A minimum assumption analysis for global 21-cm signal experiments:

Avoiding the flaws of single-spectrum analyses

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A review of Tauscher, Rapetti, & Burns, ApJ 897 132, arXiv:2005.00034

Outline

- Enumerate assumptions of all global 21-cm signal analyses
- Examine the form of assumptions for EDGES and other single spectrum analyses
- Introduce a minimum assumption analysis

Key assumptions of 21-cm global signal experiments

1. Sky-averaged radio data contains a sum of beam-weighted foreground emission and the global signal.

2. The noise of the data follows a known or estimated distribution (usually a zero-mean Gaussian distribution with covariance C).

3. The true beam-weighted foreground can be fit with the given foreground model to well below the noise level of the data. This is equivalent to $\delta^T C^{-1} \delta \ll N_c$, where N_c is the number of channels in the data and δ is the unmodeled component of the foreground.

4. The signal follows a specific form.

Form of assumptions for EDGES

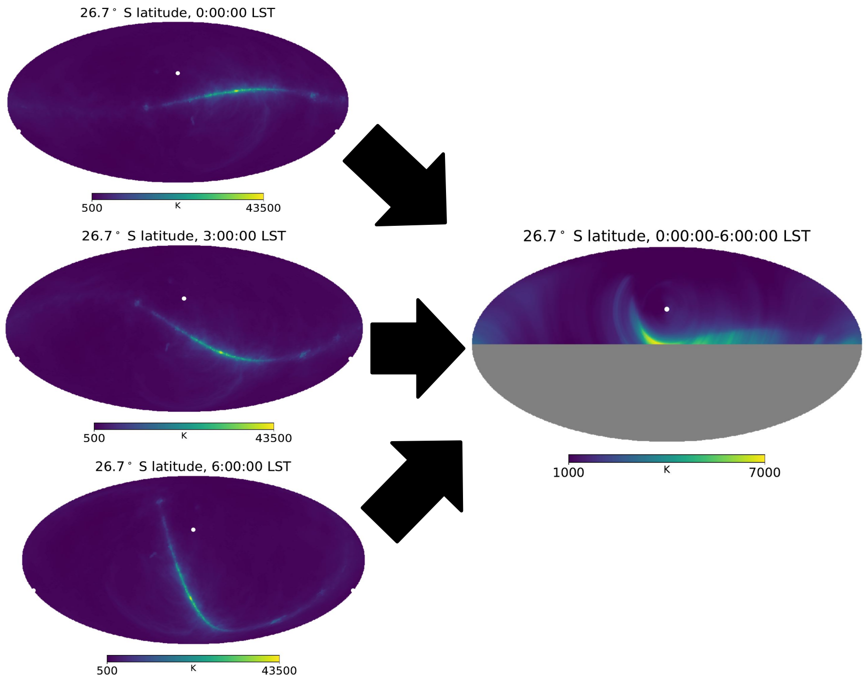
1. Sky-averaged radio data contains a sum of beam-weighted foreground emission and the global signal. **The instrument is well-calibrated.**
2. The noise of the data follows a known or estimated distribution (usually a zero-mean Gaussian distribution with covariance C). **Noise is spectrally flat and independent.**
3. The true beam-weighted foreground can be fit with the given foreground model to well below the noise level of the data. This is equivalent to $\delta^T C^{-1} \delta \ll N_c$, where N_c is the number of channels in the data and δ is the unmodeled component of the foreground. **Polynomials can fit the beam-weighted foreground down to mK level.**
4. The signal follows a specific form. **Signal follows a flattened Gaussian.**

Testing EDGES foreground modeling assumption

- We simulated beam-weighted foreground observations from the EDGES latitude for local sidereal times from 0:00 to 6:00 hr

$$T_{\mathrm{eff}}(\nu, \theta, \varphi, 0 \rightarrow 6\ \mathrm{hr\ LST}) = \int_{0\ \mathrm{hr\ LST}}^{6\ \mathrm{hr\ LST}} T(\nu, \theta, \varphi, t)\, \mathrm{d}t$$

- A flat horizon was added at zero elevation angle.
    - For simplicity, only emission from above the horizon is simulated.

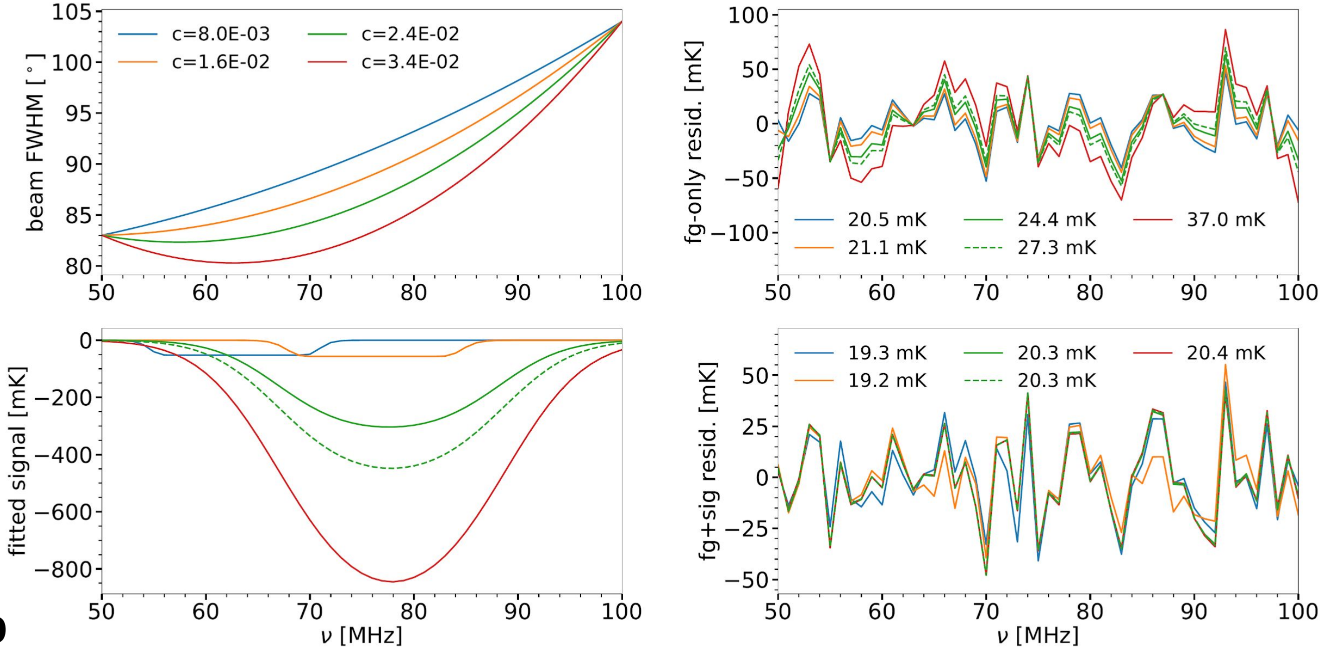


Foreground modeling bias induces signal bias

- Beam is Gaussian with spectral FWHM
- No signal is added to simulated data
- Foreground is fit by model of the form:

$$\left(\frac{\nu}{\nu_0}\right)^{-2.5}\sum_{k=0}^{5} a_k\left[\ln\left(\frac{\nu}{\nu_0}\right)\right]^k$$

- **Fitted signals can be hundreds of mK deep**



Foreground modeling issues motivate a new method

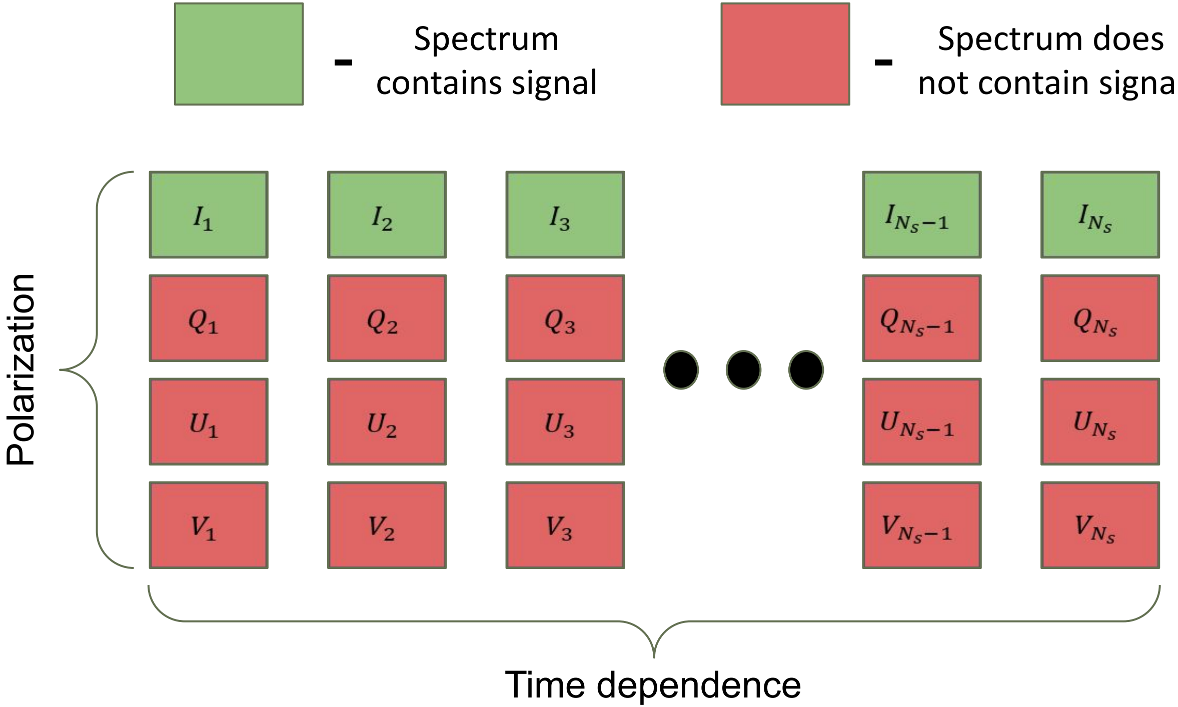
- Single spectrum nature of EDGES analysis necessarily excludes possible signal models.
  - Using multiple spectra can allow for analysis that can fit any possible signal spectrum.
- Polynomial models are not guaranteed to (and probably won't) fit the beam-weighted foreground sufficiently well in few enough terms before overlap with the signal model causes uncertainties to blow up when including a large class of possible signal models.
  - Using training sets specific to the given experimental situation can minimize the number of foreground terms necessary while also accounting for all foreseen observation effects

Form of assumptions for Minimum Assumption Analysis (MAA)

1. Sky-averaged radio data contains a sum of beam-weighted foreground emission and the global signal. **The instrument is well-calibrated.**
2. The noise of the data follows a known or estimated distribution (usually a zero-mean Gaussian distribution with covariance C). **Noise is independent and given by radiometer equation.**
3. The true beam-weighted foreground can be fit with the given foreground model to well below the noise level of the data. This is equivalent to $\delta^T C^{-1} \delta \ll N_c$, where N_c is the number of channels in the data and δ is the unmodeled component of the foreground. **Beam-weighted foreground can be fit from eigenmodes derived from a simulated training set.**
4. The signal follows a specific form. **Signal is the same in every total power spectrum.**

Logistics of using multiple spectra

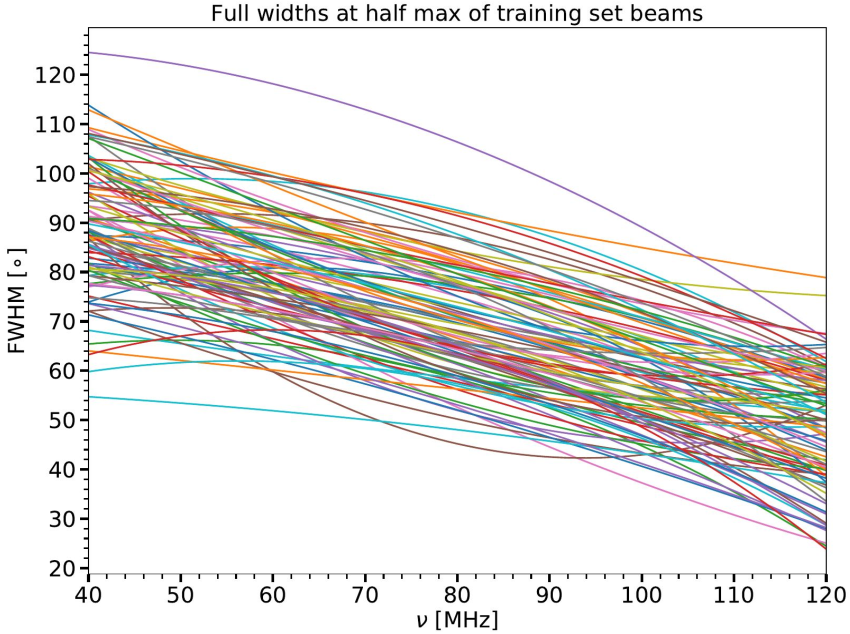
- The signal should appear the same in all total power spectra while the foreground will change as the sky seen by the antenna rotates



Making a training set to simulate the MAA

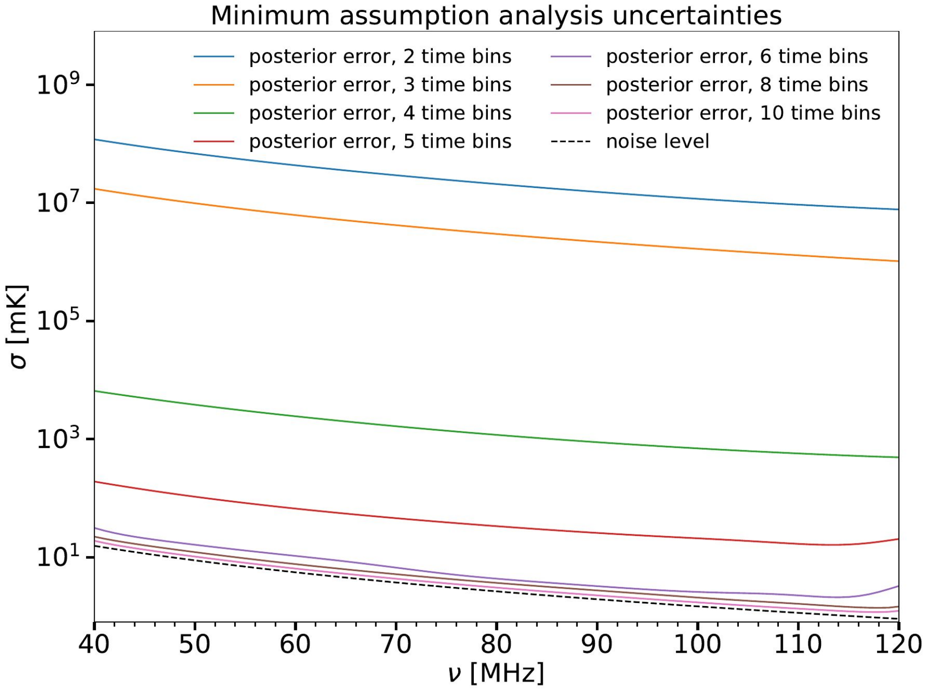
- Foreground was simulated in the same way as earlier EDGES simulations, except it was done in 100 equal LST bins

- Beam FWHM training set made using quadratic Legendre polynomials

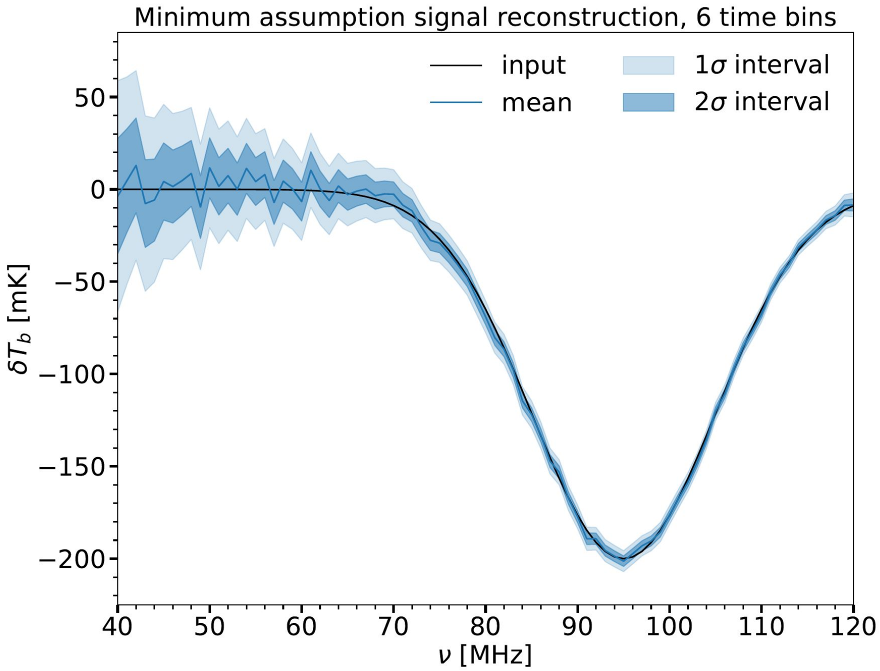


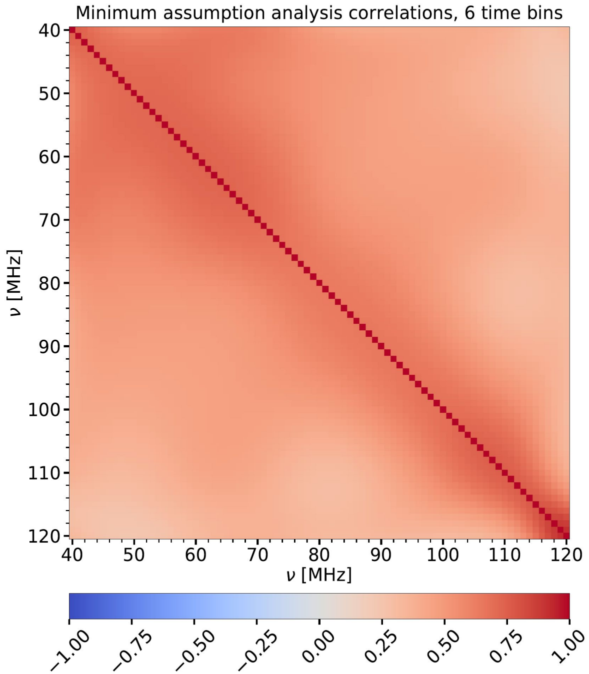
Signal uncertainties under the MAA

- MAA uncertainties are infinite with a single spectrum
- Larger number of spectra pushes uncertainties down near noise level but are harder to create an accurate training set for
- 100 hrs integration split evenly between LST bins



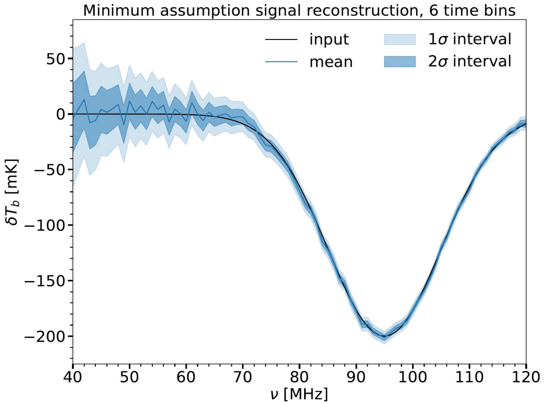
Example signal reconstruction from the MAA



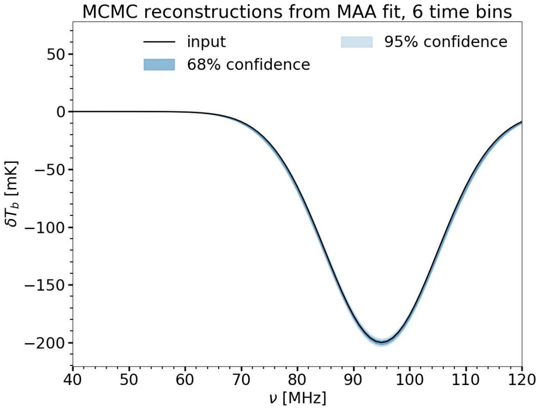


MAA is generalized foreground subtraction

- In the perfect knowledge case, foreground is removed completely with no change to data uncertainties.
- MAA allows for foreground to be removed while increasing uncertainties.
- Importantly, MAA can be followed up by an (e.g. MCMC) exploration of signal parameters that ignores foreground parameters



MCMC



Conclusions

- Common single spectrum analyses are prone to large signal biases due to:
  - Necessity of choosing a signal model that may be inaccurate w.r.t. to true signal
  - Beam-weighted foreground models are often simple, but inaccurate in simulations
- Minimum Assumption Analysis (MAA) is a rigorous alternative that:
  - Allows for any 21-cm signal spectrum
  - Fits multiple spectra at a time, utilizing time dependence to separate signal and foreground
  - Allows for follow-up signal-only MCMC with any parameterization
- The pipeline described in the previous talks by D. Rapetti and N. Bassett is in between the two extremes.
- We are currently attempting to model EDGES with these techniques, but forming a training set is difficult.

Extra slide: basic MAA equations

Ψ – 21-cm expansion matrix ($N_s N_\nu$ x N_ν matrix)

F – beam-weighted foreground basis ($N_s N_\nu$ x N_b matrix)

y – data (length-$N_s N_\nu$ vector)

C – noise covariance ($N_s N_\nu$ x $N_s N_\nu$ matrix)

Foreground projection matrix ($N_s N_\nu$ x $N_s N_\nu$ matrix) → $\Phi = F(F^T C^{-1} F)^{-1} F^T C^{-1}$

Signal posterior covariance (N_ν x N_ν matrix) → $\Delta_{21} = [\Psi^T C^{-1}(I - \Phi)\Psi]^{-1}$

Signal posterior mean (length-N_ν matrix) → $\gamma_{21} = \Delta_{21} \Psi^T C^{-1}(I - \Phi)y$