

SVD/MCMC Pipeline for Separating the Global 21-cm Signal from Foregrounds/Systematics

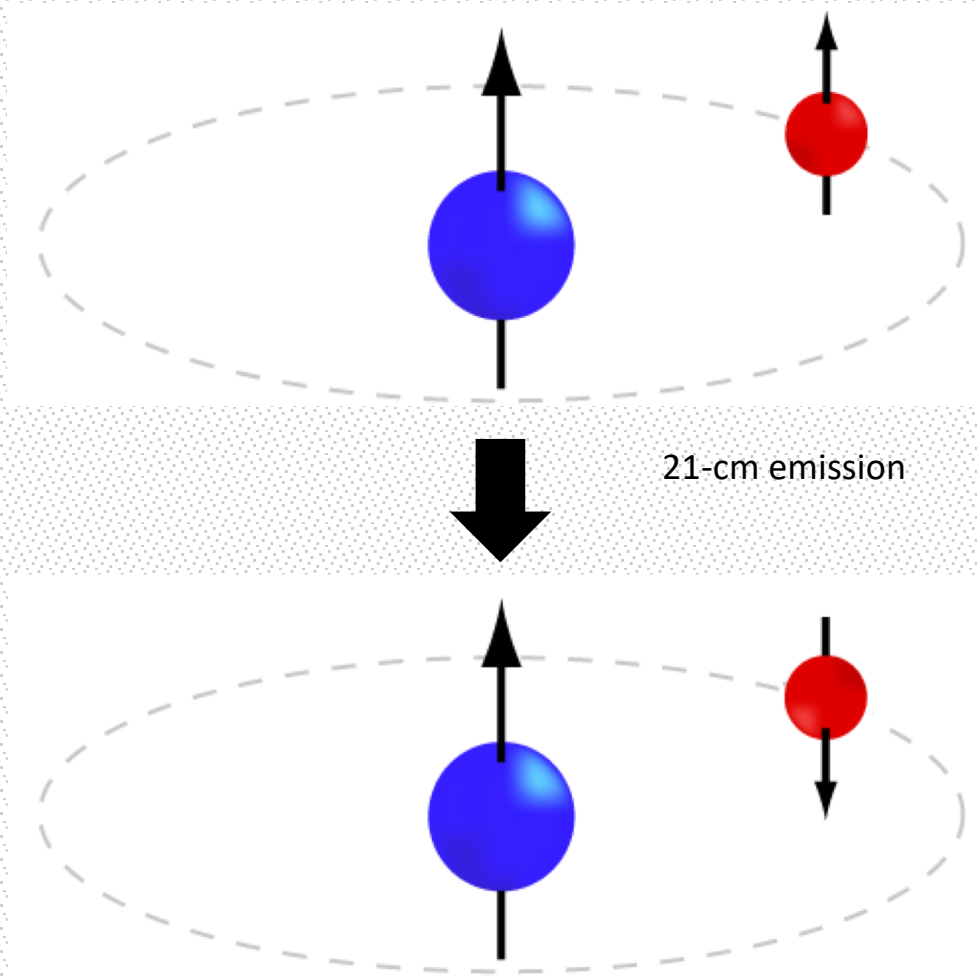
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The work presented here is in collaboration with:

Keith Tauscher (CU Boulder), Jack Burns (CU Boulder), Eric Switzer (NASA Goddard), Jordan Mirocha (UCLA), Steven Furlanetto (UCLA)

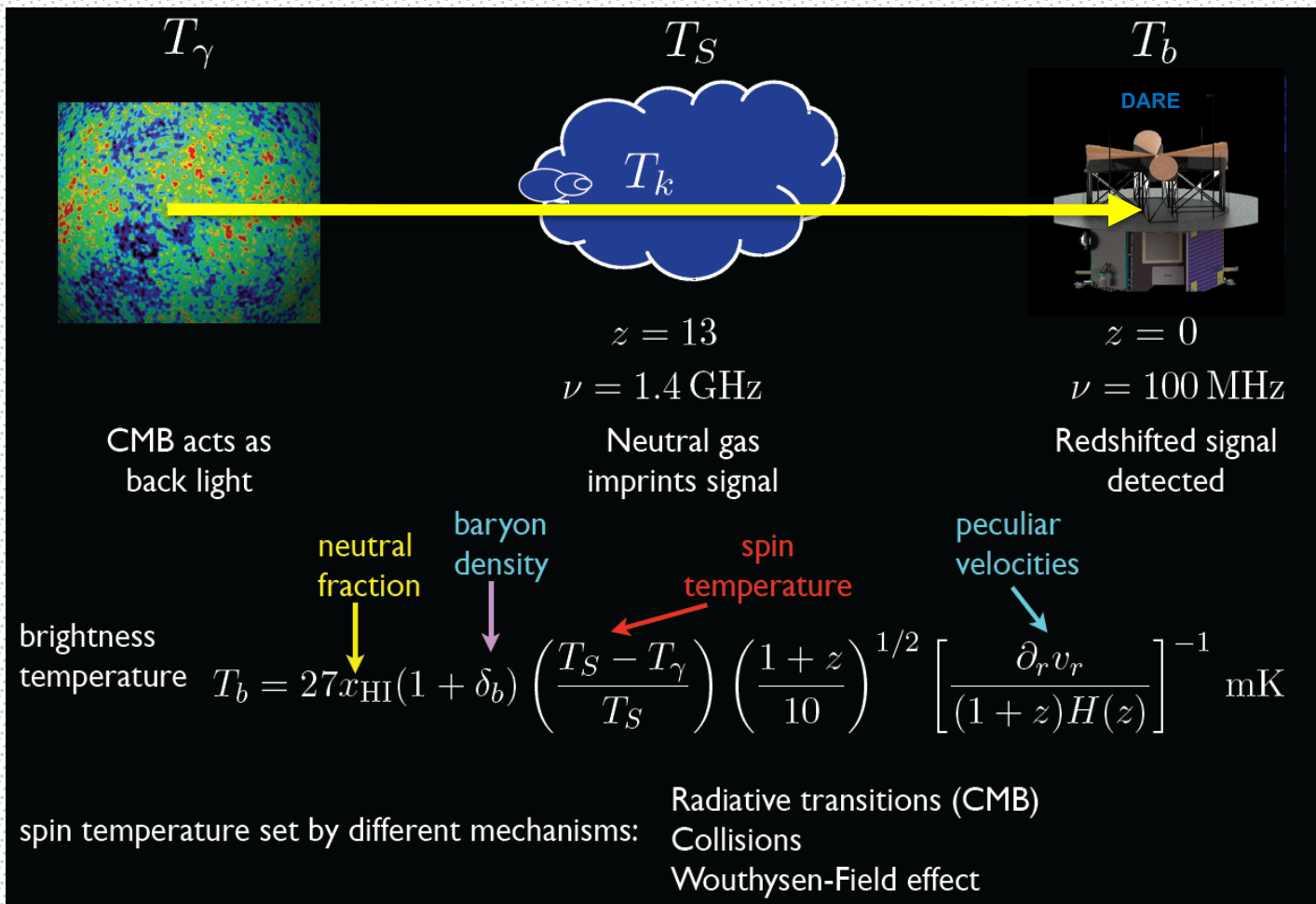
21-CM HYPERFINE TRANSITION



Images from Wikipedia

- Transition that occurs in **neutral hydrogen**.
- The proton and electron have **magnetic moments** due to their **spin**.
- The energy difference between these moments being aligned and anti-aligned is equal to the energy of a photon with a rest wavelength (frequency) of **21cm** (1420 MHz).

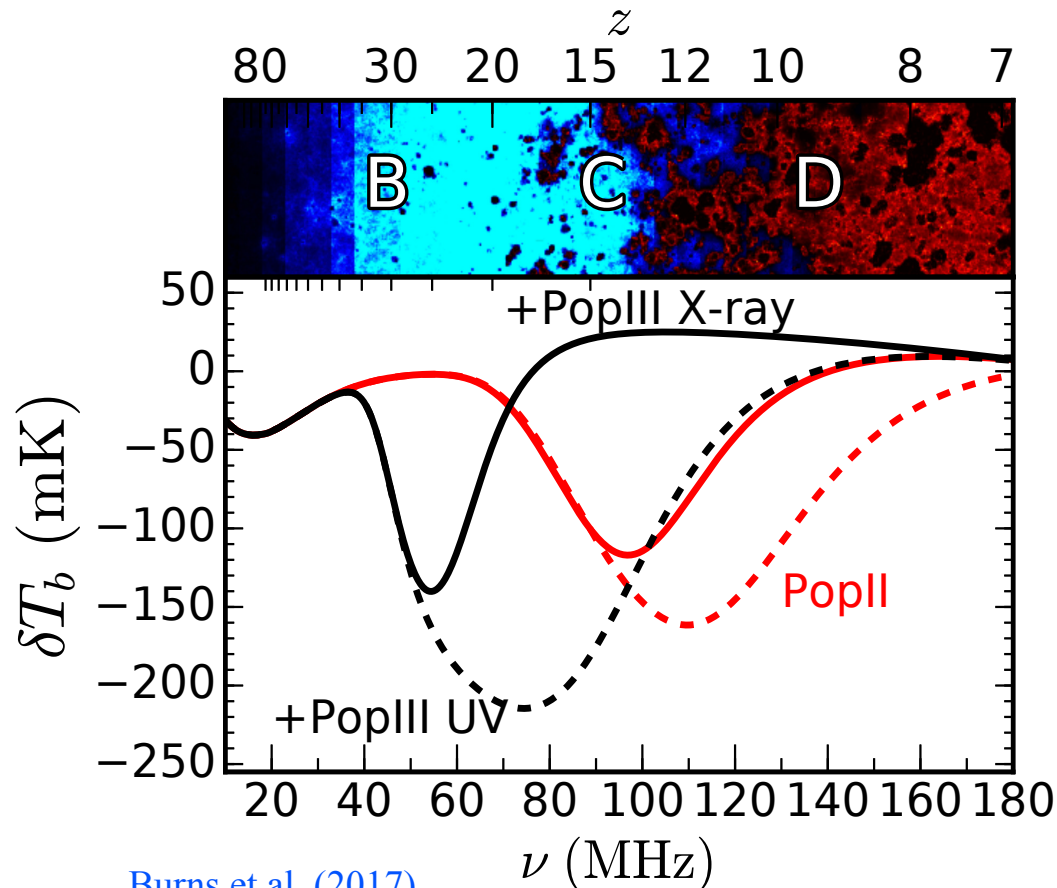
21-CM HYPERFINE TRANSITION



- The multi-wavelength radiation emitted by these **first luminous objects** shifted the **spin-flip temperature** of the intergalactic medium (IGM) gas.
- This **changed** the **strength of the 21-cm line** of HI (Madau et al. 1997, Mirocha et al. 2017) with respect to that of the Cosmic Microwave Background (CMB).

Image credit: J. Burns

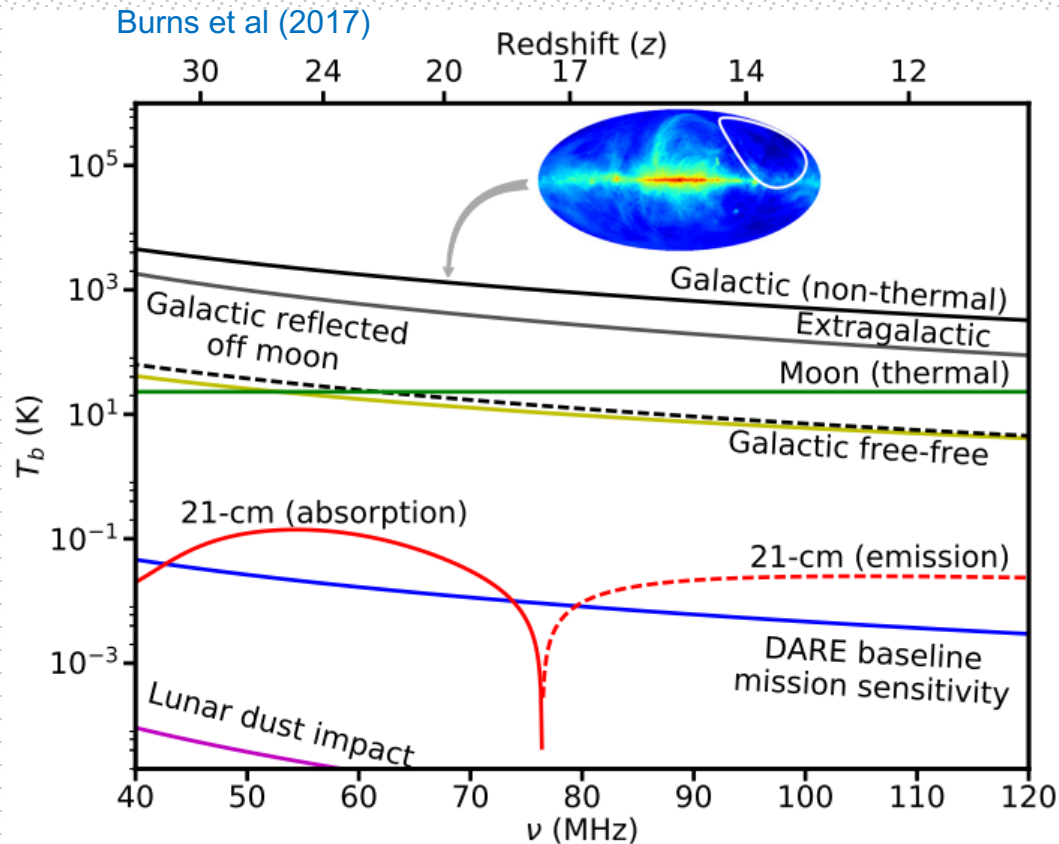
HYDROGEN COSMOLOGY



Burns et al. (2017)

- Upper panel: Evolution of a Universe's slice from early (left) to late times (right).
- Lower panel: Standard models of the global 21-cm spectrum relative to the CMB temperature; red models with metal-rich stars (Pop II), black curves assume that metal-free stars (Pop III) also occur, but only in low-mass galaxies where atomic cooling is inefficient. The dashed and solid curves differ in specific emission and stellar properties (see Burns et al. 2017 for details).
- The epochs B, C and D correspond to the ignition of the first stars, the initial accretion of black holes, and the onset of reionization, respectively.
- Figure from adapted in turn from Pritchard & Loeb (2010) using newer models from Mirocha et al. (2017).

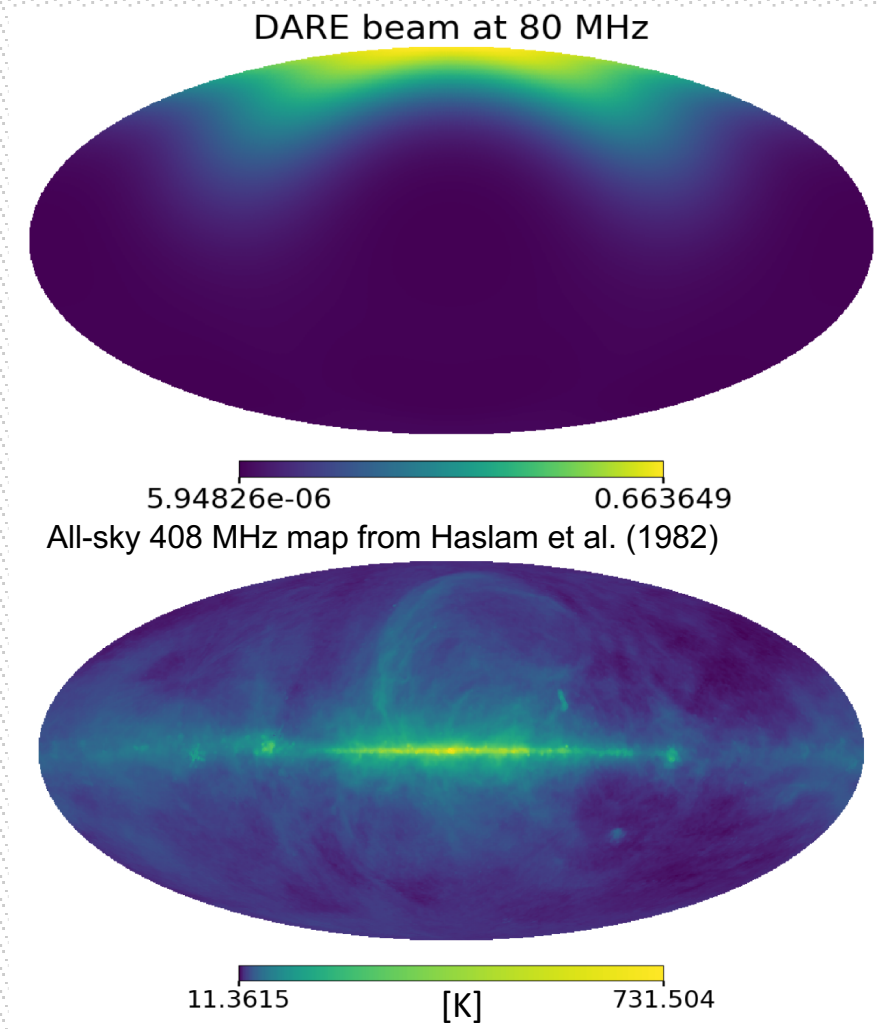
BIGGEST CHALLENGES IN MEASURING THE GLOBAL 21-CM SIGNAL



- Unavoidable (beam-averaged) **foregrounds** which are $> \sim 10^4$ times larger than the **signal**.
- **Beam chromaticity** mixes spatial and spectral structure of the foregrounds with that of the instrument.
- Also: Ionospheric effects (see Datta et al. 2016), RFI, radio Solar emissions.

As part of the solution: The pipeline utilizes the differences between the signal and foregrounds in **spatial and spectral variations** as well as **polarization**.

FOREGROUND TRAINING SET

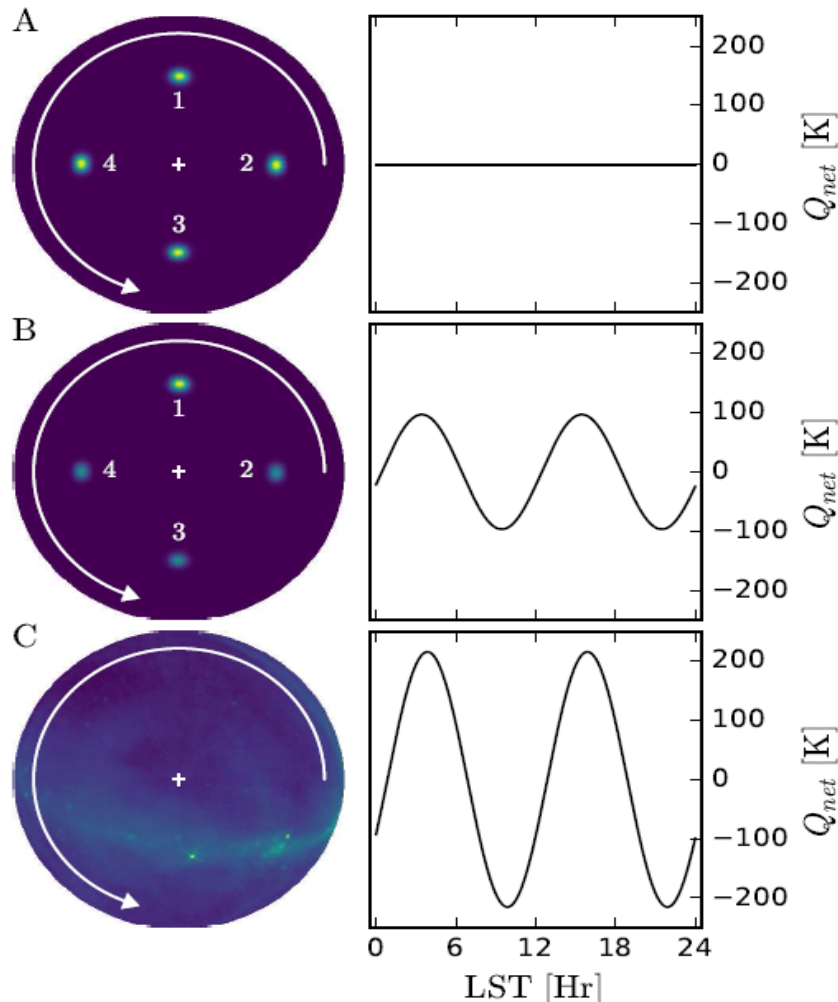


- Antenna temperature simulated convolving beam, $B(\nu, \Omega)$, and sky, $T_{sky}(\nu, \Omega)$, through

$$T_A(\nu) = \frac{\int B(\nu, \Omega) T_{sky}(\nu, \Omega) d\Omega}{\int B(\nu, \Omega) d\Omega}$$

- CST code used to model beam
- Sky maps from Guzmán et al. (2010) and Haslam et al. (1982)

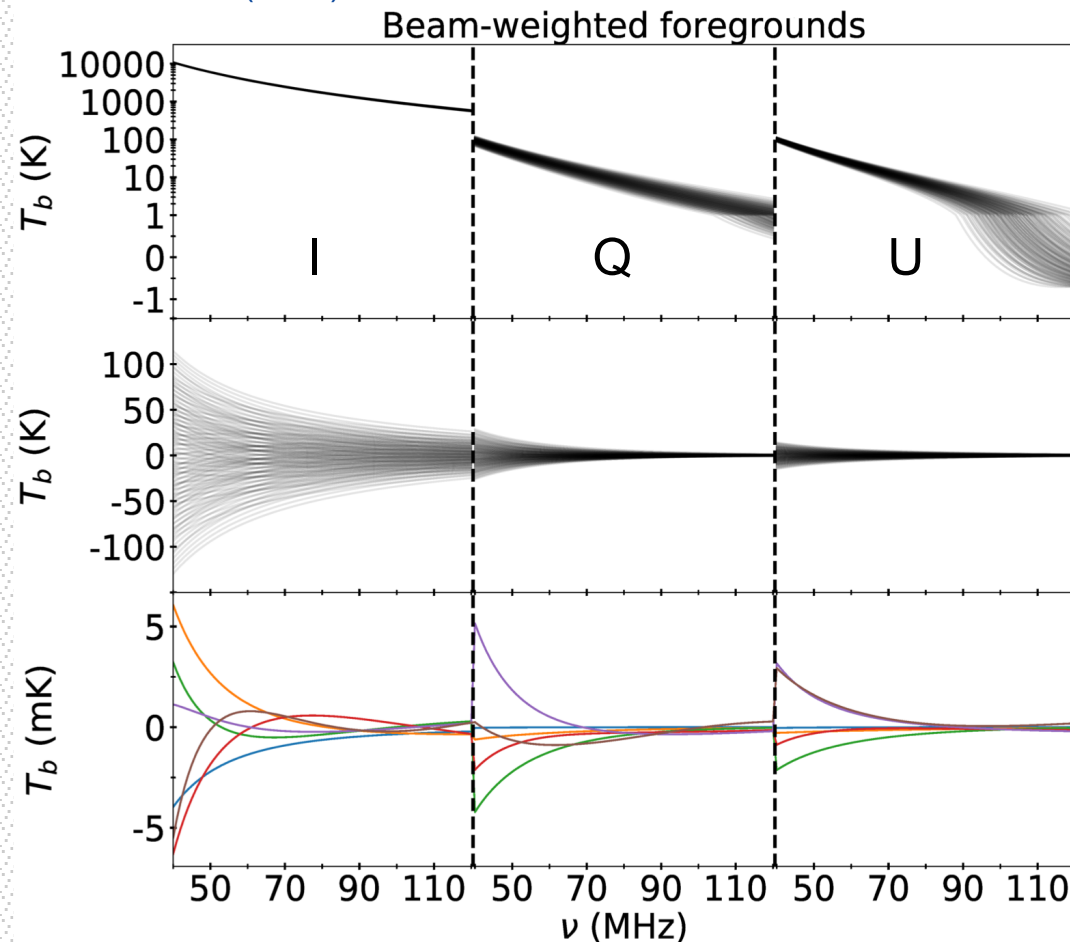
EXPERIMENTAL DESIGN: INDUCED POLARIZATION



- Left: The observed source distribution where the cross is the pointing center and the white arrow is the rotation direction of the antenna.
- Right: The net Q Stokes vector as measured by a symmetric Gaussian beam at 60 MHz.
- **Case A:** Four identical sources symmetric about the boresight resulting in a net zero polarization vector under rotation.
- **Case B:** Four sources symmetric about the boresight where source one is enhanced in intensity resulting in a non-zero net polarization vector.
- **Case C:** The real sky resulting in a non-zero net polarization vector.
- See Nhan et al. (2017) for further details.

EXPERIMENTAL DESIGN: INCLUDING STOKES PARAMETERS INTO THE LIKELIHOOD FUNCTION

Tauscher et al. (2018)

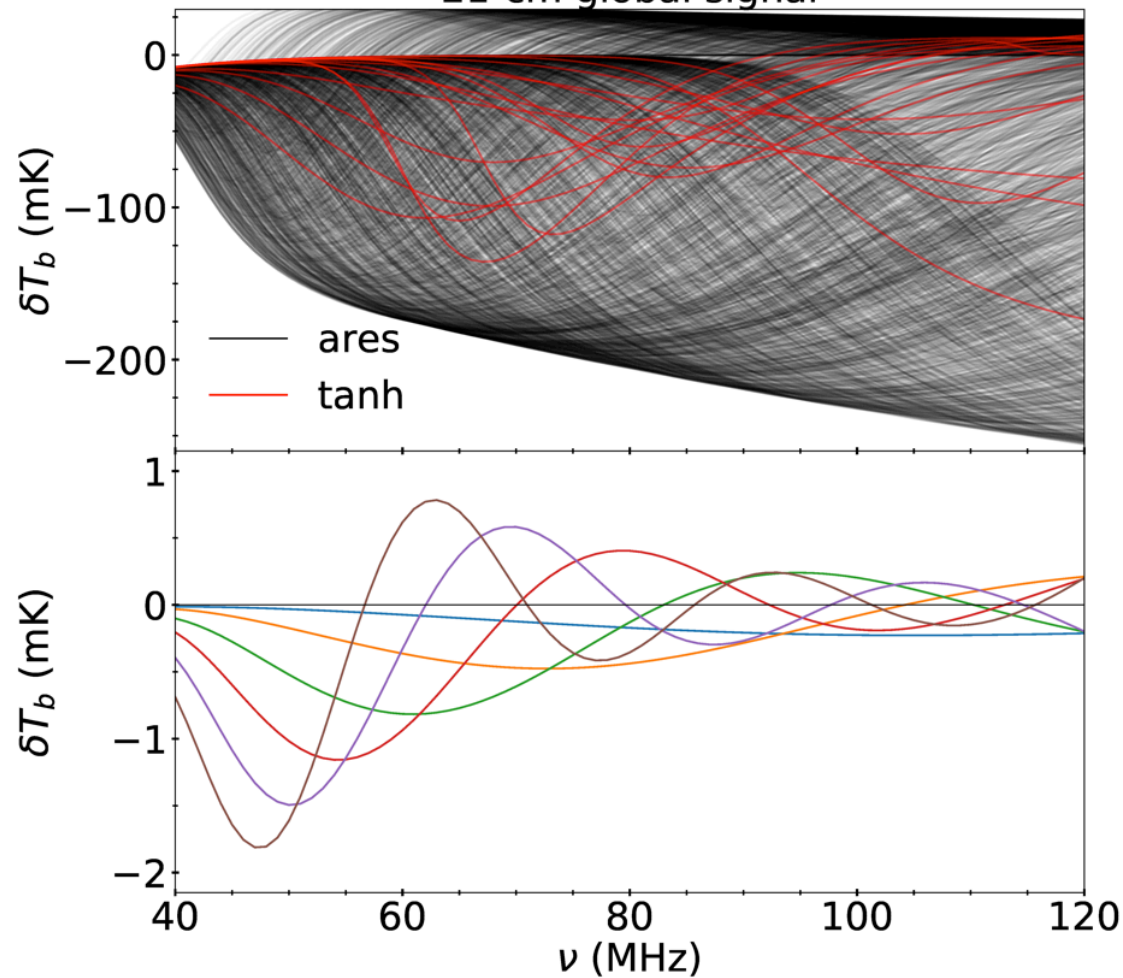


- Beam-weighted foreground training set for a single rotation angle about one of the 4 antenna pointing directions (top).
- The same training set with its mean subtracted (middle).
- The first 6 SVD basis functions obtained from the training set (bottom).
- The different rotation angles about the antenna pointing direction are part of the same training set so that SVD can pick up on angle-dependent structure and imprint it onto the basis functions.

GLOBAL 21-CM SIGNAL TRAINING SET

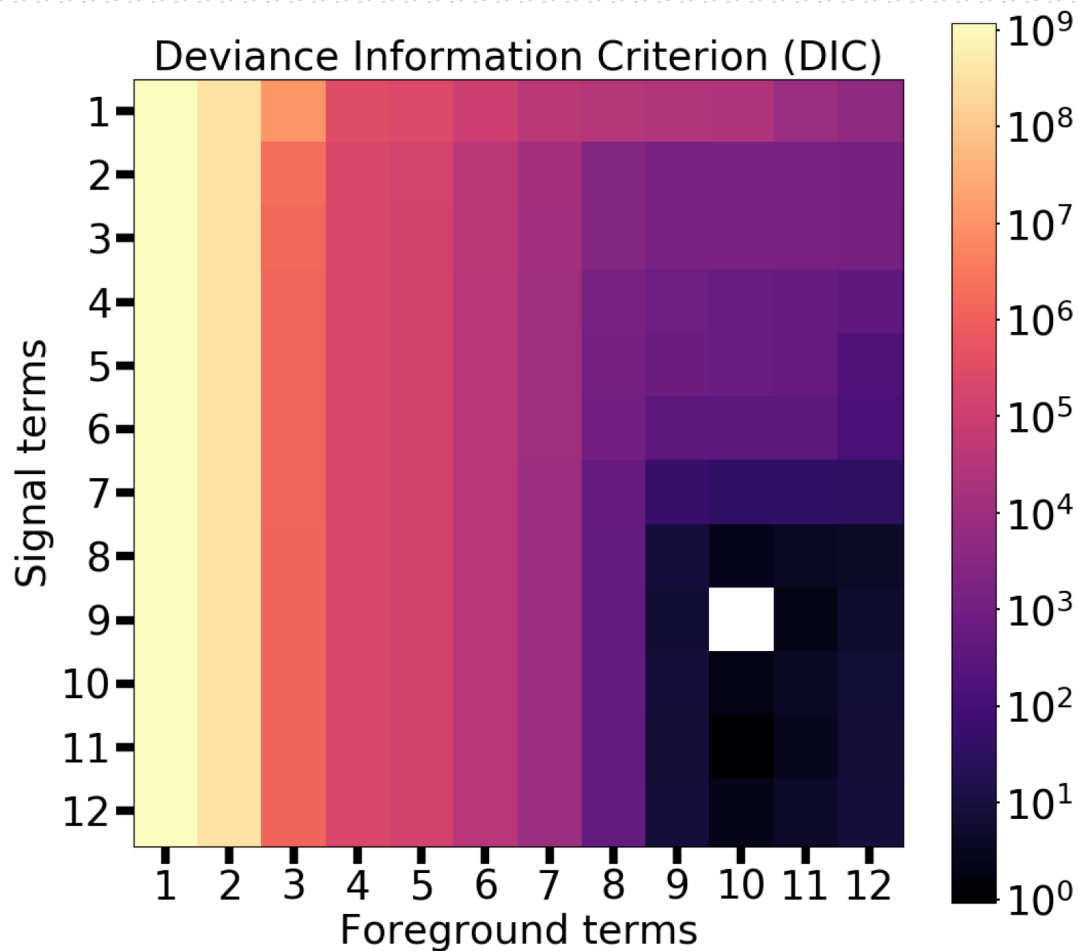
Tauscher et al. (2018)

21-cm global signal



- The signal training set used for our analysis was generated by running the [ares](#) code 7×10^5 times within reasonable parameter bounds in order to fill the frequency band.
- The top panel shows a thinned sample of that set (black curves). The [SVD modes are ordered from most to least important](#).
- The modes are normalized so that they yield 1 when divided by the noise level, squared, and summed over frequency, antenna pointing, and rotation angles about the antenna pointing.

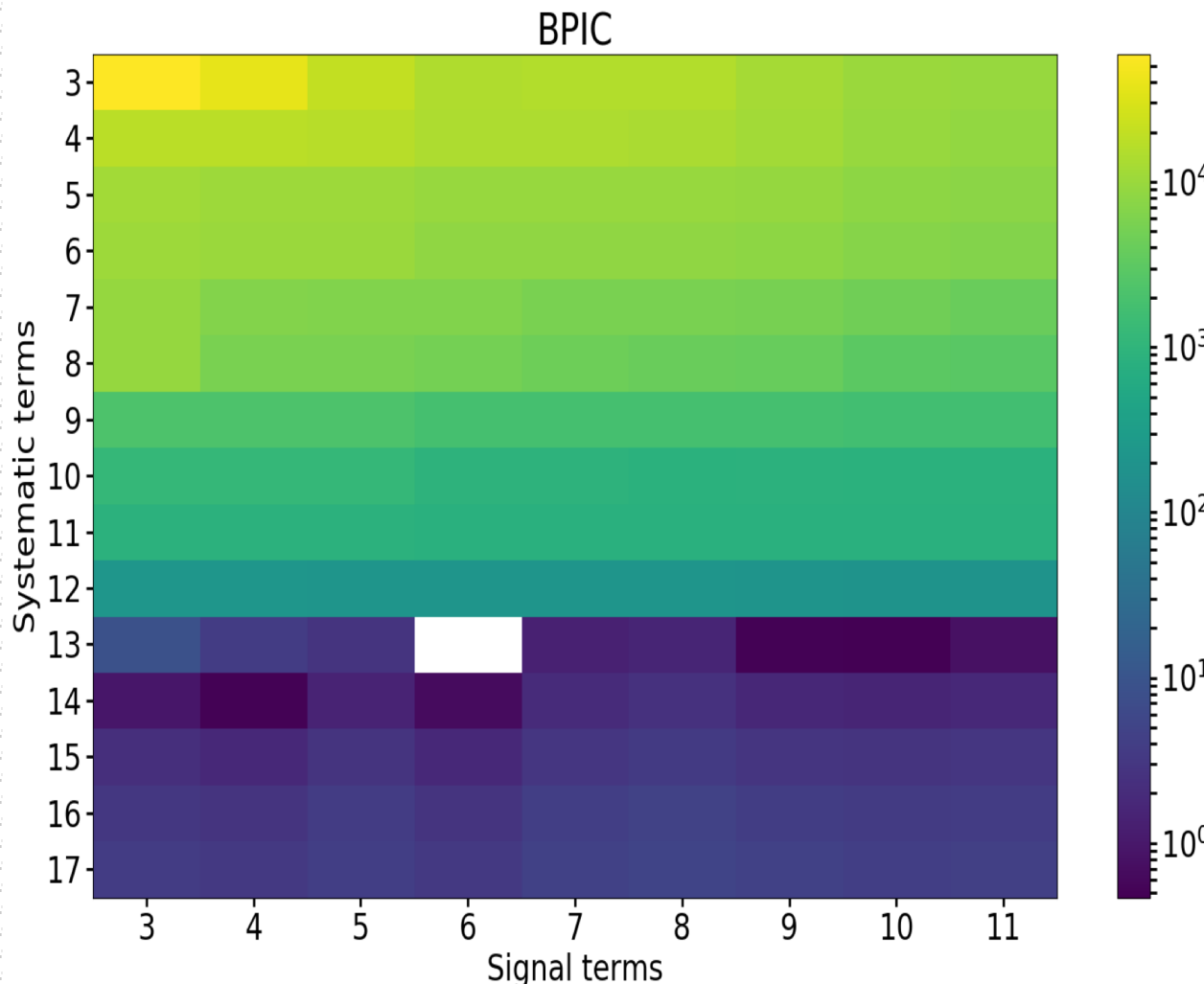
MODEL SELECTION: OPTIMIZING THE NUMBER OF SIGNAL AND SYSTEMATIC MODES



Tauscher et al. (2018)

- Grid of values of the [Deviance Information Criterion \(DIC\)](#).
$$\text{DIC} = -2 \ln \mathcal{L}_{\max} + 2p$$
- The colors indicate the difference between the DIC and its [minimal value](#), marked by the [white square](#).
- This same process can be done with [any information criteria](#) (BIC, AIC, BPIC, etc.).
- Although only a 12×12 grid is shown here, all of the information criteria were calculated over a 60×30 grid.

MODEL SELECTION: ANOTHER EXAMPLE USING BPIC



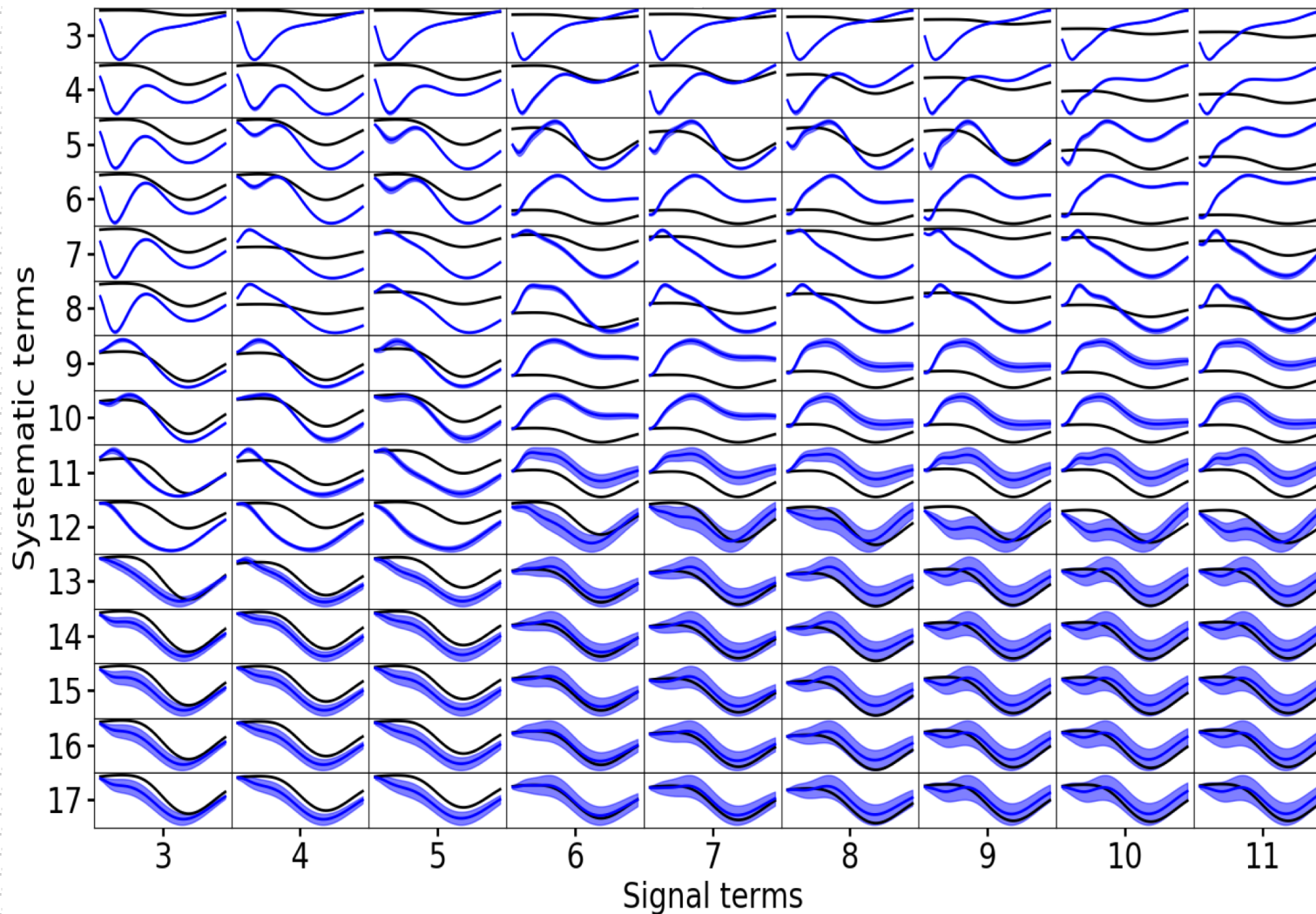
- Grid of values of the [Bayesian Predictive Information Criterion \(BPIC; Ando 2007\)](#).

$$\text{BPIC} = \delta^T \mathbf{C}^{-1} \delta + N_p + 2 \text{Tr}(\mathbf{C}^{-1} \Delta \mathbf{C}^{-1} \mathbf{D})$$

$$\Delta = \mathbf{G} \mathbf{S} \mathbf{G}^T, \quad \delta = \mathbf{G} \xi - y, \quad \text{and} \quad \mathbf{D} = [\text{diag}(\delta)]^2$$

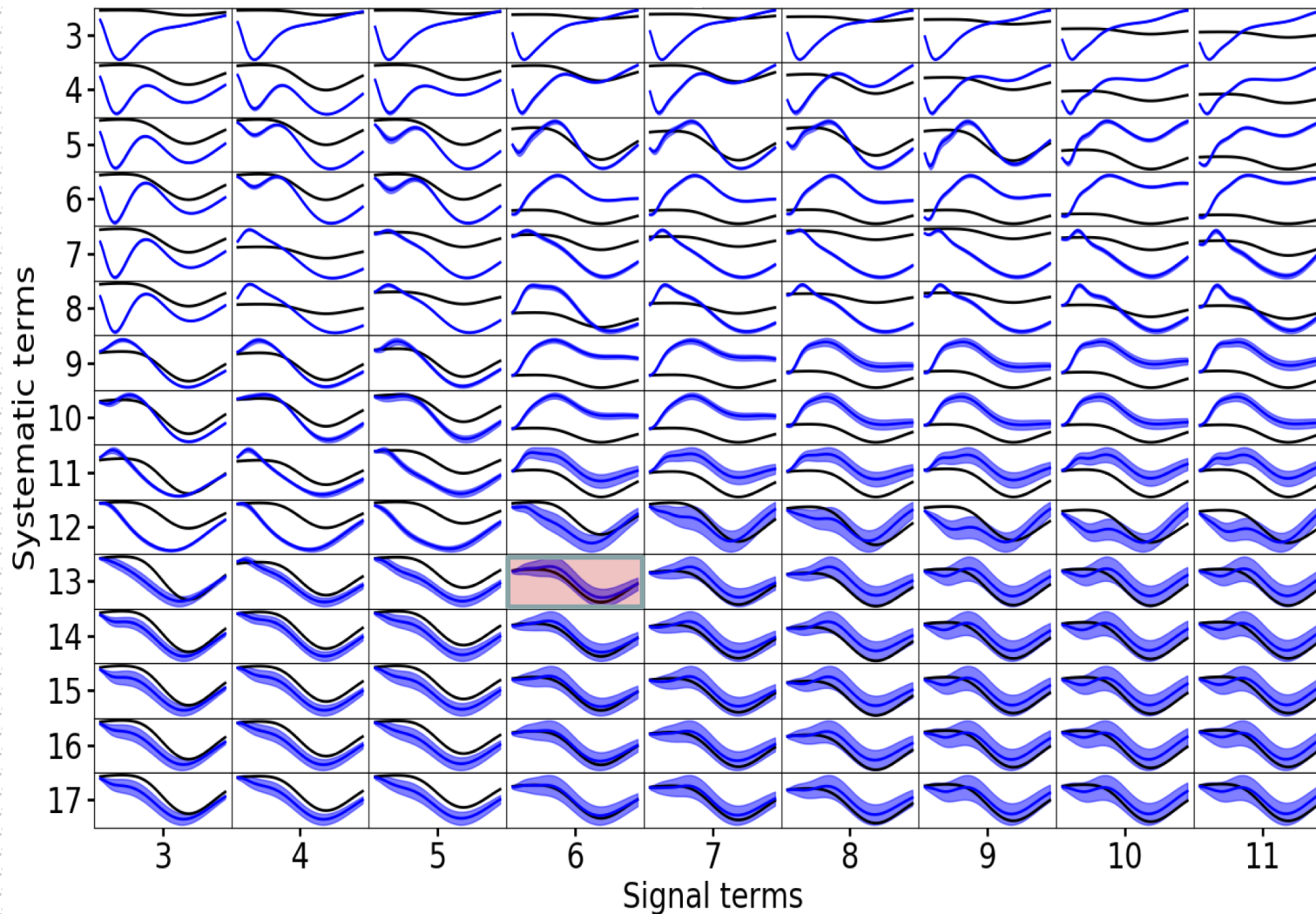
- where y is the full data vector. See further definitions in Tauscher et al (2018).
- The colors indicate the difference between the BPIC and its [minimal value](#), marked by the [white square](#).

MODEL SELECTION: ANOTHER EXAMPLE USING BPIC



Signal Extraction optimization: The black line for all panels is the input 21-cm signal. The blue bands are the pipeline reconstructions of the signal for a given number of SVD signal and systematic parameters/modes.

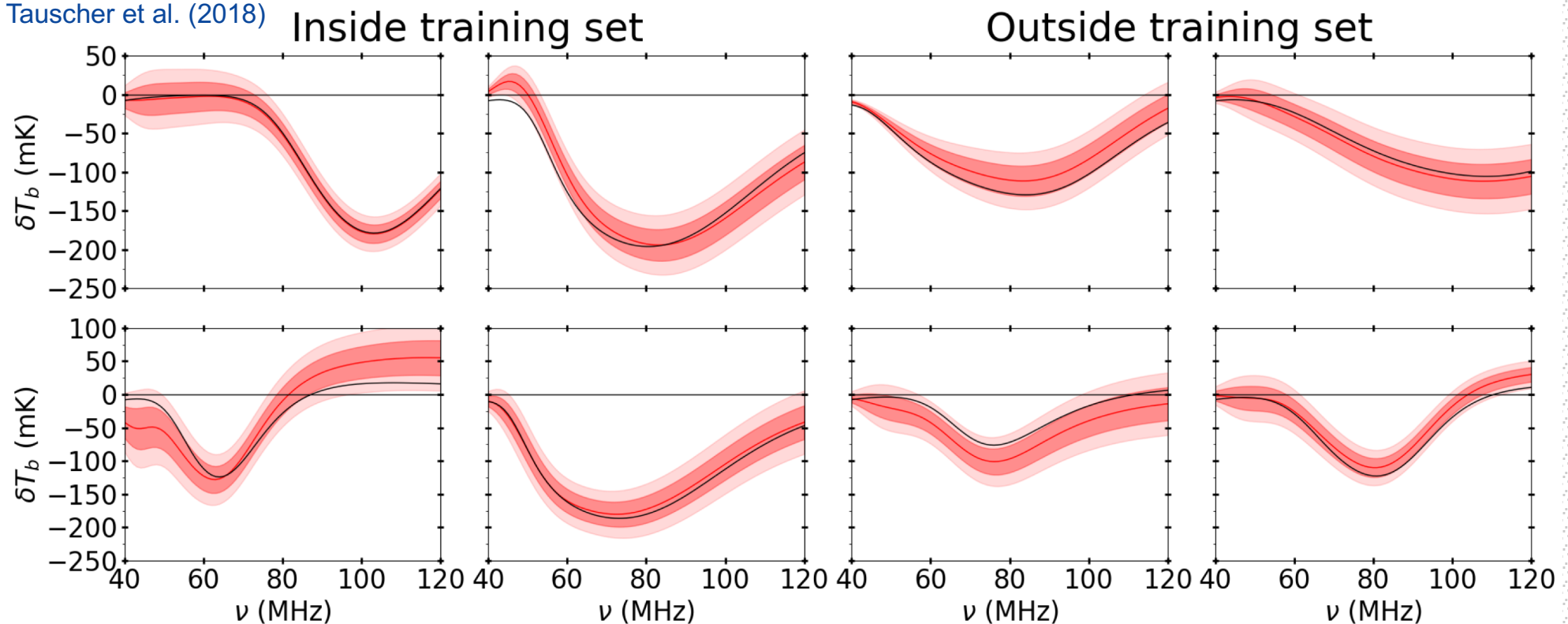
MODEL SELECTION: ANOTHER EXAMPLE USING BPIC



Signal Extraction optimization: The black line for all panels is the input 21-cm signal. The blue bands are the pipeline reconstructions of the signal for a given number of SVD signal and systematic parameters/modes.

SIGNAL EXTRACTION WITH THE CODE PYLINEX

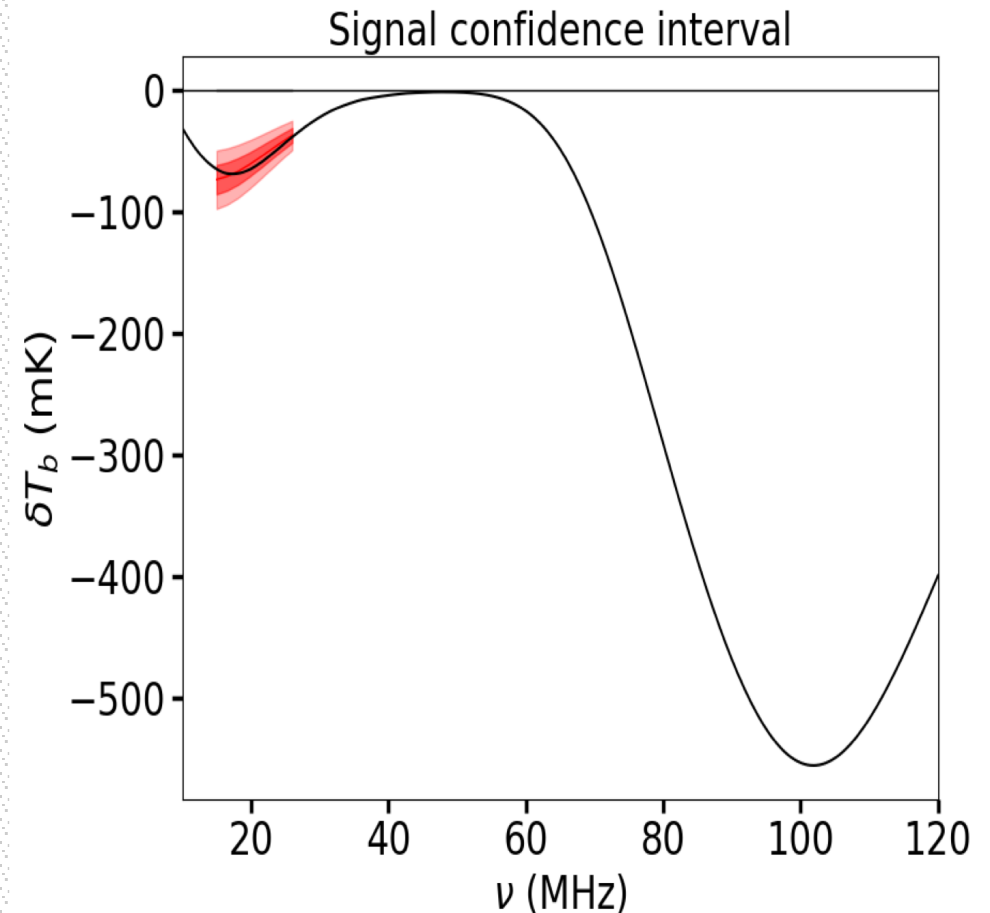
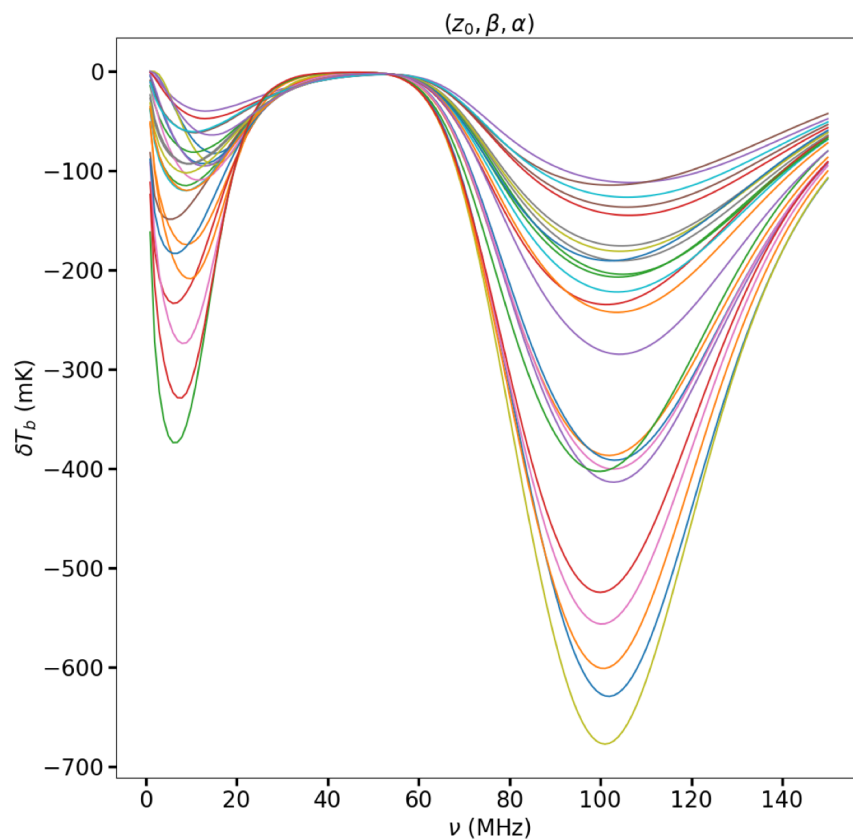
Tauscher et al. (2018)



Signal Estimates from linear models defined by SVD eigenmodes. The **black** curves show the input signals, the **red** curves the signal estimates, the **dark/light** red bands the posterior **68/95%** confidence intervals. The input signals for the 4 **left** plots came from the *ares* signal **training set**, and the 4 on the **right** from the **tanh model** (see e.g. Harker et al. 2016).

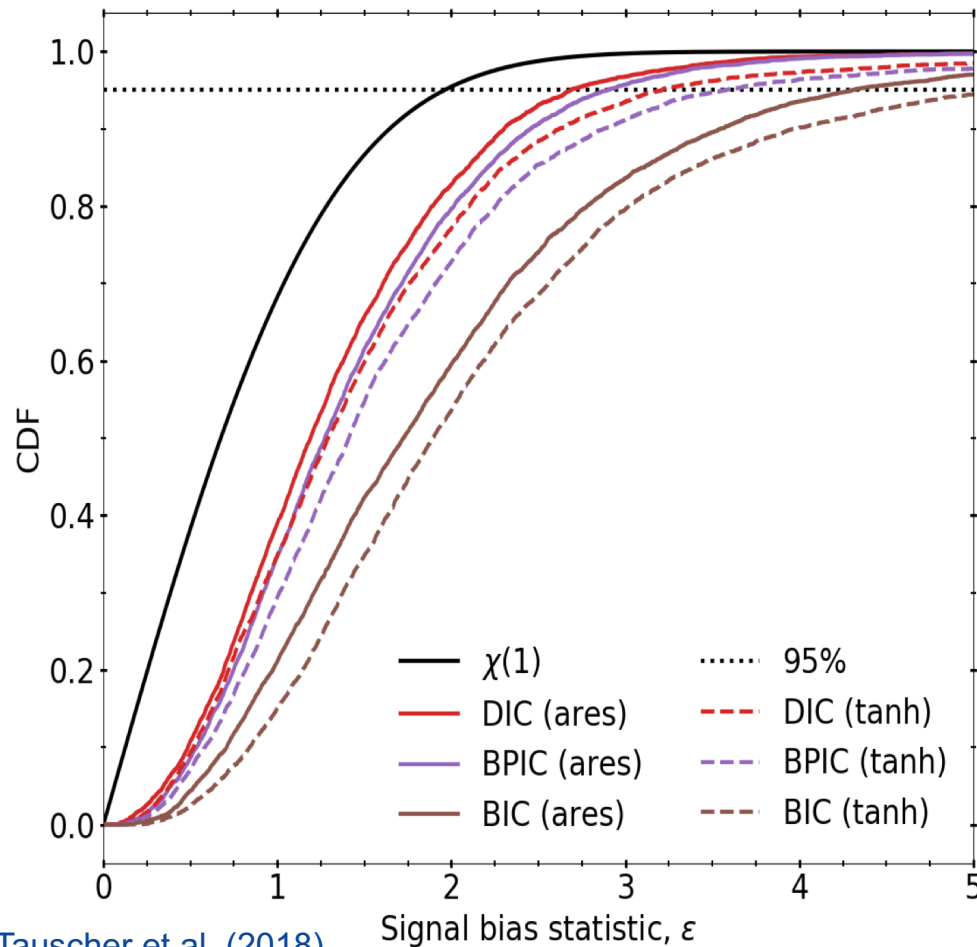
See the **pylinex** in this link: <https://bitbucket.org/ktausch/pylinex>

SIGNAL EXTRACTION WITH THE CODE PYLINEX



See the [pylinex](https://bitbucket.org/ktausch/pylinex) in this link: <https://bitbucket.org/ktausch/pylinex>

SIGNAL BIAS STATISTIC

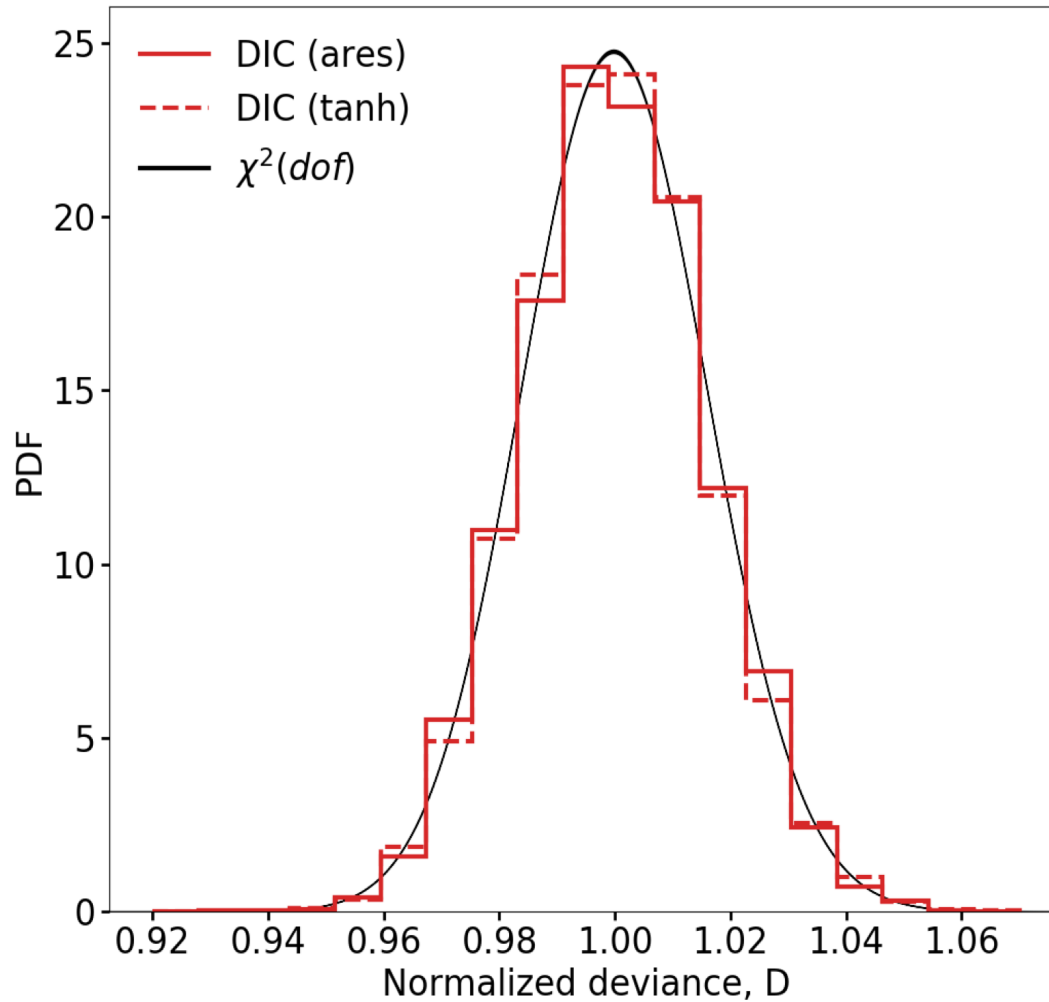


Tauscher et al. (2018)

- The signal bias statistic is a measure of the root mean square error weighted bias of the signal fit:
$$\varepsilon_{21\text{-cm}} = \sqrt{\frac{\delta_{21\text{-cm}}^T \mathbf{C}^{-1} \delta_{21\text{-cm}}}{N_\nu}}$$
- Estimate of the [Cumulative Distribution Function \(CDF\)](#) of the signal bias statistic from 5000 input simulated datasets.
- A bias statistic of ε roughly corresponds to a bias at the $\varepsilon\sigma$ level. The **solid black** reference line is for the distribution which associates 1σ with 68% confidence and 2σ with 95%.
- To guide the eye, the **dotted black** line indicates the 95% level.

NORMALIZED DEVIANCE

Tauscher et al. (2018)

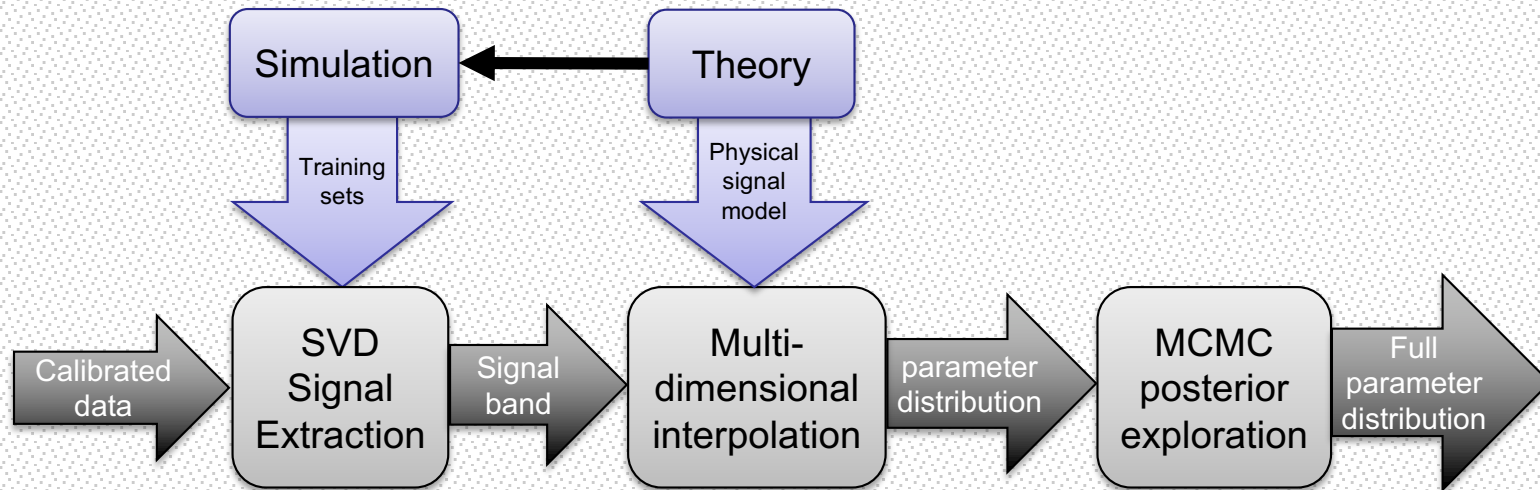


- The deviance normalized by the degrees of freedom contains information about how well the training sets fit the data:

$$D = \frac{\delta^T C^{-1} \delta}{N_{\text{dof}}}$$

- Histogram of the [Probability Distribution Function \(PDF\)](#) for 5000 values of the normalized deviance from fits with different input signals, beam-weighted foregrounds, and noise when using the DIC to choose the best model.
- [D should follow a distribution approximated by the extremely thin black region](#), which is a combination of chisquare distributions associated with the range of degrees of freedom chosen for the extractions.

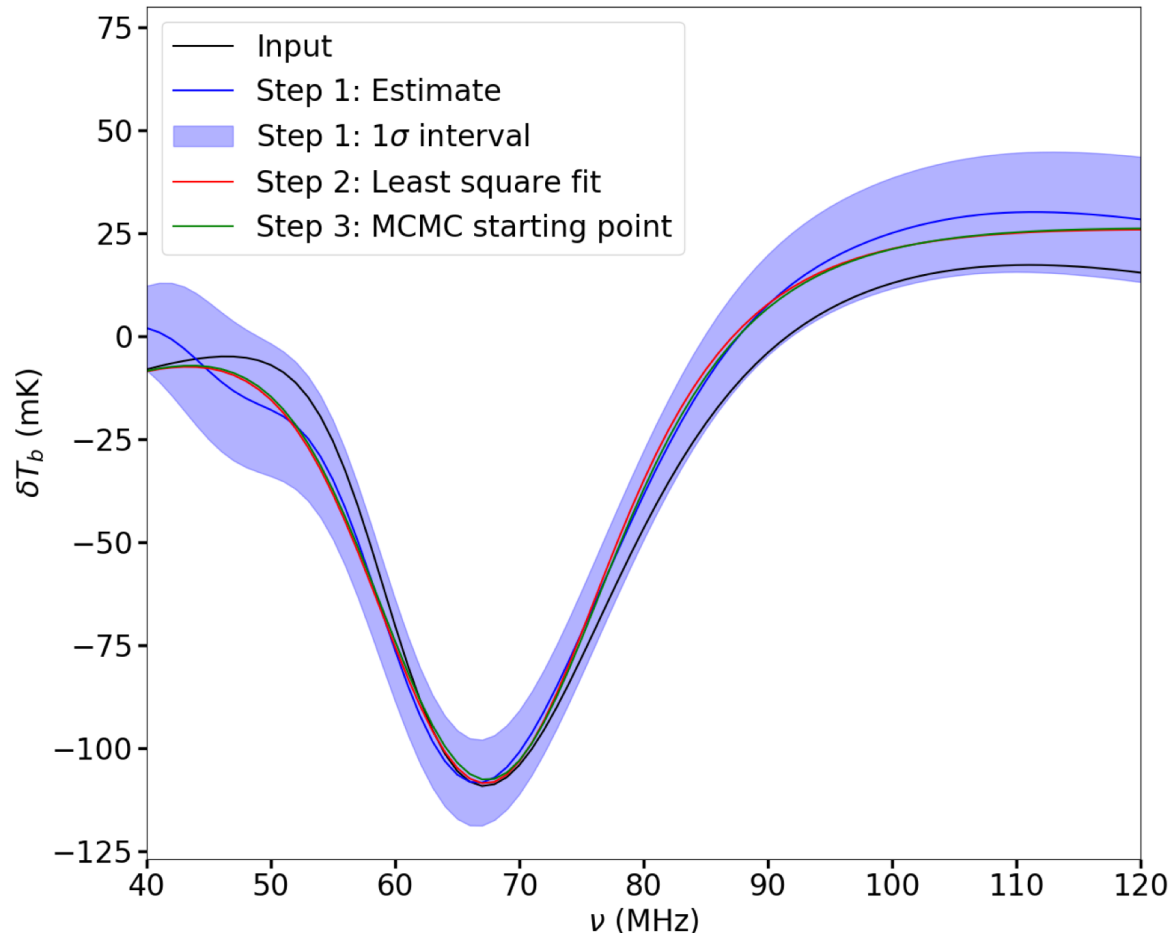
SVD/MCMC DATA ANALYSIS PIPELINE (PRELIMINARY)



- After extracting the signal in frequency space in the first step of the pipeline we need to transform this result into a constraint in physical parameter space.
- For this, we use a multi-dimensional interpolation using a Delaunay mesh for the change in parameter space and then a Markov Chain Monte Carlo search to constrain the full probability distribution.

(Rapetti et al. 2018, in preparation)

MULTI-DIMENSIONAL INTERPOLATION USING A DELAUNAY MESH (PRELIMINARY)



(Rapetti et al. 2018, in preparation)

- We generalize linear interpolation to **arbitrary input and output dimensions**.
- We use this interpolation to perform a **least square fit (red line)** using the training set.
- Importantly, note that having an **starting point (green line)** within the **estimated error (blue band)** provided by the first (very fast) step of the pipeline is crucial for the convergence of the MCMC in a vast parameter space where we do not have otherwise any prior information on the solution and its uncertainty (for the jump proposal).

CONCLUSIONS

- A challenge of extracting the global 21-cm signal is the **large foregrounds**.
- However, unlike the foregrounds, the signal is **spatially uniform**, has well-characterized **spectral features**, and is **unpolarized**.
- We benefit from these differences using our **novel approach** for **signal extraction** and **physical parameter constraints**, using an **SVD/MCMC** pipeline.
- We obtain a **highly significant** improvement by using a pioneering **induced polarization technique**.
- Our pipeline can be used for both **lunar orbit** and **lunar surface** low-frequency radio telescopes.
- We are also working on running our pipeline on current/ongoing **ground based data** from **EDGES** and **CTP** using our **Pattern Recognition/Information Criteria/MCMC** to detect the expected absorption features in the Global 21-cm spectrum.