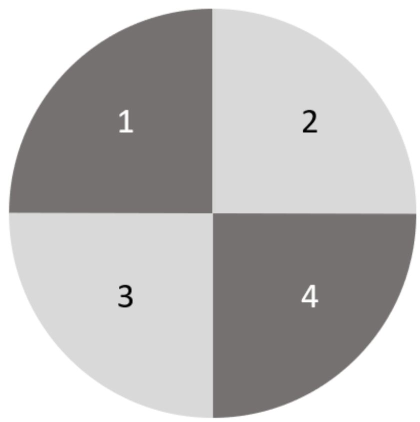
HOMEWORK 1

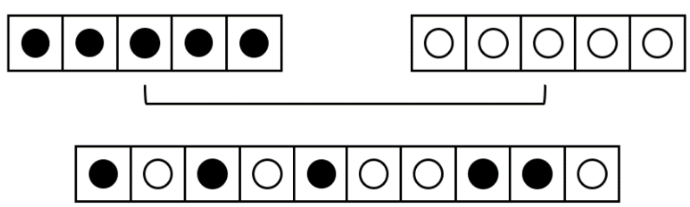
- Show all the work/derivation with neat writing (this counts for the score). Engineering paper should be used. Each student should finish the homework independently.

1. Consider a dart board below. This board is equally divided into four regions numbered from 1 to 4, corresponding to the score if the dart lands in this region. Blindfolded, you throw two darts to the board. Assuming the darts always hit the board:

   (a) What is the probability of you scoring 5 points in total?
   (b) Considering the game as a thermodynamic system and assume each of the score is a micro-state, from a statistical mechanics viewpoint, what is the entropy of this system?



2. Consider two boxes, one contains 5 black particles and the other has 5 white particles. Assuming no interaction between particles, what is the change in entropy if these two boxes are combined into one?



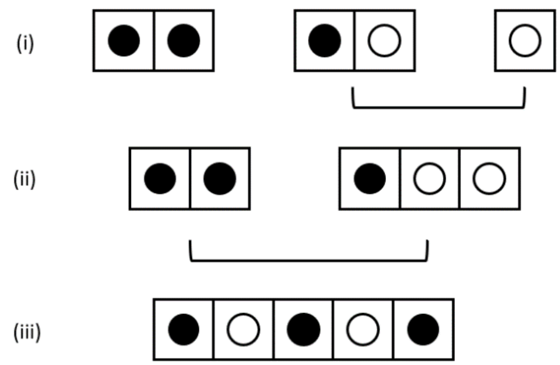
3. Considering the problem similar to the class, a box contains 3 black and 2 white particles. Assuming the interaction energy between the white and black particles is $\epsilon_{wb} = 1$ (i.e., the white and black particles repel one another).

   (a) Draw each of the ten microstates and their respective internal energy.
   (b) Find the probability of each of these microstates when $kT = 0.1, 1$ and 10.
   (c) What difference do you observe between this situation and the case studied in class, where $\epsilon_{bb} = 1$?

4. Consider three boxes shown below, let's consider following operations:

   a) Combine the latter two boxes in the way shown in (i-ii).
   b) Further combine the two boxes (from ii-iii).

Let's assume the interaction energy between black particles is $\epsilon_{BB} = 1$, and the energies for white-white or black-white interactions are zero. Compute the entropy S, the internal energy U and the Helmholtz free energy F of all these three states. Verify whether your result satisfies $\mathrm{U} - \mathrm{TS} = \mathrm{F}$. Comment on how do U, S and F change from state i to iii and explain the trends.



5. A drunk is taking a random walk on the street. Let p be the probability of the drunk taking a step to the right, and $q = 1 - p$ is the probability of taking a step to the left. If the drunk takes N steps, the probability of the drunk's position m is computed by $P(m) = C_N^{(N+m)/2} p^{(N+m)/2} q^{(N-m)/2}$.

   (a) If the drunk does not favor any direction ($p = q = 0.5$), using Matlab, plot the probability distribution P of the drunk's position as histogram at $N = 50$ steps.
   (b) If the drunk is more inclined to go right ($p = 0.6$, $q = 0.4$), using Matlab, plot the probability distribution P of the drunk's position as histogram at $N = 50$ steps.
   (c) For the above two cases, the mean of the drunk's position $\langle m \rangle$ and the standard deviation s^2 can be computed by $\langle m \rangle = \sum_{m=-N}^{N} mP(m)$ and $s^2 = \langle (m - \langle m \rangle)^2 \rangle = \sum_{m=-N}^{N} (m - \langle m \rangle)^2 P(m)$. Evaluate these two values for case (a) and (b) and compare them with the analytical solution.

Hints:

- The analytical solution for $\langle m \rangle$ and s^2 are $\langle m \rangle = N(2p - 1)$ and $s^2 = 4Np(1 - p)$.
- For this problem, m can only be an even number. For the histogram plot, use Matlab command "bar".