic idea of what's going on.* | Photon | — | — |
| MZ | Post2 Q8-9 | 8) Give a brief description of how a single photon would trigger a detector. 9) Give a brief description of how an electromagnetic wave would trigger a detector. | Use the response to Q9 as context to help code the ontology of photon in Q8. Same preface as for Q6-7. | Photon | — | — |

| Topic | Question ID | Question | Context notes | Entity | Leading to ontology | Leading to ontological structure |
|---|---|---|---|---|---|---|
| MZ | Post2 Q12 | In which experimental setup would you be able to observe interference effects? Describe in what sense photons are behaving like classical waves during this experiment. What features of the setup allow you to draw this conclusion without actually conducting the experiment? | Images of Exp 1 (1 beam splitter) and Exp 2 (2 beam splitters) | Photon | Wave | Unitary |
| MZ | Post2 Q13 | In which experimental setup would you not be able to observe interference effects? Describe in what sense photons are behaving like classical particles during this experiment. What features of the setup allow you to draw this conclusion without actually conducting the experiment? | Images of Exp 1 (1 beam splitter) and Exp 2 (2 beam splitters) | Photon | Particle | Unitary |

| Topic | Question ID | Question | Context notes | Entity | Leading to ontology | Leading to ontological structure |
|---|---|---|---|---|---|---|
| MZ | Final Q11 | In the following setup of a single photon experiment, what will be the impact of making the path from BS1 to MB longer? | Image of Mach-Zehnder interferometer with 2 beam splitters. MC answers: A) It will not impact anything about the experiment; B) No change in the observed counts at NA and NB, but it will take longer for the photon to reach NB because of the longer path; C) You will change the counts of NA and NB due to interference; D) The outcome is fundamentally uncertain | Photon | — | Unitary |

CLUSTERS OF PROMPTS: Where applicable, we clustered questions together and coded the cluster overall in order to make stronger inferences about a student's ontology. Questions were only clustered when they appeared next to each other on an assignment, and when the content of the questions fit together or built off of one another. The following table lists which questions were clustered together, and how the *cluster as a whole* was coded.

| **CLUSTERS OF PROMPTS** | | | | |
|---|---|---|---|---|
| **Topic** | **Questions in cluster** | **Entity** | **Leading to ontology** | **Leading to ontological structure** |
| DS | Post2 Q16-17 | Photon | Both particle/wave | Blended |
| T | HW9LA Q5, Q7 | Electron | Something else | Parallel or blended |
| MZ | HW6 Q16-18 | Photon | Particle or wave | Unitary |
| MZ | HW6 Q19-23 | Photon | — | Parallel or blended |
| MZ | Post2 Q12-13 | Photon | Both particle/wave | Parallel |

## B Chapter V: Tutorials for supporting ontological reasoning and associated instructor guides

This appendix includes the suite of ten modern physics tutorials we developed, in part, to support students' ontological reasoning. Each tutorial is followed by the corresponding instructor guide. All of the materials are available for download on PhysPort (https://www.physport.org/curricula/quantumentities/), and cover the following topics:

Tutorial 1: Waves, photons, and energy

Tutorial 2: Where was the photon?

Tutorial 3: Plane waves and wave packets

Tutorial 4: Particle in a box

Tutorial 5: Tunneling

Tutorial 6: Electron in a wire

Tutorial 7: Energy and position

Tutorial 8: Doppler cooling

Tutorial 9: Zeeman effect

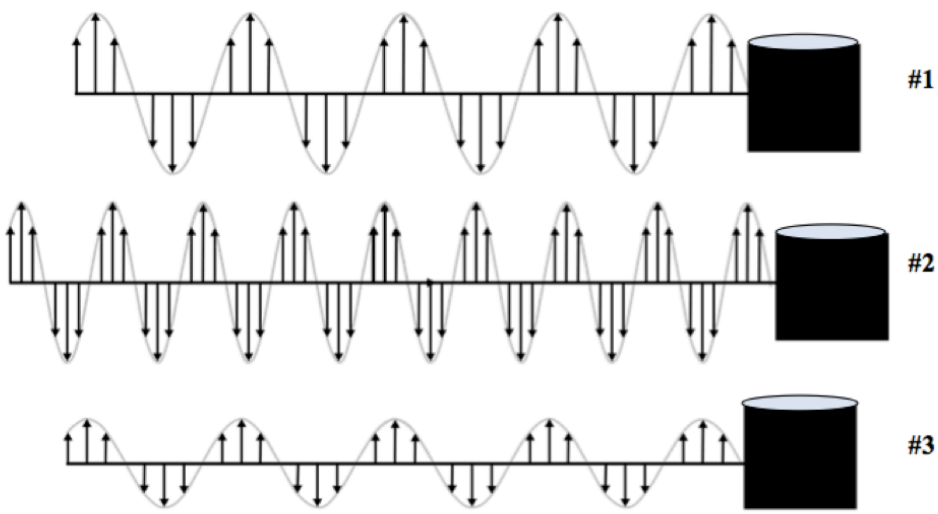
Tutorial 10: LEDs

***Tutorial 1:*** Name ____________________________ Section _____

# Waves, photons, and energy

In each of the three situations below, a light beam is shining on a black barrel full of water. The width of the beam is the same in all 3 cases. The barrels absorb the light energy, heating up in the process (much like the way you feel hot when standing in the sunlight).

Light is often modeled as electromagnetic waves as shown in the diagram below. In #1 and #2, the light waves have the same amplitude, while #3 has a smaller amplitude. The light waves in #1 and #3 have the same frequency, and #2 has a higher frequency.




1. Thinking of light as an electromagnetic wave, in which case(s) does the barrel heat up the fastest? Slowest? Explain your reasoning.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## *Waves, photons, and energy*

2. As you might have seen in your quantum or modern physics class, light can also be modeled as consisting of photons. How would you draw the same three situations using the photon model? (There are many good ways to do this, with no single right answer.)
   Try drawing this individually first, and then discuss with your group.

**#1**

**#2**

**#3**

3. Thinking of light as photons, in which case(s) does the barrel heat up the fastest? Slowest? Explain your reasoning.

4. Now let's focus on some of the specifics of trying to depict the three cases in terms of photons. In which of the three cases does each individual photon carry the *most energy*? The second most energy? The least energy? Why?

5. In which of the three cases are there the *most photons* per second reaching the barrel? The second most photons per second? The least? Why?


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6. In which of the three cases are the photons *fastest*? Slowest? Explain your reasoning.

✯ *Consult an instructor before you proceed.*

7. Are your answers to questions 4, 5, and 6 consistent with one another? Explain why or why not.

8. Do you still agree with the diagrams that you drew?

9. Let's revisit question 3. What does the photon model say about which barrel would heat up the fastest and the slowest?


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## *Waves, photons, and energy*

10. Did you get the same results using the EM wave and the photon models? If so, will the EM wave and photon models **always** give you the same results? If not, how do you reconcile them?

✯ *Consult an instructor.*


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### *Instructor Guide 1:*

# Waves, photons, and energy

**Instructional goals:**

- Comparison of the EM wave and photon models of light
- Guide students through productive discussions about the reconciliation of classical and quantum ideas
  - Why must the results from the two models be reconciled in this instance? Can they always be reconciled?
- Guide students in thinking about light energy in a particular physical situation

**EM wave model:**
We expect that students doing this tutorial will know that in this model of light, the power goes as the square of the amplitude. Thus, they should be able to conclude that barrels 1 and 2 heat up at the same rate, faster than barrel 3.

**Photon model:**
We expect that students doing this tutorial will have learned something about photons in class, and will know that $E=hf$. We expect that students will recognize that the higher frequency of #2 means higher energy. This may lead them to conclude (in question 3) that according to the photon model, barrel 2 heats up faster than 1 and 3 which heat up at the same rate. However, the subsequent questions will guide the students in reconciling the results of the two models.

When these questions were given to focus groups, some students were convinced that the photon model implied that barrel 2 would heat up faster than 1 and 3, and that this was the correct answer. There was a sense that because photons were the new, quantum mechanical thing that they were learning about, this had to be correct, and the classical theory no longer applied. These ideas led to debates and productive discussions among students about whether or not classical and quantum theories had to agree. The questions of this tutorial are designed to guide students through this process of reconciliation. Note: The language of "classical" and "quantum" is not used in this tutorial, to avoid this perception that quantum is new and therefore correct. Instead of classical vs. quantum, this tutorial presents more of a wave-particle dichotomy. Students should reason that the empirical result of the EM wave model (barrels 1 and 2 heat up at the same rate, faster than 3) is correct, and that the photon model should give a consistent answer. There is more energy per photon hitting barrel 2, but barrel 1 has more photons hitting it per second, such that the total power hitting barrels 1 and 2 is equal (and more than that hitting barrel 3). This can be reasoned conceptually or mathematically. (The idea that, for a given amplitude, higher-frequency light means **fewer** photons per second, can be confusing to students because of how they are accustomed to thinking about frequency.)

At the checkpoint after question 6, you can check on the raw ideas that the students are bringing in to answer each of the questions, and point them toward the right direction if they're ignoring any of the major pieces that they'll need for reconciliation, but it's ok if they haven't fully reconciled the models at this point – they'll get a chance to do that afterwards.


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**_Tutorial 2:_** Name ____________________________ Section _____

# Where was the photon?

For this tutorial, you'll need a computer with the PhET **"Quantum Wave Interference"** simulation, available at http://phet.colorado.edu/en/simulation/quantum-wave-interference

1. Run the simulation, and set up a double-slit experiment with a single photon. What happens?

2. What happens when you run the experiment many times?

A single photon is shot towards the slits and detected at the point shown on the screen.

3. What can you say about where the photon was just before it was detected?


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4. Here are some answers that other people have given to that question. Which (if any) do you agree with? Which do you disagree with? Explain why.
    a. It was located just in front of where it was detected.
    b. It was spread out evenly in the space in front of the screen.
    c. It was spread out in a non-even pattern in the space in front of the screen.
    d. It was spread out evenly through all space.
    e. Other: ____________________

✯ *Consult an instructor before you proceed.*

5. Does this photon have a definite speed before it is detected?

6. Can you say anything about the energy of the photon, either before or after it is detected?


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7. Can you say anything about the intensity of the light in this situation?

8. Think back to the photoelectric effect, and what happened there when you changed the intensity of the light. How does the effect of intensity compare here?

9. Can you compare the role of intensity here to the heating-the-barrels problem in Tutorial 1?

✯ *Consult an instructor.*


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## *Instructor Guide 2:*
# Where was the photon?

This tutorial involves thinking about the physical interpretation of the double-slit experiment with single photons. A number of the prompts are interpretation questions, and thus there is not necessarily one correct answer. This tutorial requires that each group have a computer, to run the PhET simulation.

**Instructional goals:**

- Engage students in a productive discussion about the physical interpretation of a quantum phenomenon
- Allow the students to explore the PhET simulation and use it as a tool for thinking about a quantum phenomenon
- Guide students in thinking about the particle and wave characteristics of a single photon
- Guide students in thinking about the position-momentum uncertainty principle

**PhET simulation:** The sim has two tabs: the first is the “High Intensity” tab in which a beam of light (or electrons, neutrons, or atoms) is shined on the double slits; the second is the “Single Particles” tab in which you can fire a single photon (or electron, neutron, or atom) and watch it go through the slits and land on one spot on the screen. The students should largely be using the second tab for this tutorial. However, comparison of the first and second tabs can be helpful for students comparing the particle and wave aspects. For example, #2 asks the students to fire many photons and report on the result. After firing about 50 photons, the students should see an interference pattern emerge on the screen. They can compare this with the pattern that results from the beam of light in the first tab.

**Uncertainty principle:** Some of the students who did these questions in focus groups had not yet covered the uncertainty principle in class. During the discussions, they talked about when they could know the position of the photon, and when they could know something about the energy. We see these as productive discussions leading up to instruction about the uncertainty principle. This prompt was revisited at the end of the semester, at which point the students were able to connect the uncertainty principle (and the equation) to this physical situation. If students working through this tutorial have already learned about the uncertainty principle, we expect that they will make this connection, and that the double-slit experiment with a single photon will give them a chance to apply the mathematical relationship they have learned.

**Multiple choice question (#4):** As noted above, this is an interpretation question and thus there could be multiple correct answers. We identify “c” as the correct answer—before detection, we do not know the position of the photon, but we do know that some places on the screen have a higher probability for the photon to land there. If the students have learned about or have some notion of a wavefunction at the time they work through this tutorial, we expect that they will use this information to help them answer this question. However, knowledge of a wavefunction is not necessary to complete this tutorial.


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**Comparing the role of intensity (#7-9):** The goal of these comparisons is to get students to realize that we have to think carefully about models in relation to intensity. In the photoelectric effect, intensity matters but not in the way one might expect. Intensity corresponds to number of photons, but below the threshold frequency there will be no current regardless of the intensity. In the heating of the barrels question, intensity is proportional to the square of the amplitude of the electric field. In reconciling the EM wave and photon models in Tutorial 1, students should have come to the conclusion that intensity $\sim E^2 \sim Nhf$, where $N$ is the number of photons per second and $f$ is the frequency. For the double-slit experiment, intensity will mean more photons and brighter fringes, more defined fringes, or faster appearance of the interference pattern.


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***Tutorial 3:*** Name ______________________ Section _____

# Plane waves and wave packets

1. What is the difference between a plane wave and a wave packet?

2. *Try this individually first, and then compare with the other members of your group:*
   Draw a plane wave and a wave packet.

3. For which one (plane wave or wave packet) is the position ($x$) most well-defined?
   Explain your reasoning.

4. For which one (plane wave or wave packet) is the momentum ($p$) most well-defined?
   Explain your reasoning.


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5. Are your answers to #3 and #4 consistent with the uncertainty principle?

✯ *Consult an instructor before you proceed.*

6. What is(are) the difference(s) between a blue photon and a red photon?

7. How would you describe these differences in terms of (a) plane waves and (b) wave packets? Which description do you like better?

8. Imagine you have two photon generators, one for blue photons and one for red photons, and they are equidistant from a screen. At time $t$=0, a single blue photon and a single red photon are shot out by the generators. Which one will hit the screen first?


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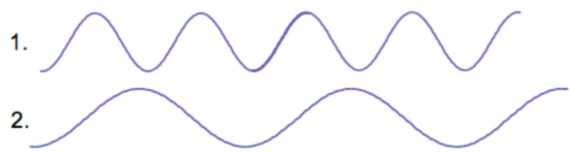
9. The wave-like graphs labeled #1 and #2 describe the wavefunctions for two **electrons**. (Note: We are talking about electrons here, not photons.) Which of the two free electrons have more kinetic energy? #1, #2, or do they both have the same energy?




10. In thinking about these questions, what kinds of assumptions were you making about what a photon is (or isn't), or what an electron is (or isn't)?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### *Instructor Guide 3:*

# Plane waves and wave packets

**Instructional Goals:**

- Guide students in thinking about representations of plane waves and wave packets
- Guide students in thinking about the uncertainty principle in the context of plane waves and wave packets
- Have students engage in productive discussions about photons (and electrons)

We expect that students doing this tutorial are familiar with the terms *plane wave* and *wave packet*. They should be able to draw these. The tutorial questions are designed to help students think more deeply about the physical meaning of plane waves and wave packets.

**Questions 6-7:** We expect students to identify frequency as the difference between a red and blue photon (blue has a higher frequency than red). When we gave this question to focus groups, students talked about representing a red and a blue photon with two plane waves of different frequencies. Then, the discussion went in circles as they tried to describe it in terms of wave packets. We believe that discussions like these in which students are struggling to try and articulate a description in terms of a wave packet can be productive. However, we don't want the students to get too frustrated here. If a facilitator sees that the discussion is becoming unproductive, we recommend they have the students move on to the next question.

**Question 8:** Possible answers the students might give for this question are as follows:

- They get there at the same time because they both travel at the speed of light.
- Blue has a higher frequency and thus higher energy, so it gets there faster.
- (We've seen students thinking about photons as moving in sinusoidal paths, which may be influenced by a common way of drawing photons. Based on this, we've seen students say:) Blue has a higher frequency, so blue photons are oscillating (in the transverse direction) faster; since both blue and red photons have the same overall speed (the speed of light), this means the red photons will get to the other side faster.

**Question 9:** Note here that we have switched from photons to electrons. This is important because in the case of the free electrons, the higher the frequency, the more kinetic energy, and thus the faster the electron. [We expect that students are familiar with the deBroglie wavelength at this point, $\lambda = h/p$] It might be useful to have the students compare this result to that of the photons in question 8 (in this case, higher frequency does not mean faster).


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**_Tutorial 4:_** Name ____________________________ Section _____

# Particle in a box

## A. The quantum particle in a box

Consider the "particle in a box": a quantum system where one particle (for example, let's say an electron) is confined between positions 0 and $L$. This can be in 1, 2, or 3 dimensions, but for now, let's focus on the 1-dimensional system.

1. Let's say the system is in the ground state ($n = 1$). Sketch a graph of the wavefunction.

2. How would you explain the physical meaning of the wavefunction?

3. Why isn't the ground state $n = 0$? That is, why isn't it possible for the particle to have zero energy?


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4. If you were to measure the position of the particle at some point in time, what position(s) would you expect to measure? Why? If you repeated this experiment, would you get the same measurement every time?

5. If you were to measure the energy of the particle at some point in time, what would you expect to measure? Why? If you repeated this experiment, would you get the same measurement every time?

6. If you were to measure the speed of the particle at some point in time, what would you expect to measure? Why? If you repeated this experiment, would you get the same measurement every time?

7. Is your answer to #6 (speed) consistent with your answers to #4 (position) and #5 (energy)? If not, how do you reconcile them?


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8. Ok, the world isn't really 1-dimensional, and infinite potentials don't really exist. The particle in a box (PIAB) is a simplified model. But are there real-world physical systems for which the PIAB is a useful approximation (even if it's not exact)? What are they?

✯ *Consult an instructor before you proceed.*

## B. Standing wave on a string

Now let's put aside the PIAB for now, and think about some systems in classical physics that might have some similar properties. First, let's think about a standing wave on a string. (Just a regular old string, not anything from quantum mechanics.)

9. Does it make sense to talk about the energy of this system? What physical properties does it correspond to?

10. Can we define a "speed" for the wave? If so, what would that mean? If not, why not?

11. What is the relationship between the energy (in #9) and the speed (in #10)?

✯ *Consult an instructor before you proceed.*


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## C. Classical particle in a box

Now let's consider a classical particle, moving in 1 dimension inside an actual box. (Again, this isn't the same as the quantum "particle in a box.")

12. What would the "ground state" look like for the classical particle? (I.e., what is the least possible energy that the particle could have, and what would the particle be doing in that case?)

13. Suppose the particle has some energy, and is bouncing back and forth. If you were to measure the position of the particle at some point in time, what position(s) would you expect to measure? Why? If you repeated this experiment, would you get the same measurement every time?

14. Graph the probability of measuring the particle at each position, as a function of position. How does this compare to the graph you drew in #1?


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15. If you were to measure the energy of the particle at some point in time, what would you expect to measure? Why? If you repeated this experiment, would you get the same measurement every time?

16. If you were to measure the speed of the particle at some point in time, what would you expect to measure? Why? If you repeated this experiment, would you get the same measurement every time?

17. Is your answer to #16 (speed) consistent with your answers to #13 (position) and #15 (energy)? If not, how do you reconcile them?

## D. Quantum particle in a box, revisited

Returning to the quantum "particle in a box"...

18. Is it more like a classical particle, a classical wave, both, or neither? Why?

19. In what ways is it more like a classical particle? In what ways is it more like a classical wave? Both? Neither?


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## *Instructor Guide 4:*

# Particle in a box

The purpose of this tutorial is for students to think through different models for the nature of a quantum particle, and think about when it is useful to think about a quantum particle as (classically) particle-like, wave-like, or neither. Here we have students think about this not only in the typical examples used to teach wave-particle duality (e.g. the double-slit experiment, as in Tutorial 2), but in a scenario that is usually taught quantitatively, with less attention to the nature of quantum entities.

Familiarity with the particle in a box (PIAB) is a prerequisite, as is basic understanding of wavefunctions (of which the PIAB is one of the first examples in many typical quantum courses). However, this tutorial need not be used immediately after the PIAB is taught in class. We have seen in the pilot testing that it can still have utility when used later on; even if the class has moved on to more complicated quantum systems, students will still find a number of the tutorial questions nontrivial.

**Instructional Goals:**

- Get students thinking about the relationship between quantum particles and analogous classical models (particles and waves)
- Have students generate similarities and differences between quantum particles, classical particles, and classical waves

**Part A. Quantum particle in a box**

1) This is a warmup question; we expect that this will be mostly recall and will not lead to much discussion.

2) This question will also draw mostly on compiled knowledge, and is not likely to spur extensive discussion.

3) This will be a novel question for many students, and we have seen students go in a number of directions with this. One type of answer is that $n = 0$ actually is a valid solution to the Schrödinger equation that satisfies the boundary conditions, but it's not normalizable (because $\psi(x) = 0$ everywhere) so it's not physically possible. Another type of answer is that if the particle has zero energy, then it has zero momentum and thus no uncertainty in the momentum, but it also has finite uncertainty in its position (because the particle is confined to the box), which violates the uncertainty principle (unlike a plane wave solution for a free particle, where the momentum is known definitely but nothing at all is known about the position, so the uncertainty principle is not violated). We're not concerned about whether students get to one of these specific answers or lines of reasoning; what's more important is that this question should get them thinking about what physical principles apply to a quantum particle. (Students who have had relativity, especially within the same modern physics course, might also bring up rest energy.)


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4) This is a relatively straightforward question; students should find that the result is probabilistic, and related to the distribution they drew in #1. The result will not be the same every time.

5) This one should also be straightforward: there is a fixed energy for the ground state, and this should be the result every time. However, we have seen students convince themselves that the energy is dependent on position, and therefore also depends on the probability distribution, so this question may not be simple for everyone.

6) After students have been primed with #4 and #5, this question may throw them for a loop, since they're not used to talking about "speed" in a quantum context (they're used to talking about momentum and energy instead). Even if the students decide to translate "speed" back into "momentum", it's still a challenging question even for experts. A classical particle in the same potential would maintain a constant speed (since $V$=0 everywhere inside the box)... or would it? As one student asked, "Does it slow down at the edges?" (If the walls are truly infinitely steep, then a classical particle hitting the wall would experience infinite force and infinite acceleration, suggesting that the infinite well model can break down if we look too closely.) If the potential is zero inside the box (so that all the energy is kinetic energy), it would seem that if the energy of the ground state is constant, then so is the momentum, and the wavefunction in momentum space looks like two delta functions at $\pm\sqrt{(2mE)}$... or does it? If you Fourier-transform that back to position space, you get a standing wave that extends to all space, not one that is confined to the box. Again, the infinite well makes things complicated. We don't expect students to think through all of these issues (the purpose of discussing them here is just to make clear that this is not a trivial question), but we hope this question will lead to productive discussions about what can and can't be assumed.

7) Students' responses will differ depending on how they answered 4, 5, and 6, but if they find that the answers are not consistent, we hope they will go back and reconsider.

8) This is open-ended. There are multiple possibilities of real-world systems where a particle is confined in some way.

**Part B. Standing wave on a string**

The next section shifts gears into classical wave mechanics, and this may be the part of the tutorial where students' background knowledge is the shakiest, since for many students, it's neither something they have been learning recently (like quantum mechanics) nor something they've reinforced in multiple contexts (like classical particle mechanics).

9) Students should connect the energy to how fast the string is moving up and down, and therefore to the frequency and amplitude of the wave.

10) There's not necessarily a unique "speed" that can be defined for this wave, and therefore we've seen students respond to "Can we define" with "No, we can't". As long as they've


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thought through all the possibilities, we think this is an acceptable conclusion, but it's also acceptable if they said "yes" and picked one measure of speed. One way to think about it is the wave speed associated with the string (which equals $\lambda f$), though nothing is actually traveling at this speed (since it's a standing wave). Another way to think about it is the speed that the string is moving up and down (though this isn't really a wave speed).

11) If they answered #10 with the wave speed, then there is no relationship between the speed and the energy (a standing wave with zero amplitude has no energy, but the wave speed is unchanged). If they answered it with the transverse speed of the string, then the energy is directly proportional to this speed (squared).

Going back to the PIAB here (which the students aren't explicitly asked to do), there are some similarities and some differences. The eigenstates of the PIAB look like standing wave solutions, and the energy of the PIAB is related to the frequency of the standing wave. (The amplitude is the same for all the states, because of normalization.) However, there is no relationship between the "speed" of the PIAB (by any of the definitions that students may have come up with in #6) and the wave speed of a standing wave on a string.

**Part C. Classical particle in a box**

Finally, the students are asked about a classical particle, which will probably be the easiest section for them.

12) The particle can be at rest, with no energy (in contrast to the quantum particle in #3).

13) Any position in the box is possible, with equal probability, and the measurements will not be the same each time.

14) Unlike the graph in #1, the probability distribution will be flat inside the box. But similar to the graph in #1, the distribution will be zero outside the box.

15) As in #5, the energy should be constant, and equal to whatever it started as. The students might (as in #6) get into questions about whether the particle slows down at the edges or bounces back instantaneously.

16) "Speed" for a classical particle probably won't be a confusing concept like "speed" for a quantum particle. The students will probably find that the speed is constant, though again they may get into questions about the properties of the box.

17) As in #7, they should find that their answers are consistent, or else go back and reconsider.

**Part D. Quantum particle in a box, revisited**

18-19) These questions are the "punch line" of the tutorial. This is where we hope students will synthesize the results from the three sections of the tutorial. We expect that they will find that


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there are some ways that the PIAB is like a classical particle, a classical wave, both, and neither. (The question in #18 but which it is “more like” is subjective, and there isn’t one right answer.) This is an opportunity for discussion about which models are useful under what circumstances.


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***Tutorial 5:*** Name ____________________________ Section _____

# Tunneling

In this tutorial, we'll be exploring quantum tunneling. We'll first examine the behavior of classical particles, electromagnetic waves, and quantum particles at potential boundaries. Then we'll compare and contrast the quantum particle's behavior to that of the classical particle and electromagnetic wave.

## A. Classical ball

1. Imagine a ball with energy $E$ rolls towards a hill with a height such that the ball's potential energy would be $V_0$ at the top of the hill. Describe the motion of the ball, assuming $E < V_0$ and no friction.

2. If you measure the position of the ball at random times, what positions might you measure, and with what (approximate) probabilities?

3. If you measure the total energy of the ball at various times, what might you measure?


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4. Now, instead of rolling a single ball with energy $E$, we roll many balls with a spread of energies around $E$. Is it possible to find a ball to the right of the hill?

✯ *Consult an instructor before you proceed.*

## B. Electromagnetic wave

5. Now consider a short pulse of electromagnetic wave incident on the left side of a partially reflecting piece of glass, sitting at the origin. The glass has a reflectivity of 25% and is lossless. Suppose we fire the pulse and after it hits the glass, we take some measurements.

    a. If you measure the strength of the field at various positions along the x-axis, to the left and right of the origin, what might you find?

    b. If you took these two measurements simultaneously, could you measure the field in both places at the same time?


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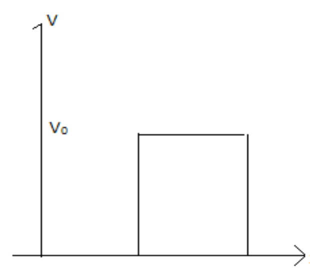
c. Measuring the amplitude is really the same as measuring the energy of the pulse, because the energy depends on the amplitude squared and a few constants. Is it possible to measure the energy of the pulse as having split (i.e. you find energy in more than one place at once)?

✯ *Consult an instructor before you proceed.*

## C. Quantum particles

6. Imagine now that we have a beam of quantum particles coming in from the left that encounter a potential boundary, as shown. All particles have the same energy, E, where $E<V_0$.




   a. What are the forms of the different pieces of the wavefunction in each of these three regions?

   b. Sketch what you think the wavefunction of a particle looks like.

   c. Where might we be able to find particles? How would you find or describe the probability of finding particles in each of these regions?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   d. Are we likely to find any particles to the right of the barrier, i.e. particles that have tunneled?

7. If a particle is found to the right of the barrier, how does its energy compare to its original energy, *E*?

8. Suppose we change the particle beam to now have a *spread* of energies around E.
   a. How might this change where we can find particles?

   b. Will tunneling occur in both situations, with all particles having energy E and with particles having a spread in energies around E?


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i. If no, why not?

ii. If it will, does the amount of tunneling change between these two situations?

9. We've now considered three different types of objects encountering a potential barrier, a classical particle, an electromagnetic wave and a quantum particle. (Hint: For the questions below, think about the quantum particle before *and* after we measure it)

   a. In what ways is the behavior of the quantum particle similar to the classical particle? In what ways is it different?

   b. How is the quantum particle similar to *and* different from the electromagnetic wave?

   c. Imagine you have a friend who's taken some intro physics and E&M but has not yet learned much about quantum mechanics. How might you describe what happens when a quantum particle tunnels?

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## *Instructor Guide 5:*

# Tunneling

**Instructional goals**

- Distinguish the behavior between classical particles, classical waves and quantum particles at potential boundaries
  - o Guide students in seeing that the behavior of the quantum particle is not well described by either particle or wave alone, but a mix of the two.
  - o Have students consider an ontological blend of a classical particle and EM wave as a way to model tunneling of a quantum particle.
- Guide students through two common conceptual difficulties around tunneling: the incorrect ideas that
  - o Tunneling itself is a result of a spread in energies of an ensemble of particles.
  - o The particle loses energy after it tunnels.

**Part A. Classical particle**
1-3) We anticipate that these questions will be relatively straightforward for the students. The key points here are that energy is conserved and the ball is never found to the right of the hill.

4) We have intentionally left the "spread in energy around *E*" as unspecified. The goal here is to get students thinking about cases where the spread would allow a ball to roll over the barrier, i.e. $E > V_0$. This question will likely lead to clarification prompts from students, which might be best managed through asking the students to consider different cases, a case where Emax<V and a case where Emax>V.

**Part B. Electromagnetic wave**

5) The idea of reflectivity being 25% may be unclear to students. We want to eventually have them discover that the pulse will be split: some is transmitted and some is reflected (we're assuming no losses). If a student asks what we mean, go ahead and tell them it means that 25% is reflected, but have them come up with a few possibilities about where the rest of the pulse goes. If they come up with an idea about losses, tell them that the mirror is lossless and ask them where the rest of the pulse may go.

These questions should provide a contrasting case to the classical particle. We want them to see that the wave actually splits into two parts, and that we can find energy from the pulse on either side of the boundary.

**Part C. Quantum particle**

6) This question allows students to piece together the wavefunction of the particle and consider if the particle is tunneling.


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## *Tunneling*

Students should remember that the probability of finding the particle in a region is proportional to the magnitude squared of the wavefunction.
They may have difficulty remembering that the wavefunction does not go to zero in the boundary, but exponentially decays, making it possible to find the particle in the boundary.

7) This question gets at a common difficulty that students have, the idea that the particle's energy is somehow used up from tunneling.

8) This is again a question where the spread is intentionally not well-specified and the students' answers can be qualitative.

We want them to realize that more/less energy makes it more/less likely that a particle will tunnel. We don't expect the spread in energies to even out the probability of tunneling to match the previous case, where all particles have the same energy.

Students may be tempted to grasp this idea of "evening out." If they come to this conclusion, you may prompt them to think further about what exactly is the probability that the particle tunnels. They won't be able to find this exactly but you can remind them that the amplitude in the barrier is proportional to $(V\text{–}E)^{-1/2}$, then ask how the amplitude the wavefunction to the right of the barrier depends on the height of the barrier.

9) Considering the similarities and differences. One thing to get students to consider is the differences between the quantum particle before and after measurement.

a) Students might find that the quantum particle is similar in that we measure it to be in one place, same as the classical particle.
b) It's different in that the quantum particle can tunnel and be found to the right of the boundary!
c) It's similar before measurement, when the quantum particle is described by a wave that is present everywhere. It's also similar in that it can go through the boundary. It's different in that we will only measure the energy of the particle to be in one place, whereas the energy of the electromagnetic wave will actually split.


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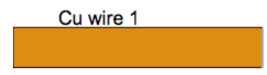
## *Tutorial 6:*

Name ____________________________ Section _____

# Electron in a wire

### Part I

An electron is in a copper wire, as shown to the right.




1. Draw the potential energy felt by the electron due to the wire, as a function of the position along the wire.

2. Of the quantum systems you've learned about so far, would any provide a good model for the potential felt by the electron in the copper wire? Why or why not?

3. Draw the ground state wavefunction of the electron.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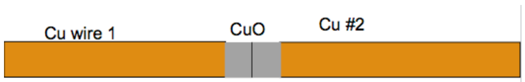


4. Now a second copper wire is brought close to the first, with a small insulating gap in between. Is there a quantum system that you've learned about that is a good model for this two-wire setup? Explain your thinking.

5. Draw what you think the wavefunction and potential energy of the electron look like, explaining key features of your diagrams.


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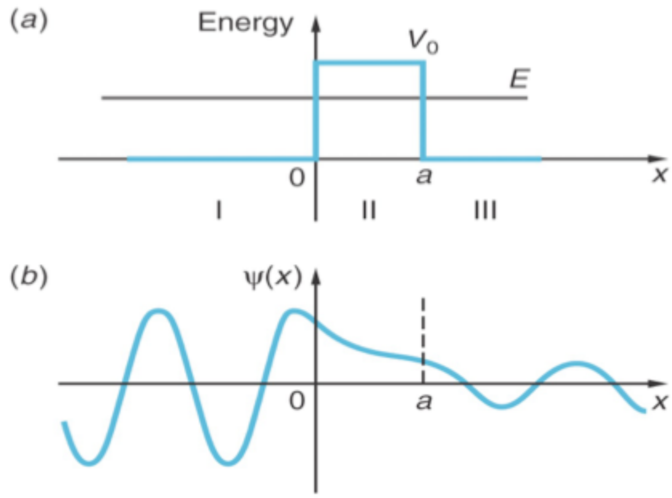
✯ *Consult an instructor, who will give you the next page of the tutorial.*

**Part II**




6. The potential energy and wavefunction of the electron look something like what is shown in the diagram above, now treating the wires as infinite. If you did not draw something similar, can you now explain why this is the correct picture?

7. Qualitatively describe what this wavefunction tells us about the electron. What is oscillating, and what is decaying?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☆ *Consult an instructor before you proceed.*

8. Where is the electron? Explain your reasoning.

9. Is the electron in the space between the wires? What is happening there?

10. Draw an energy diagram similar to what is shown in (a) on the previous page, this time showing how the total energy and kinetic energy vary with position along the wire.


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11. What can you say about the kinetic energy of the electron between the wires? Explain your reasoning.

12. The electron can possibly exhibit quantum tunneling in this two-wire system.
Explain what that means here.

13. Do you need the second wire for this phenomenon of tunneling to occur?
Is it possible in the single wire case?


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## *Instructor Guide 6:*
# Electron in a wire

**Instructional goals**

- Guide students in applying what they've learned about quantum tunneling from Tutorial 5 to a physical system, the copper wire
- Consider the conditions for quantum tunneling
- Consider how the potential, kinetic, and total energies vary in different potentials

The student is first asked to consider an electron in a copper wire. Then, an additional copper wire is brought close to the first wire. The electron we are considering will have a higher probability density in the left wire, as we know the electron started in the left wire. To ensure that students have the correct potential energy and associated wavefunction, we give them the potential energy and wavefunction of the two-wire system (at the beginning of Part II). **Therefore, we recommend giving students Part I of the tutorial (the first double-sided page) by itself, then giving them Part II (the other pages) after they finish Part I.**

**Part I**

We want to be careful to not give too much scaffolding on the first page, as we'll give students the answers to these questions and ask them to reason about why these answers are correct. These questions should be used to allow students to guess and explore their own thinking

1-2) A potential barrier provides a good model for the electron in the copper wire. This assumes that the wire extends to $\pm\infty$. If students are having trouble getting started, a nice foothold idea is to ask them where the electron "wants to be." If they think that the electron wants to leave the wire, explore this idea to show the students that it doesn't really make sense. They should reason that the electron wants to be in the wire. A nice follow-up to this prompt, if they still need guidance, would to be ask students what does "where the electron wants to be" indicate about the potential energy felt by the electron.

3) Potential wavefunctions should be a sine wave inside the wire, and show some "leaking" (exponential decay) into the area outside of the wire. Students will see the wavefunction on the next page, so it's okay for them to have some errors here.

4-5) This question again should go without much intervention by the instructor. The previous questions should provide enough scaffolding, and if the students don't get the answer, they will have a chance to reason backwards through these problems later from the answers.


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**Part II**

Once the students finish Part I, you may pass out Part II. Allow them to retain their completed Part I so that they may compare their answers.

6) One specific thing students may struggle with is why the wavefunction is larger to the left of the barrier than to the right of the barrier. Here, it is important to remind them of the context of the problem: we start with an electron in *one* wire and then a second wire is brought close to the first wire. So we know the electron starts in the left wire, making the probability density on the left larger.

- Make sure that students attend to all three regions of the potential, indicating that the wavefunction is oscillating outside of the potential barrier and decaying within it.

7) This question gets students to consider where the electron can be found and with what relative probabilities. We see that the electron can be found in any of the regions with the highest probability on the left.

- This question gets at a potential misconception that the electron itself is moving in an oscillating pattern. This may be a good point to check that students are on the right track and understand that it's the probability density that is oscillating and decaying.

8) There are a number of directions that the students can take with this question. They may consider where the electron exists before measurement, or places where we can possibly measure the particle to be.

- If the student considers where the electron is before measurement: This may be a good opportunity to prompt the student to think about whether the electron has a determinant position before measurement. Another broad prompt may be "what exactly *is* the wavefunction," as some students may consider the wavefunction to *be* the electron.
- We believe both directions could be beneficial for students to pursue, but it is up to what direction you encourage. It is, of course, possible to have students consider both.

9) Again, the language is ambiguous as in question 8. Allow the student to explore the language, then steer them in the direction you want them to go, if appropriate. Here, we want the students to see that the electrons can be found in the classically forbidden region. The probability density is decaying with distance, but is still nonzero.

10-11) The key point here is the students seeing the kinetic energy is negative!

12-13) Because the potential energy is finite, there's some leaking into this classically forbidden region, i.e. there's some possibility of finding the particle here.


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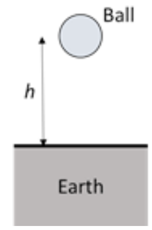
**Tutorial 7:** Name ___________________________ Section _____

# Energy and position

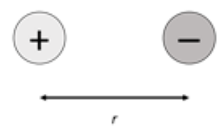
## A. Classical Physics

1. Let's consider the first example of potential energy you probably learned: gravitational potential energy near the Earth's surface. As a thrown ball goes up and down, what happens to the potential energy of the Earth-ball system?




2. In this example, do higher and lower potential energies match up with higher and lower positions?

3. Now consider a system of two opposite charges.
   a. Does a larger separation correspond to a higher or lower potential energy?




   b. If the negative charge were fixed in place and the positive charge were free to move, would the positive charge move to a higher or lower potential energy?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## B. Quantum Physics

In this section, we'll think about energy and position in quantum mechanics.

4. Consider the hydrogen atom. One student described the relationship between the total energy of the electron and position by saying "The electron is in a higher energy state when the electron is farther from the nucleus." Do you agree or disagree? Why?

5. The hydrogen atom is similar to the situation in #3. Are there similarities and/or differences in the claims that we can make about the relationship between energy (or potential energy) and position, in #3 and #4?

6. When the electron in a hydrogen atom "falls to a lower energy level", does this correspond to a physical falling motion? Why or why not?

✯ *Consult an instructor before you proceed.*


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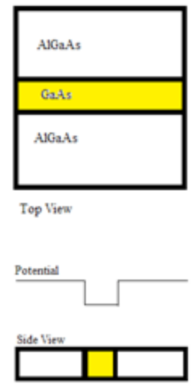
## *Energy and position*




For this next part, your group will need a computer. Run the **"Quantum Bound States"** PhET simulation, at http://phet.colorado.edu/en/simulation/bound-states

7. Quantum wells can be created by sandwiching thin layers of semiconducting materials. A common device uses a 10-nm layer of gallium arsenide (GaAs) between two layers of 100-nm-thick aluminum gallium arsenide (AlGaAs). An electron can be excited into the conduction band of the semiconductor by a laser. In the conduction band, the electron can be modeled by a particle in a finite square well.

   Open the (finite) **Square Well** tab in the PhET sim. Figure out all the ways in which you can increase the difference between energy levels.

8. For a given well width and height, look at the wavefunctions corresponding to different energy levels. For which energy levels does a greater percentage of the wavefunction "leak" out of the well?

9. For the square well potential, is there a relationship between the position of the particle and its energy? If so, describe the relationship and the type of position you are considering.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## *Energy and position*

10. Now open **Two Wells (Molecular Bonding)**. Two-well potentials can be created by adding additional semiconductor layers to the single-well device. Draw what you think the new device may look like, and the resulting potential.

11. As you did for the single square well, play with the physical dimensions of the potential and describe how the energy levels change. Are there *new* parameters that you didn't encounter with the single square well?


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## *Energy and position*

12) Explore **Many Wells (Band Structure)**. Describe the way the structure of the energy levels changes as you adjust the different parameters of the semiconductor.

13) For the two- and many-well potentials, do particles obey the same energy versus position relationship that you found in #9? Why or why not?

14) The Many Wells simulation shows that if we adjust characteristics of the semiconductor, we can get energy levels that separate into bands. (This simulation is a nice model of a crystal lattice.) If you wanted an electron in the lattice to be able to move easily between the wells, which energy band would you put it in? Explain your reasoning.

✯ *Consult an instructor.*


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*Instructor Guide 7:*

# Energy and position

This tutorial was originally developed to lead into Tutorial 10 (LEDs). However, it can stand on its own and can still be used even if you're not using the LED tutorial (which has more prerequisites, since it expects that students have some familiarity with semiconductors). If you **are** using the LED tutorial, it is recommended to do this tutorial first, since it introduces students to the Quantum Bound States simulation in general, and to using it to model an electron in a lattice in particular.

**Equipment:**

- A computer for each group, with the PhET "Quantum Bound States" simulation: https://phet.colorado.edu/en/simulation/bound-states

**Prerequisites:**

- General familiarity with wavefunctions, including the hydrogen atom
- No prior experience with multiple wells is necessary

**Instructional goals:**

- Help students explore the relationship between energy and position in classical and quantum physics, while distinguishing between position as a metaphor for energy (e.g. "energy levels") and actual physical position
- Give students the opportunity to distinguish between classical particles with a definite position, and quantum particles with a probability distribution for position
- Have students use simulations to explore quantum systems that cannot be easily solved analytically, such as a multiple-well system
- Model solid materials with quantum wells

**Part A (Classical Physics)**

1-2) Students should see that in this case, higher and lower positions match up with higher and lower energies. We use gravitational potential energy as an implicit analogy when we talk about potential "wells", or even when we talk about "higher" and "lower" energy.

3) Students may forget that the potential energy of this system is negative and approaches zero at infinity. So they may be tempted to say that the potential energy is highest when the position is "lowest" (i.e., when the distance between the charged particles is smallest), when the potential is actually lowest here. Part 3b should remind students that objects experience forces from high to low potential energy, thereby indicating they may need to reconsider part 3a if they answered that higher position is lower potential.


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## *Energy and position*

**Part B (Quantum Physics)**

4) This question gets at the probabilistic nature of quantum measurements. Students may draw on a "shell" model (in which a particular energy level represents a particular distance from the nucleus) and agree with the statement, without getting into the probability issues. Even if students address the probabilistic nature, they may be tempted to say things like "in general, but not necessarily." If they give this type of hesitant answer, it may be beneficial to prompt more thinking by the students, asking them to expand on their ideas. "So where might you find the electron?" "Will you always find it in the same place?"

5) This question should get students comparing the deterministic nature of classical mechanics to the probabilistic nature of quantum mechanics.

6) Students may hold on to classical notions quite strongly, and describe the electron as physically falling from one orbit to the other. If they invoke this classical picture, you might ask them whether the electron has a well-defined position before measurement and if we are actually measuring its position in this scenario.

7) This task is primarily present as a way to get students to explore the simulation independently, without giving them too many instructions. Students should find that they can increase the difference between energy levels by increasing the height of the well, decreasing the width of the well, or decreasing the mass of the particle.

8-9) Higher-energy particles have a higher degree of leaking. So, the higher the energy, the more likely it is you will find the particle further from the center of the well.

10-11) The new device would look similar to the first, with additional layers of AlGaAs and GaAs. With multiple wells, you can now change the distance between the wells. This might be a good opportunity to keep students grounded in the physical situation, by asking what changing this new parameter would mean physically. Adjusting the distance between the wells can be accomplished by changing the thickness of the center AlGaAs layer.

12) Students may now see that adjusting these parameters separates the energy levels into bands. The students may not be able to find the same types of relationships as above, because now adjusting parameters such as the well width will make some energy levels closer and others farther. If students seem insistent on repeating the same type of analysis that they did earlier with the single well, prompt them to think more qualitatively and ask them what changing these parameters does to the band structure.

13) The two- and many-well potentials should have the same general energy-position relationship: we see greater leaking with higher energy levels.

14) Students should see from the previous question that this band would be the higher energy band.


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***Tutorial 8:*** Name ____________________________ Section _____

# Doppler cooling

1. How does an ambulance sound speeding towards you, as compared to sitting still or moving away?

2. Suppose we have an atom that can move along the horizontal dimension, with laser light coming in from the right and from the left.

   a. If the atom moves to the right, how do the wavelengths of each laser change, as seen by the atom?

   b. What about if the atom moves to the left?


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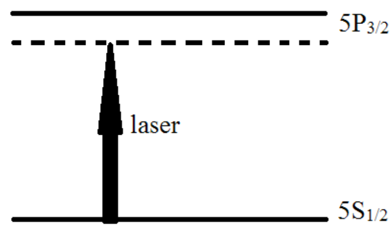
3. Suppose the atom in #2 has an energy level structure, as shown to the right. The arrow shows the energy of the photons produced by the lasers. This laser is "red-detuned" from the atomic transition because photons from the laser have a lower frequency and energy, hence a longer or more-red wavelength, than the atomic transition.




   Draw an energy level diagram of the atom when it is moving to the right, showing the photon energy of both lasers, as seen by the atom.

4. Does motion along the axis change the likelihood of excitation of the atom by photons from one of the lasers? If so, which laser?

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5. If the atom is moving to the right and absorbs a photon, how does the atom's momentum change, if at all? (*Hint:* A photon carries momentum.)


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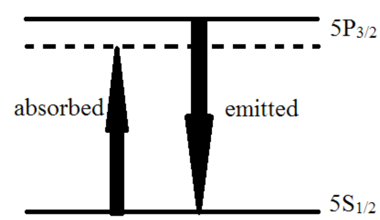
6. After absorbing the photon, the atom will then emit a photon, with an energy equal to the $5P_{3/2} \rightarrow 5S_{1/2}$ transition, in a random direction. Describe how the momentum of the atom changes after many, many cycles of absorption from the right laser and random emission.

7. The diagram at right shows the energy absorbed and then emitted by the atom in one cycle *as seen by a motionless observer*. So in each cycle, the atom emits more energy than it absorbs. How can you reconcile this with the conservation of energy?




8. Let's pull things together. Describe what happens to an atom that is moving in one dimension between two red-detuned laser beams that are shining in opposite directions (as shown in the diagram in question #2).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9. Now let's try to generalize to three dimensions. Instead of having an atom that is confined to move in one dimension, it is now allowed to move in all three. How could additional lasers be arranged and tuned so that the atom loses speed no matter which way it's moving?

10. If you had a large number of atoms in the system you described in #9, how does the temperature of the atoms change as they undergo many cycles of absorption and emission?

11. Atoms in the system described in #9 are called an optical molasses. Why do you think physicists chose that name?

✯ *Consult an instructor.*


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## *Instructor Guide 8:*

# Doppler cooling

This tutorial is designed to introduce students to the concept of laser cooling, specifically Doppler cooling. By the end of the tutorial, the students should recognize that counter-propagating, red-detuned laser beams may be used to create an optical molasses, or a cloud of cool atoms. This tutorial can be used in conjunction with Tutorial 9 (Zeeman Effect), where students see that a magnetic field can then be used to provide spatial confinement of the cooled atoms. Taken together, the laser and magnetic-field configurations described in Tutorials 8 and 9 form a Magneto-Optical Trap (MOT). Doppler cooling can be used to cool atoms for atomic clocks. In creating Bose-Einstein Condensates, scientists usually create a cloud of cooled atoms with a MOT.

In the tutorial, some of the big ideas students will focus on are the following:

- Doppler shift of sound and light
- Atomic energy levels and transitions between levels
- Laser-matter interactions

**Doppler Shift**

1) Students should recognize that the sound waves from the ambulance are shifted to a higher frequency when coming towards the observer, and a lower frequency when moving away.

2) This is the same question, but on the Doppler shift for light. The lasers are blue-shifted when the atom is moving towards them, and red-shifted when moving away.

**Doppler Shifted Energy levels**

3) There are many way to represent the energy levels, but many students choose to draw diagrams similar to the one shown. Students should see that one laser (right) gets shifted up in energy and the other laser gets shifted down.

4) Students should see that the laser that the atom is moving towards is shifted closer to the atomic resonance.

**Momentum Considerations**

5) The photon's momentum is absorbed by the atom. The atom gets a kick in the direction of the laser's propagation, essentially slowing the atom down *in the direction* antiparallel to the laser's propagation. The change in velocity that the atom receives is called the recoil velocity and can be calculated using the momentum of the photon and mass of the atom. The recoil velocity is related to the minimum temperature that is achievable through Doppler cooling.

6) Each cycle, the atom is slowed in its direction of motion through absorption and then receives a kick in a random direction through emission. After many cycles, there is a large slowing effect


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in the direction of motion, while the kicks that the atom gets through reemission should average out to no net change.

**Energy Considerations**

7) One important consideration is that energy and energy conservation are frame-dependent! This can be a difficult concept for students, as energy conservation is one of the tools used most frequently in physics. In the lab frame, the atom does absorb a photon of lower frequency than what it re-emits, leaving some small amount of energy unaccounted for. This energy comes from the atom itself; kinetic energy is being taken from the atom and given to the second photon.

8) The atom is slowed in its direction of motion, in both directions.

9) Three orthogonal pairs of counter-propagating, red-detuned beams.

10) The atoms are losing kinetic energy, so the temperature decreases.

11) Answers may vary. A molasses is something that moves slowly.


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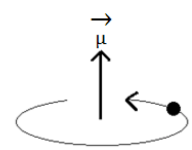
***Tutorial 9:*** Name ______________________ Section _____

# Zeeman effect

In this tutorial, we will investigate how a magnetic field and a pair of lasers might be used to confine atoms in a small region of space.

1. Moving charges experience a force in a magnetic field. The potential energy of the interaction is $U = -\boldsymbol{\mu}\cdot\mathbf{B}$, where $\boldsymbol{\mu}$ is the magnetic moment of the charge. It is proportional to the charge and points in the direction perpendicular to the plane of motion.




   a. If we have a charge that is forced to move in a circle, what happens to the charge if we turn on a magnetic field that points in, for example, the positive z-direction?

   b. Does your answer above depend on what direction the magnetic moment points in? Explain why or why not.

2. Atoms have angular momentum, which we can investigate by modeling the atom as an electron orbiting a nucleus. If we apply a magnetic field in the positive z-direction, what orientation of the atom has the lowest potential energy? (I.e., in what direction should the magnetic moment point?)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3. Thinking of the atom as a charge orbiting the nucleus is a classical model for a quantum system. How accurately do you think this model describes the physical situation?

4. Is there any value in using the classical model, even if we see that it breaks down when we consider that an accelerated charge would radiate away energy? Explain your reasoning.

5. For angular momentum of $l=1$, the magnetic quantum number $m$ can take on values of -1, 0 and 1. (The magnetic quantum number tells us the projection of the angular momentum vector in a given direction, say z.) Which of these states ($m$ = -1, 0, 1) gives maximum, zero and minimum potential energy, given a magnetic field that points in the positive z-direction?


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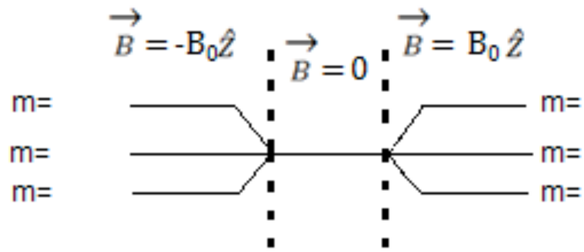
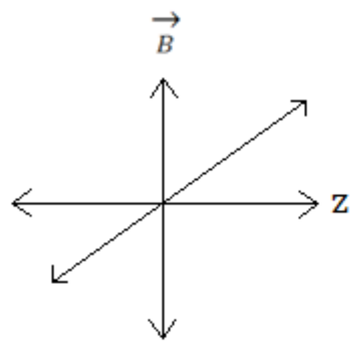
6. In the absence of a magnetic field, the three levels would have the same energy. Fill in the energy level diagram of the three magnetic levels splitting in the two cases shown. On the left, the magnetic field is in the -z direction and on the right it is in the +z direction. $B_0$ is a constant.




7. Suppose we have the linear magnetic field shown in the graph. Describe how the energy levels change as you move away from the origin. Is it the same going in the +z direction as the -z direction?




✯ *Consult an instructor before you proceed.*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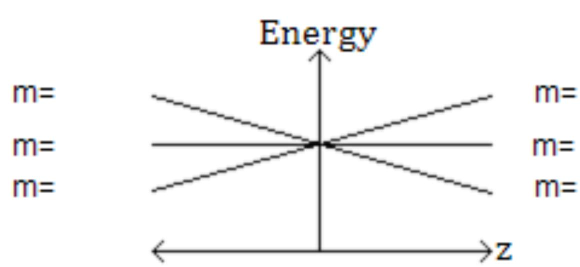
8. Time for a quick detour into polarization. (Polarization tells us which direction the electric field of an electromagnetic wave points in.) Light can have many polarizations, such as horizontal, vertical or circular. With circular polarization, the E-field traces out a circle as the wave propagates. This rotation can be clockwise or counterclockwise. We'll call these $\sigma^+$ (or $\sigma^-$), respectively. When an atom absorbs or emits a $\sigma^+$ photon, the electronic transition must satisfy $\Delta m= +1$. Similarly, absorption or emission of a $\sigma^-$ photon allows a transition with $\Delta m = -1$.
   Which types of polarization ($\sigma^+$, $\sigma^-$, or none) can enable the following transitions?
   a. $l=0, m=0 \rightarrow l=1, m=+1$
   b. $l=0, m=0 \rightarrow l=1, m=0$
   c. $l=0, m=0 \rightarrow l=1, m=-1$
   d. $l=1, m=+1 \rightarrow l=0, m=0$
   e. $l=1, m=0 \rightarrow l=0, m=0$
   f. $l=1, m=-1 \rightarrow l=0, m=0$

9. Imagine we have the magnetic field described in question 7, one that varies linearly with z-position. We can adapt our energy level diagram from problem 6 to the diagram shown below. Label the magnetic *m* levels on the diagram.





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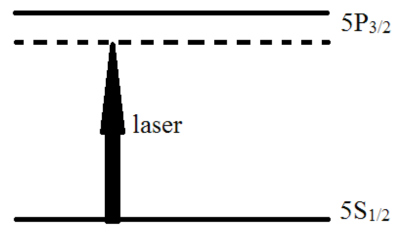


10. Suppose we have a laser with an energy level that is shown by the dotted line on the diagram. The atom will absorb a photon from the laser if the energy of the photon is close to the energy of an appropriate transition of the atom.
    If we have an atom in the ground state that is to the right of the origin, what kind of polarization ($\sigma^+$ or $\sigma^-$) is it more likely to absorb?
    What about a ground state atom to the left of the origin?

✯ *Consult an instructor before you proceed.*

11. A photon carries a momentum of $\hbar k$, where $k = 2\pi\lambda$. If a stationary atom absorbs a photon, where does the photon's momentum go? Explain your reasoning.

12. If the photon in the previous question is coming in from the right, what is the velocity of the atom (magnitude and direction) after the collision? This is called a *recoil velocity*.


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13. Now suppose we have the same linear magnetic field described in question 7, and two lasers coming in from the right and left. The one from the left is $\sigma^+$ while the one from the right is $\sigma^-$, and both carry photons with the energy depicted in the diagram with question 10.
Describe what happens to ground state atoms at different positions along the z-axis. (Note: in this situation, the atoms can only move along the z-axis.)

14. When the atom absorbs a photon, it quickly re-emits another photon in a random direction.
    a. Imagine the atom is to the right of the origin. What is the effect of many, many cycles of absorption and reemission?

    b. What happens to an atom that is to the left of the origin after many cycles?

15. It's possible that an atom that originally started to the right of the origin ends up moving to the left past the origin after going through many cycles of reemission. Can this atom move back towards the origin? Explain your reasoning.


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16. Our goal was to confine atoms to a region near the origin. Have we accomplished this? Why or why not?

17. Can we adapt this system to confine atoms in three dimensions? Explain your trapping setup.

✯ *Consult an instructor.*


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## *Instructor Guide 9:*

# Zeeman effect

This tutorial is designed to introduce students to the Zeeman effect. The students will see that a magnetic field can then be used to provide spatial confinement of cooled atoms. This tutorial should be used in conjunction with Tutorial 8 (Doppler cooling). Taken together, the laser and magnetic-field configurations described in tutorials 8 and 9 form a Magneto-Optical Trap (MOT). Doppler cooling can be used to cool atoms for atomic clocks. In creating Bose-Einstein Condensates, scientists usually create a cloud of cooled atoms with a MOT.

In the tutorial, some of the big ideas students will focus on are the following:

- Motion of charged particles in magnetic fields
- Models of atomic orbits
- Quantized angular momentum
- Polarization
- Momentum transfer

**Motion in magnetic fields**

1a-b) This may be a bit of a trick question, as we don't say what plane the atom rotates in. The point of this question is that the students should see that this matters. If students do not reach this point for whatever reason, they can move on to the next question without the facilitator providing too much help, as 1b asks the students directly whether the relative orientation matters. In any case, the rotating charge will experience a torque, which will force the charge's magnetic moment to align with the magnetic field.

- Students may struggle with the concept that a single charge can be modeled as experiencing a torque. The facilitator may initiate the idea of an electron attached to the end of a string, with the other end fixed in place. The electron rotates quickly in a circle while in the presence of a weak magnetic field. Ask the students to imagine how the plane of rotation would change in response to this field; it should slowly tilt towards the the direction of the magnetic field.

2) The potential energy of the interaction is lowest when the magnetic moment and magnetic field are aligned. This means the magnetic moment should point in the positive z-direction.

- To check for comprehension, a facilitator might ask what this magnetic moment would imply for motion,  e.g. asking the students what plane the rotation lies in and what direction it goes. A magnetic moment in the positive z-axis corresponds to counterclockwise motion in the x-y plane.

3) The students' answer to this question will not likely influence their work in the rest of tutorial. This question provides a point for students to discuss some of the interpretive issues in quantum mechanics. Some students may see the classical model as a good description. Students may also look ahead and choose an answer based on #4. The facilitator should choose whether they want to further discuss this point with students.


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4) The facilitator should use their judgment on how to approach this problem with students. We would likely expect students to come up with the response that the model (like the Bohr model) is useful in some ways, but maybe not all.

**Quantized Angular Momentum**

5) The states *m* = -1, 0, 1 would correspond to maximum, zero and minimum energies, respectively. The state *m*=+1 is the state where the projection on the z-axis is positive, so the angular momentum vector is above the x-y plane. Students may lose track of the negative sign in the equation for potential energy, thinking that the parallel configuration yields a maximum energy. Have the students check to make sure their answers are consistent.

6) The states *m* = -1, 0, 1 would be maximum, zero and minimum energies on the right side of the diagram. The states *m* = +1, 0, -1 would be maximum, zero and minimum energies on the left side of the diagram.

7) The states would be linear in energy with respect to the z-axis. The *m*=-1 state would have a positive slope, the *m*=+1 state would have a negative slope and the *m*=0 state would remain horizontal. By #7, students should have come to see that energy levels will shift in response to the magnetic field, but they may struggle with the idea that the energy now depends on position along the z-axis. However, students will need to fill a graph showing the linear tilts of the energy levels in #9. So if they are unable to get the linear tilt, they will be given it in #9. There may be some confusion here about what the axes of the graph represent; specifically, the vertical axis now represents the magnitude of the magnetic field (rather than energy as in #6).

**Polarization**

8) Students may struggle with circular polarization. With circular polarization, the electric field vector traces out a helix around the axis of propagation.

- a. $l=0, m=0 \rightarrow l=1, m=+1$
  - $\Delta m = +1$, $\sigma^+$
- b. $l=0, m=0 \rightarrow l=1, m=0$
  - $\Delta m = 0$, none
- c. $l=0, m=0 \rightarrow l=1, m=-1$
  - $\Delta m = -1$, $\sigma^-$
- d. $l=1, m=+1 \rightarrow l=0, m=0$
  - $\Delta m = -1$, $\sigma^-$
- e. $l=1, m=0 \rightarrow l=0, m=0$
  - $\Delta m = 0$, none
- f. $l=1, m=-1 \rightarrow l=0, m=0$
  - $\Delta m = +1$, $\sigma^+$


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9) The *m*=-1 state has the positive slope, the *m*=+1 state has the negative slope and the *m*=0 state is horizontal.

10) The students should be looking for the energy levels that are shifted *down*, closer to the wavelength of the laser. On the right side of the origin, the atom's *m*=-1 energy level is shifted closer to the energy of the laser. An atom here is unable to absorb $\sigma^-$, as $\sigma^-$ would can only drive a transition with $\Delta m$= -1, but *m*=-1 is the lowest magnetic state available. Hence, an atom is more likely to absorb a $\sigma^+$ photon and transition from *m*=-1 to *m*=0. To the left of the origin, an atom is likely to absorb a $\sigma^-$, and transition from *m*=+1 to *m*=0.

**Momentum Transfer**

11) The photon's momentum is transferred to the particle, by conservation of momentum. Throughout the tutorial, we draw both on the wave-like and particle-like characteristics of light. For some students, it may be difficult to discuss light as an electromagnetic wave with polarization and then consider what momentum a photon can carry.

12) The momentum of the atom after absorption would be equal to the momentum of the photon before. To find velocity, divide by mass, $v = \hbar km$. This velocity points in the direction the photon was moving.

13) When atoms are to the right of the origin, the *m*=+1 state is closer to the laser's energy, making it more likely that the atom can absorb a $\sigma^-$ photon coming in from the right side. On the left side, the *m*=-1 state is closer to the laser's energy, which makes it more likely that the atom can absorb a $\sigma^+$ photon coming in from the left side.

**Atomic Confinement**

14a-b) If the atom is to the right (left) of the origin, the atom gets many kicks towards the origin. The kicks the atom receives from re-emission average out to zero.

15) Once the atom is to the left of the origin, it is closer in resonance to the laser coming in from the left, making it more likely that the atom will get kicked back towards the center.

16-17) In order to confine atoms in three dimension, we would have three orthogonal pairs of counter-propagating lasers. Once students reach this point in the tutorial, have them try to draw connections to the previous tutorial. A facilitator might ask whether the temperature of the atom cloud has changed or how the speed of individual atoms changes over time.


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## *Tutorial 10:*

Name ___________________________ Section _____

# LEDs

The 2014 Nobel Prize in Physics was awarded to Isamu Akasaki, Hiroshi Amano, and Shuji Nakamura, "for the invention of efficient blue light-emitting diodes [LEDs] which has enabled bright and energy-saving white light sources." In this tutorial we will try to understand the significance of this discovery.

1. Previously, only red and green LEDs were available. Why did the blue LED make white LED light sources possible?

2. LEDs of different colors are made of different semiconductor materials. For example, the original infrared LEDs used gallium arsenide (GaAs), while blue LEDs use gallium nitride (GaN). (Forming GaN crystals was an engineering challenge, which is one reason why it took longer to develop blue LEDs.) What properties of the materials could result in emitting different colors of light? In other words, what controls the emitted light color?

For the rest of the tutorial, your group will need a computer.
Run the **"Semiconductors"** PhET sim, at http://phet.colorado.edu/en/simulation/semiconductor

3. Come up with all the possible ways to create a current in the circuit. (Note: "n-type" semiconductors are "doped" with atoms that contain extra electrons, while "p-type" semiconductors are doped with atoms that contain fewer electrons, or more "holes".) What arrangements make a steady current and what makes a temporary current?


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4. Did you create an LED? How do you know? (The simulation does not show photon emission directly, so you'll need to figure out whether it's happening.)

5. Can you tinker with some of the parameters to increase the brightness of the emitted light? If so, explain how you know that the brightness increased.

6. In this simulation, you cannot change the frequency of the emitted light. What part of the simulation depicts the frequency of the light emitted? What features of the simulation would need to be changed in order to make the frequency adjustable?

7. What is the physical meaning of the aspect of the simulation that depicts the frequency of emitted light?

✯ *Consult an instructor before you proceed.*


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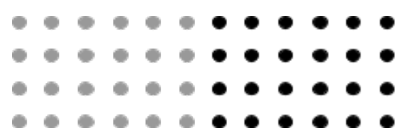
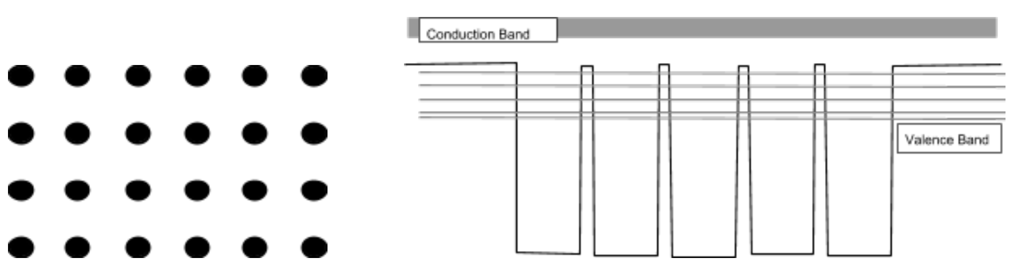
8. Based on how electrons are depicted in this simulation, are they treated more like classical particles, classical waves, quantum things, some combination, or something else? Explain your thinking.

In an LED, electrons move through two sections of the same semiconducting material. One section is p-type and the other is n-type. (*Note*: Doping does not significantly change the energy levels of the material.) In terms of nuclear lattices this might look something like the diagram below (gray being one type of doping and black being the other):



Electrons in a *single* semiconductor lattice (within either the gray *or* the black zone above) can be modeled as being in a potential of many quantum wells. This produces the band structure for electrons shown by the horizontal lines. The electrons in the "conduction band" are unbound, i.e., have energies greater than the top level in the potential well.





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9.

a. Draw a potential diagram for an LED connected to a battery. (Hint: Referring back to the Semiconductors sim may help indicate how to distinctly draw the p- and n-type lattices.)

b. Which features of electron motion that you observed in the sim can be explained by your representation? Which ones are not?


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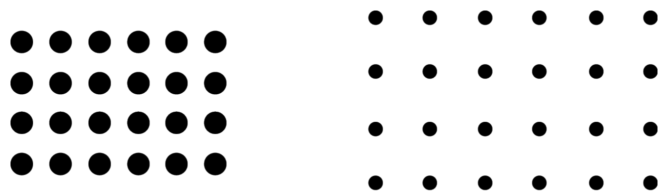
10. Consider two semiconductor lattices, as shown below. Draw (1-D) potential energy diagrams for an electron in each of these two lattices and discuss any similarities and differences between them.



Lattice 1 Lattice 2

11. When used in an LED, predict which lattice would produce light of higher frequency. Explain your reasoning.

✯ *Consult an instructor.*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Run the **“Quantum Bound States”** PhET sim, at http://phet.colorado.edu/en/simulation/bound-states , and go to the **Many Wells (Band Structure)** tab.

12. The purple line represents the potential. How does this simplified model of a potential correspond to the potential experienced by an electron in a lattice? (Where are the nuclei?)

13. As you can see, the allowed energy levels are grouped together into bands. In how many different ways can you change the gap between energy bands?

14. How do these changes to the band structure affect the frequency of the light emitted by the LED? For instance, what changes to the band structure increase the frequency of the light?

15. In Question 10, if the crystal with the larger atoms corresponds to gallium arsenide (GaAs), and the crystal with the smaller atoms corresponds to gallium nitride (GaN), which crystal, when used in an LED, emits light of higher frequency? How do you know?


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16. When an electron transitions from the n-type side to the p-type side, is there a sense in which the electron physically falls as it transitions to a lower energy level? Why or why not?

17. To understand the differences between LEDs of different colors, we used different models of electrons.
    a. When did you use a particle model? And what were the benefits or disadvantages you found?
    b. When did you use a wave model? And what were the benefits or disadvantages you found?
    c. Do you have a preference? Why or why not?


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## *Instructor Guide 10:*
# LEDs

This tutorial builds on Tutorial 7 (Energy and position), so it is recommended to do that one first, since it gets students thinking about modeling an electron in a lattice of atoms, and introduces them to using the Quantum Bound States simulation to help them reason about energy levels and wavefunctions of quantum systems with complicated potentials.

What this tutorial does **not** do is provide a general introduction to semiconductors and diodes. Rather, it assumes that students have some background on these topics, and therefore it may or may not be appropriate to use in a given course, depending on the curriculum and the students' prior experience.

**Equipment:**

- A computer for each group, with the PhET "Semiconductors" and "Quantum Bound States" simulations:
  http://phet.colorado.edu/en/simulation/semiconductor
  http://phet.colorado.edu/en/simulation/bound-states

**Prerequisites:**

- Tutorial 7
- General background on semiconductors and diodes, including familiarity with p-n junctions

**Instructional Goals:**

- Guide students to understand what is involved in engineering the band gap for semiconductor materials
- Coordinating multiple models for electrons in a circuit, including a semi-classical model and a more "quantum" model

**Introduction**

1) Producing white light requires the combination of red, green, and blue. If red and green were already available, blue completed the combination. (Some students may think this is too obvious and be looking for something more complicated, but you can assure them that this is all it is.)

2) The frequency of the emitted light (= energy of the photons) corresponds to the band gap of the semiconductor material. (Strictly speaking, the gap between the conduction band of the n-type material and the valence band of the p-type material.) Students may or may not get this yet, but if they don't, they'll have a chance to think about this further when they run the simulations.

**Semiconductors sim (Semi-classical model)**

3) To create a current in the circuit, a battery voltage needs to be turned on, and dopants have to be placed in both segments. Specifically, if both segments are p-type or both are n-type, this results in a steady current. If the p-type segment is connected to the positive terminal of the


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battery and the n-type to the negative terminal (forward bias), this also results in a steady current if the battery voltage is high enough. The reverse configuration (reverse bias) results in a temporary current.

4) The forward-biased circuit might be an LED, because the electron drops down to a lower energy and might emit a photon (though we can't know for sure without knowing more about the material).

5) The brightness of the light corresponds to the number of photons emitted per unit of time. By increasing the voltage, we can increase the current (number of electrons passing through per unit of time), which means that more photons are emitted.

6-7) The frequency of the light corresponds to the band gap between the conduction band of the n-type material and the valence band of the p-type material. In order to change this, we would need to be able to change the material (or the amount of doping), which is not provided as an option.

8) Students will respond in different ways to this, and it's more valuable here that they go meta and think about these issues than that they come up with a particular answer. We think this simulation treats electrons mostly as classical particles: they are depicted as little balls with definite position and velocity. The one "quantum" element of the simulation is the discrete energy levels, which obey the exclusion principle (depicting electrons as fermions).

**Modeling a Lattice**

9a) This is challenging, and we've had experts disagree on how best to produce this representation. Again, here we're not necessarily interested in the students reaching a certain correct answer, but we have seen students use this as an opportunity for productive reasoning. Referring back to the Semiconductors sim, students may come up with an energy drop at the p-n junction, and (especially if they tried turning on the electric field when playing with the Quantum Bound States sim in the previous tutorial) may come up with a slanted "floor". One issue that may be a stumbling block: when we talk about a "potential" in quantum mechanics, we actually mean potential **energy**, not electric potential, and since an electron is negatively charged, higher voltage translates to lower potential energy. **Because it can be unclear what the correct representation is here, if this tutorial is being used as a graded assignment, we suggest excluding this question (as well as part b). It can lead to productive reasoning in a low stakes setting focused on sense making, but we want to avoid students spending too much time searching for a "correct answer".

9b) The sim represented the electrons moving around the circuit, but the potential energy diagrams that students are drawing here don't really include any kind of time evolution. However, it might still be possible to infer the direction of the current, and the behavior of electrons at the junction (probably from looking at the potential energy diagram in a semi-classical way).


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10) The point of this question is to help students recognize that the size of the dots corresponds to the width of the wells, and the spacing between the dots corresponds to the spacing between the wells. Making these connections between representations is crucial to understanding the remainder of the tutorial.

11) The students may recall from using the simulation in Tutorial 7 (or may be able to figure out) that energy spacing between bands is proportional to spacing between wells and inversely proportional to well width. Therefore, lattice 2 (with larger spacing between wells, and smaller wells) will result in higher-frequency light, since there is a larger band gap. But if students don't get this now, they'll have another chance later in the tutorial, so they can move on.

**Quantum Bound States simulation**

12) The nuclei are represented by the wells (since electrons are attracted to them), though we have greatly simplified the interaction by using square wells.

13) To increase the separation between bands you can: increase the separation between wells, decrease the width of the wells, or decrease the electric field. (To decrease the separation between bands, do the opposite of those.)

14) A larger band gap means higher-frequency light.

15) As discussed above in #10, lattice 2 (GaN) emits light of higher frequency. (This information is also given at the beginning of the tutorial!) Some sources suggest that the lattice spacing of GaAs is actually larger than that of GaN, which does not directly map onto the diagrams in #10. If students look this up (or know this), they may have a productive discussion about how the band gap can be larger either from larger well spacing or smaller wells but both criteria are not necessary. That is, even though GaAs has a larger lattice spacing, the band gap of GaN is larger because the wells are much smaller than those of GaAs.

16) This question follows up on Tutorial 7, with the discussion of the relationship between energy and position. The expected answer is "It's complicated" - we're now talking about a wavefunction for the electron that extends over all space. The "falling" that we see in the Semiconductors sim is metaphorical ("falling" to a lower energy), but if we try to map this onto position, it's not so simple. But this may lead to good discussions even if they're unresolved.

17) Students were using a (mostly) particle model when using the first simulation, and a (mostly) wave model when using the second simulation, and these both contributed different pieces to the puzzle. Of course, they may or may not have a preference for one or the other.


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## C Chapter VI: Protocol for individual interviews bearing on epistemological comparisons

The questions below served as a guide for interviewers in the 30-minute semi-structured interviews with seven individual students from the CU Modern Physics course in Semester 1. Not all of the questions were asked in every interview. Question 3 asks students how the Modern Physics course changed the way they think about physics. If a student responded to this question by comparing epistemological aspects of classical and quantum physics, we did not code it as spontaneous due to the comparative nature of the question.

1. How did the semester go for you overall? (class and focus group)
2. What aspects of the course did you like or not like?
3. Has this course changed the way you think about physics? How?
4. How do you see the roles of calculation vs. conceptual reasoning in quantum mechanics?
5. What was your favorite thing (content area) you learned from class?
6. What was the most challenging thing for you in this class? (content-wise)
7. In the following situations, which of the interpretations do you feel like you are using? Why?
    a. Double slit experiment (with beam of light, single photon, electrons)
    b. Photoelectric effect
    c. Electron transition in an atom
    d. Aspect experiments with one/two beam splitters
    e. Shining light on barrels to heat them
8. What did you think about the focus group?
9. What were the most/least productive aspects of the focus group for you?
10. Did your participation in this group affect your thinking in the class?

## D Chapter VI: Bifurcated survey results: $3 \times 3$ matrices

In this appendix, we include the percentages of students in each permutation of responses for each item on each administration of the survey, where D, N, and A denote the responses of Disagree, Neutral, and Agree. Items with statistically significant classical-quantum splits (via the Bhapkar test) are indicated in boldface.

| Item | Survey | *N* | Percentage of each permutation | | | | |
|---|---|---|---|---|---|---|---|
| **Item 6:** Knowledge in [classical / quantum] physics consists of many disconnected topics. | **Sem1 Pre** | 118 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 30.51 | 21.19 | 5.08 |
| | | | | N | 1.69 | 16.10 | 4.24 |
| | | | | A | 7.63 | 6.78 | 6.78 |
| | Sem1 Post | 114 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 46.49 | 13.16 | 8.77 |
| | | | | N | 9.65 | 0.88 | 2.63 |
| | | | | A | 8.77 | 4.39 | 5.26 |
| | Sem2 Pre | 21 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 23.81 | 14.29 | 14.29 |
| | | | | N | 9.52 | 9.52 | 19.05 |
| | | | | A | 4.76 | 0.00 | 4.76 |
| | Sem2 Post | 22 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 45.45 | 13.64 | 18.18 |
| | | | | N | 0.00 | 0.00 | 0.00 |
| | | | | A | 9.09 | 0.00 | 13.64 |
| | **Sem3 Pre** | 43 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 41.86 | 32.56 | 13.95 |
| | | | | N | 4.65 | 2.33 | 0.00 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | A | 0.00 | 0.00 | 4.65 |
| | Sem3 Post | 51 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 50.98 | 15.69 | 7.84 |
| | | | | N | 5.88 | 5.88 | 3.92 |
| | | | | A | 3.92 | 3.92 | 1.96 |
| | Sem4 Post | 40 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 60.00 | 12.50 | 10.00 |
| | | | | N | 7.50 | 0.00 | 2.50 |
| | | | | A | 5.00 | 0.00 | 2.50 |
| **Item 23:** In doing a [classical / quantum] physics problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem. | **Sem1 Pre** | 118 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 41.53 | 22.88 | 13.56 |
| | | | | N | 2.54 | 9.32 | 3.39 |
| | | | | A | 2.54 | 1.69 | 2.54 |
| | **Sem1 Post** | 113 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 31.86 | 16.81 | 23.89 |
| | | | | N | 5.31 | 3.54 | 3.54 |
| | | | | A | 8.85 | 4.42 | 1.77 |
| | **Sem2 Pre** | 21 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 47.62 | 19.05 | 23.81 |
| | | | | N | 0.00 | 0.00 | 0.00 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | A | 0.00 | 0.00 | 9.52 |
| | **Sem2 Post** | 22 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 40.91 | 22.73 | 18.18 |
| | | | | N | 4.55 | 0.00 | 4.55 |
| | | | | A | 0.00 | 4.55 | 4.55 |
| | **Sem3 Pre** | 43 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 55.81 | 20.93 | 11.63 |
| | | | | N | 2.33 | 2.33 | 2.33 |
| | | | | A | 0.00 | 2.33 | 2.33 |
| | **Sem3 Post** | 51 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 50.98 | 21.57 | 13.73 |
| | | | | N | 5.88 | 3.92 | 0.00 |
| | | | | A | 0.00 | 1.96 | 1.96 |
| | **Sem4 Post** | 40 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 50.00 | 20.00 | 15.00 |
| | | | | N | 5.00 | 2.50 | 0.00 |
| | | | | A | 0.00 | 5.00 | 2.50 |
| **Item 28:** Learning [classical / quantum] physics changes my ideas about how the world works. | Sem1 Pre | 118 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 0.85 | 2.54 | 2.54 |
| | | | | N | 0.85 | 8.47 | 11.02 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | A | 4.24 | 8.47 | 61.02 |
| | **Sem1 Post** | 108 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 2.78 | 1.85 | 21.30 |
| | | | | N | 3.70 | 1.85 | 21.30 |
| | | | | A | 1.85 | 2.78 | 48.15 |
| | Sem2 Pre | 21 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 4.76 | 0.00 | 0.00 |
| | | | | N | 0.00 | 9.52 | 19.05 |
| | | | | A | 0.00 | 14.29 | 52.38 |
| | Sem2 Post | 21 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 0.00 | 0.00 | 4.76 |
| | | | | N | 0.00 | 0.00 | 14.29 |
| | | | | A | 0.00 | 4.76 | 76.19 |
| | **Sem3 Pre** | 43 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 0.00 | 2.33 | 6.98 |
| | | | | N | 0.00 | 2.33 | 9.30 |
| | | | | A | 0.00 | 25.58 | 53.49 |
| | **Sem3 Post** | 49 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 0.00 | 4.08 | 20.41 |
| | | | | N | 0.00 | 2.04 | 16.33 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | A | 2.04 | 2.04 | 53.06 |
| | **Sem4 Post** | 40 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 5.00 | 2.50 | 7.50 |
| | | | | N | 2.50 | 0.00 | 22.50 |
| | | | | A | 0.00 | 5.00 | 55.00 |
| **Item 35:** The subject of [classical / quantum] physics has little relation to what I experience in the real world. | **Sem1 Pre** | 117 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 49.57 | 16.24 | 9.40 |
| | | | | N | 0.85 | 9.40 | 1.71 |
| | | | | A | 3.42 | 5.13 | 4.27 |
| | **Sem1 Post** | 108 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 51.85 | 16.67 | 11.11 |
| | | | | N | 2.78 | 5.56 | 1.85 |
| | | | | A | 3.70 | 1.85 | 4.63 |
| | **Sem2 Pre** | 21 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 66.67 | 19.05 | 9.52 |
| | | | | N | 0.00 | 0.00 | 4.76 |
| | | | | A | 0.00 | 0.00 | 0.00 |
| | **Sem2 Post** | 22 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 50.00 | 9.09 | 27.27 |
| | | | | N | 0.00 | 0.00 | 0.00 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | A | 0.00 | 4.55 | 9.09 |
| | Sem3 Pre | 43 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 44.19 | 18.60 | 23.26 |
| | | | | N | 2.33 | 2.33 | 2.33 |
| | | | | A | 2.33 | 2.33 | 2.33 |
| | Sem3 Post | 51 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 54.90 | 19.61 | 25.49 |
| | | | | N | 0.00 | 0.00 | 0.00 |
| | | | | A | 0.00 | 0.00 | 0.00 |
| | **Sem4 Post** | 40 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 40.00 | 12.50 | 42.50 |
| | | | | N | 0.00 | 0.00 | 2.50 |
| | | | | A | 2.50 | 0.00 | 0.00 |
| **Item 40:** If I get stuck on a [classical / quantum] physics problem, there is no chance I'll figure it out on my own. | **Sem1 Pre** | 118 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 36.44 | 20.34 | 11.86 |
| | | | | N | 0.85 | 16.95 | 5.08 |
| | | | | A | 2.54 | 0.00 | 5.93 |
| | **Sem1 Post** | 104 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 47.12 | 19.23 | 19.23 |
| | | | | N | 0.96 | 3.85 | 0.96 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | A | 3.85 | 1.92 | 2.88 |
| | **Sem2 Pre** | 21 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 42.86 | 28.57 | 19.05 |
| | | | | N | 0.00 | 9.52 | 0.00 |
| | | | | A | 0.00 | 0.00 | 0.00 |
| | **Sem2 Post** | 22 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 27.27 | 31.82 | 22.73 |
| | | | | N | 0.00 | 0.00 | 13.64 |
| | | | | A | 0.00 | 0.00 | 4.55 |
| | **Sem3 Pre** | 43 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 51.16 | 18.60 | 6.98 |
| | | | | N | 0.00 | 4.65 | 16.28 |
| | | | | A | 0.00 | 0.00 | 2.33 |
| | **Sem3 Post** | 51 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 54.90 | 11.76 | 15.69 |
| | | | | N | 0.00 | 7.84 | 1.96 |
| | | | | A | 0.00 | 0.00 | 7.84 |
| | **Sem4 Post** | 40 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 57.50 | 15.00 | 10.00 |
| | | | | N | 2.50 | 2.50 | 7.50 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | A | 2.50 | 2.50 | 0.00 |
| **Item 41:** It is possible for physicists to carefully perform the same measurement in a [classical / quantum] physics experiment and get two very different results that are both correct. | **Sem1 Pre** | 118 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 1.69 | 8.47 | 29.66 |
| | | | | N | 2.54 | 16.95 | 12.71 |
| | | | | A | 1.69 | 3.39 | 22.88 |
| | Sem1 Post | -- | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | -- | -- | -- |
| | | | | N | -- | -- | -- |
| | | | | A | -- | -- | -- |
| | **Sem2 Pre** | 21 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 0.00 | 4.76 | 23.81 |
| | | | | N | 0.00 | 19.05 | 23.81 |
| | | | | A | 4.76 | 0.00 | 23.81 |
| | **Sem2 Post** | 22 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 0.00 | 0.00 | 50.00 |
| | | | | N | 0.00 | 4.55 | 22.73 |
| | | | | A | 0.00 | 0.00 | 22.73 |
| | **Sem3 Pre** | 43 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 2.33 | 13.95 | 41.86 |
| | | | | N | 0.00 | 4.65 | 18.60 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | A | 0.00 | 2.33 | 16.28 |
| | Sem3 Post | 51 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 5.88 | 5.88 | 54.90 |
| | | | | N | 0.00 | 5.88 | 7.84 |
| | | | | A | 3.92 | 1.96 | 13.73 |
| | Sem4 Post | 40 | | | Quantum | | |
| | | | | | D | N | A |
| | | | Classical | D | 2.50 | 0.00 | 67.50 |
| | | | | N | 0.00 | 5.00 | 12.50 |
| | | | | A | 5.00 | 0.00 | 7.50 |

# E Chapter VII: Protocols for focus groups on epistemological views of classical versus quantum physics

In this appendix we include the protocols for the two rounds of focus groups conducted with students in two different semesters of an upper division QM1 course (CU 3220-4 and CU 3220-7 from Chapter VII). The protocol for each semester had three parts, with slight variations between the two semesters. We used the students' conversations from parts one and three in Semester 4 and parts one and two in Semester 7 for our study of students' domain-specific epistemologies.

**Protocol for focus groups with QM students (CU 3220-4)**

**Part One**

A. Please take 1-2 minutes to answer the following 7 questions by yourself.

**Classical/Newtonian Questions:** For the first 3 questions, please think about your previous physics courses on motion, electrical phenomenon, etc., (what we are calling "classical" physics) in the items below.

| | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|
| The subject of classical physics has little relation to what I experience in the real world. | ○ | ○ | ○ | ○ | ○ |
| In doing a classical physics problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem. | ○ | ○ | ○ | ○ | ○ |
| It is possible for physicists to carefully perform the same measurement in a classical/Newtonian physics experiment and get two very different results that are both correct. | ○ | ○ | ○ | ○ | ○ |

**Quantum Questions**: For the next 3 questions, please think about past/upcoming courses on modern physics and/or quantum physics.

| | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|
| The subject of quantum physics has little relation to what I experience in the real world. | ○ | ○ | ○ | ○ | ○ |
| In doing a quantum physics problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem. | ○ | ○ | ○ | ○ | ○ |
| It is possible for physicists to carefully perform the same measurement in a quantum physics experiment and get two very different results that are both correct. | ○ | ○ | ○ | ○ | ○ |

| | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| When not being observed, an electron in an atom still exists at a definite (but unknown) position at each moment in time. | ◯ | ◯ | ◯ | ◯ | ◯ |

B. Now, please discuss these 7 questions with your group members. For each question:

a) What did you answer, and WHY?

b) Can you come to consensus?

**Part Two**
Let's revisit part of the tutorial you did week 4 of this semester.

1. A) We often write $\boldsymbol{S}_z|+> = \hbar/2|+>$. In words, we say "Acting the $\boldsymbol{S}_z$ operator on $|+>$ gives $^{\hbar}/_{2}$ times $|+>$." In both technical and ordinary/colloquial language – how do you interpret each of the individual terms and symbols that appear in your equation?

B) Repeat part A but now with $\boldsymbol{S}_z$ acting on an arbitrary (normalized) ket, $|\psi> = a|+> + b|->$.
(Don't spend TOO much time on interpretation just yet, we want your initial opinions)

C) Now consider a conversation about the result of JUST part B above:

Student A says: "I think this formula tells you the experimental result of measuring the z-component of spin on some (generic) incoming state $|\psi>$."

Student B says: "I disagree. I think it tells you what the resulting quantum state is after you measure the z-component of spin. That's different from an experimental result (which would be a number, not a state)."

Student C says: "I think you are both over-interpreting this equation – it's just a mathematical relation."

Discuss the above opinions, where/how do you agree or disagree?

You can move on after you have considered all 3 claims above - you'll revisit this conversation on the next page!

Suppose we act $\boldsymbol{S_z}$ on the state $|\psi> = a|+> + b|->$
D) What kind of OBJECT is $\boldsymbol{S_z}|\psi>$? Is it a bra? a ket? an operator? a number? Something else?

E) Let's pick a very particular $|\psi> = |+>_X$. Re-write $|+>_X$ as a column vector in the $\boldsymbol{S_z}$ basis, act $\boldsymbol{S_z}$ on it, and see what you get. What kind of object is it? Is it normalized?

Is the equation you have written an "eigenequation"?

If you do normalize the state on the right side, what physical state does that represent? Does it have any obvious or direct physical connection to what would happen if you actually measured $\boldsymbol{S_z}$ on your starting state?

F) Do you think a measurement of $\boldsymbol{S_z}$ on a particle in state $|+>_X$ should have a determined (formulaic) value? (That is, can you predict the experimental value in advance?)
What about the final state after the measurement, do you think there is any formula predicting that in advance of the measurement?

Please revisit the "student discussion" at the bottom of the previous page. Are your responses still the same? Can you see why I would strongly **disagree** with the claim that $\boldsymbol{O}|\psi>$ means "a measurement of $\boldsymbol{O}$ on state $|\psi>$"?

**Part Three**
**Now let's switch gears a bit and reflect on your experiences in this course and with learning quantum mechanics in general.**

Please discuss with your group members:

1) What was your experience like this semester? (What were you feeling at the beginning of the semester, and how are you feeling now? What changed, and why?)

2) Did you have any "aha moments" in the course?

3) Was there anything you were particularly proud of in this course? Did you have any experiences that made you feel proud?

4) What were the challenges for you in this course? Did you have any experiences that were particularly challenging?

**Protocol for focus groups with QM students (CU 3220-7)**

**Part One**

Please discuss with your group members:

1) What was your experience like this semester?

2) What are your thoughts on the role of math in this course? Did this course focus more on math, concepts, or both equally? What do you think the focus should be for a quantum class?

3) What role did math play in your learning in this course?

4) Has this course changed the way you think about physics? In what ways?

5) What role does intuition play in quantum mechanics? Can you think of an example from this course where your intuition helped you make sense of the physics?

6) What did you think of the use of simulations in this class?

**Part Two**

A. Please take 1-2 minutes to answer the following 10 questions by yourself.

**Classical/Newtonian Questions:** For the first 5 questions, please think about your previous physics courses on motion, electrical phenomenon, etc., (what we are calling "classical" physics) in the items below.

| | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|
| The subject of classical physics has little relation to what I experience in the real world. | ◯ | ◯ | ◯ | ◯ | ◯ |
| In doing a classical physics problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem. | ◯ | ◯ | ◯ | ◯ | ◯ |
| Knowledge in classical physics consists of many disconnected topics. | ◯ | ◯ | ◯ | ◯ | ◯ |
| Learning classical physics changes my ideas about how the world works. | ◯ | ◯ | ◯ | ◯ | ◯ |
| If I get stuck on a classical physics problem, there is no chance I'll figure it out on my own. | ◯ | ◯ | ◯ | ◯ | ◯ |

**Quantum Questions**: For the next 5 questions, please think about past/upcoming courses on modern physics and/or quantum physics.

| | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|
| The subject of quantum physics has little relation to what I experience in the real world. | ◯ | ◯ | ◯ | ◯ | ◯ |
| In doing a quantum physics problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem. | ◯ | ◯ | ◯ | ◯ | ◯ |
| Knowledge in quantum physics consists of many disconnected topics. | ◯ | ◯ | ◯ | ◯ | ◯ |
| Learning quantum physics changes my ideas about how the world works. | ◯ | ◯ | ◯ | ◯ | ◯ |
| If I get stuck on a quantum physics problem, there is no chance I'll figure it out on my own. | ◯ | ◯ | ◯ | ◯ | ◯ |

B. Now, please discuss the following 5 **classical/quantum** paired items with your group members. For each question:

a) What did you answer, and why?

b) Can you come to consensus?

| | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|
| The subject of **classical** physics has little relation to what I experience in the real world. | ○ | ○ | ○ | ○ | ○ |
| The subject of **quantum** physics has little relation to what I experience in the real world. | ○ | ○ | ○ | ○ | ○ |

| | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|
| In doing a **classical** physics problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem. | ○ | ○ | ○ | ○ | ○ |
| In doing a **quantum** physics problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem | ○ | ○ | ○ | ○ | ○ |

| | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|
| Knowledge in **classical** physics consists of many disconnected topics. | ○ | ○ | ○ | ○ | ○ |
| Knowledge in **quantum** physics consists of many disconnected topics. | ○ | ○ | ○ | ○ | ○ |

| | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|
| Learning **classical** physics changes my ideas about how the world works. | ◯ | ◯ | ◯ | ◯ | ◯ |
| Learning **quantum** physics changes my ideas about how the world works. | ◯ | ◯ | ◯ | ◯ | ◯ |

| | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|
| If I get stuck on a **classical** physics problem, there is no chance I'll figure it out on my own. | ◯ | ◯ | ◯ | ◯ | ◯ |
| If I get stuck on a **quantum** physics problem, there is no chance I'll figure it out on my own. | ◯ | ◯ | ◯ | ◯ | ◯ |

**Part Three**

The following are questions from the QMCA you took last Monday. Respond and discuss with your group members.

For questions **27-30**, the normalized wave function for a spin-1/2 particle at time $t = 0$ is

$$|\psi\rangle = C_1 |+\rangle + C_2 |-\rangle \ .$$

where $|+\rangle$ and $|-\rangle$ are eigenvectors of the $\hat{S}_z$ operator with eigenvalues $\pm\hbar/2$ respectively.

**27.** Immediately after a measurement of $S_z$, what is the state of the system?

**A.** $C_1 \frac{\hbar}{2} |+\rangle - C_2 \frac{\hbar}{2} |-\rangle$

**B.** $\frac{\hbar}{2} |+\rangle - \frac{\hbar}{2} |-\rangle$

**C.** Undetermined, but it is either $\frac{\hbar}{2} |+\rangle$ or $-\frac{\hbar}{2} |-\rangle$

**D.** Undetermined, but it is either $|+\rangle$ or $|-\rangle$

**28.** Immediately after a measurement of $S_z$ on the state $|\psi\rangle = C_1 |+\rangle + C_2 |-\rangle$ , the system is in the state $\hat{S}_z |\psi\rangle$.

**A.** True

**B.** False

**29.** There exists a single definite value for the z-component of spin associated with the state $|\psi\rangle = C_1 |+\rangle + C_2 |-\rangle$ .

**A.** True

**B.** False

**30.** For this state, $|\psi\rangle = C_1 |+\rangle + C_2 |-\rangle$, there is a certain probability that an $S_z$ measurement on this state $|\psi\rangle$ will yield a value $-\hbar/2$.

If the constant $C_2$ is replaced with $C_2 e^{i\theta_2}$ (with $\theta_2$ a real number, $0 < \theta_2 < 2\pi$), what happens to the probability that a measurement of $S_z$ will result in $-\hbar/2$?

**A.** The probability will not change

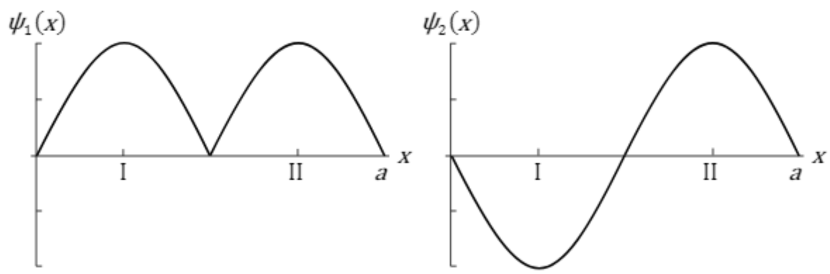
**B.** The probability will change

**C.** The probability depends on the particular value of $\theta_2$

**33.** Consider a particle confined to a region $0 < x < a$ . At $t = 0$ it is in a state given by

$$\psi(x) = \frac{1}{\sqrt{2}}(\psi_1(x) + \psi_2(x))$$

where $\psi_1(x)$ and $\psi_2(x)$ are normalized wave functions as shown below. At $t = 0$, what is the approximate probability of finding the particle in region I, $P_I$?




**A.** $P_I = \frac{1}{\sqrt{2}}$

**B.** $P_I = \frac{1}{2}$

**C.** $P_I = 0$

**D.** None of the above.

## F Chapter VII: Bifurcated CLASS survey results from full dataset

Table F.1: Classical-quantum splits on the bifurcated CLASS questions. The p-values are determined with the Bhapkar test and corrected using the Holm-Bonferroni method. Items with significant splits at the $\alpha = 0.05$ level are indicated in boldface.

| CLASS Item # | Item | Survey | $N$ | p-value |
|---|---|---|---|---|
| 6 | Knowledge in [classical/quantum] physics consists of many disconnected topics. | **CU 2130-1 Pre** | 118 | **1.8E-04** |
| | | CU 2130-1 Post | 114 | 0.58 |
| | | UMD 420-2 Pre | 21 | 0.17 |
| | | UMD 420-2 Post | 22 | 0.22 |
| | | CU 2130-2 Pre | 85 | **5.1E-05** |
| | | CU 2130-2 Post | 78 | 0.07 |
| | | CU 2170-2 Pre | 72 | 0.09 |
| | | **CU 2170-2 Post** | 76 | **7.0E-03** |
| | | **CU 2130-3 Pre** | 43 | **4.0E-05** |
| | | CU 2130-3 Post | 51 | 0.43 |
| | | CU 3220-4 Post | 40 | 0.45 |
| | | **CPP 401-4 Post** | 30 | **0.03** |
| | | CPP 499 Post | 17 | 0.49 |
| | | **CSUF Post** | 17 | **0.03** |
| | | CU 2130-6 Post | 68 | 0.75 |
| | | **CU 2130-7 Pre** | 110 | **6.0E-03** |
| | | **CU 2130-7 Post** | 94 | **6.0E-03** |
| | | **CU 2170-7 Pre** | 73 | **0.04** |
| | | **CU 2170-7 Post** | 52 | **6.0E-03** |
| | | CU 3220-7 Pre | 21 | 0.61 |
| | | CU 3220-7 Post | 42 | 0.73 |
| | | **CU 2130-8 Pre** | 114 | **0.01** |
| | | CU 2130-8 Post | 105 | 1 |
| | | **CU 2170-8 Pre** | 148 | **9.7E-07** |
| | | **CU 2170-8 Post** | 103 | **0.02** |

| CLASS Item # | Item | Survey | $N$ | p-value |
|---|---|---|---|---|
| 23 | In doing a [classical/quantum] physics problem, if my calculation gives a result very different from what I'd expect, I'd trust the calculation rather than going back through the problem. | CU **2130-1 Pre** | 118 | **4.4E-08** |
| | | **CU 2130-1 Post** | 113 | **7.0E-04** |
| | | **UMD 420-2 Pre** | 21 | **e.0E-03** |
| | | **UMD 420-2 Post** | 22 | **0.03** |
| | | **CU 2130-2 Pre** | 85 | **2.7E-09** |
| | | **CU 2130-2 Post** | 77 | **1E-06** |
| | | **CU 2170-2 Pre** | 72 | **1.4E-07** |
| | | **CU 2170-2 Post** | 76 | **8.8E-07** |
| | | **CU 2130-3 Pre** | 43 | **2.0E-03** |
| | | **CU 2130-3 Post** | 51 | **3.0E-03** |
| | | **CU 3220-4 Post** | 40 | **3.0E-03** |
| | | CPP 401-4 Post | 30 | 0.37 |
| | | **CPP 499 Post** | 17 | **0.04** |
| | | CSUF Post | 17 | 0.37 |
| | | **CU 2130-6 Post** | 68 | **4.1E-09** |
| | | **CU 2130-7 Pre** | 110 | **4.5E-10** |
| | | **CU 2130-7 Post** | 94 | **3.0E-04** |
| | | **CU 2170-7 Pre** | 72 | **1.2E-06** |
| | | **CU 2170-7 Post** | 51 | **1.8E-05** |
| | | **CU 3220-7 Pre** | 21 | **5.0E-03** |
| | | **CU 3220-7 Post** | 42 | **6.5E-06** |
| | | **CU 2130-8 Pre** | 114 | **1.2E-11** |
| | | **CU 2130-8 Post** | 105 | **1.6E-09** |
| | | **CU 2170-8 Pre** | 148 | **8.3E-15** |
| | | **CU 2170-8 Post** | 103 | **8.9E-16** |
| 28 | Learning [classical/quantum] physics changes my ideas about how the world works. | CU 2130-1 Pre | 118 | 0.98 |
| | | **CU 2130-1 Post** | 108 | **2.1E-08** |
| | | UMD 420-2 Pre | 21 | Undefined |
| | | UMD 420-2 Post | 21 | 0.97 |
| | | CU 2130-2 Pre | 85 | 1 |
| | | CU 2130-2 Post | 78 | 1 |
| | | **CU 2170-2 Pre** | 72 | **1.0E-03** |
| | | CU 2170-2 Post | 76 | 0.19 |
| | | **CU 2130-3 Pre** | 43 | **0.03** |
| | | **CU 2130-3 Post** | 49 | **4.0E-05** |

| CLASS Item # | Item | Survey | $N$ | p-value |
|---|---|---|---|---|
| | | **CU 3220-4 Post** | 40 | **0.01** |
| | | CPP 401-4 Post | 30 | 0.16 |
| | | CPP 499 Post | 17 | 0.59 |
| | | CSUF Post | 17 | 0.16 |
| | | **CU 2130-6 Post** | 68 | **4.3E-05** |
| | | CU 2130-7 Pre | 110 | 0.27 |
| | | **CU 2130-7 Post** | 93 | **7.2E-06** |
| | | CU 2170-7 Pre | 73 | 0.15 |
| | | CU 2170-7 Post | 52 | 0.77 |
| | | CU 3220-7 Pre | 21 | 0.51 |
| | | **CU 3220-7 Post** | 42 | **0.01** |
| | | CU 2130-8 Pre | 114 | 0.12 |
| | | **CU 2130-8 Post** | 105 | **7.5E-10** |
| | | **CU 2170-8 Pre** | 148 | **0.03** |
| | | **CU 2170-8 Post** | 103 | **0.03** |
| 35 | The subject of [classical/quantum] physics has little relation to what I experience in the real world. | **CU 2130-1 Pre** | 117 | **7.9E-06** |
| | | **CU 2130-1 Post** | 108 | **7.0E-04** |
| | | **UMD 420-2 Pre** | 21 | **0.03** |
| | | **UMD 420-2 Post** | 22 | **4.0E-03** |
| | | CU 2130-2 Pre | 85 | **<1.0E-05** |
| | | **CU 2130-2 Post** | 78 | **< 1.0E-05** |
| | | **CU 2170-2 Pre** | 72 | **1.8E-10** |
| | | CU 2170-2 Post | 76 | **<1.0E-05** |
| | | **CU 2130-3 Pre** | 43 | **3.0E-04** |
| | | **CU 2130-3 Post** | 51 | **3.2E-09** |
| | | **CU 3220-4 Post** | 40 | **8.9E-09** |
| | | **CPP 401-4 Post** | 30 | **1.7E-07** |
| | | **CPP 499 Post** | 17 | **3.0E-04** |
| | | **CSUF Post** | 17 | **1.7E-07** |
| | | **CU 2130-6 Post** | 67 | **5.3E-08** |
| | | **CU 2130-7 Pre** | 110 | **3.3E-09** |
| | | **CU 2130-7 Post** | 94 | **2.6E-12** |
| | | **CU 2170-7 Pre** | 73 | **1.3E-09** |
| | | **CU 2170-7 Post** | 52 | **2.6E-09** |
| | | **CU 3220-7 Pre** | 21 | **4E-12** |

| CLASS Item # | Item | Survey | $N$ | p-value |
|---|---|---|---|---|
| | | **CU 3220-7 Post** | 42 | **6.2E-07** |
| | | **CU 2130-8 Pre** | 114 | **1.5E-08** |
| | | **CU 2130-8 Post** | 105 | **6.5E-09** |
| | | **CU 2170-8 Pre** | 105 | **<1E-13** |
| | | **CU 2170-8 Post** | 103 | **2.5E-10** |
| 40 | If I get stuck on a [classical/quantum] physics problem, there is no chance I'll figure it out on my own. | **CU 2130-1 Pre** | 118 | **2.8E-08** |
| | | **CU 2130-1 Post** | 104 | **1.6E-08** |
| | | **UMD 420-2 Pre** | 21 | **2.0E-04** |
| | | **UMD 420-2 Post** | 22 | **9.6E-08** |
| | | **CU 2130-2 Pre** | 85 | **7.6E-06** |
| | | **CU 2130-2 Post** | 78 | **6.6E-08** |
| | | **CU 2170-2 Pre** | 72 | **4.5E-08** |
| | | **CU 2170-2 Post** | 76 | **1.3E-09** |
| | | **CU 2130-3 Pre** | 43 | **7.6E-06** |
| | | **CU 2130-3 Post** | 51 | **2.0E-04** |
| | | **CU 3220-4 Post** | 40 | **0.04** |
| | | **CPP 401-4 Post** | 30 | **0.03** |
| | | CPP 499 Post | 17 | 0.59 |
| | | **CSUF Post** | 17 | **0.03** |
| | | **CU 2130-6 Post** | 68 | **3.5E-11** |
| | | **CU 2130-7 Pre** | 110 | **4.7E-14** |
| | | **CU 2130-7 Post** | 92 | **1.3E-15** |
| | | **CU 2170-7 Pre** | 73 | **0.01** |
| | | **CU 2170-7 Post** | 52 | **1.3E-06** |
| | | CU 3220-7 Pre | 21 | 0.13 |
| | | **CU 3220-7 Post** | 42 | **2.0E-03** |
| | | **CU 2130-8 Pre** | 114 | **6E-10** |
| | | **CU 2130-8 Post** | 105 | **<1E-13** |
| | | **CU 2170-8 Pre** | 148 | **4.4E-16** |
| | | **CU 2170-8 Post** | 103 | **<1E-13** |

| CLASS Item # | Item | Survey | $N$ | p-value |
|---|---|---|---|---|
| 41 | It is possible for physicists to carefully perform the same measurement in a [classical/quantum] physics experiment and get two very different results that are both correct. | CU 2130-1 Pre | N/A | N/A |
| | | CU 2130-1 Post | N/A | N/A |
| | | **UMD 420-2 Pre** | 21 | **0.01** |
| | | **UMD 420-2 Post** | 22 | **7.3E-13** |
| | | **CU 2130-2 Pre** | 85 | **<1E-13** |
| | | **CU 2130-2 Post** | 78 | **1.5E-13** |
| | | **CU 2170-2 Pre** | 72 | **1.5E-14** |
| | | **CU 2170-2 Post** | 76 | **<1.0E-05** |
| | | **CU 2130-3 Pre** | 43 | **<1.0E-13** |
| | | **CU 2130-3 Post** | 51 | **4.1E-12** |
| | | **CU 3220-4 Post** | 40 | **<1.0E-13** |
| | | **CPP 401-4 Post** | 30 | **3.1E-06** |
| | | **CPP 499 Post** | 17 | **4.1E-08** |
| | | **CSUF Post** | 17 | **3.1E-06** |
| | | **CU 2130-6 Post** | 68 | **<1.0E-13** |
| | | **CU 2130-7 Pre** | 110 | **1.6E-13** |
| | | **CU 2130-7 Post** | 94 | **<1.0E-13** |
| | | **CU 2170-7 Pre** | 73 | **7.2E-12** |
| | | **CU 2170-7 Post** | 52 | **<1.0E-13** |
| | | **CU 3220-7 Pre** | 21 | **5.9E-11** |
| | | **CU 3220-7 Post** | 42 | **<1.0E-05** |
| | | **CU 2130-8 Pre** | 114 | **<1.0E-13** |
| | | **CU 2130-8 Post** | 105 | **<1.0E-13** |
| | | **CU 2170-8 Pre** | 148 | **2.0E-15** |
| | | **CU 2170-8 Post** | 103 | **<1.0E-13** |