HOMEWORK 5

- Show all the work/derivation with neat writing (this counts for the score). Engineering paper should be used. Each student should finish the homework independently.

1. Prove the following statement:

   For small deformation, the Young's modulus E of a neo-Hookean material can be approximated by $3nkT$, where n is the chain concentration and kT is the thermo energy.

2. Consider two balloons that have the same Young's modulus E, balloon A has a wall of thickness h_A while the wall thickness of balloon B is $h_B = 2h_A$.

   a) Based on the class note, plot the pressure (P)-volume (V) relationship of these two balloons **in one plot**.

   b) Let's consider these two balloons are connected and are inflated from the same source (Fig. 1). Based on the $P - V$ curves you plotted in question (a), qualitatively describe the inflation process of this system.

   (Hints: i. In this experiment, the pressure inside these two balloons is the same.

   ii. Write your description based on following thoughts: are these two balloons inflated simultaneously over the inflation history? If yes, which balloon is inflated faster? If not, identify at what time you would see a different inflation state. Would one balloon grow while the other shrinks? Explain the physical reasons.)

   c) If there is a pressure sensor at the inlet (Fig. 1a), what is the pressure (P)- volume (V) curve that you would obtain from this sensor?

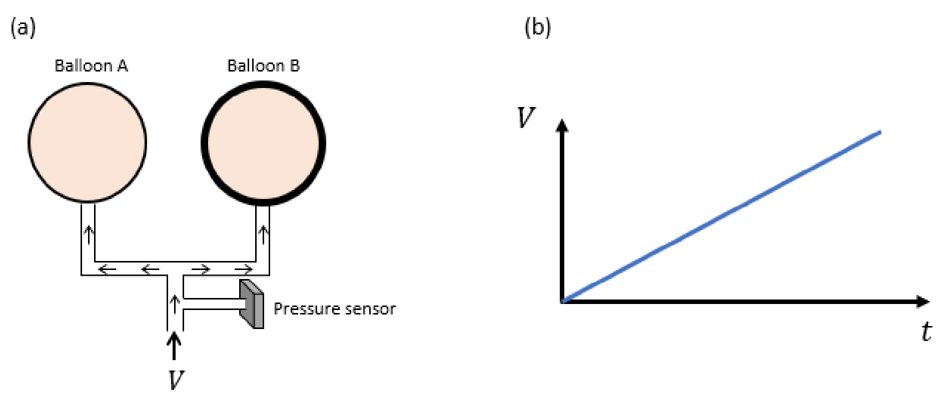

Figure 1

3. Consider the relaxation test of a sample made of dynamic network of detachment rate k_d. At $t = 0$, this sample is instantly stretched to a state, where the normalized covariant becomes $\boldsymbol{\mu} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$. After then, the deformation is fixed ($\boldsymbol{L} = \boldsymbol{0}$) to let the stress relax.

   a) Knowing that $\boldsymbol{\mu}$ changes in time by $\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\mu}\boldsymbol{L}^T + \boldsymbol{L}\boldsymbol{\mu} + k_d(\boldsymbol{I} - \boldsymbol{\mu})$, derive the equation for $\boldsymbol{\mu}$ as a function of time.
   b) Based on your result in (a), describe how does the stress along the vertical direction (σ_{22}) change in time. (Hint: for uniaxial tension, $\sigma_{11} = \sigma_{33} = 0$)

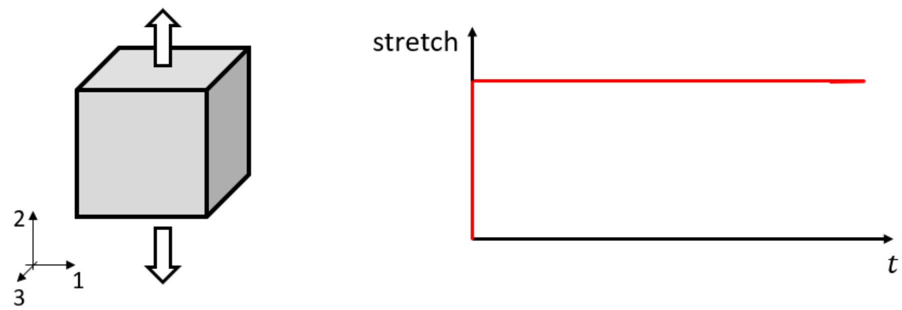

Figure 2