Robustly Constraining the Global 21-cm Signal using Pattern Recognition and Bayesian Inference

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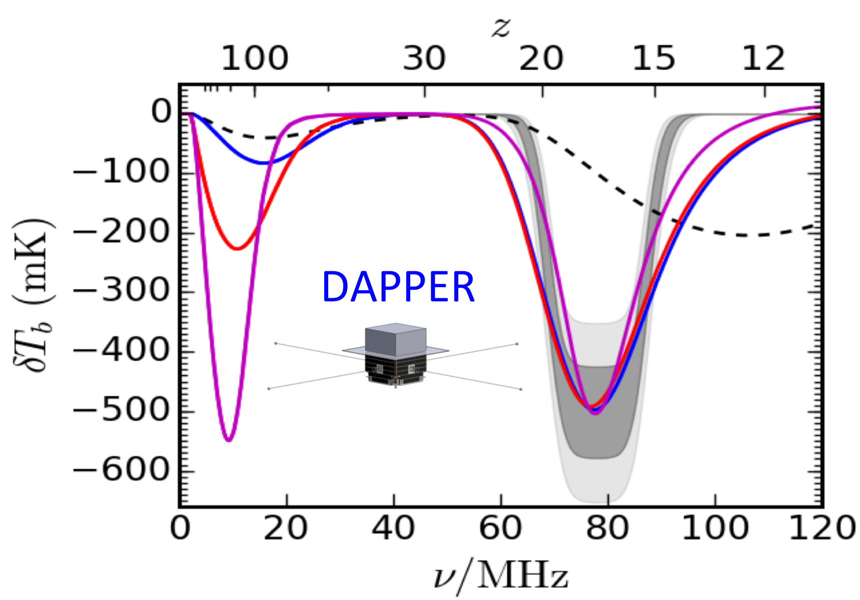
OUTLINE OF THE CHALLENGES

- Accurately & precisely extracting & constraining the small global 21-cm signal from within a foreground 4-6 orders of magnitude larger.

- Limited available information on both the signal and the systematics of the experiment, including potential overlaps between the spectral shapes of the signal and systematics.



The Dark Ages Polarimeter PathfindER (DAPPER) is a NASA SmallSat mission concept for hydrogen cosmology from the radio pristine lunar far side (see Burns' talk earlier).

78 MHz trough reported by EDGES in gray.

Standard astrophysical model **(black, dashed curve) inconsistent with EDGES data.**

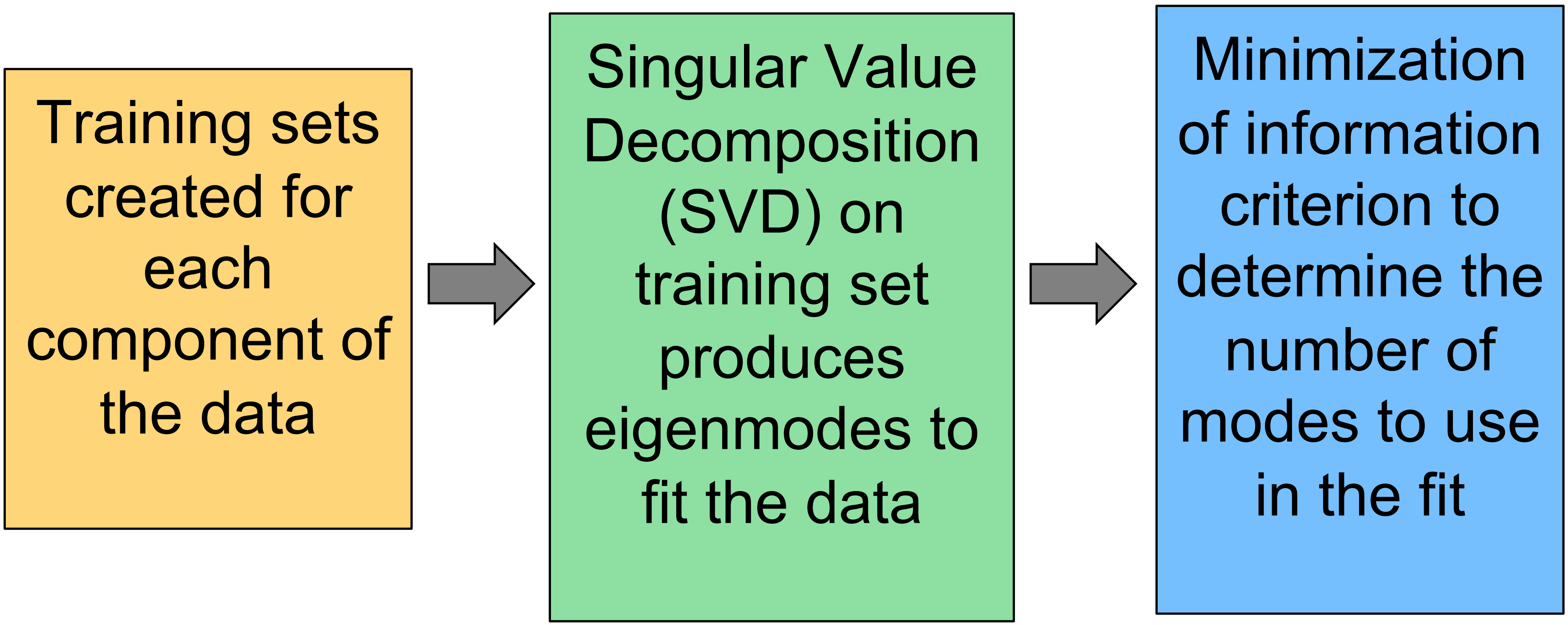
Exotic physics models of the **Dark Ages** trough consistent with the EDGES signal (color curves; credit J. Mirocha).

OUR CURRENT SOLUTIONS: PIPELINE PUBLICATIONS

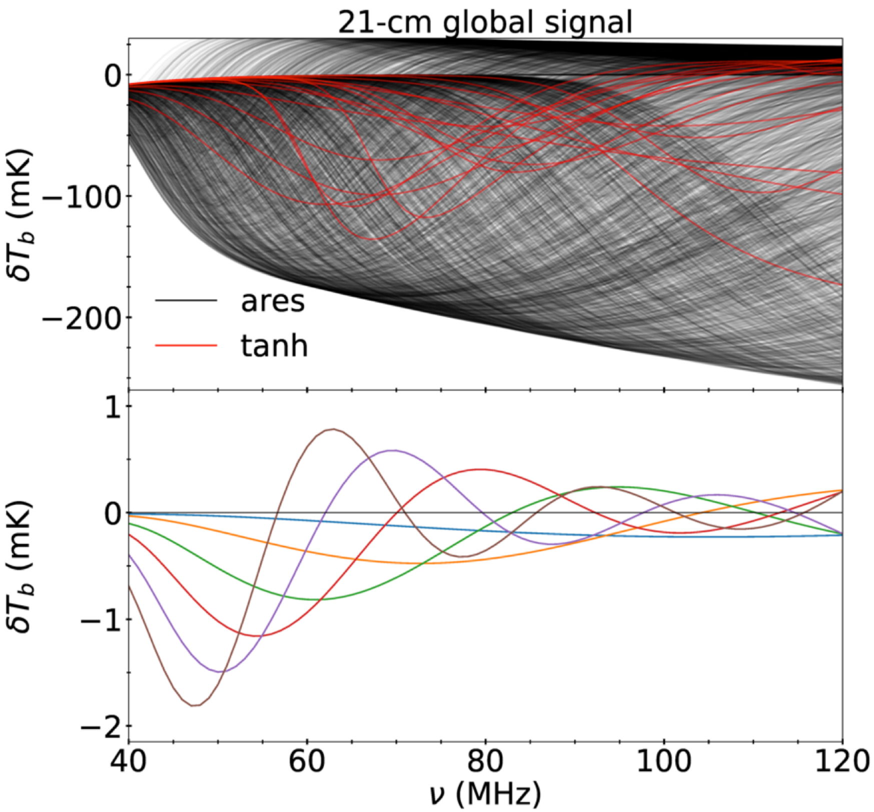
- Pipeline series Paper I, II, III (published), and IV (in preparation):
  - Paper I: Global 21 cm Signal Extraction from Foreground and Instrumental Effects. I. Pattern Recognition Framework for Separation Using Training Sets (2018, ApJ, 853, 187)
  - Paper II: Global 21 cm Signal Extraction from Foreground and Instrumental Effects. II. Efficient and Self-consistent Technique for Constraining Nonlinear Signal Models (2020, ApJ, 897, 174)
  - Paper III: Global 21 cm Signal Extraction from Foreground and Instrumental Effects. III. Utilizing Drift-scan Time Dependence and Full Stokes Measurements (2020, ApJ, 897, 174)

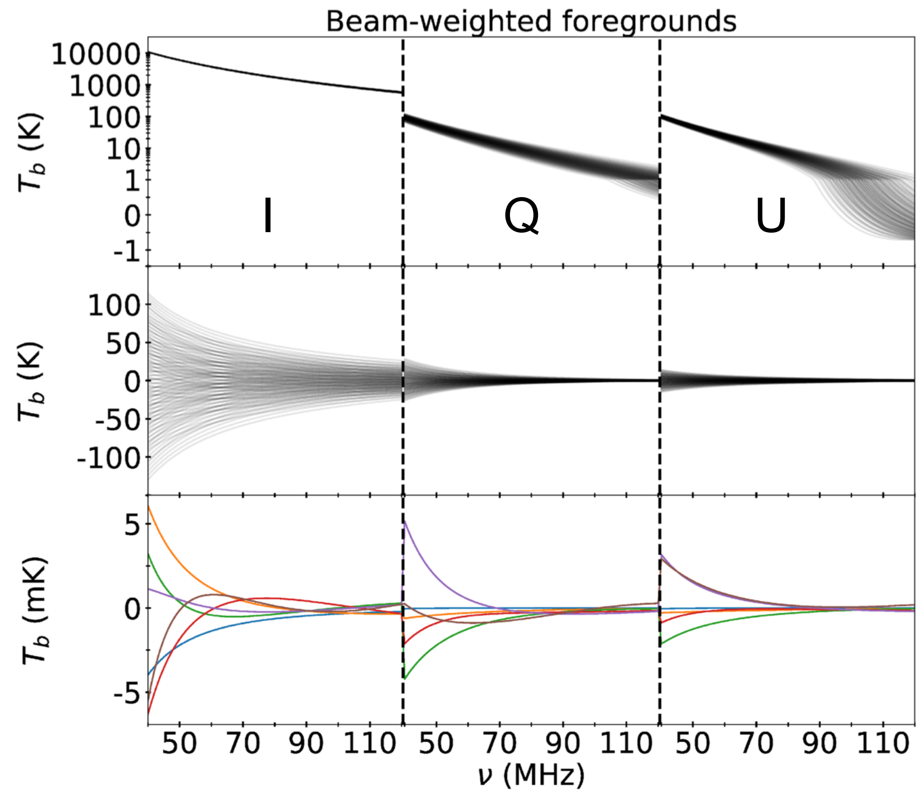
- Minimum Assumption Analysis (MAA) paper (see Tauscher’s talk later):
  - Formulating and Critically Examining the Assumptions of Global 21 cm Signal Analyses: How to Avoid the False Troughs That Can Appear in Single-spectrum Fits (2020, ApJ, 897, 132)

SCHEMATIC VIEW OF THE PATTERN REGONITION SEGMENT (PAPER I)



DATA COMPONENT TRAINING SETS AND SVD MODES



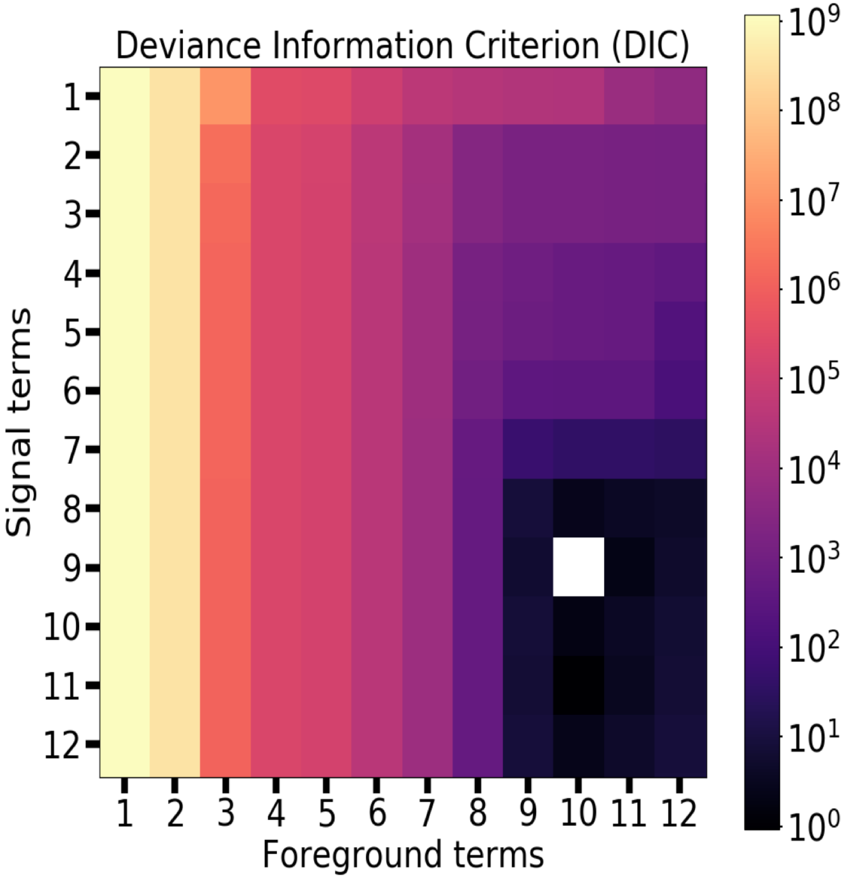


Tauscher, Rapetti, Burns, Switzer (2018; Paper I)

- The bottom panels show the first six SVD eigenmodes obtained from the training sets above in black.
- SVD modes ordered from most to least important. For foreground modelling, see Hibbard's talk tomorrow.
- For an overall linear model, we choose each component's number of SVD modes using an information criteria.

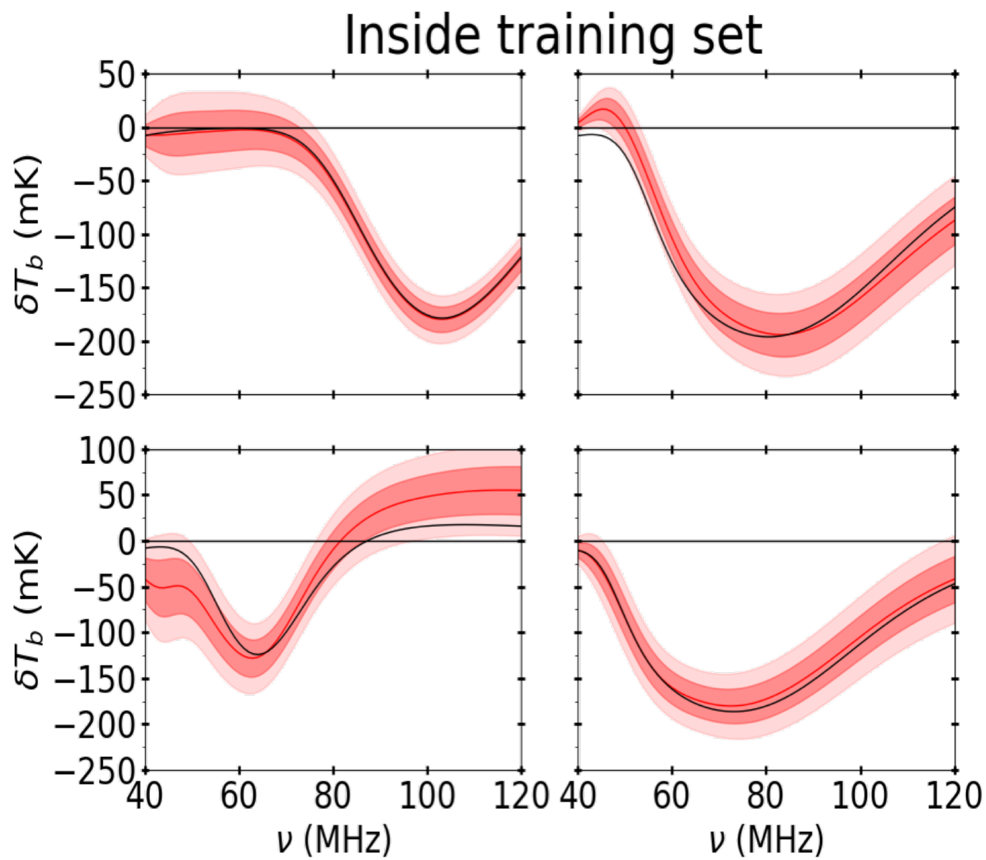
MODEL SELECTION (NUMBER OF MODES) AND LINEAR, ANALYTICAL SIGNAL EXTRACTION

Tauscher, Rapetti, Burns, Switzer (2018; Paper I)



$$\mathrm{DIC} = -2\ln\mathcal{L}_{\max} + 2p$$

(See also Bassett's talk afterwards)



Black curves: Input signals; red: estimates; dark/light red bands: 68/95% confidence intervals

JOINT MCMC FIT OF NONLINEAR SIGNAL MARGINALIZING OVER LINEAR BEAM-WEIGHTED FOREGROUND

- In Paper II (Rapetti, Tauscher, Mirocha & Burns, 2020), we present a new technique converting spectral constraints into constraints on any given nonlinear signal parameter space.

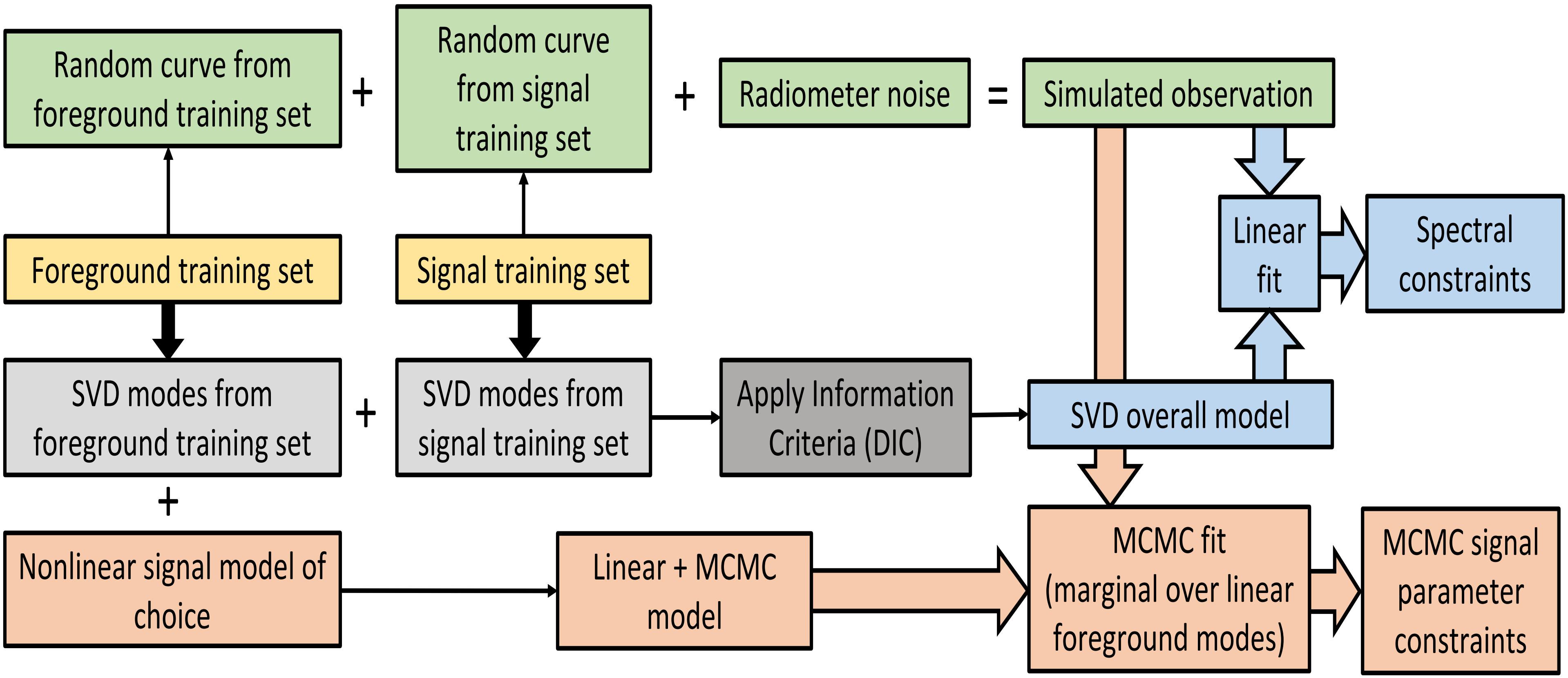
- We analytically find a joint linear fit of the signal and systematics (currently, beam-weighted foreground) and used as starting point (mean and covariance) for a simultaneous, nonlinear Markov Chain Monte Carlo (MCMC) fit.

- At each step of the MCMC fit, we marginalize over the coefficients to the SVD foreground modes. This allows us to straightforwardly use a large number of foreground parameters, while efficiently exploring the signal parameter space.

- This calculation is exact and provides a natural separation of linear nuisance parameters without a need for a parametric model and nonlinear signal parameters to be numerically sampled. A similar separation between linear and nonlinear parameters can be performed in receiver modeling (Paper IV, in prep.).

CONSTRAINING SINGAL PARAMETERS (PAPER II)

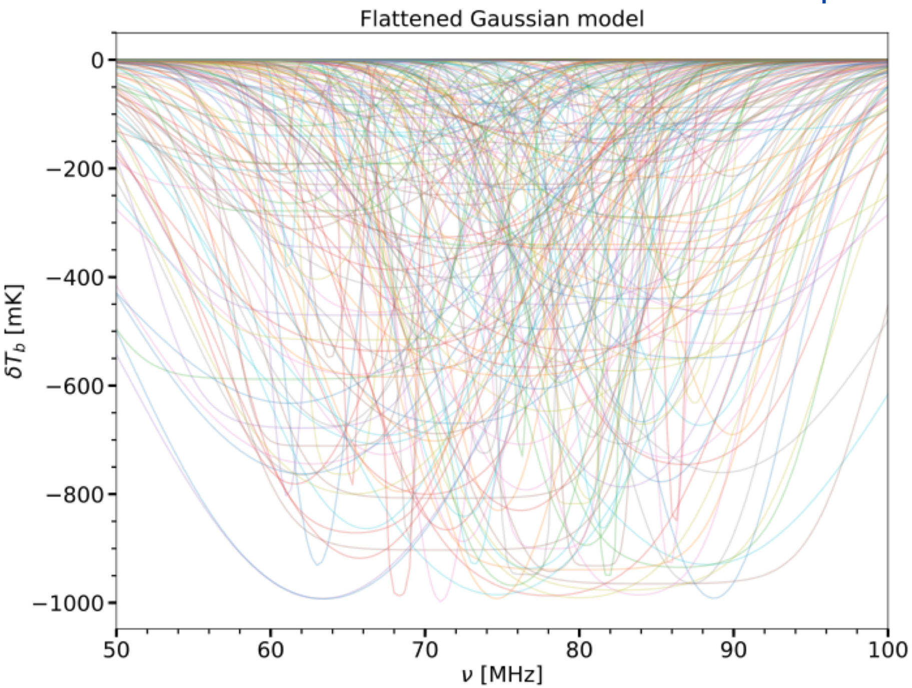
Rapetti et al. (Paper II)

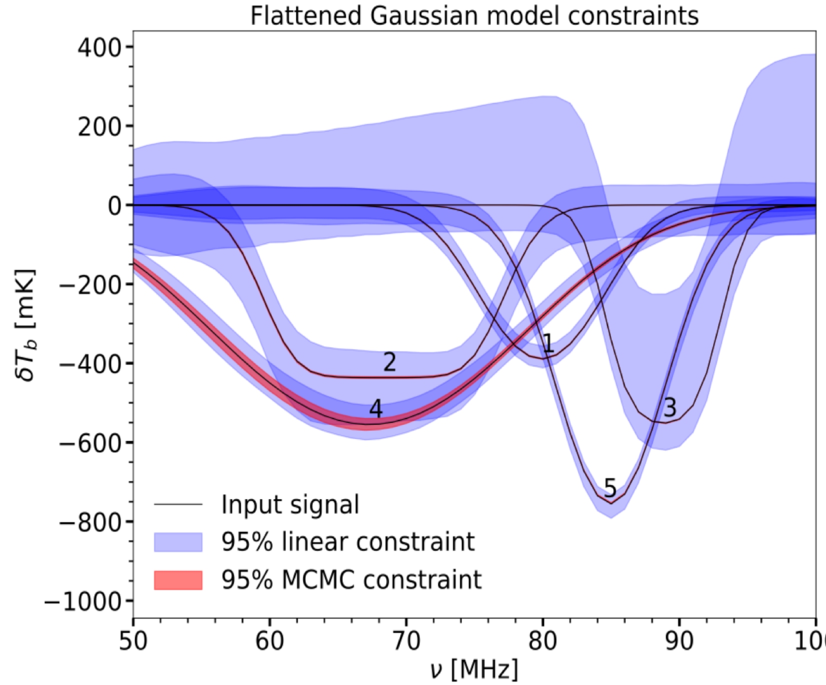


Find the code pylinex in this link: https://bitbucket.org/ktausch/pylinex

GLOBAL 21-CM SIGNAL MODEL: FLATTENED GAUSSIAN

Rapetti et al. (Paper II)





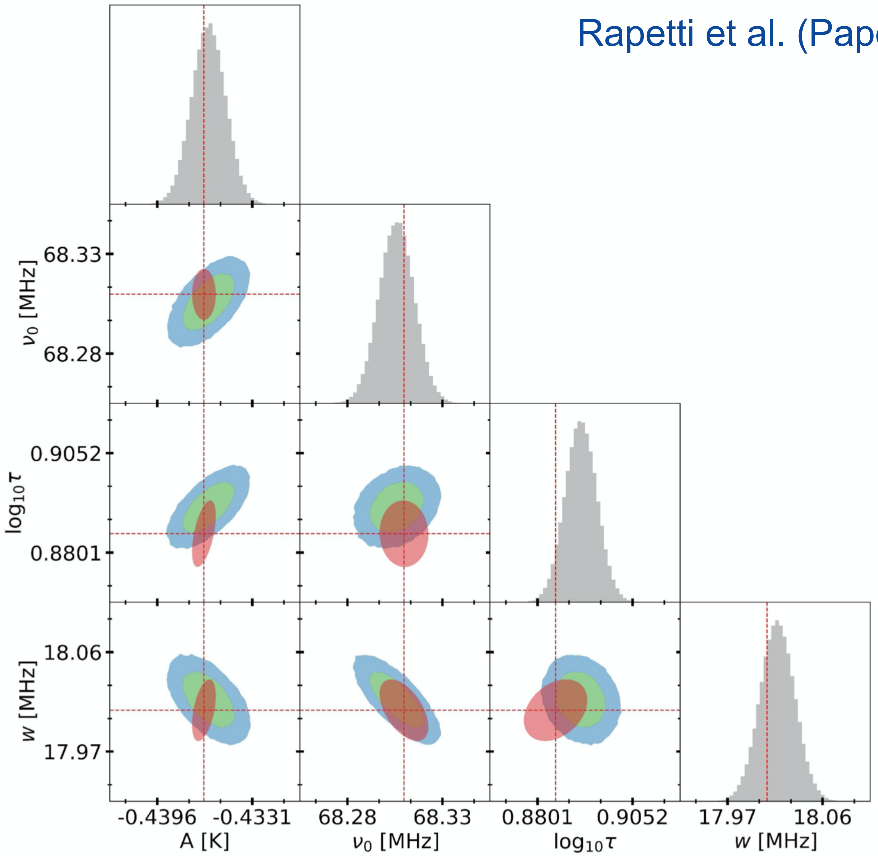
Sample of 200 curves from the training set for the flattened Gaussian model.

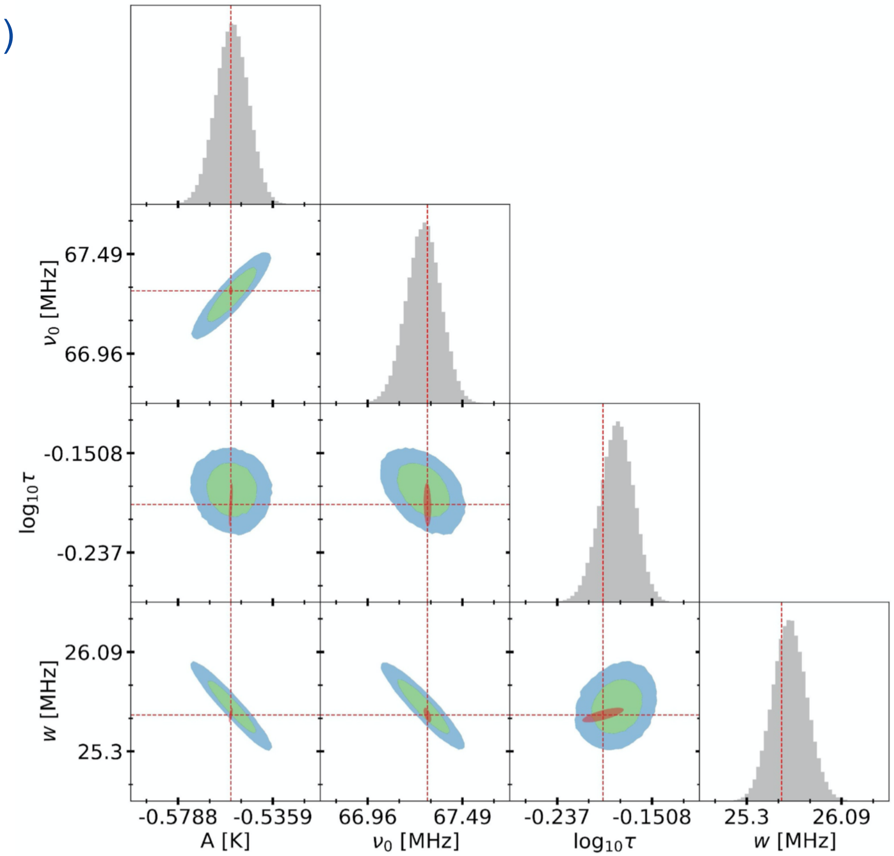
A uniform (-1, -0.1) K
ν_0 uniform (60, 90) MHz
w uniform (1, 30) MHz
τ exponential (1)

- 5 random cases successfully recovered
- 95% C.I. in blue for the linear fit (SVD signal & foreground modes) and red for the MCMC fit (nonlinear signal model & SVD foreground terms marginalized over)
- In the linear fits, the 95% C.I. correspond to 8.75σ.

FLATTENED GAUSSIAN MODEL:
FULL MCMC PARAMETER CONSTRAINTS

Rapetti et al. (Paper II)

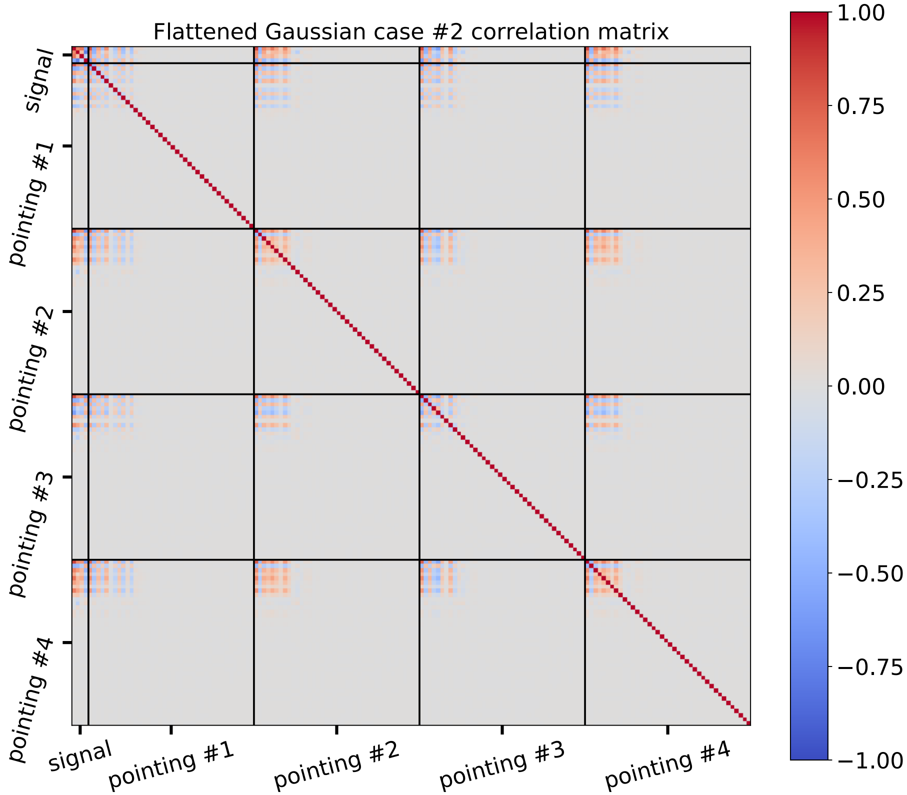


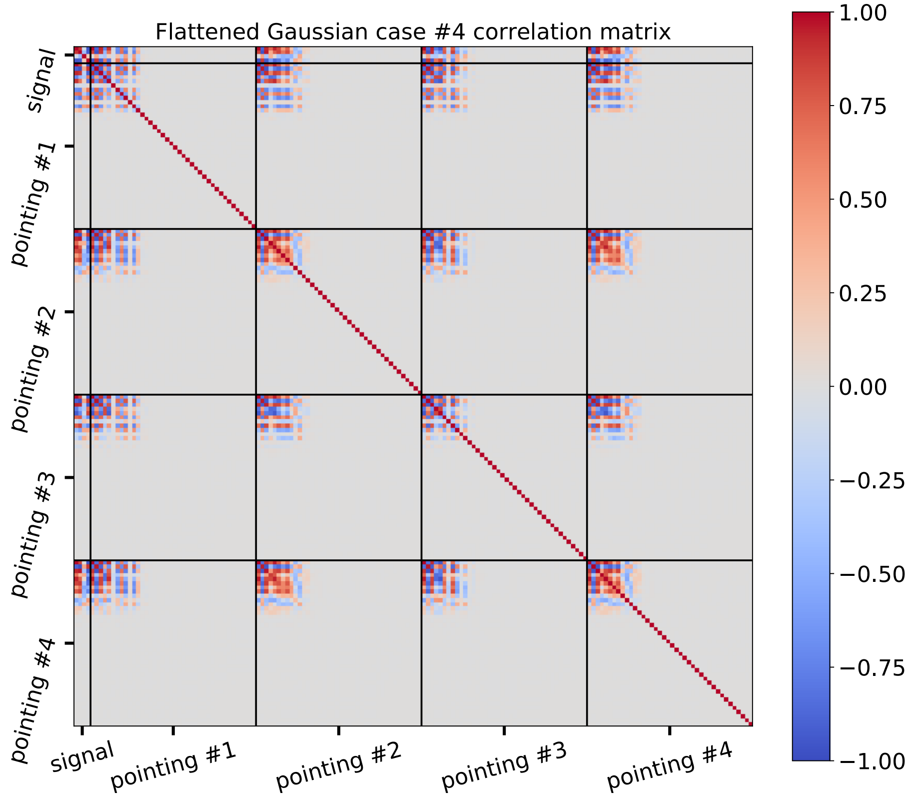


- 1D (gray) and 2D (68/95%) MCMC posterior parameter constraints.
- Red contours: represent 95% errors for statistical noise alone.
- Red, dashed lines: mark the input parameters.
- In case FG4 (right) the systematics clearly play a larger role than in case FG2 (left).

FLATTENED GAUSSIAN MODEL: CORRELATION MATRICES

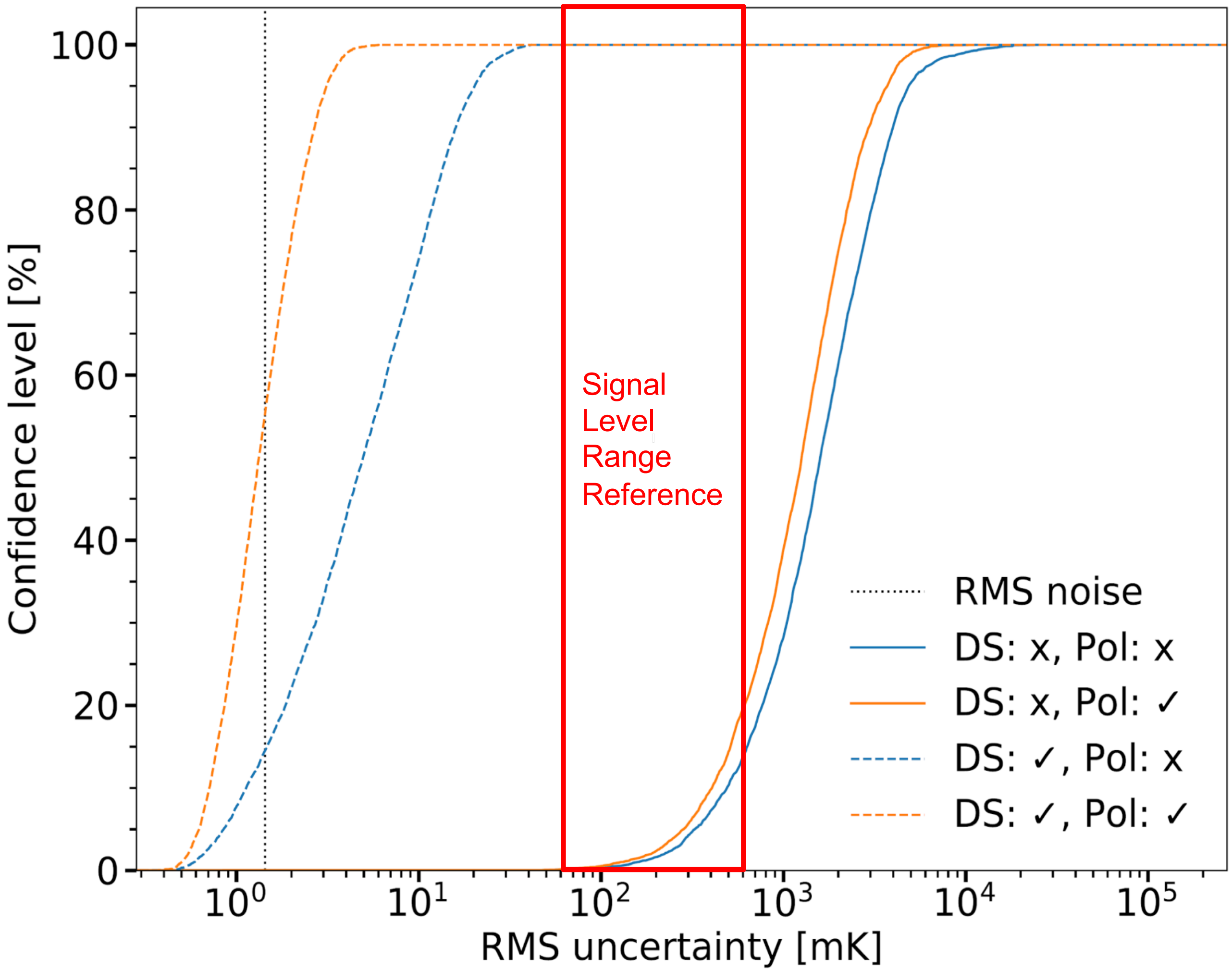
Rapetti et al. (Paper II)





- Correlation matrices for the MCMC fits of FG2 (left) and FG4 (right).
- Matching the previous slide, these show stronger correlations for FG4 versus FG2.
- 4 signal & 160 (40 per 4 pointings) foreground parameters. Our marginal MCMC reduced the MCMC parameters from 164 to 4.
- Most foreground parameters (above a certain mode order) have negligible correlations (in gray), becoming thereby irrelevant thanks to the down-weighting power of foreground priors.

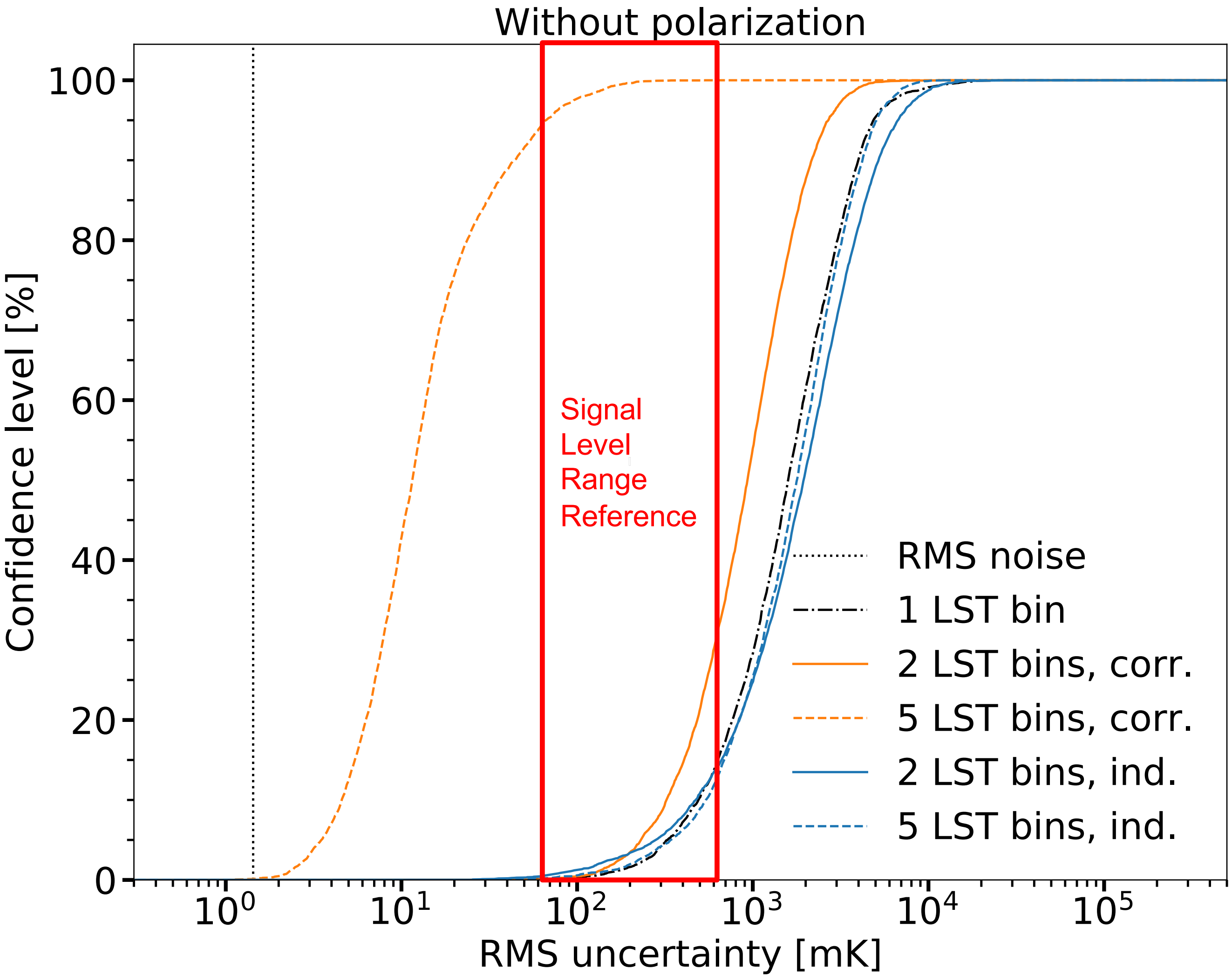
MEASUREMENT STRATEGY EFFECTS ON UNCERTAINTIES (PAPER III)



Tauscher et al. (Paper III)

- Each CDF for 5000 fits
- DS: Drift-scan time dependence (Dashed lines: 25 bins in LST)
- Pol: Full polarized Stokes measurements (in blue)
- Both time dependence and polarization measurements provide marked benefits

UTILIZING THE CORRELATIONS BETWEEN SPECTRA



Tauscher et al. (Paper III)

- Without polarization
- Orange: Correlated LST bins
- Blue: Independent LST bins
- Benefiting from correlations between LST bins is critical for best constraints

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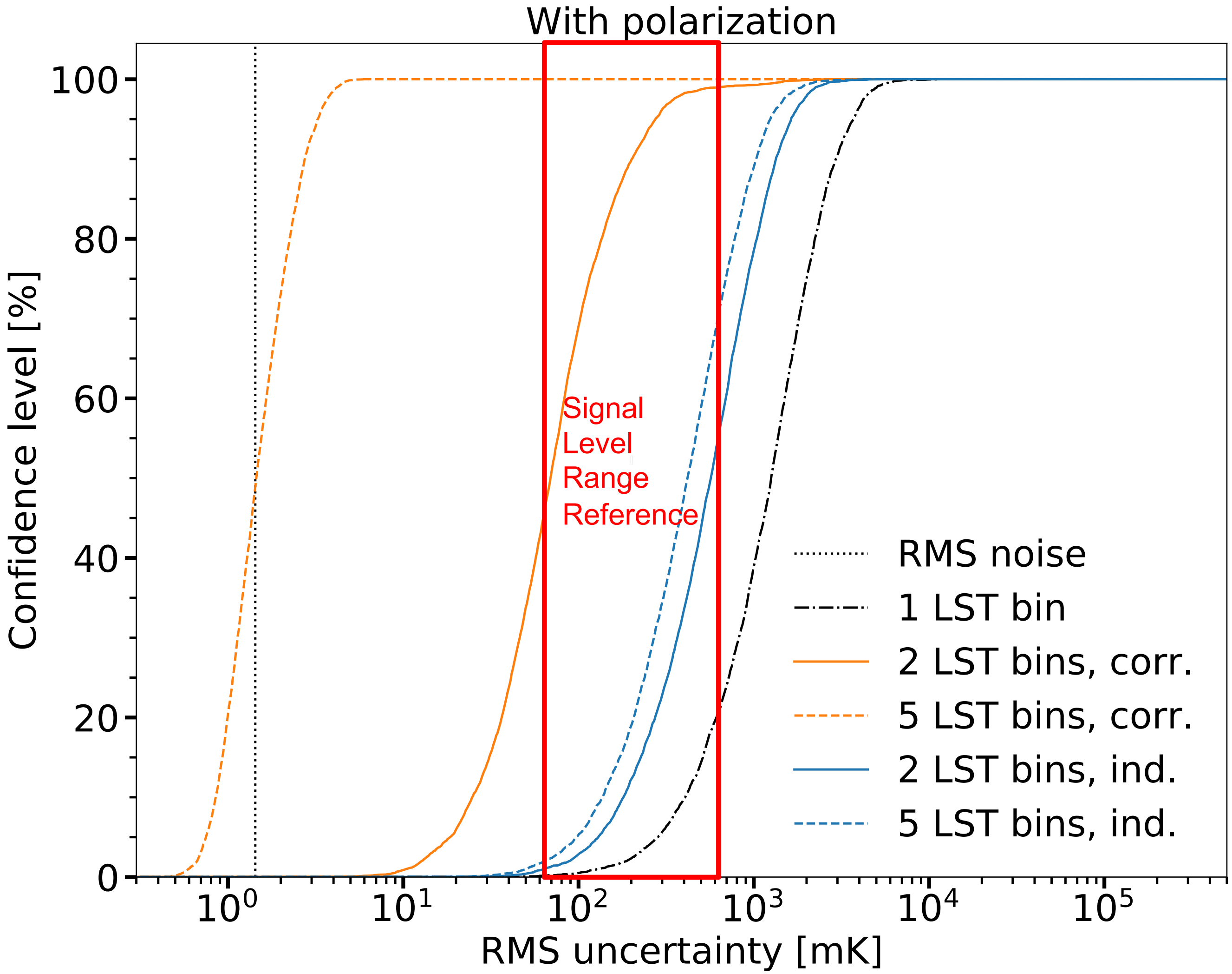

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BENEFITS OF OUR METHODOLOGY

- We can use a large number of correlated spectra at once, create models specifically suited for a given experimental dataset, form training sets arbitrarily complex (avoiding the necessity for smooth, phenomenological foregrounds), and include beam effects directly into the model (instead of having to remove them).
- SVD factorizes a training set providing the optimal vector basis to fit it.
- We employ a linear, fast, analytic methodology to separate the global 21-cm signal from systematics, properly accounting for their potentially large overlaps, to estimate the starting point of a full MCMC search of any chosen nonlinear signal model.
- We utilize the linear coefficients of the SVD beam-weighted foreground model to properly and efficiently (in terms of convergence) incorporate this modeling by marginalizing over these generally large number of parameters at each step of our MCMC signal calculation.
- We benefit from the use of correlated foreground spectra via multiple sky views or Stokes parameters to differentiate between foreground & signal, significantly lowering uncertainties.
- Our statistically rigorous pipeline is able to extract the global 21-cm signal while modeling signal & systematics using detailed training sets from theory, simulations and observations.