1. Fabrication of Scanning Probes and Suspended Microdevices

The scanning thermal microscopy (SThM) probes required for this work were custom-fabricated. Fabrication of the SThM probes follows the basic approach outlined in our previous work\(^9\) on nanoscale-resolution scanning probe thermometry. All SThM probes feature a Au-Cr thermocouple that is integrated into the distal end of the probe’s tip. Upon fabrication of the basic probe, subsamples were specifically modified to enable the experiments presented in this work. Probes with dielectric surface materials were coated with SiO\(_2\) or SiN by depositing 100 nm of PECVD SiO\(_2\) or SiN (non-stoichiometric), respectively, on pristine SThM probes. Au-coated probes were prepared by first depositing 100 nm of PECVD SiN on the SThM probes, followed by a 30 nm layer of Au deposited by electron-beam (e-beam) evaporation. The role of the SiN layer is to electrically isolate the outer Au layer from the Cr layer of the thermocouple. Scanning electron microscope images of the probes, obtained after using them in our experiments, are shown in Figs. S1, S2 & S3.

The key steps for fabricating the suspended microdevices, used in high-resolution Au-Au eNFRHT measurements, are shown in Fig. S4. Briefly, the device is made from a Silicon-On-Insulator (SOI) wafer that has a 500 \(\mu\)m-thick Si substrate, a 1 \(\mu\)m-thick buried oxide (BOX) layer and a 10 \(\mu\)m-thick Si device layer (Step 1). A 500 nm-thick, low stress LPCVD SiN (non-stoichiometric) film is first deposited on the wafer for electrical insulation of subsequent metal patterns on the device layer (Step 2), followed by the deposition of a 2 \(\mu\)m-thick layer of low-temperature LPCVD SiO\(_2\) (Step 3). A 100 nm-thick layer of Au is deposited on the SiO\(_2\) layer using e-beam evaporation (Step 4). Using the same mask, the Au layer is etched followed by the etching of the SiO\(_2\) layer using reactive-ion etching (RIE), leaving a square pad from which Au-Au eNFRHT measurements are subsequently made (Step 5). A Pt resistance heater-thermometer with its electrical connections is then patterned on the SiN layer using successive lift-off processes (Step 6). The device profile is formed by RIE through the SiN layer and the Si device layer from the front side of the wafer, stopping at the BOX layer (Step 7). Subsequently, the SiN layer and the SiO\(_2\) layer on the back of the wafer are selectively etched using RIE to open a window for further processing (Step 8). The device is suspended by etching through the Si substrate from the backside using deep reactive-ion etching, stopping at the BOX layer (Step 9). Finally, the BOX layer is removed using a buffered hydrofluoric acid (BHF) etch (Step 10).
Figure S1. Schematic cross-section of the tip of the probe and Scanning Electron Microscope (SEM) images of the SiO₂ coated scanning probe used in our experiments. a, Schematic cross-section of the SiO₂-coated tip. b-d, SEM images of the fabricated probes. The cantilevered scanning thermal probe (b), the tip with the metallic Au and Cr wires (c), a magnified view of the tip (d) and the tip’s apex with the spherical thermocouple portion (e) are shown. Images as shown in (e) were used to estimate the tip diameter (~450 nm).
Figure S2. Schematic cross-section of the tip of the probe and SEM images of the SiN-coated scanning probe used in our experiments. a, Schematic describing the cross-section of the SiN coated tip. b-d, SEM images of the fabricated devices. The cantilevered scanning thermal probe (b), the tip (c), a zoomed image of the tip (d), and the apex, which contains the spherical portion of the tip (e), are shown. From the image shown in (e) the tip diameter was estimated to be ~350 nm.
Figure S3. Schematic cross-section of the tip of the probe and SEM images of the Au-coated scanning probe used in our experiments. **a**, Schematic describing the cross-section of the Au-coated tip. **b-d**, SEM images of the fabricated devices. The cantilevered scanning thermal probe (**b**), the tip (**c**), a zoomed image of the tip (**d**), and the apex, which contains the spherical portion of the tip (**e**), are shown. From the image shown in (**e**) the tip diameter was estimated to be ~900 nm.
Figure S4. Schematic describing the steps in the fabrication of the suspended microdevice. Step 1, SOI wafer, where the buried oxide layer (BOX) is shown. Step 2, deposition of a SiN insulator. Step 3, deposition of LPCVD oxide. Step 4, Au deposition. Step 5, patterning of Au and oxide. Step 6, patterning of Pt resistance thermometer and electrical connections. Step 7, RIE to form the device profile. Step 8, backside etching via RIE. Step 9, DRIE to suspend the device. Step 10, removal of the BOX layer via a BHF etch. Additional details regarding this microdevice fabrication can be found in section 1 of the SI.

2. Substrate Preparation

In order to prepare the dielectric substrates used in the experiments we deposited 100 nm of silicon dioxide (SiO$_2$) or silicon nitride (SiN) on silicon substrates (PECVD). From atomic force microscopy (AFM) analysis the RMS roughness of the SiO$_2$ and SiN surfaces was found to be 0.2 nm and 0.4 nm, respectively (see Fig. S5a & b for AFM images). In estimating this RMS roughness we used data from 200 nm $\times$ 200 nm AFM scans: These dimensions were chosen because they were large enough to represent the area over which eNFRHT measurements were taken. The Au-coated substrates used in the experiments were prepared by deposition of 100 nm of Au on mica using e-beam evaporation. The RMS roughness of the samples prepared using this approach is 0.5 nm. Finally, the RMS roughness of the Au layer (100 nm thick) on the suspended microdevice was also characterized using AFM and found to be ~0.9 nm. Since the surface roughness of all the devices used in our experiments was substantially smaller than that of the probes, the roughness of the surfaces was considered negligible in all our computational analysis.
Figure S5. Atomic force microscopy images of surface roughness. a, Surface topography of a 100 nm-thick SiO$_2$ film deposited on a Si substrate (RMS roughness 0.15 nm). b, Surface topography of a 100 nm thick SiN film deposited on a Si substrate (RMS roughness 0.4 nm). c, Surface topography of a 100 nm thick Au film deposited on mica (RMS roughness 0.5 nm). d, Surface topography of a 100 nm thick Au deposited on the suspended microdevice (RMS roughness 0.9 nm). All the scanning areas are 200 nm × 200 nm in size.

3. Ultra-Low Noise Measurement Environment

The experiments described in this work were performed in a UHV-scanning probe instrument housed in an ultra-low noise facility where the ground vibrations were attenuated to meet the stringent NIST-A criterion$^{33}$. The temperature of the facility was controlled to vary <0.1 K about a chosen set point (294 K). The acoustic noise in the measurement chamber in which our scanning probe microscope is operated, is below 30 dB SPL (sound pressure level in dB with respect to a 20 μPa reference level) in the frequency range from 60 – 100 Hz, and declines
monotonically from this value with increasing frequency throughout the entire acoustic range. At 1 kHz, for example, the SPL decreases to \(-15\) dB.

4. Displacement of the SThM Probe Towards Heated Substrate and Measurement of Thermoelectric Voltage Output

In unmodulated experiments the SThM probe was displaced towards the heated substrate at a rate of 0.5 nm/s, and the thermoelectric voltage output from the SThM probe was measured in a bandwidth of 5 Hz. In modulated measurements the temperature of the microdevice was modulated by 5 K at 18 Hz when the gap size was larger than 6 nm. At smaller gap sizes the amplitude was reduced to 2.5 K to attenuate mechanical deflections from bimaterial effects to \(<0.3\) nm (see section 11). The gap size in the unmodulated measurements was referenced to the point at which snap-in occurs. This was found to be adequate for accurately quantifying gap size as the mechanical drift in the experiments during the time of the measurement (100 s) was negligibly small (\(~0.1\) nm). However, in the modulated experiments it took \(~1000\) s to acquire each data point in Fig. 3d of the manuscript, necessitating measurements that lasted over 12 hours in total during which drift is significant. Therefore, we performed individual gap size calibrations for each data point \((i.e.\) every 1000 s period) by displacing the probe towards the microdevice until contact was made. This enabled direct measurements of the gap size for each data point shown in Fig. 3d. In all experiments, the measured thermoelectric voltages were related to the temperature rise via the calibrated\(^9\) Seebeck coefficient \((16.3 \, \mu\text{V/K})\) of the Au-Cr junction.

5. Quantification of the Distance at which Snap-in Occurs

Snap-in is a sudden contact between the tip and the substrate, which takes place when the tip at the end of the SThM cantilever, is brought into close proximity (maximally a few nanometers or less) of a (planar) surface/substrate. Snap-in occurs primarily due to instabilities that arise from electrostatic and near-field forces that have a non-linear distance dependence. Snap-in limits the smallest gaps at which we are able to perform eNFRHT measurements.

The snap-in distance was quantified using approaches developed in the past in the field of scanning probe microscopy\(^34\). Briefly, we performed experiments where we monitored the deflection of the cantilever using an optical scheme when the gap size between the tip and the
substrate was systematically reduced. Figure S6 presents a representative trace of the deflection signal (photodiode signal) from one such experiment. As illustrated in the insets of the figure, the cantilevered portion of the probe is initially in an unbent state. When the probe snaps-in the cantilever bends downwards due to the attractive forces between the tip and the substrate. Upon further displacement of the probe, the cantilever is returned to its “unbent state”. The additional displacement required to reach this unbent state represents the snap-in distance as indicated in Fig. S6. From measurements like these, the snap-in distance in experiments with SiO2, SiN and Au probes (where the temperature was not modulated) was estimated to be 1.9 ± 0.4 nm, 2.4 ± 0.3 nm, 2.2 ± 0.5 nm, respectively. The snap-in distance for modulated Au-Au measurements was similarly estimated to be 2.8 ± 0.2 nm.

Figure S6. Quantification of snap-in distance. A typical trace showing the deflection of the cantilever of the probe as it is displaced towards the substrate. Cantilever deflection is measured via an optical signal that is quantified by a photodiode detector (not shown).
Figure S7. Measurement of thermal resistance of the SThM probes. a. Schematic of the measurement approach employed to determine the thermal resistance of the probes. b-d, Measured tip temperature rise ($\Delta T_P = T_P - T_R$) as a function of the heat current ($Q_P$) for SiO$_2$ (b), SiN (c) and Au (d) coated probes. Insets show the measured thermal resistance ($R_P$).

6. Thermal Resistance Measurements of Scanning Probes

In order to experimentally measure the thermal resistance of the SThM probes we followed a procedure similar to that described in recent work from our group$^{22}$. Briefly, we employed suspended calorimeters (Fig. S7a) with embedded platinum heater-thermometers and input known quantities of heat into the suspended region. This resulted in a temperature rise that could be accurately measured using the embedded platinum thermometers. Subsequently, we placed each of the SThM probes used in the experiments in contact with the heated suspended calorimeter (at a pre-determined contact force). This resulted in heat loss through the point-contact between the tip and the calorimeter that in turn is reflected as a lower temperature rise of
the calorimeter. The decrease in the temperature of the calorimeter was again quantified via the Pt thermometer. The difference in the measured temperatures before and after contact formation enabled us to determine the heat flow/input \( (Q_P) \) through the SThM probe. Further, using the thermocouple embedded in the tip of the SThM probe we also simultaneously measured the temperature rise \( (\Delta T_P) \) of the tip upon contacting the calorimeter. This enabled us to readily determine the thermal resistance \( (R_P) \) of the SThM probe using: \( R_P = \Delta T_P / Q_P \). Figures S7b, c & d show the measured temperature rise of SiO\(_2\), SiN and Au coated probes, respectively, as a function of \( Q_P \). From these data the thermal resistance of the probes \( (R_P) \) was readily obtained and is shown in the insets of Fig. S7b, c & d. Please see Ref. 22 for more details of this thermal resistance measurement scheme.

7. Noise Characterization of Thermoelectric Voltage of SThM Probes

The thermoelectric voltage output from the thermocouple embedded in the SThM probe has noise contributions from Johnson noise, low frequency temperature drift and other sources. This noise was quantified by computing the power spectral density (PSD, Fig. S8) of the output from the thermocouple using a SR760 FFT analyzer from Stanford Research Systems. The signal was first amplified using a two stage custom built amplifier with a gain of \( 10^4 \) prior to signal processing. The measured PSD in Fig. S8 clearly shows that at low frequencies noise increases rapidly with decreasing frequency. In unmodulated measurements which employ a bandwidth of 5 Hz, the noise-limited temperature resolution is estimated to be \( \sim 15 \) mK. It can also be seen that the temperature resolution is greatly improved when modulated measurements are performed at a frequency above 10 Hz (where the noise spectrum becomes flat). Specifically, if the temperature of the hot substrate is modulated at 18 Hz and the thermoelectric voltage is measured in a bandwidth of 0.8 mHz, the temperature resolution \( (\Delta T_{\text{Res}}) \) can be as high as \( \sim 20 \) µK. Once the temperature resolution is known, the corresponding heat flow resolution \( (Q_{\text{Res}}) \) is obtained via 
\[
Q_{\text{Res}} = \Delta T_{\text{Res}} / R_P .
\]
For the Au-coated probes which have a resistance of \( R_P = 0.7 \times 10^6 \) K/W, an unmodulated temperature measurement results in a heat flow resolution of \( \sim 22 \) nW, while a resolution as high as 30 pW can be achieved with 18 Hz temperature modulation. Further, since the gap thermal conductance is given as the ratio between heat flow and its driving temperature difference, one can also estimate the thermal conductance resolution. Specifically, a thermal
conductance as small as 220 pW/K can be detected when an unmodulated temperature differential of ~100 K is applied, whereas thermal conductances as small as 6 pW/K can be detected when modulated temperature differentials of ~5 K are applied.

Figure S8. Noise characteristics of the probe thermoelectric voltage signal (referenced to the input). Measured PSD of the thermoelectric voltage output from a Au-coated probe. The red plot shows the PSD in nV/Hz$^{1/2}$, whereas the blue line presents the PSD in units of nW/Hz$^{1/2}$.

8. Modulation of the Temperature of the Microdevice and Measurement of the Amplitude of Temperature Modulation

The microdevice temperature was modulated sinusoidally (18 Hz) at an amplitude of 5 K by supplying a sinusoidal electric current (amplitude 0.49 mA, frequency 9 Hz) into the serpentine Pt line integrated into the microdevice. A correspondingly smaller current (0.35 mA) was applied in measurements at smaller gaps, which employed a 2.5 K temperature differential. All temperature oscillations were quantified by measuring the voltage oscillations at $3f = 27$ Hz across the Pt line as described in our previous work$^{24}$.

9. Frequency Response of the Suspended Microfabricated Devices and the SThM Probes

The frequency response of the suspended microdevice was characterized following an approach described in our previous work on calorimetry$^{24}$. Briefly, we systematically increased the frequency ($f$) of the sinusoidal electric current input into the integrated resistance heater-thermometers embedded into the microdevice while maintaining a constant amplitude (400 µA).
The temperature fluctuations of the device at $2f$ were measured by monitoring the voltage fluctuation at $3f$.

Figure S9a shows the normalized amplitude of temperature oscillations as a function of the frequency of temperature oscillations ($2f$) for the suspended microdevice. It can be seen that the temperature response is relatively constant up to a frequency of 20 Hz, clearly showing that temperature can be readily modulated at frequencies below this cut-off. In order to measure the thermal frequency response of the SThM probe we focused a laser beam at the very end of the cantilevered portion of the probe and modulated the laser power sinusoidally at a range of frequencies while maintaining a constant amplitude. The oscillating tip temperature was then recorded via the embedded thermocouple. Figure S9b shows that the measured frequency response of the probe remains flat until a heating frequency of about 10 Hz and drops by ~13% at 18 Hz, which was systematically accounted for in our data analysis.

![Figure S9a](image1.png)

![Figure S9b](image2.png)

**Figure S9.** Frequency response of the suspended microdevice and the SThM probes. Normalized amplitude of temperature oscillations as a function of frequency for the microdevice (a) and the Au-coated SThM probe (b). The amplitudes of temperature oscillations were normalized to peak values, which occur at the lowest measurement frequency.

**10. Characterization of the Thermal Conductance of Suspended Microdevices**

The thermal conductance of the suspended microdevices was characterized using an approach similar to that described in our previous work on picowatt resolution calorimetry. In order to measure the thermal conductance we input known amounts of heat into the suspended region of the device and measured the corresponding temperature rise. The relationship between the power
input \((Q_{in})\) and the measured temperature rise \((\Delta T)\) is shown in Fig. S10 and can be used to readily estimate the thermal conductance of the beams of the device \((G_{beams})\) via \(G_{beams} = Q_{in} / \Delta T\), which is about 175 \(\mu\)W/K. Please see ref. 24 for more details on the scheme used for characterization of the thermal conductance.

**Figure S10. Thermal conductance of the suspended microdevice.** The relationship between the temperature rise of the suspended region and the power dissipated is given (experiments were performed in UHV conditions to eliminate contributions from conduction through air). The slope of the plotted line represents the conductance of the beams of the suspended microdevice.

11. Characterization of the Thermally-Induced Deflections of Suspended Microdevices

Sinusoidal heating of the suspended microdevice results in both periodic temperature oscillations as well as thermally induced mechanical deflections of the suspended region due to bimaterial effects. These mechanical oscillations, if large, can potentially impact the smallest gap sizes that can be achieved between the tip and the suspended device. In order to quantify these thermally induced deflections we first placed a SThM probe in good mechanical contact (contact force of \(\sim 150\) nN) with the suspended device. The laser light reflecting off the probe (see Fig. 1) into a quadrant photodiode provides the feedback signal to a control loop (control loop bandwidth > 1 kHz). The control loop supplies a feedback voltage to the piezoelectric tube onto which the SThM probe is mounted and acts to displace the piezoelectric tube and the SThM probe by an amount equal to the deflection of the microdevice so as to maintain a constant probe deflection. Figure S11 shows the displacement of the piezoelectric tube when the microdevice temperature
was modulated by 12 K at 18 Hz. It can be seen that the amplitude of the displacement is ~1.5 nm, which in turn implies that the sensitivity is ~0.12 nm/K (=1.5 nm/12 K). Data obtained at lower temperature modulations (not shown) indeed confirms this result. We note that in our experiments, the amplitude of temperature oscillations employed for measurements in small gaps (<6 nm) is limited to 2.5 K. This corresponds to mechanical oscillations of amplitude ~0.3 nm which are significantly smaller than the snap-in distance (~3 nm) observed in the modulated Au-Au measurements. Hence the effect of mechanical oscillations is negligible.

Figure S11. Characterization of the thermally induced mechanical deflections of suspended microdevices. The measured piezoelectric displacement under feedback control, when the temperature of the microdevice was sinusoidally (18 Hz) modulated by 12 K. From this data it can be readily inferred that the amplitude of deflection of the microdevice is ~0.12 nm/K.

12. Measured Dielectric Properties of Silicon Nitride

Modelling of near-field radiative heat transfer requires the frequency-dependent complex dielectric function of materials as a key input. It is well-known that even for nominally identical materials, the dielectric functions may differ due to variations in actual material compositions and microstructures as a function of preparation methods and conditions. The use of different dielectric functions for the same nominal material could lead to significant discrepancies among calculated near-field heat fluxes, analogous to what has been reported in the case of Casimir force\textsuperscript{35} calculations.
While there is relatively little uncertainty in the composition of SiO$_2$ and Au films the stoichiometry of SiN films is sensitive to deposition conditions. Hence the frequency-dependent complex dielectric function of the SiN films deposited on the substrates used in our measurements was characterized using two Woollam™ spectroscopic ellipsometers (VUV-VASE and IR-VASE). Three angles of incidence (55°, 65° and 75°) were used and a wavelength range from 137 nm to 40 µm was covered. These measurements were performed to accurately account for the fact that the properties (porosity, chemical composition, dielectric function, etc.) of SiN films could vary significantly with different deposition methods and parameters. The dielectric function obtained from these measurements and subsequent analysis$^{36}$ is shown in Fig. S12b.

![Dielectric Functions](image)

**Figure S12. Dielectric functions.** a-c, Real and imaginary parts of the dielectric functions employed in our simulations as a function of energy for SiO$_2$, SiN, and Au.

13. Formalism Employed for Computing Radiative Heat Transfer

In order to model the radiative heat transfer for the configurations experimentally studied by us we employed the fluctuating-surface-current (FSC) formulation of the heat transfer problem that has been recently put forward by one of us in collaboration with others$^{13,25}$. This novel approach is based on the surface-integral-equation (SIE) formulation of classical electromagnetism and allows direct application of the boundary element method (BEM). In this method the electromagnetic scattering problem is solved by considering a set of linear equations involving a number of surface unknowns (fictitious surface currents in the surfaces of the objects). The FSC-BEM combination allows describing the radiative heat transfer between bodies of arbitrary shape, and can provide numerically exact results within the framework of fluctuational electrodynamics. In practice, we use the implementation of this approach provided in the open-source SCUFF-EM
software package\textsuperscript{26,37} which was developed by one of us. This code makes use of the BEM to discretize the surfaces of the bodies into triangular elements or panels and the surface currents in each element are described by piecewise low-degree polynomials. In particular, SCUFF-EM employs the so-called RWG\textsuperscript{38} basis of vector-valued polynomial functions defined on a mesh of triangular panels. This basis is suitable to deal with arbitrary geometries and yields results that converge with increasing resolution (smaller triangles). Further technical details can be found in refs. 13 and 25.

In our theoretical approach we assume that the radiative heat transfer can be well described in terms of dielectric functions that only depend on frequency or energy (local approximation). For our calculations we took the dielectric function of SiO\textsubscript{2} and Au from Palik\textsuperscript{39}, and Ordal et al.\textsuperscript{40}, respectively. The dielectric function for SiN was characterized by us as described in section 12 above. All dielectric functions used in this work are shown in Fig. S12 for the energy range relevant for heat transfer in our experiments.

14. Tip-Substrate Geometries and Convergence of Simulations

In order to provide a quantitative description of our experiments, we considered tip-substrate geometries like the one shown in Fig. S13a. The tip has a conical shape and ends in a spherical cap. The angle of the cone and the radius of the spherical cap have been obtained in every case from the SEM images of the experimental probes. For the cases discussed in the manuscript the tip radii were 450 nm for Au, 225 nm for SiO\textsubscript{2}, and 175 nm for SiN. The height of the tip was chosen to be 3 \( \mu \)m for Au and 1.3 \( \mu \)m for SiO\textsubscript{2} and SiN. The substrate was modelled by a finite disk of radius 4 \( \mu \)m for Au and 2 \( \mu \)m for SiO\textsubscript{2} and SiN, and thickness 2 \( \mu \)m for Au and 1 \( \mu \)m for SiO\textsubscript{2} and SiN. The dimensions of both the tip and the substrate were carefully chosen so that there are no finite-size effects. Moreover, as explained in the manuscript, we have also simulated the roughness of our probes by including random Gaussian-correlated noise in the tip profile. To be precise, the maximum protrusion height on the tip of the probes was chosen to be 10 nm, and the correlation length between protrusions was chosen to be 17 nm.

A key issue in our numerical simulations is the choice of the mesh of triangular panels for the RWG-BEM approach. To obtain accurate results, the size of the triangular panels must be comparable to the (local) gap size or smaller. This was accomplished by employing a non-uniform grid that was finer at both the tip apex and in the centre of the plate (Fig. S13a).
Figure S13. Simulated tip-substrate geometry and numerical convergence. a, Example of the tip-substrate geometries employed in our numerical simulations. The tip has a conical shape and ends in a spherical cap, while the substrate is modelled as a thick disk. The solid white lines correspond to the mesh of triangular panels used in the BEM calculations. The right inset shows a blow-up of the tip apex region. Here, $R$ is the tip radius, and $d$ is the gap size (or distance between the tip and the substrate). b, Example of a convergence test of the numerical results upon refinement of the mesh of triangular panels for a SiO$_2$ tip-substrate geometry with a tip radius of 200 nm and a gap size of 4 nm. In this case, the tip was assumed to not have any roughness. The different curves correspond to different numbers of basis functions employed in the BEM calculations as indicated in the figure. Notice that the two curves for the largest numbers of basis functions lie on top of each other illustrating the excellent convergence of our results. The reservoir temperatures in this calculation were assumed to be 310 K for the tip and 425 K for the substrate.
The convergence of our results was checked, for every combination of materials and every tip-substrate distance, by progressively refining the mesh, i.e. by reducing the size of the triangles and increasing the number of basis functions in the BEM calculations. In Fig. S13b we show an example of our convergence tests for the case of a SiO$_2$ tip-substrate geometry. It can be seen that the results progressively converge to a single solution (or spectral conductance) as the number of basis functions is increased. For calculations that included the presence of tip roughness, we have defined a smaller spherical cap around the tip apex where the triangular panels were chosen to have an equal size. The same was done in the substrate for a circular region around its centre. Again, the size of these triangles was checked to be sufficiently small to obtain converged results. We note that the convergence of the results with the size and the number of triangles depends on the material of choice. For metals (Au) the convergence is faster than for polar dielectrics (SiN, SiO$_2$) due to the different physical mechanism that dominates the NFRHT. In metals, NFRHT is governed by total internally reflected waves and does not depend critically on the distance between the tip and the plate. In contrast, NFRHT in polar dielectrics is dominated by surface phonon polaritons (SPhPs), electromagnetic waves whose dispersion relation is very sensitive to the distance between the two objects. Thus, the simulations for polar dielectrics require much more refined grids with considerably higher number of basis functions, i.e. with much smaller triangular panels.

15. Spectral Calculations of eNFRHT for SiN Gaps

For completeness, we report here the results for SiN that were not included in the manuscript. In particular, we show in Fig. S14a the spectral heat conductance for a SiN tip-substrate geometry for three different gap sizes. As one can see, the major contribution to heat transfer comes from an energy region around 0.12 eV, which corresponds to the energy of the transverse optical phonons in this material. As in the case of SiO$_2$, the coupling of optical phonons to electromagnetic waves gives rise to SPhPs that dominate the NFRHT. For this reason, the NFRHT rapidly decays with the gap size, very much like in the SiO$_2$ case and at odds with the Au case. An important conceptual difference between SiO$_2$ and SiN is that for SiN the real part of the dielectric constant never becomes negative, see Fig. S12. This means in practice that, even in the energy region where the optical phonons exist, SiN behaves as a lossy dielectric material. However, one can show that it is still possible to have surface electromagnetic waves in this
material with properties that are similar to those of SPhPs. The surface waves in the interface between a dielectric like vacuum and a lossy dielectric like SiN are often referred to in the literature as Zenneck waves\textsuperscript{41}.

The similarities between SiN and SiO\textsubscript{2} are also evident in the spatial distribution of the surface Poynting vector, as illustrated in Fig. S14b. As in the SiO\textsubscript{2} case, the radiative heat transfer is very much concentrated at the tip apex as a consequence of the fact that NFRHT is dominated in this case by surface electromagnetic waves with very small penetration depths. Again, this is clearly at variance with the observations in the Au case (Fig. 4d).

![Figure S14. Spectral conductance and spatial distribution of the Poynting-flux for SiN. a, Spectral conductance as a function of the energy for a SiN tip-sample geometry for three different gap sizes. The tip radius is 175 nm and the reservoir temperatures are 310 K for the tip and 425 K for the substrate. Notice the logarithmic scale in the vertical axis. b, Surface-contour plot showing the spatial distribution of Poynting-flux pattern on the surface of the bodies for the SiN tip-substrate geometry of panel a with a gap of 1 nm. The colour scale has units of W/(K·eV·m\textsuperscript{2}) and the plot has been computed at an energy of 0.12 eV, which corresponds to the maximum of the spectral conductance. The right inset shows the corresponding surface heat flux on the substrate, while the left inset displays the entire tip-sample geometry simulated along with the mesh used in the calculations.]

16. Role of the Tip Roughness in the Calculations of eNFRHT

As explained both in the manuscript and in section 14, in our simulations we have taken into account the roughness present in our experimental probes. For completeness, we illustrate here the impact of the tip roughness. Towards this goal, in Fig. S15 we compare the results with and without roughness for the three materials investigated in this work (SiO\textsubscript{2}, SiN and Au).
Figure S15. Role of the tip roughness. a, Computed near-field radiative conductance as a function of the gap size for a SiO$_2$ tip-sample geometry. The tip radius is 225 nm, and the reservoir temperatures are 310 K for the tip and 425 K for the substrate. The solid blue line corresponds to the average result obtained for 15 different tips featuring a roughness of 10 nm, while the blue shaded region represents the corresponding standard deviation. The red solid line corresponds to the result for an ideal tip without any roughness. The black dashed line corresponds to the result for an ideal tip without any roughness. The black dashed line corresponds to the result obtained using the proximity approximation, see text. b, The same as in panel a, but for a SiN tip-sample geometry. In this case the tip radius is 175 nm, and the reservoir temperatures are 310 K for the tip and 425 K for the substrate. c, The same as in panel a, but for a Au tip-sample geometry. In this case the tip radius is 450 nm, and the reservoir temperatures are 300 K for the tip and 301 K for the substrate.
From this comparison, one can draw two main conclusions: (i) the presence of roughness tends to reduce the radiative heat conductance and (ii) the roughness has a larger impact on the conductance of polar dielectrics. The first property is a simple consequence of the fact that the gap size is defined as the shortest tip-sample distance (following our AFM experiments). Thus, the presence of roughness effectively leads to an increase in the average tip-sample distance, as compared with the ideal tip with no roughness. This fact leads naturally to a reduction of the radiative heat transfer at a given gap size. Notice also that the impact of the roughness is obviously larger for the smallest gaps, where it can lead to a reduction of the conductance on the order of a factor of two, while it is negligible when the gap size becomes larger than the natural scale of the roughness (10 nm in our case). Furthermore, the larger impact of the roughness in the case of SiO₂ and SiN, as compared with Au, is again due to the fact that in polar dielectrics the radiative heat transfer is much more localized in the tip apex due to the excitation of SPhPs with very short penetration depths.

To conclude, it is interesting to compare the results for the ideal tips (with no roughness) with those obtained using the so-called proximity or Derjaguin approximation⁴², which is frequently used to estimate the radiative heat transfer in complex geometries. For this purpose, we assume that the tips can be modelled by spheres of the same radius and compute the NFRHT between a sphere and an infinite plate. Within the proximity approximation, this calculation is done by assuming that the sphere is sliced into a series of infinitesimal annuli of different radii and the conductance between every annulus and the substrate is computed using the results for the NFRHT in a plate-plate geometry of the corresponding material. A more detailed description of this approximation can be found in section IIIA of the supplementary information of ref. 6. The results obtained with this approximation for the three materials considered in this work are shown in Fig. S15 (black dashed lines). It can be seen that this simple approximation provides a very good estimate of the gap-size dependent NFRHT in the case of the ideal tips. Therefore, it is clear that for ideal tips the proximity approximation can be used as a good first approximation to estimate the radiative heat transfer.
References: