

**Thermal Management of Hypersonic Vehicles with Electron  
Transpiration Cooling Systems**

by

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Thesis directed by Prof. Iain D. Boyd

Electron transpiration cooling (ETC) is an emerging thermal management concept for hypersonic vehicles that converts surface heat into electrical energy through thermionic emission. In ETC, electrons emitted from a high-temperature leading edge carry away heat and are subsequently collected on a downstream surface, completing an electrical circuit. This dissertation presents a comprehensive analysis of the ETC system, incorporating the physics of emission and collection as well as the interactions with a hypersonic environment. Governing physics of thermionic emission in a hypersonic environment are examined and a novel model for an ETC system is presented and validated against experimental data. Parameter studies of the model identify physical configurations of ETC systems that maximize cooling, as well as the hypersonic flight regimes best suited for an ETC system. Inclusion of ETC to a hypersonic trajectory optimization framework highlights the influence an ETC system can have on the performance of hypersonic vehicles. Finally, incorporation of the ETC model in a two-temperature hypersonic flow solver determines the role of the hypersonic flowfield on the operation of ETC systems. Results of this dissertation support continued research into ETC. ETC substantially reduces surface temperatures along a hypersonic vehicle for reasonable power requirements. The reductions in surface temperatures are shown to enable access to new hypersonic flight regimes and improve the performance of hypersonic vehicles. Electron collection is found to be a key limitation to ETC technology, as emission currents exceed those recovered downstream.

*To my family.*

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*“Your future hasn’t been written yet! No one’s has.*

*Your future is whatever you make it. So make it a good one.”*

*Doc Brown, “Back to the Future Part III”*

# Chapter 1

## Introduction

### 1.1 The Challenges of Hypersonic Flight

In the 123 years since the Wright brothers' first flight in 1903, the complexity and performance of aerospace vehicles have grown enormously. Today, aerospace vehicles operate over a wide range of flight speeds, from subsonic vehicles such as the Boeing 787 (Fig. 1.1(a)), to supersonic platforms such as the F-35 (Fig. 1.1(b)), and even interplanetary spacecraft such as the Orion Multi-Purpose Crew Vehicle (Fig. 1.1(c)) that experience hypersonic flight during atmospheric re-entry. The span of this velocity range is a triumph of modern aerospace engineering, as each flight regime contains distinct aerodynamic, thermodynamic, and design challenges. It is therefore useful to begin by distinguishing the major velocity regimes that characterize atmospheric flight.

When classifying flight regimes (e.g., subsonic, supersonic, hypersonic), it is useful to use the Mach Number,  $M$ , defined as

$$M = \frac{v}{a} \tag{1.1}$$

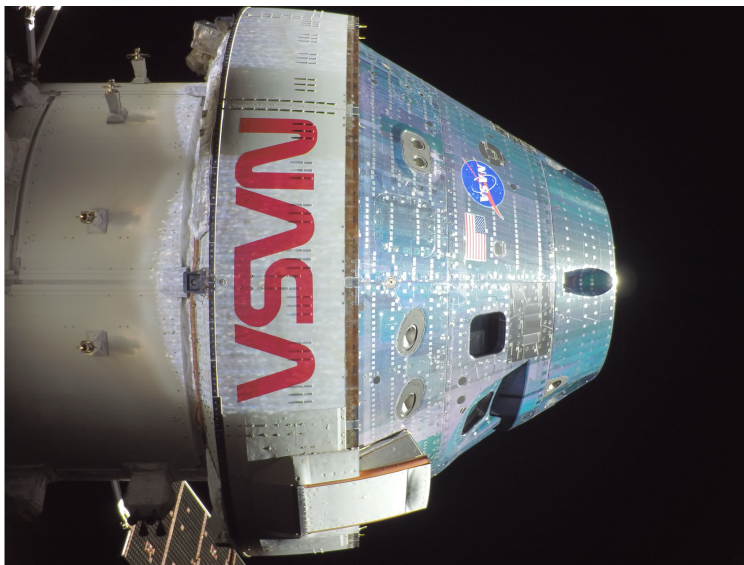
which is a function of speed,  $v$ , and the local speed of sound  $a$ . The Mach number captures the measure of fluid compressibility in the environment while also serving as a useful reference to the sonic speed of the fluid ( $M = 1$ ). For  $M < 0.8$ , the entire flowfield around an object is likely less than the speed of sound and the flow is classified as subsonic. In this regime, pressure disturbances can propagate over the entire domain and can be treated elliptically. For  $M \lesssim 1$ , acceleration of



(a)



(b)



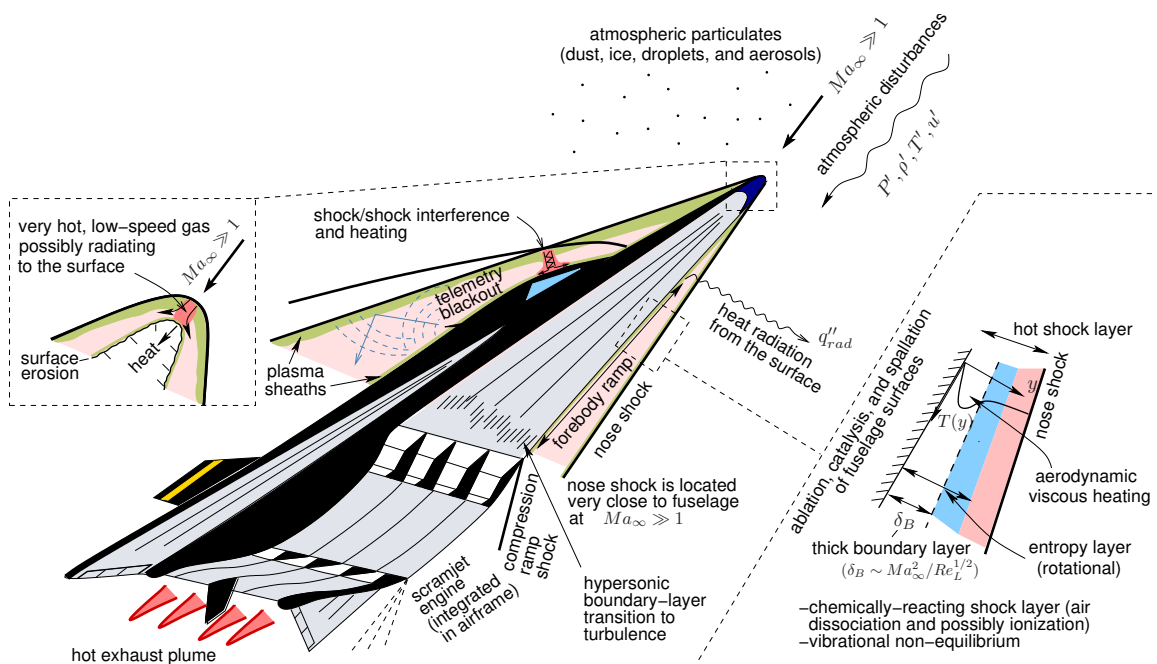
(c)

**Figure 1.1:** (a) Boeing 787-8 ANA Airlines. Photo by Kiefer, via Wikimedia Commons, licensed under CC BY-SA 2.0. (b) Lockheed Martin F-35A. Photo by U.S. Air Force (c) Orion Multi-Purpose Crew Vehicle during the Artemis II mission. Photo by NASA.

the flow due to favorable pressure gradients along the surface of a body may lead to regions of both subsonic and supersonic conditions. This is the transonic regime, which can result in shockwave formation along the body surface, leading to large increases in drag, boundary-layer separation, and loss of control authority [105, 40]. In the supersonic regime,  $M > 1$ , shockwaves move upstream to form either bow or attached shocks near the leading edge, and a large portion of the domain becomes hyperbolic to pressure disturbances. A common definition of the hypersonic regime can be denoted as speeds above  $M = 5$ . This definition is often satisfactory as a rule of thumb when classifying a flight regime, but it also obscures the reasons as to why a distinction occurs between supersonic and hypersonic flight. Unlike how the transonic through supersonic regimes are denoted by changes in compressibility effects through the behavior of shockwaves, hypersonic flight is identified with the introduction of new physical phenomena that are not significant in the supersonic regime.

Due to the significant kinetic energy at these high speeds, flows in a hypersonic regime depart from ideal gas assumptions (e.g., constant ratio of the specific heats,  $\gamma$ ) that are commonly used in the subsonic and supersonic regimes. In particular, this is due to changes in the state of the post-shock environment caused by chemical changes (dissociation, ionization) as well as thermodynamic effects (activation of vibrational and electronic energy modes). Detailed discussion on real-gas effects can be found in Chapter 6, however several notional effects in the hypersonic regime are illustrated in Fig. 1.2. These effects make the behavior of hypersonic environments nonlinear and complicate the determination of integrated quantities such as the drag or lift of a vehicle. As an example, the first Space Shuttle mission, STS-1, experienced a larger-than-expected nose-up moment during re-entry and the on-board flight control system needed to bring the body flap close to its maximum deflection angle to maintain control. This was later found to be the result of real-gas effects and a reduction in the effective value of  $\gamma$  in the hypersonic regime [26, 67].

One of the most important consequences of hypersonic flight is the extreme aerodynamic heating caused by the interaction of the flowfield with a vehicle's surface. Despite the fact that atmospheric temperatures are typically low, compression of the freestream through a shockwave at hypersonic speeds can lead to gas temperatures on the order of thousands of Kelvin which



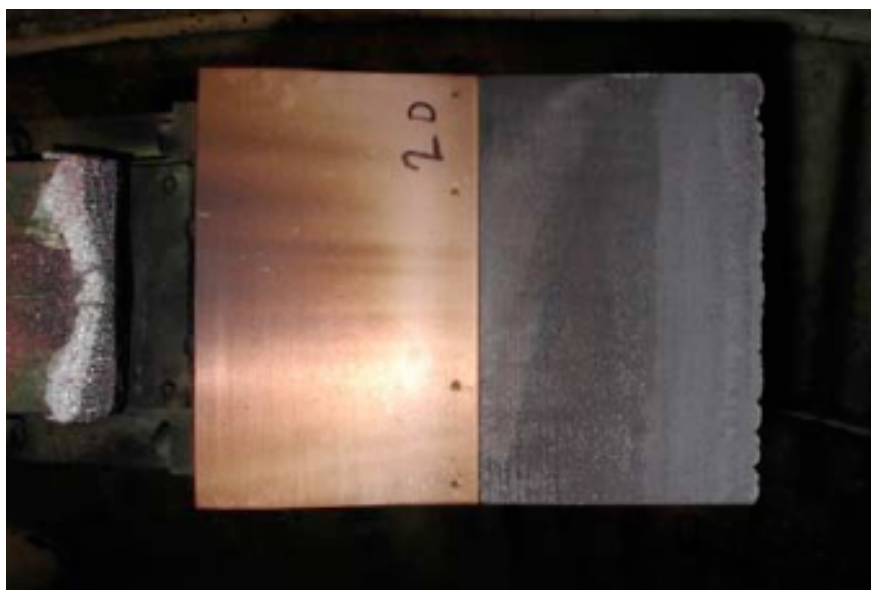
**Figure 1.2:** Depiction of various hypersonic gasdynamic phenomena from Urzay [146]

impart significant heat fluxes to the vehicle. Consequently, hypersonic vehicles are often subjected to significant thermal design constraints. These constraints tend to be present at the nose or leading edges of the vehicle, where severe heat fluxes and limited internal volume makes thermal management a challenge for vehicle designers [82, 136].

To protect the vehicle's internal structure, a thermal protection system (TPS) is often employed. However, there are a limited number of approaches to TPS design for hypersonic vehicles. Ablative TPSs, while robust in handling high heat fluxes, experience changes in surface geometry as material is burned away. This can result in large uncertainties in their time-dependent aerodynamic performance, significantly complicating their analysis [150], as well as the need to be replaced after every flight. Radiative TPSs, such as the X-43's reinforced carbon-carbon (RCC) leading edge, have been observed to be well-suited for temperatures below 2000 K [148]. However, at high hypersonic environments, inter-material stresses and high surface temperatures can cause even state-of-the-art materials to fail [32]. Figures 1.3(a) and 1.3(b) illustrate this by showing thermal damage observed on the X-15 after flight and the RCC material used on the leading edge of the X-43. As a result, advanced thermal management techniques for hypersonic vehicles continue to be of interest to the hypersonic community [130, 65, 135].



(a)



(b)

**Figure 1.3:** (a) Thermal damage along the leading edge of the X-15 [29]. (b) Erosion of the X-43's RCC leading edge after arc-jest testing [98]



## 1.2 Electron Transpiration Cooling as a Thermal Protection System

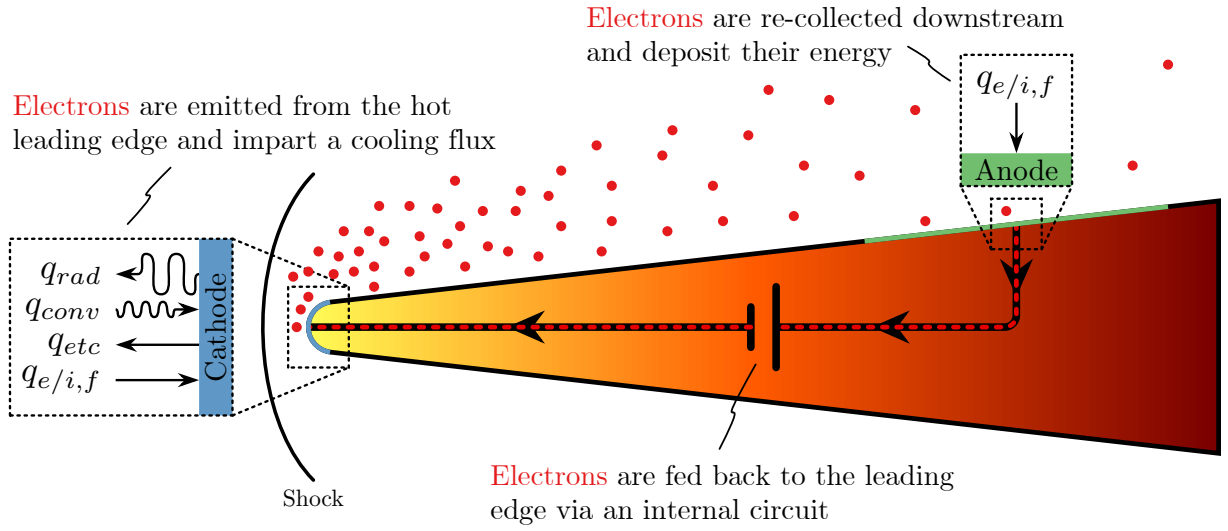
In this section, a novel TPS for hypersonic vehicles called electron transpiration cooling, or ETC, is introduced. Section 1.2.1 provides a conceptual description of an ETC system and identifies several characteristics that make ETC an interesting technology for solving thermal challenges present in hypersonic flight. Sections 1.2.2 and 1.2.3 then review the computational and experimental literature regarding ETC.

### 1.2.1 A Conceptual Description of ETC Systems

A conceptual diagram of an ETC system integrated into a hypersonic leading edge is shown in Fig. 1.4. At the stagnation point, hypersonic vehicles are subjected to extreme heat fluxes due to convection from the flowfield,  $q_{conv}$ , as well as heating from flowfield ions and electrons due to being accelerated in the plasma sheath,  $q_{if}, q_{e,f}$ . A reusable TPSs, such as RCC, balances these heat fluxes via radiative cooling,  $q_{rad}$ .

An ETC system utilizes a thermionic material at the leading edge (i.e. cathode) that emits electrons into the bulk flowfield when heated. Notably, the emission of electrons imparts a cooling effect upon the leading edge,  $q_{etc}$ . Downstream of the leading edge, an anode surface is placed for the purpose of re-collecting electrons from the flowfield. The re-collection process raises the wall temperature at the anode by introducing similar sheath-driven heat fluxes to the anode surface. Finally, the collection electrons are returned to the leading edge through an internal voltage source, thus completing an electrical circuit and ensuring a sustained supply of electrons for the cathode.

A natural comparison can be made between ETC and a traditional transpiration cooling system. In a traditional transpiration system, a working fluid (e.g. water, propylene glycol) is held in an internal reservoir onboard the hypersonic vehicle. The fluid is then injected at the stagnation point to aid in surface cooling. Cooling is expected to scale with the evaporation rate of the fluid, and the activation time and weight of the system will scale with the volume of the reservoir. However, these systems often require complicated piping or a porous material at the



**Figure 1.4:** Diagram of a proposed ETC system. A leading edge experiences convective heating and radiative cooling fluxes,  $q_{conv}, q_{rad}$ , as well as electron and ion heating due to the plasma sheath,  $q_{e,f}, q_{i,f}$ . Via a thermionic material, electrons are emitted from the leading edge and impart a cooling flux,  $q_{etc}$ . Downstream, electrons are collected, depositing a similar plasma sheath heat flux to the surface, and fed back to the leading edge via an internal circuit.

stagnation point in order to transfer the working fluid to the freestream-facing surface.

An ETC system functions as an electrical analog to the traditional transpiration cooling system. Electrons serve as the working fluid and cooling scales with the total current in the system. The activation time and weight of the system is primarily driven by the energy storage capabilities in the voltage source. Owing to the high heat capacity of electrons, large energy densities of power sources, and no requirement of a porous material, ETC systems offer several advantages over the traditional transpiration cooling system.

When used as a thermal protection system, ETC is expected to positively impact the performance of hypersonic vehicles. This stems from the inherently multidisciplinary nature of hypersonic flight, where aerothermodynamics, structures, materials, propulsion, and trajectory are strongly coupled, such that changes in one domain can significantly influence overall vehicle performance [122, 106]. By providing additional surface cooling, ETC may enable operation of hypersonic vehicles at higher velocities and for longer durations [136]. Reduced surface temperatures also relax thermal constraints on structural and internal components, potentially increasing allowable payload or fuel fractions. Collectively, these effects expand the feasible design space for hypersonic vehicles.

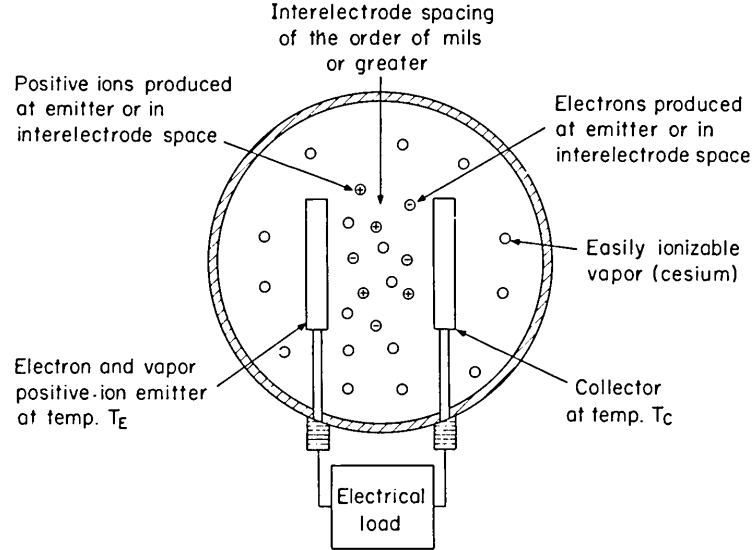
### **1.2.2 A Brief History of Thermionic Emission**

The first observation of thermionic emission predates the discovery of the electron (1897), where, in 1853, Edmond Becquerel observed that an electric current existed between two high-temperature electrodes in air [35]. Over the next 50 years, the phenomenon Becquerel observed was closely studied, where one of the first practical uses was developed by Thomas Edison as an early electronic diode [39].

In the 20th century, continued study of thermionic emission by Richardson [118] led to a mathematical form of a thermionic emission law (Eq. 2.1). Richardson was given credit with coining the term “thermionic emission”, and was eventually given the 1928 Nobel Prize in Physics for his contributions. In addition, improvements to Edison’s diode, namely by John Ambrose Fleming, led to the creation of the first form of vacuum tubes, originally for detecting radio waves

[42]. Fleming's invention laid the groundwork for the commercial vacuum tube which offered a simple way of controlling electrical current through amplification, rectification, and signal mixing. In fact vacuum tubes became crucial to the development of electrical technologies over the first half of the century and a common component in radio, televisions, and radar systems.

As early semiconductors began to replace vacuum tubes in the later half of the 20th century, thermionic emission was mostly studied in the context of thermionic power converters [10, 61, 62], with significant contributions by Ned Rasor and Rasor Associates [115, 114, 113, 27]. An illustration of a type of thermionic energy converter is shown in Fig. 1.5. These are small devices that convert heat into useful electrical power output, useful for their simple design and lack of moving mechanical parts. In one of the first aerospace-specific uses of thermionic technology, Sandia investigated the application of thermionic technology on hypersonic re-entry vehicles for the purposes of power generation in Sandia's plasma arc tunnel. This led to several technical reports, papers and a thesis over the 1960s [140, 141, 142, 80, 142, 79].



**Figure 1.5:** Illustration of a cesium diode configuration of a thermionic energy converter from Hatsopoulos and Gyftopoulos [61]

### 1.2.3 A Review of the Computational and Experimental State-of-the-Art

Only recently has thermionic emission emerged as a thermal management concept for hypersonic vehicles. One of the first conceptual studies was performed by Uribarri and Allen [145] in 2015 in an effort to combat the severe heating present at sharp leading edges and stagnation regions. This work contained one of the first uses of the acronym “ETC”, and established the physical phenomena governing ETC systems, even suggesting that thermionic cooling could rival or exceed radiative cooling at high temperature.

To explore the promises of ETC, several computational studies leveraging higher-fidelity modeling followed. Predominately, these studies modeled the emitting surface only (i.e. no collecting anode or voltage source) to focus on understanding the dynamics of thermionic emission in hypersonic environments. Early studies by Alkandry, Hanquist, and Boyd [7] and Gibbons et al. [48] incorporated idealized thermionic emission physics into computational fluid dynamics (CFD) solvers. These studies analyzed select hypersonic flight geometries and conditions and demonstrated that electron emission can significantly cool a surface, with one case exhibiting a 40% reduction in surface temperature.

Through the work of Hanquist, Hara, and Boyd [59, 57, 60, 55] the idealized emission physics were replaced with those incorporating space-charge effects. Space-charge effects, discussed in detail in Chapter 2, are a critical aspect of ETC systems in that they restrict the level of cooling an ETC system can achieve. These studies showed that, despite the presence of space-charge effects, thermionic emission can still successfully cool a hypersonic surface. Continued work studying space-charge effects has produced improved models from the earlier studies [121], as well as informative simulations resolving space-charge dynamics via particle in cell [147] and finite element frameworks [46].

Multiple approaches to measuring thermionic cooling have been explored, but conclusive evidence of the existence of ETC in a representative hypersonic environment is yet to be found. Studies with shock-tube geometries have been partially successful in measuring emissive currents,

but short test times and test-to-test variability of the tunnel conditions have occluded trends in ETC performance [104]. Perhaps more promising are approaches with inductively coupled plasma (ICP) torches [91, 94, 78, 31], which allow for extended test durations and high plasma densities. These studies have produced current-voltage traces of ETC systems, however limited material choices and complications with surface diagnostics have made directly capturing the effect of ETC on surface temperature unclear at the time of the present work. Between these approaches, it is evident that designing a successful experimental setup for quantifying ETC performance is not trivial.

A key challenge in advancing the state of the art in ETC systems lies in validating computational models against experimental measurements. A major reason for this difficulty is that prior computational studies have typically modeled only the emissive cathode, while neglecting the collecting anode and internal voltage source. This reduces the scope of predictions the models can achieve through two aspects.

The first aspect is due to the fact that ETC is inherently a system-level process. For steady operation, electrons emitted from the cathode must be recollected downstream at a rate sufficient to sustain continued emission. Models that neglect the anode therefore implicitly assume an unlimited supply of thermionic electrons, which is not realistic. For a physical ETC system, electrons are not unlimited, and the collecting anode must recover current equal to that emitted by the cathode in order to close the circuit. Importantly, all experimental ETC configurations include an anode of some kind. The second aspect is that omission of the internal voltage source limits how naturally the emission physics can be represented. Without a voltage source, features of thermionic emission must be explicitly enforced rather than emerging from the operating conditions of the coupled system. Likewise, without both the anode and voltage source, it is not possible to compute system-level quantities such as the total circuit current. This is a significant limitation, because total current is one of the primary diagnostics reported in experiments. For example, the studies of Meyers [91] and Bak [15] both report measurements of the total ETC current, but such data cannot be directly used for validation with computational models that only contain the emissive cathode.

As a result, previous validation efforts have faced substantial challenges. Hanquist [56] at-

tempted to validate the current measured by the Sandia experiments, however this comparison had to be framed in terms of the thermionic current only, which comprises only a component of the total current that was measured. This, coupled with large uncertainties in material emissivity, freestream conditions, and work function, complicated the validation attempt. Similarly, Campbell et al. [28], in collaboration with the University of Vermont ICP facility [99], reported difficulty drawing definitive conclusions because of uncertainties in the freestream environment and the absence of collected current predictions.

More recent efforts have begun to address these limitations by incorporating additional components of the ETC system. Andrienko et al. [9] included both cathode and anode surfaces within a CFD framework and investigated the effect of thermionic emission in a hypersonic flowfield. This work provided some of the first estimates of hypersonic plasma properties across an ETC configuration, including electric field strengths and locations of Joule heating. However, the anode was treated only as an electric field boundary condition and did not absorb electrons, and the study considered cesium injection at the cathode, which differs from the ETC configuration considered in Fig. 1.4. Analytical work by Boyer [23] also modeled both thermionic emission and electron collection using one-dimensional models. When applied to hypersonic geometries, that framework showed that ETC could provide substantial cooling, but only passive ETC configurations were examined, with no applied voltage source.

Overall, the primary gaps in the ETC literature stem from a lack of system-level modeling. Currently, few models can predict system quantities, such as total current, which limits the ability to use emerging experimental data for validation. In addition, broader scaling relationships, such as the influence of geometry, material properties, and flight conditions on ETC performance, remain largely unexplored and have been explicitly identified as open research needs [56, 103]. Finally, the transport of emitted electrons through the flowfield to the collecting anode has not been adequately characterized, leaving uncertainty in whether sufficient current closure can be achieved in practical systems. These gaps motivate the present thesis, which focuses on ETC as a coupled system involving electron emission, transport, collection, and electrical closure, and seeks to connect

computational predictions with experimentally measurable behavior.

### 1.3 Scope and Outline of Dissertation

The goal of this dissertation is to advance the understanding of ETC systems for use as a TPS. While several potential TPS applications for ETC exist (e.g. a cooling method for the interior surfaces of fusion reactors), this dissertation specifically focuses on its use in a hypersonic context. Over the coming chapters, ETC systems are studied across multiple levels of fidelity. This begins with the fundamental plasma physics that govern thermionic emission and eventually concludes with high-fidelity modeling of a hypersonic flowfield with thermochemical nonequilibrium and electric fields. The structure of this dissertation is described below; the chapters are organized so that each stage supplies the physical and modeling foundation needed for those that follow.

Chapter 2 introduces the governing physics underlying ETC, including thermionic emission, the dynamics of the plasma sheath, and the transport of charged particles between a bulk plasma and a material surface. Detailed expressions for current densities and surface energy fluxes are presented. A particular importance is placed on revisiting assumptions from previous ETC studies and the inclusion of kinetic effects in these expressions. The foundational principles in this chapter establish the groundwork that is used throughout the remainder of the dissertation.

Chapter 3 presents a numerical methodology for a one-dimensional model of an ETC system. The one-dimensional model extends upon previous work on ETC considering solely an emitting cathode with the inclusion of a collecting anode as well as internal circuitry. Analysis of the model continues with a parameter sweep of the ETC system where several optimal configurations are identified that maximize cooling and efficiency. The performance of the ETC model is then validated against a recent experimental study where suggestions for future ETC experiments are discussed. Notably, validation efforts require the computation of the total current within the ETC circuit. This is a novel capability of the one-dimensional model and represents one of the first times such an analysis has been completed.

Chapter 4 investigates the performance of ETC systems over a range of hypersonic environ-



ments. This is enabled by the creation of a post-shock model to estimate the plasma composition surrounding a hypersonic vehicle. Contours of ETC performance as a function of altitude and velocity are presented and the role of space-charge effects on thermionic cooling is identified. Power requirements of ETC are then evaluated, and the chapter concludes with the identification of flight regimes most favorable for ETC operation.

Chapter 5 explores the implications of ETC at the vehicle level. The one-dimensional model introduced in Chapter 3 is coupled to an aerothermal framework suitable for hypersonic trajectory analysis and optimization. Next, a trajectory optimization problem is introduced and applied to a canonical hypersonic vehicle. Optimal trajectories of the vehicle are computed both with and without the presence of an ETC system, and changes in the resulting objective due to ETC are discussed in detail. By embedding the ETC system within this framework, the cooling benefits of ETC are analyzed in the broader context of thermal constraints, trajectory design, vehicle performance, and electrical power requirements.

Chapter 6 presents a higher-fidelity treatment of ETC through a computational hypersonic flow solver. Boundary conditions are developed to represent the interaction of emissive and collecting surfaces with the surrounding plasma and applied to a hypersonic leading edge. The effects of ETC on surface temperatures, plasma potential structures, and electrode currents are shown and the implications on the design of a feasible ETC system are discussed.

Finally, Chapter 7 provides a summary of the work within this dissertation as well as the novel contributions to the current understanding of ETC systems. The dissertation concludes with recommendations and suggestions for future work in both experimental and computational contexts.

## Chapter 2

### Governing Physics of Thermionic Emission in a Hypersonic Plasma

The following section explores the key physical processes in an ETC system that dictate thermionic emission, the emergence of various operational modes, and the parameters that govern the cooling effect on a surface. By establishing these foundational principles, this chapter provides the theoretical groundwork necessary for the numerical models derived in subsequent chapters.

The chapter is organized as follows. In Section 2.1, a physical description of an ETC system is given, including the thermionic emission process within the material and the formation of a plasma sheath at the material-fluid interface. Section 2.2 continues with a detailed description on the dynamics and role of the plasma sheath in ETC systems. Finally, Section 2.3 describes how thermionic emission and the physics of the plasma sheath influence surface heat fluxes.

#### 2.1 Physical Description of ETC Systems

Electron transpiration cooling is analogous to an evaporative cooling process, where the phase change of a working fluid is leveraged to keep material temperatures low. Rather than evaporating a typical fluid (e.g. water), ETC relies on the “evaporation” of electrons out of the material through a process called thermionic emission. Much like evaporation, ETC depends on two principle characteristics: the material work function (which parallels the latent heat of vaporization), and emission current (vaporization rate). The emission current depends on the surface temperature but also the ambient electron density (vapor pressure). With this analogy in mind, we devote this section to the physics that underpins thermionic emission and its relation to ETC.

As a material is heated, the energy distribution of electrons in the material lattice begins to widen. A portion of the electrons at the tail end of the distribution has energy higher than the binding potential of the material (i.e. the work function) and escapes from the material lattice into the surrounding environment. Richardson [118] described the thermionic current density emitted into a vacuum,  $J_R$ , as a function of the wall temperature,  $T_w$ , material work function, WF, and a material specific constant  $A_R$  in units of  $A/(m^2 \cdot K^2)$ . We show this relationship in Eq. 2.1, also known as temperature limited emission.

$$J_R = A_R T_w^2 \exp\left(\frac{-eWF}{k_B T_w}\right) \quad (2.1)$$

While Eq. 2.1 is a useful representation of an ideal limit for the maximum emission current, it does not account for environmental effects nor electron depletion within the emissive materials that may be important to the effectiveness of an ETC system. Thus, the true thermionic current from a surface is bounded from above by the temperature limited value :  $J_{etc} \leq J_R$ .

In a hypersonic environment, the transformation of kinetic to internal energy in the post-shock region may result in the formation of a weakly ionized plasma. The physics of the formation of this plasma and a detailed discussion of it's influence on ETC performance can be found in Chapter 6. We define the potential of the bulk plasma as  $\phi_0$  and assume a quasineutral composition ( $n_e = n_i$ ).

When the plasma interacts with a surface, such as a hypersonic vehicle, an analog to a boundary layer forms called a plasma sheath. The sheath arises due to the disparity between the electron and ion masses ( $m_e \ll m_i$ ), which causes the local electron thermal flux to be much higher relative to ionic thermal fluxes at a given temperature. As the vehicle surface naturally insulates itself from the plasma, it takes a potential,  $\phi_w < \phi_0$ , to counter the high electron flux such that ion and electron current densities are balanced at the surface.

Similar to a fluid boundary layer which contains the velocity gradient between the wall and the freestream, the plasma sheath contains the change in potential between that of the wall and bulk plasma. We show a diagram of the plasma sheath in Fig. 2.1, where  $\lambda_D$  is the Debye length,

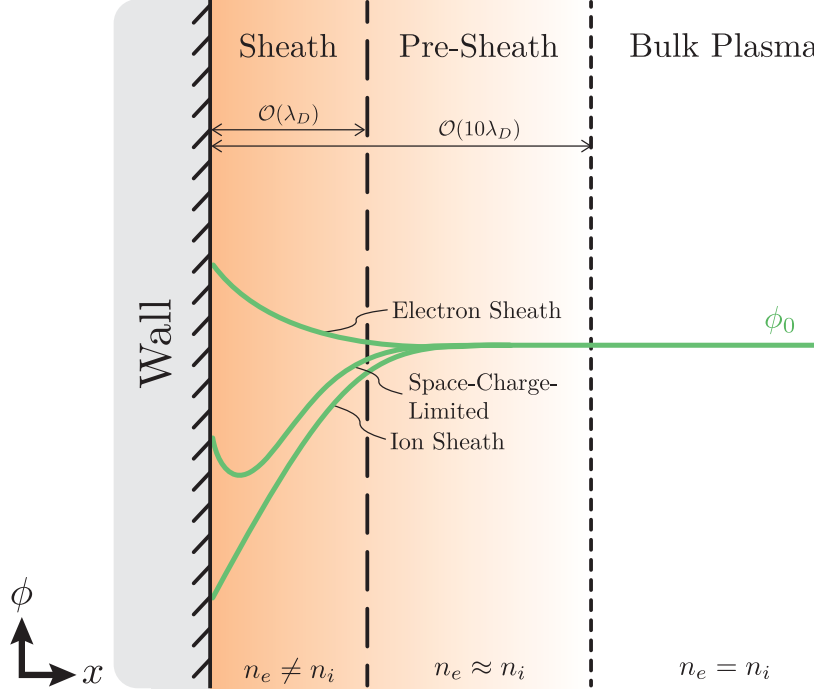
defined as

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_b T_e}{n_e e^2}} \quad (2.2)$$

where  $\epsilon_0$  is the permittivity of free space and  $e$  is the electron charge. For a quasineutral plasma (or similar substances with mobile charge carriers), charged particles re-arrange themselves to screen out any electric fields within itself. The Debye length represents a characteristic length within the plasma over which electrostatic effects decay by a factor of  $\exp(1)$ , and thus, quantifies the scale of charge interactions. As the plasma sheath is defined by the change in potential between the wall and the plasma, it has a height above the wall on the order of a few multiples of  $\lambda_D$  and is in fact sometimes referred to as a “Debye sheath”. Other characteristics within the sheath include sharp gradients in charged species density, velocity, plasma potential, as well as a violation of quasineutrality ( $n_e \neq n_i$ ). For weakly ionized plasmas common to high-hypersonic flow environments, the plasma sheath is typically  $\mathcal{O}(10^{-8}\text{--}10^{-5})$  m.

In addition to the sheath, Fig. 2.1 shows the plasma pre-sheath, which begins a few multiples of  $\lambda_D$  away from the wall. Within the pre-sheath, the potential has mostly returned to that of the bulk plasma, the plasma is quasineutral, and gradients in potential and density are smooth. However, the gradients are not zero and a small drop in the plasma potential occurs over the pre-sheath relative to the bulk plasma. The existence of this behavior is discussed in Riemann [119], where it is shown that the role of the pre-sheath is to accelerate ions from the bulk plasma to a sonic condition at the pre-sheath-sheath interface. Depending on the collisionality of the pre-sheath, the pre-sheath-bulk boundary can be difficult to quantify [43]. In this work, we use the term pre-sheath to broadly refer to the accelerating electric field between the bulk plasma and the sheath.

The properties of the bulk plasma determine the characteristics of the sheath while, in turn, the sheath’s potential structure controls the rate of thermionic emission from the surface. This feedback between the environment and the emission process, analogous to vapor pressure effects limiting evaporative cooling, makes understanding the behavior of the sheath integral to under-



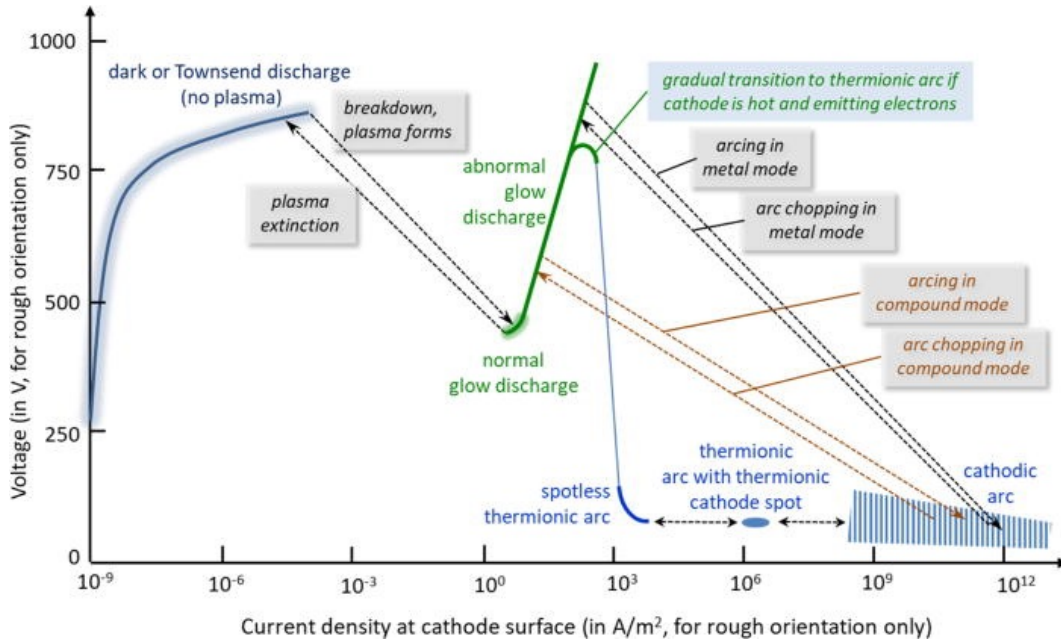
**Figure 2.1:** Qualitative diagram of length scales relevant to ETC with a colored gradient representing the violation of plasma quasineutrality. Curves (—) denote several expected potential distributions of the plasma sheath.

standing the effectiveness of an ETC system. However, due to the plasma sheath’s non-neutral composition, non-Maxwellian behavior, and small length scale, resolving the sheath in CFD is a challenging and computationally expensive endeavor. The following section leverages the use of simplifying assumptions and sheath models to capture the role of the plasma sheath with an ETC system.

Finally, it is useful to consider thermionic emission within the broader context of gas discharge physics. In a general sense, thermionic emission into a plasma can be viewed as a form of gas discharge. Gas discharges have been extensively studied and are commonly categorized into modes such as “glow”, “corona”, and “arc” discharges. Traditionally, discharge modes are identified using current–voltage characteristics [111]. However, this representation can be difficult to relate to complex geometries and variable operating conditions. When considering an ETC system, attempts

to identify a discharge mode becomes further complicated as the bulk plasma is sustained by a combination of thermionic emission and the surrounding hypersonic flowfield, rather than solely by electric field driven ionization as in conventional discharges.

To address these limitations, Anders [8] worked to better classify gas discharge modes according to specific electron production mechanisms. By recasting the traditional current–voltage diagram in terms of current density, different discharge modes are more easily identified. The resulting plot, shown in Fig. 2.2, identifies a distinct thermionic discharge mode at current densities on the order of  $10^6$  A/m<sup>2</sup>.



**Figure 2.2:** Generalized current-voltage characteristics using current density instead of current as the independent variable. Reproduced from [8] under the Creative Commons Attribution 4.0 license

For ETC systems in hypersonic environments, current densities are expected to be dominated by thermionic emission. Based on typical material temperatures and work functions (discussed in Section 2.3 and illustrated in Fig. 2.7), these currents fall in the range of  $\mathcal{O}(10^4\text{--}10^8)$  A/m<sup>2</sup>. Corresponding electrode potentials are predicted to be relatively low, on the order of  $\mathcal{O}(1\text{--}10)$  V, as discussed in Chapter 6. Within the framework proposed by Anders [8], these conditions place

ETC operation within the thermionic arc regime.

## 2.2 The Role of the Plasma Sheath in ETC Systems

If the plasma Debye length is significantly smaller than the mean free path of ion-neutral and electron-neutral collisions ( $\lambda_D \ll \lambda_{e,n}, \lambda_{i,n}$ ), the sheath can be assumed to be collisionless. Previous studies of plasma environments in hypersonic conditions reported mean free paths,  $\lambda_{e,n}$  and  $\lambda_{i,n}$ , on the order of  $\mathcal{O}(10^{-5}\text{--}10^{-3})$  m, supporting the validity of the collisionless sheath assumption [59, 74]. Importantly, it was noted that the collisionless assumption breaks down at low plasma densities, corresponding to velocities below 4 km/s. However, Chapter 4 demonstrates that ETC is not particularly effective in this lower velocity regime. Therefore, a collisionless sheath is assumed in the present work.

The collisionless sheath assumption implies that the flux of charged particles is conserved in a sheath, and thus, properties computed at the sheath edge can represent conditions at the wall-sheath interface. Specifically, defining  $V_{i,se}$  and  $V_{e,se}$  as ion and electron velocities at the sheath edge, current densities of charged particles from the flowfield to the surface are defined in Eqs. 2.3 and 2.4, where  $n_{e,se} = n_{i,se}$  is the quasineutral plasma density at the sheath edge.

$$J_{e,f} = en_{e,se}V_{e,se} \quad (2.3)$$

$$J_{i,f} = en_{i,se}V_{i,se} \quad (2.4)$$

Considering the pre-sheath, a measure of its collisionality can be determined via the mean free paths of Coulomb collisions between two charged species,  $\lambda_{z,z}$ , and charge-neutral collisions,  $\lambda_{z,n}$ , where  $z$  is taken to be  $i$  or  $e$  for ions or electrons respectively [12]. Specifically, we consider the ratio  $\lambda_{z,z}/\lambda_{z,n}$ , where values of less than one indicate the pre-sheath is collisional to the  $z-z$  interaction while values of greater than one indicate the pre-sheath is collisionless. The Coulomb collision length  $\lambda_{z,z}$  can be computed as  $\lambda_{z,z} = v_{th,z}/\nu_{z,z}$ , where  $v_{th,z} = \sqrt{k_b T_z/m_z}$  is a thermal velocity and  $\nu_{z,z}$  is the collision frequency. Equations 2.5 and 2.6 show expressions for  $\nu_{z,z}$ , where

$\ln \Lambda$  is the Coulomb Logarithm and  $\mu$  is a ratio of the ion to proton mass [21]. Additionally, the charge-neutral mean free path can be computed via Eq. 2.7 where  $\sigma_{z,n}$  is the neutral collisional cross section, taken to be  $5 \times 10^{-20} \text{ m}^2$  for electron-neutral collisions [70] and  $1 \times 10^{-18} \text{ m}^2$  for ion-neutral collisions [107].

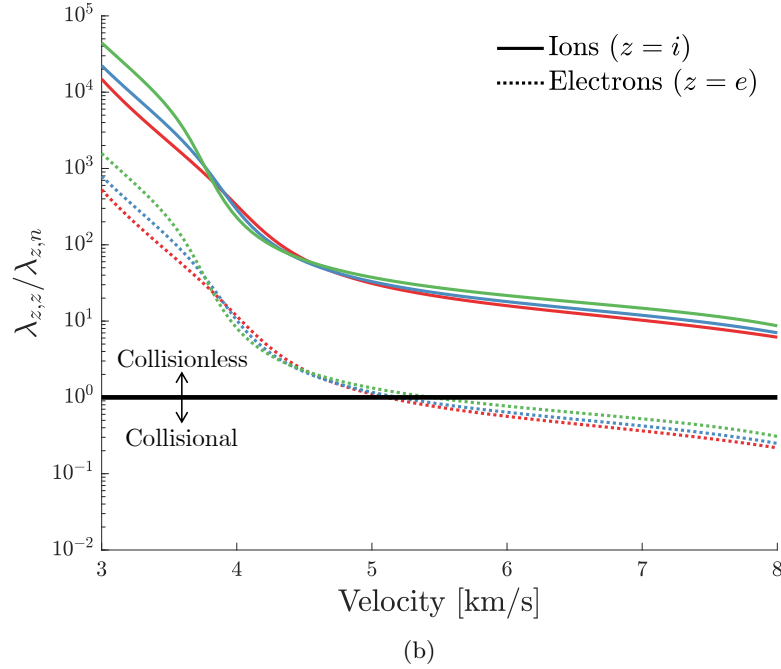
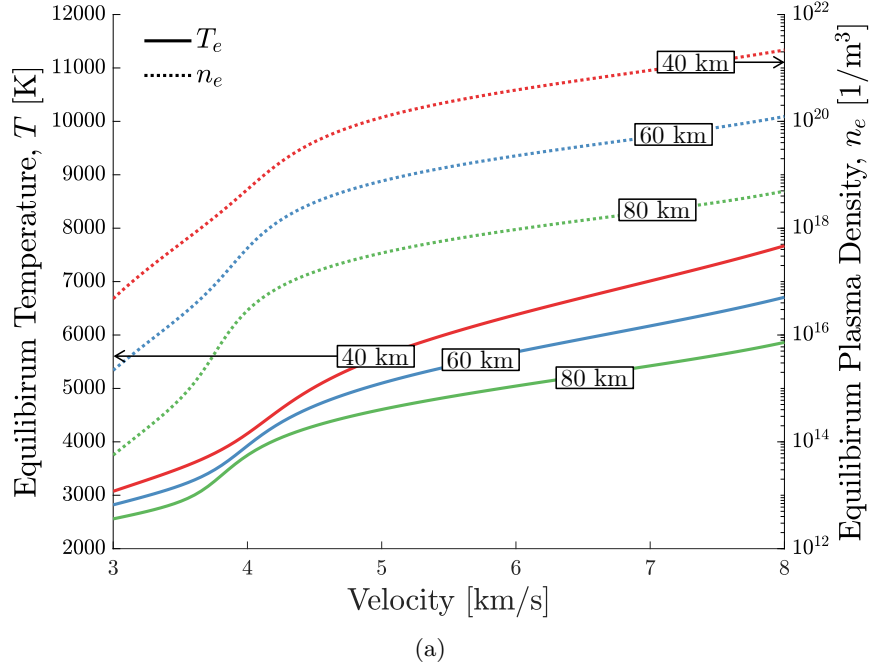
$$\nu_{e,e} = 2.91 \times 10^{-6} \left( \frac{n_e \ln \Lambda}{T_e^{3/2}} \right) \quad (2.5)$$

$$\nu_{i,i} = 4.80 \times 10^{-8} \left( \frac{n_i \ln \Lambda}{\mu^{1/2} T_i^{3/2}} \right) \quad (2.6)$$

$$\lambda_{z,n} = \frac{1}{(n_i + n_n) \sigma_{z,n}} \quad (2.7)$$

To attain an estimate of pre-sheath collisionality in hypersonic environments, an atmospheric model for a hypersonic post-shock plasma is created, details of which are described in Chapter 4. Given a velocity and altitude, the model computes a post-shock temperature and chemical composition assuming thermochemical equilibrium. Figure 2.3 shows values of the post-shock temperature,  $T$ , and electron density,  $n_e$  for altitudes of 40, 60, and 80 km and velocities ranging from three to eight km/s. Assuming a value for the Coulomb Logarithm of  $\ln \Lambda \approx 15$ , Fig. 2.3 uses the predicted values of  $T$  and  $n_e$  to compute the ratio of  $\lambda_{z,z}/\lambda_{z,n}$  over the same conditions. We note that as  $\lambda_{i,i}/\lambda_{i,n}$  is largely greater than one, the pre-sheath is considered to be collisionless for ions. For electrons,  $\lambda_{e,e}/\lambda_{e,n}$  varies from being collisionless to collisional depending on the hypersonic environment. However, Sec 3.3 will show that an ETC system requires  $n_e \gtrsim 1 \times 10^{18} \text{ 1/m}^3$  to have a significant cooling effect. Thus, from Fig. 2.3, the present work considers electrons to be moderately collisional.



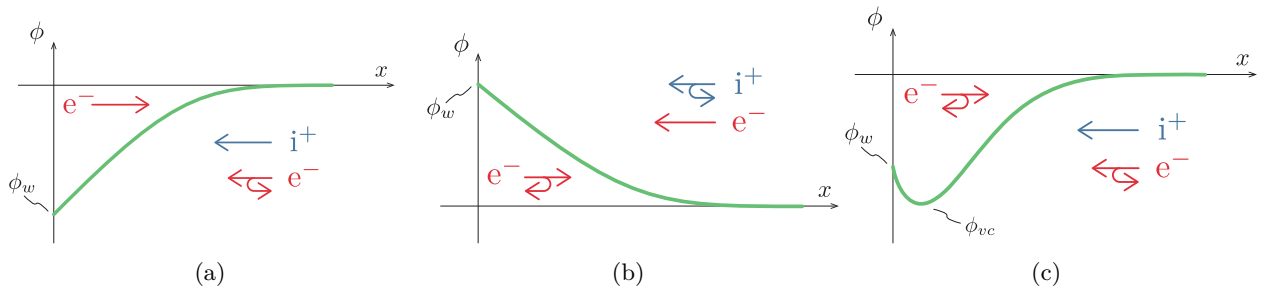


**Figure 2.3:** Profiles of (a) equilibrium temperatures and plasma density and (b) pre-sheath collisionality  $\lambda_{z,z}/\lambda_{z,n}$ , as a function of freestream velocity. Line colors correspond to three altitudes of 40 km, 60 km, and 80 km.

### 2.2.1 Ion Sheaths

Figure 2.4 illustrates three categories of sheath shapes evaluated in the present work, namely the classic ion sheath, the electron sheath, and the space-charge limited (SCL) sheath. We first consider the ion sheath (Fig. 2.4(a)), which is the natural sheath structure that arises due to the difference in ion and electron mobilities. When encountering the ion sheath, all ions directed towards the sheath at the sheath edge are accelerated to the surface. Conversely, only high-energy electrons from the flowfield have sufficient energy to overcome the decreasing potential and reach the wall. If a wall is not actively biased, the wall potential takes a value known as the floating potential,  $\phi_f$ , relative to the plasma such that flowfield ions and electrons impact the wall at the same rate and no net current is imparted to the wall.

If one assumes electrons are Maxwellian and are isothermal through the sheath, a Boltzmann relation can be used to obtain the velocity of electrons entering the sheath. Under the assumption of cold ions, Bohm [20] showed that the velocity of ions entering the sheath is bounded from below by the plasma acoustic speed,  $c$ , known as the “Bohm criterion”. Equations 2.8 and 2.9 show the classical relations for determining the charged species velocities at the sheath entrance, and have been used in previous numerical ETC studies [59, 28].



**Figure 2.4:** Dynamics of charged species interacting with a/an (a) Ion sheath, (b) Electron sheath, (c) Space-charge limited sheath containing a virtual cathode  $\phi_{vc}$ .

$$V_{e,se} = \sqrt{\frac{k_b T_e}{2\pi m_e}} \exp\left(\frac{e\phi_w}{k_b T_e}\right) \quad (2.8)$$

$$V_{i,se} \geq c_{bohm} \equiv \sqrt{\frac{k_b T_e}{m_i}} \quad (2.9)$$

Equations 2.8 and 2.9 are derived from a simplified two fluid description that is typical of low-temperature plasmas. Inherent to these expressions are two critical assumptions: Maxwellian electrons and a cold ion population (i.e.  $T_i \ll T_e$ ). In the context of hypersonics, where ionization is thermally mediated (in contrast to low-temperature plasmas generated with electric and magnetic fields), these assumptions may not hold. For example, due to thermal relaxation of the bulk plasma in the post-shock region it is reasonable to assume that the electron temperature is similar to the ion translational temperature,  $T_i$ , and we can expect a contribution of  $T_i$  to the plasma acoustic speed. Recent analytical and computational work revisiting these assumptions have shown that kinetic effects can have a strong influence at the sheath boundary [12, 3, 127, 125, 143]. Importantly, experimental findings using laser-induced-florescence have observed the depletion of high wall-normal velocities in the electron velocity distribution function (EVDF) near the sheath edge [128], as well as subsonic ion velocities [54] in apparent violation of the classical Bohm criterion. As the sheath's structure critically mediates the emission from a surface we depart from Eqs. 2.8 and 2.9 and implement a more nuanced sheath theory that captures some of this non-idealized behavior.

This work leverages a sheath theory by Loizu et al. [84] which accounts for the observed depletion of high wall-normal particle velocities at the sheath edge via a truncation operator on a Maxwellian velocity distribution function (VDF). This effect is particularly relevant when sheath potentials are above the floating potential of the plasma, which is expected to occur at the collecting anode of an ETC system. For ion sheaths, Loizu et al. [84] defined an EVDF at the sheath-pre-sheath interface as a truncated Maxwellian of the following form

$$f_e(v_x, \eta) = \frac{1}{v_{th,e} I(\eta) \sqrt{2\pi}} \exp\left(-\frac{v_x^2}{2v_{th,e}^2}\right) H(v_x + v_{||,c}) \quad (2.10a)$$

$$I(\eta) = [1 + \operatorname{erf}(\sqrt{\eta})] / 2 \quad (2.10b)$$

where  $v_{th,e} = \sqrt{k_b \bar{T}_e / m_e}$  is the electron thermal velocity,  $v_{||,c} = \sqrt{2\eta} v_{th,e}$  is a truncation speed due to the sheath potential,  $H$  is the Heaviside step function, and  $\eta = -e\phi_w / (k_b \bar{T}_e)$  is a normalized wall potential which takes the potential at the pre-sheath-sheath transition to be zero. The bar notation (e.g.  $\bar{T}_e$ ) indicates parameters defining the full Maxwellian in Eq. 2.10a aside from the truncation operator.

Taking the zeroth and first moments of Eq. 2.10a, we derive Eqs. 2.11 and 2.12 which show expressions for the electron density,  $n_{e,se}$  and velocity at the sheath edge,  $V_{e,se}$ . We note that, for strongly biased walls, as  $\eta \rightarrow \infty$ , the expression  $I(\eta) \rightarrow 1$  and Eq. 2.12 reverts to its simplified form (Eq. 2.8). In this case, the wall potential is so strong that almost all flowfield electrons are reflected and the EVDF at the sheath edge is nearly Maxwellian. However, for weak potentials where  $\eta \rightarrow 0$ , spatial gradients in  $T_e$  and electron fluid stresses in the wall normal direction become non-negligible and Eq. 2.12 is modified due to the truncation of the EVDF.

$$n_{e,se} = \frac{\bar{n}_e}{2} [1 + \operatorname{erf}(\sqrt{\eta})] \quad (2.11)$$

$$V_{e,se} = \frac{v_{th,e}}{I(\eta) \sqrt{2\pi}} \exp(-\eta) \quad (2.12)$$

To determine the effect of EVDF truncation on the ion velocity, Loizu et al. [84] take fluid equations for steady-state conservation of mass for ions and electrons, and conservation of momentum for ions. We show these expressions in Eqs. A.1 through A.3 in Appendix A. Substituting Eq. 2.12 into the conservation equations and solving for  $V_{i,se}$  at the sheath-pre-sheath interface, the ion velocity at the sheath edge,  $V_{i,se}$ , is obtained. We present the final expression in Eq. 2.13a, where  $\tau = T_i / T_e$ ,  $\gamma_z$  is the polytropic coefficient at the sheath entrance where  $z$  is taken to be  $i$

or  $e$  for ions or electrons respectively, and the function  $\kappa(\eta)$  represents the effect of the truncated EVDF. For large values of  $\eta$ , Eq. 2.13a reduces to Eq. 2.9 with an additional ion temperature term. Conversely, when sheath potentials are weak, the ion velocity is reduced due to the effect of the truncated EVDF. Further details on this derivation can be found in Appendix A, as well as Loizu et al. [84].

$$V_{i,se} = \sqrt{\frac{k_b T_e}{m_i}} \sqrt{\frac{1}{1 + \kappa(\eta)} + \gamma_i \tau \left( \frac{n_{e,se}}{n_e} \right)^{\gamma_i - 1}} \quad (2.13a)$$

$$\kappa(\eta) = \frac{\exp(-\eta)}{2I(\eta)\sqrt{\pi\eta}} \quad (2.13b)$$

We note that the value of  $\gamma_i$  in Eq. 2.13a, is dependent on local plasma conditions in the pre-sheath and difficult to compute without access to an ion velocity distribution function (IVDF). As the goal of the present work is to inform the construction of ETC boundary conditions for fluid solvers, we must rely on a reasonable guess for  $\gamma_i$ . Values for the ion polytropic index have been taken to be  $\gamma_i = 1$  for isothermal flow, and in the case of adiabatic flow the index can be related to the number of degrees of freedom for the species,  $\nu$ , where  $\gamma_i = (\nu + 2)/\nu$  [158, 133]. When ion-ion collisionality is weak, we can consider ions to be expanding towards the wall in one direction and thus take  $\gamma_i = 3$ . Conversely, if ion-ion collisionality is dominant in the pre-sheath, collisions will couple motion towards the wall to directions parallel to the wall, increasing the degrees of freedom to three and decreasing the polytropic coefficient to  $\gamma_i = 5/3$ . Considering that low ion collisionality is predicted in Fig. 2.3 for hypersonic regimes of interest, the current work takes  $\gamma_i = 3$ . We provide a summary of expected values of  $\gamma_i$  in Table 2.1, based on collisionality parameters in the pre-sheath.

Equations 2.12, and 2.13a provide kinetically-informed expressions for particle velocities through an ion sheath. At strong wall potentials, the kinetically-informed expressions reduce to a form similar to the classical expressions (Eqs. 2.8 and 2.9). However, when the wall potential is weak or above the plasma floating potential, kinetic effects become significant, and Eqs. 2.12, and 2.13a diverge from their classical counterparts. In the context of ETC systems, kinetic effects are

**Table 2.1:** Expected values of the polytropic index,  $\gamma_z$ , based on collisionality in the pre-sheath, where  $z$  is taken to be  $i$  or  $e$  for ions or electrons respectively.  $\lambda_{z,z}$  and  $\lambda_{z,n}$  are the mean free path for  $z - z$  and  $z$ -neutral collisions, respectively.

| Pre-Sheath Collisionality       | Polytropic Coefficient, $\gamma_z$ |
|---------------------------------|------------------------------------|
| $\lambda_{z,z} < \lambda_{z,n}$ | 5/3                                |
| $\lambda_{z,z} > \lambda_{z,n}$ | 3                                  |

expected at the collecting anode, where the wall potential of the anode must be biased above the plasma potential to collect a net electron current and sustain the emissive cathode. Furthermore, since the emitting cathode wall potential is electrically coupled to that of the collecting anode via the internal circuit, accurately capturing this behavior is important for modeling a complete ETC system. We note that the kinetic effects of ions in the pre-sheath on sheath fluxes was also investigated, however it was found that Bohm's treatment of a monoenergetic ion source was sufficiently accurate, with kinetic effects not significantly altering species fluxes [13].

### 2.2.2 Electron Sheaths

If a voltage source is connected to a plasma-facing surface, the wall potential  $\phi_w$  can be set to a desired value. This is relevant to the collecting anode, where in order to collect a net current of electrons, the wall potential needs to be above the floating potential,  $\phi_f$ , or even positive relative to the plasma potential,  $\phi_0$ . Figure 2.4(b) shows a monotonically increasing electron sheath resulting from  $\phi_w > \phi_0$ . Compared to the ion sheath, the behavior of charged particles reverses: flowfield electrons are accelerated towards the wall by the increasing potential while only the most energetic ions overcome the rise in potential and make it to the wall.

While literature on electron sheaths is lacking compared to the well studied ion sheath, recent work has provided insights into the electron sheath, including the existence of an electron pre-sheath [157, 126]. The electron pre-sheath was shown to serve a similar role as the ion pre-sheath: accelerating flowfield electrons to a sonic speed at the sheath entrance. Under the assumption of Maxwellian ions at the sheath edge, simple expressions for species velocities can be found

$$V_{e,se} \geq \sqrt{\frac{k_b(T_e + T_i)}{m_e}} \quad (2.14)$$

$$V_{i,se} = \sqrt{\frac{k_b T_i}{2\pi m_e}} \exp\left(\frac{e\phi_w}{k_b T_i}\right) \quad (2.15)$$

However, akin to the ion sheath, numerical and theoretical investigations have shown that kinetic effects provide a more complicated relationship with the wall potential, particularly when near the plasma potential [126, 125]. Loizu et al. [84] present an electron sheath analog to Sec. 2.2.1, assuming that the IVDF takes the form of a truncated Maxwellian and modeling the electron pressure via a polytropic law  $-\partial(n_e k_b T_e)/\partial x$  with a coefficient for electrons,  $\gamma_e$ . Setting the determinant of the system to zero and assuming gradients are much larger compared to source terms at the sheath edge, this leads to an expression for ion and electron velocities at the sheath edge as

$$V_{i,se} = \frac{v_{th,i}}{\sqrt{2\pi}I(|\eta|/\tau)} \exp(-|\eta|/\tau) \quad (2.16)$$

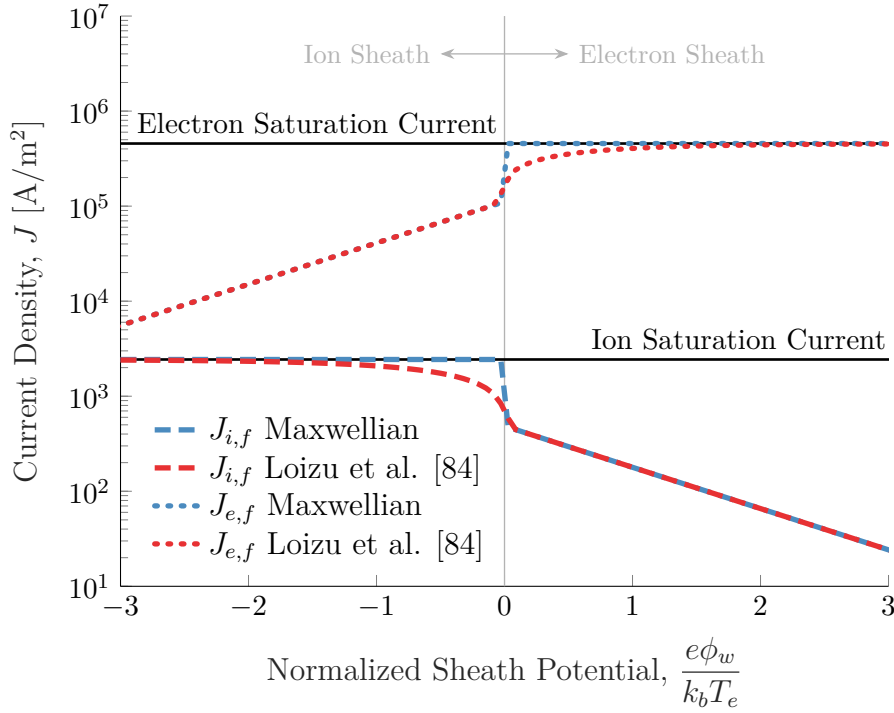
$$V_{e,se} = v_{th,e} \sqrt{\frac{\tau}{1 + \kappa(|\eta|/\tau)} + \gamma_e \left(\frac{n_{e,se}}{n_e}\right)^{\gamma_e - 1}} \quad (2.17)$$

where  $v_{th,i} = \sqrt{k_b T_i/m_i}$  is the ion thermal velocity,  $|\eta|/\tau = e\phi_w/(k_b \bar{T}_i)$  is a normalized potential at the pre-sheath-sheath interface, and  $\gamma_e$  is the electron polytropic coefficient.

For hypersonic environments of interest, it is predicted in Fig. 2.3 that electron-electron collisions are dominant ( $\lambda_{e,e}/\lambda_{e,n} < 1$ ), and thus, we assume electrons to be expanding in three degrees of freedom, taking  $\gamma_e = 5/3$ , however this assumption has yet to be validated experimentally. For this reason, it is recommended that future work investigates the dynamics of the polytropic coefficient in the presence of electron sheaths for hypersonic environments. Similar to the ion polytropic coefficient, we summarize values for  $\gamma_e$  in Table 2.1.

We highlight the effect of truncation in Fig. 2.5 by plotting the flowfield ion and electron current densities,  $J_{i,f}, J_{e,f}$ , as a function of the sheath potential in both the ion sheath and electron sheath regimes. The Maxwellian expressions (Eqs. 2.8, 2.9, 2.14, 2.15) are computed along with the

expressions capturing VDF truncation (Eqs. 2.12, 2.13a, 2.16, 2.17). At weak potentials near zero, the kinetically-informed expressions allow for a smooth transition between the ion and electron sheath modes. This behavior is closer to reality and allows for improved numerical stability when evaluating current densities at weak potentials.



**Figure 2.5:** Ion and electron current densities  $J_{i,f}$ ,  $J_{e,f}$  through a sheath as a function of the normalized sheath potential. Kinetically informed expressions from Loizu et al. [84] have a smooth transition between ion and electron sheaths. Current densities are computed for  $T_e = T_i = 2000$  K,  $n_e = 1 \times 10^{19} \text{ 1/m}^3$ , and  $m_i = 4.78 \times 10^{-26} \text{ kg}$ .



### 2.2.3 Space-Charge-Limited Sheaths

For low levels of thermionic emission, the sheath is monotonic in the shape of the ion sheath and the full Richardson current escapes into the bulk plasma. However, as thermionic emission is increased, the electric field at the wall slowly decreases and eventually become zero. Physically this decrease in electric field happens because the sheath cannot accelerate the electrons away from the surface as fast as they are emitted, causing a buildup of negative charge near the surface. At this point, any further increase in thermionic emission results in a non-monotonic sheath distribution and the formation of a virtual cathode  $\phi_{vc}$ .

We show a diagram of this sheath distribution with the virtual cathode in Fig. 2.4(c). The local potential well attributed to the virtual cathode reflects a portion of thermionic electrons back towards the surface, and thus, Richardson's current is no longer realized. Additionally, any increase in thermionic emission is countered with an increase in the observed potential drop between the wall and the virtual cathode  $\phi_{vc} - \phi_w$ , such that the total thermionic current through the sheath  $J_{etc}$  is relatively constant. An ETC system in this configuration is considered to be operating in a space-charge-limited mode.

Takamura et al. [137] derived 1D expressions for space-charged limited sheaths with emissive surfaces, taking into account the formation of the virtual cathode. Expressions for current densities through the sheath are shown below in Eqs. 2.18 through 2.20, along with definitions for  $\beta_3^*$ ,  $\beta_2^*$ ,  $\beta_1^*$ ,  $\beta_0^*$ ,  $E$ ,  $F$ ,  $A$ ,  $C$ ,  $M$ , and  $\Phi_{vc}$ , as described in Takamura et al. [137].

$$J_{etc} = en_{e,se} \frac{G^* \sqrt{-\Phi_{vc}}}{1 + AG^*} \sqrt{\frac{2k_b T_e}{m_e}} \quad (2.18)$$

$$J_{i,se} = en_{e,se} M \sqrt{\frac{k_b T_e}{m_i}} \quad (2.19)$$

$$J_{e,se} = en_{e,se} \sqrt{\frac{2k_b T_e}{m_e \pi}} \left( \frac{\exp(\Phi_{vc})}{2(1 + AG^*)} \right) \quad (2.20)$$

where  $\exists! G^* \in \{G_1, G_2, G_3\}$  s.t.  $G^* \in \mathbb{R}^+$

and  $\{G_1, G_2, G_3\}$  are solutions to

$$\beta_3^* G^3 + \beta_2^* G^2 + \beta_1^* G + \beta_0^* = 0$$

$$\beta_3^* = EF^2 - 2FA^2 + 2\Phi_{vc}A^3 \quad (2.21a)$$

$$\beta_2^* = F^2 + 2EF(\exp(\Phi_{vc}) - 1) - 2A^2(\exp(\Phi_{vc}) - 1) - 4AF + 6\Phi_{vc}A^2 \quad (2.21b)$$

$$\beta_1^* = 2(F - 2A)(\exp(\Phi_{vc}) - 1) + E(\exp(\Phi_{vc}) - 1)^2 - 2F + 6\Phi_{vc}A \quad (2.21c)$$

$$\beta_0^* = (\exp(\Phi_{vc}) - 1)^2 - 2(\exp(\Phi_{vc}) - 1) + 2\Phi_{vc} \quad (2.21d)$$

$$E = C(A - 1) \quad (2.21e)$$

$$F = -\frac{A}{C} + 2\Phi_{vc} + \sqrt{\frac{-\pi\Phi_{vc}}{C}} \quad (2.21f)$$

$$A = \sqrt{-\pi C\Phi_{vc}} \exp(-C\Phi_{vc}) \times \operatorname{erfc}\left(\sqrt{-C\Phi_{vc}}\right) \quad (2.21g)$$

$$C = \frac{T_e}{T_{ew}} \quad (2.21h)$$

$$M = \sqrt{\frac{1 + AG^*}{1 + EG^*}} \quad (2.21i)$$

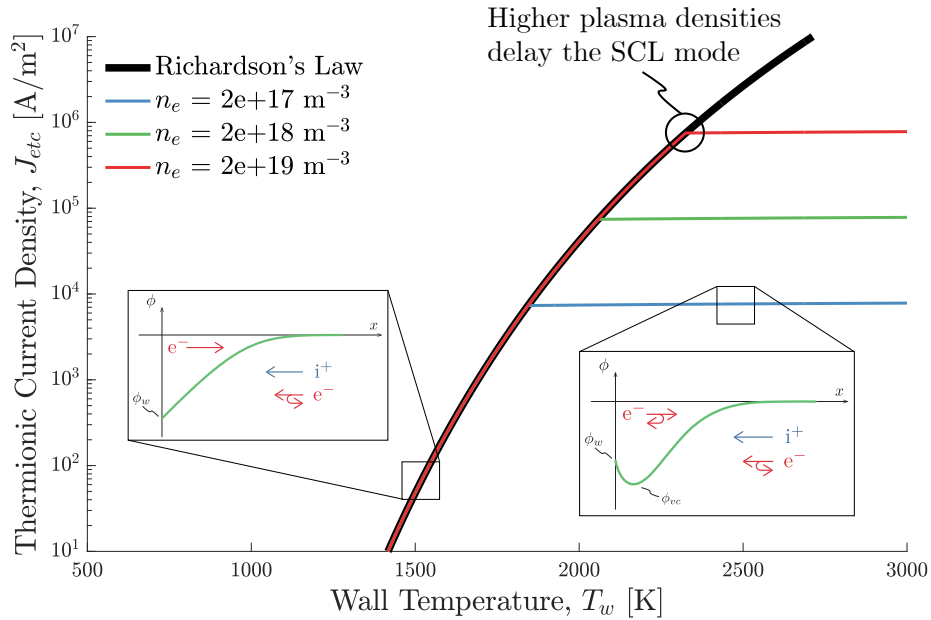
$$\Phi_{vc} = \frac{e\phi_{vc}}{k_b T_e} \quad (2.21j)$$

$$\phi_w = -\left[\ln\left(\frac{J_{etc}}{J_R}\right) \frac{T_w k_b}{e} - \phi_{vc}\right] \quad (2.21k)$$

We note that Eqs. 2.18 to 2.20 from Takamura et al. [137] do not contain kinetic corrections such as VDF truncation as presented in Sec. 2.2.1 and 2.2.2. Modifications to Takamura's theory to include the influence of truncated has been previously completed, showing improvements to sheath current predictions at wall potentials above the plasma potential [60]. However, it is demonstrated in Chapter 3 that at the operating regime where thermionic cooling is most effective, the wall potential of the emissive surface is below the local plasma potential and the VDF is not expected

to be truncated. Thus, the kinetic corrections to Takamura's theory are not explicitly required to gauge the cooling capacity of the ETC system at its ideal operating point.

To highlight the effect of SCL sheaths, we compute expected thermionic current from a cathode surface,  $J_{etc}$ , in Fig. 2.6. When operating in a Richardson-limited mode, the sheath above the emissive cathode takes the form of an ion sheath. All thermionic electrons are accelerated into the bulk plasma and  $J_{etc}$  follows Richardson's Law (Eq. 2.1). However, at a critical current density threshold, the system enters an SCL mode and a virtual cathode forms to restrict the total thermionic current to a near constant value. Importantly, the threshold for this transition depends strongly on the bulk plasma density. Higher plasma densities delay the onset of SCL behavior, allowing greater thermionic current to be sustained.



**Figure 2.6:** Thermionic current density,  $J_{etc}$ , for varied wall temperatures,  $T_w$ . Four curves are shown, including Richardson's Law (—), and three SCL cases with increasing plasma densities (—) (—) (—). All SCL cases follow Richardson's Law at low current densities before reaching a saturation point.

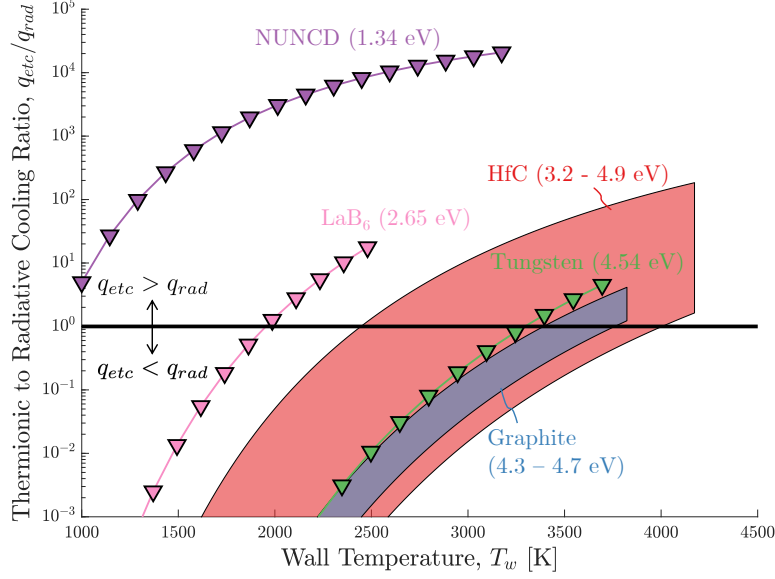
### 2.3 Sheath Heat Fluxes

For an ETC system, we are ultimately interested in the cooling flux carried by thermionic electrons as well as the heat flux from flowfield electrons and ions. The ETC heat flux can be thought of as an Ohmic process due to the product of a current and voltage. In this case, the ETC current is determined by the thermionic emission current while the ETC voltage is comprised of a material work function and thermal energy component. Assuming the thermionic electrons take a half-Maxwellian distribution, we show the ETC energy flux  $q_{etc}$  in Eq. 2.22. Note that the thermionic particle flux,  $\Gamma_{etc} = J_{etc}/e$ , is defined to be positive *away* from the wall, making  $q_{etc}$  a *cooling* flux.

$$q_{etc} = \Gamma_{etc} (eWF + 2k_b T_w) \quad (2.22)$$

Figure 2.7 highlights the potential of ETC by showing the ratio of thermionic cooling to radiative cooling,  $q_{etc}/q_{rad}$ . Here,  $q_{etc}$  is computed using Richardson's Law (Eq. 2.1), while  $q_{rad}$  is computed using the Stefan-Boltzmann law,  $q_{rad} = \epsilon\sigma T_w^4$ , where  $\sigma = 5.670 \times 10^{-8} \text{ W}/(\text{m}^2\text{-K}^4)$ . Several materials exhibit ratios greater than unity before reaching their melting points, indicating that thermionic cooling can exceed radiative cooling. In particular, lanthanum hexaboride and hafnium carbide demonstrate broad temperature ranges where thermionic cooling dominates, due to their high melting temperatures, relatively low work functions, and high emissivities - properties favorable for ETC cathodes. Nitrogen-incorporated ultra nanocrystalline diamond (NUNCD) shows particularly strong performance, although it is not currently mass manufacturable [77]. Material properties used in this analysis are summarized in Appendix B.

Regarding the heat flux from flowfield ions,  $q_{i,f}$ , it is expected that, upon surface impact, the ion neutralizes with the wall and deposit an ionization energy,  $E_i$ , flux. This charge neutralization pulls an electron from the material lattice, similarly to emission, and introduce a negative work function component. Additionally, we use a sheath heat transmission coefficient,  $\kappa_i$ , to describe the heat flux due to the IVDF in the form of  $\kappa_i k_b T_i$ . For electron sheaths, although the IVDF is



**Figure 2.7:** Curves of the ratio of thermionic cooling to radiative cooling,  $q_{etc}/q_{rad}$ , for various materials, computed until their melting point. All materials are labeled with the expected value of their work function and use a Richardson's constant of  $A_R = 120 \times 10^4 \text{ A}/(\text{m}^2\text{-K}^2)$ .

expected to be a truncated Maxwellian, the truncation does not occur on the wall-facing side of the IVDF and the one-way heat flux follows a stationary Maxwellian,  $2k_b T_i$ , where  $\kappa_i = 2$ . However, for ion and SCL sheaths, ions are accelerated in the pre-sheath to a Bohm condition, which tends to distort and narrow the IVDF from a Maxwellian. The extent of this distortion is dependent on plasma conditions such as the degree of ion-ion collisionality, which raises  $\kappa_i$  by broadening the IVDF. In this work, we take a recommended value of  $\kappa_i = 5/2$  to account for some level of collisionality as discussed in Stangeby [133]. Final expressions for  $q_{i,f}$  are shown in Eq. 2.23

$$q_{i,f} = \begin{cases} \Gamma_{i,f} (-eWF + \kappa_i k_b T_i + eE_i) & \text{Electron Sheath} \\ \Gamma_{i,f} (-eWF + \kappa_i k_b T_i + eE_i - e\phi_w) & \text{Ion and SCL Sheaths} \end{cases} \quad (2.23)$$

$$q_{e,f} = \begin{cases} \Gamma_{e,f} (eWF + \kappa_e k_b T_e + e\phi_w) & \text{Electron Sheath} \\ \Gamma_{e,f} (eWF + \kappa_e k_b T_e) & \text{Ion and SCL Sheaths} \end{cases} \quad (2.24)$$

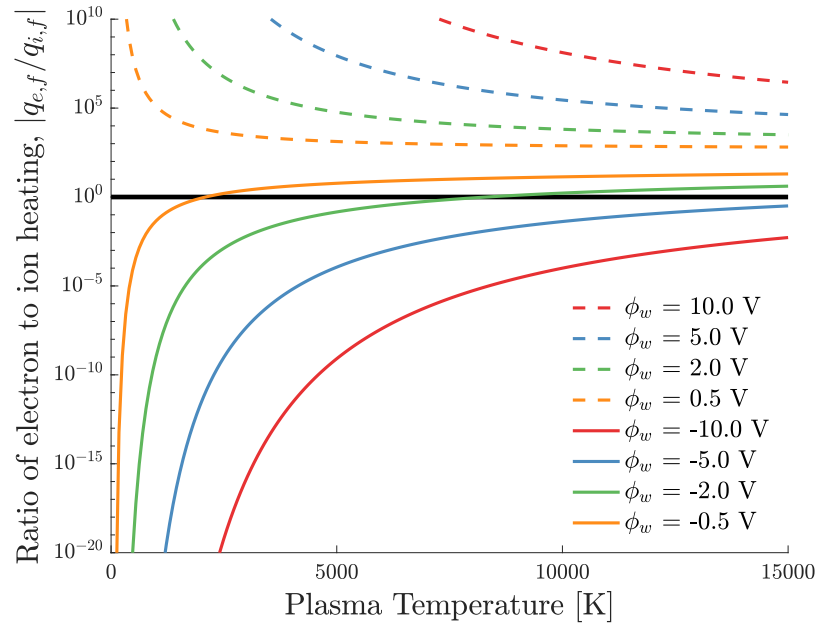
**Table 2.2:** Summary of VDF types and sheath heating coefficients for the sheath models considered in this work.

| Sheath   | IVDF                 | EVDF                 | $\kappa_i$ | $\kappa_e$ |
|----------|----------------------|----------------------|------------|------------|
| Electron | Truncated Maxwellian | Drifting Maxwellian  | 2          | 7/2        |
| Ion/SCL  | Non-Maxwellian       | Truncated Maxwellian | 5/2        | 2          |

The heat flux due to the flowfield electrons,  $q_{e,f}$ , has similar components to  $q_{etc}$ , where flowfield electrons that impact the surface are absorbed into the material lattice and pay a WF component to the total heat flux. For ion and space-charge limited sheaths, the electron EVDF takes the form of a truncated Maxwellian at the sheath-edge, leading to a sheath heat transfer coefficient of  $\kappa_e = 2$ . For electron sheaths, flowfield electrons are accelerated towards the wall and the EVDF is distorted to a degree based on the bulk plasma conditions. However, it is expected that a high electron collisionality in the pre-sheath likely lowers the degree of EVDF distortion compared to the IVDF distortion in an ion sheath. In fact, recent works investigating the electron pre-sheath have indicated that for collisional electrons, the EVDF may take the form of a drifting Maxwellian at the sheath edge [157, 126]. Assuming the EVDF is a drifting Maxwellian yields a heat transfer coefficient of  $\kappa_e = 7/2$  to the total electron heat flux [133], which we show in Eq. 2.24. Table 2.2 summarizes values of  $\kappa_s$  for both ion, SCL, and electron sheaths.

We note that previous studies with ETC have neglected the heat flux from flowfield electrons, arguing that the heat flux from flowfield ions are dominant [56]. To estimate the relative importance of electron heating onto a surface, Hanquist [56] computed estimates of electron and ion heating,  $q_{e,f}$  and  $q_{i,f}$ , for non-emissive sheaths as a function of the wall potential,  $\phi_w$  and electron temperature. For wall potentials of  $\phi_w \in \{1, 2, 5, 10\}$  V and electron temperatures up to 15000 K, and the ratio of electron to ion heating,  $|q_{e,f}/q_{i,f}|$ , was found to be much less than one, indicating that electron heating is small relative to ion heating.

We revisit this study using the ion and electron heating expressions in Eqs. 2.23 and 2.24. We show profiles of  $|q_{e,f}/q_{i,f}|$  in Fig. 2.8 for both positive and negative wall potentials and observe that the ratio is sometimes observed to be greater than one, indicating that electron heating is

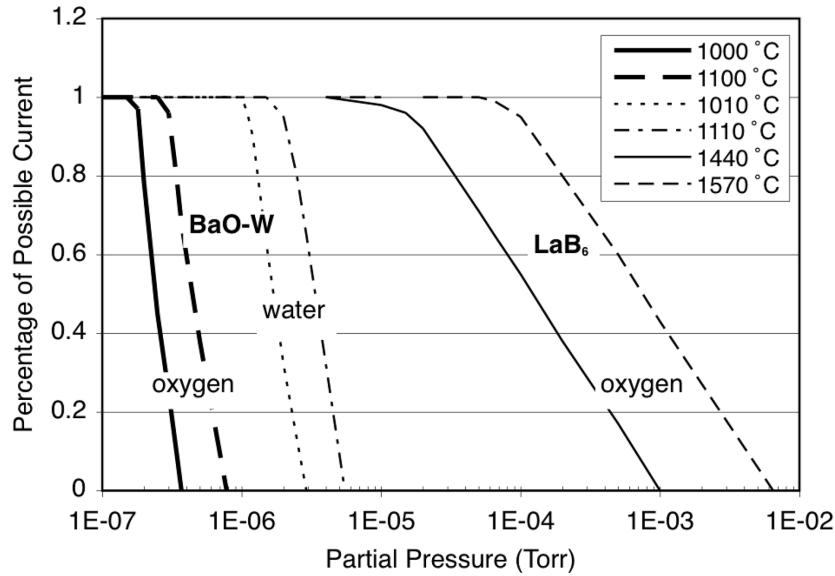


**Figure 2.8:** Absolute value of electron to ion sheath heating,  $|q_{e,f}/q_{i,f}|$  from Eqs. 2.23 and 2.24 using  $WF = 4.0$  eV and  $E_i = 15.76$  eV. Electron heating is not negligible, particularly for potentials that are positive, and close to 0 V.

not negligible. In particular electron heating seems to be dominant at weak, and positive wall potentials. We note that our result is at odds with this previous work due to an  $m_i/m_e$  inversion in the earlier study. Regarding an ETC system, the collecting anode requires a weak or positive bias to collect a net electron current and fuel the emitting cathode, making the modeling of  $q_{e,f}$  an important component to effective ETC analysis.

## 2.4 Cathode Poisoning

Cathode surfaces are influenced by their operating environment. One important degradation mechanism is known as “cathode poisoning”. Poisoning occurs when gases or vapors absorb onto the cathode surface, forming a dipole layer that modifies the effective work function, reduces thermionic emission, and can shorten cathode lifetime [89, 88, 71]. Common poisoning agents include oxygen, water vapor, and carbon dioxide. Since ETC depends on sustained thermionic emission to cool a surface, poisoning poses a practical concern for the use of ETC systems.



**Figure 2.9:** Percentage of the possible thermionic emission current for BaO-W (barium-oxide coated tungsten) and LaB<sub>6</sub> (lanthanum hexaboride) cathodes for various surface temperatures as a function of the partial pressure of oxygen and water vapor. Adapted from Goebel et al. [52].

Figure 2.9, adapted from [52], shows the percentage of possible current for two thermionic



materials (lanthanum hexaboride and barium-oxide coated tungsten) as a function of the partial pressure of a poisoning species. Cathode poisoning becomes more severe as the partial pressure of the poisoning agent increases, while resistance to poisoning improves as surface temperatures increase. Similar trends are well documented in cathode/thermionic literature [44, 45, 52, 11, 156, 71].

Cathode poisoning effects suggest that ETC operation in an Earth atmosphere may be especially challenging due to the presence of oxygen. While significant water vapor does not exist at high altitudes [69], the vapor pressure at sea level is large enough to potentially cause poisoning in experimental facilities. Severe poisoning reduces emission and weakens the thermionic cooling effect. In contrast, for aerocapture or entry missions in atmospheres such as Titan, or the ice giants, the dominant atmospheric species are not expected to produce the same poisoning behavior.

The primary objective of this thesis is to identify the system-level behavior of ETC systems and quantify their idealized performance. Accordingly, the present work neglects detailed surface chemistry and poisoning effects in order to isolate the interactions among thermionic emission, electron collection, electrical closure, and hypersonic environments. Thus, the following chapters can be interpreted as describing the ideal behavior of ETC systems in the absence of material degradation effects. Suggestions for future work on cathode poisoning for ETC systems can be found in Section 7.3.

## Chapter 3

### A Model for an ETC System

In the previous chapter, the key physical processes governing an ETC system were identified and defined. In particular, the role of the plasma sheath in an ETC system was discussed and expressions for both sheath current and sheath heat fluxes were presented.

The following chapter uses these expressions to develop a model of an ETC system consisting of an emissive cathode, a (non-thermionic) collecting anode, and an internal circuit. By incorporating all system components, the model enables estimation of *system-level* properties such as total current and the power requirements of the internal voltage source. In contrast to prior ETC studies that rely on computationally expensive CFD simulations, the model is inexpensive to evaluate and allows for the rapid exploration of ETC systems across a wide range of configurations. Leveraging these capabilities, this chapter explores the ETC design space by quantifying the sensitivity of system performance to configuration parameters and identifying ideal operating conditions.

This chapter is structured as follows. First, in Section 3.1, the methodology and governing equations of the one-dimensional model are described. In Section 3.2, metrics of cooling and efficiency are introduced and computed via the one-dimensional model for a reference case. Using these metrics, a parameter study is performed in Section 3.3, and ideal configurations for maximizing efficiency and cooling are identified. Additionally, in Section 3.4, the one-dimensional model is extended to two dimensions and validated against experimental measurements of an ETC system. The chapter concludes with insights into potentially favorable ETC system configurations and suggestions for future ETC experiments.

### 3.1 One-Dimensional Model Formulation

We begin by noting that the ETC system can be analyzed by recognizing its similarity to an electrical circuit. Figure 3.1 shows a potential diagram of the circuit where the vertical axis represents changes in electric potential,  $\phi$ , and the total current,  $I_s$  is defined to be positive in the clockwise direction. Superscripts of  $( )^c$  and  $( )^a$  denote values taken at the anode or cathode, respectively. Starting at the emissive cathode (bottom left), valence electrons are held within the cathode surface but are shared between the material lattice. Material heating broadens the electron energy distribution such that a fraction of electrons can overcome the cathode work function  $WF^c$  and escape from the surface. Emitted electrons travel through the cathode sheath,  $\phi_w^c$ , and escape into the bulk quasineutral plasma. In the inter-electrode space, plasma resistivity and changes in composition results in a change  $\Delta\phi$  in the plasma potential. For a hypersonic flowfield, compositional changes may arise due to thermochemical relaxation processes and fluid expansion along the vehicle body. At the anode, flowfield electrons travel through the anode sheath,  $\phi_w^a$ , and deposit an anode work function  $WF^a$  component as they are re-absorbed into the material lattice. Finally, electrons traveling back to the cathode encounter any applied voltage across the electrodes,  $V_s$  within the internal circuit.

Within the circuit of Fig. 3.1, voltage and current must be conserved. Defining the cathode and anode surface areas as  $A^c, A^a$  the total current through each electrode is the sum of the ion, electron, and thermionic currents traveling through the surface. We show the system residual,  $\mathbf{R}(\mathbf{w})$ , in Eq. 3.1, where the unknowns are the sheath potentials at the cathode and anode,  $\mathbf{w} = [\phi_w^c, \phi_w^a]^T$ . We note that internal resistance has been neglected.

$$\mathbf{R}(\mathbf{w}) = \begin{bmatrix} WF^c - \phi_w^c + \Delta\phi_p + \phi_w^a - WF^a - V_s \\ A^c \left( J_{i,f}^c + J_{etc}^c - J_{e,f}^c \right) + A^a \left( J_{i,f}^a - J_{e,f}^a \right) \end{bmatrix} \quad (3.1)$$

Solving the above system for  $\mathbf{R}(\mathbf{w}) = \mathbf{0}$  yields sheath potentials at the anode and cathode,



as well as the total current traveling through the ETC system,  $I_s$ . Required power to run the system can be computed with the applied voltage as  $P_s = I_s V_s$ . Values of  $I_s$  are particularly interesting as internal currents are being directly measured at experimental ETC facilities [91, 31, 104]. Additionally, in contrast to previous ETC studies where sheath potentials have been explicitly set, this approach allows for sheath potentials to naturally respond to a specified bias between the anode and cathode via an internal voltage source (e.g. a battery). This behavior better represents a practical ETC system for a hypersonic vehicle.

We note two assumptions inherent in this model. Firstly, surface properties (e.g. plasma density and temperature) over each electrode surface are considered to be spatially uniform. For a realistic hypersonic geometry, such as Fig. 1.4, these properties vary over the electrode surfaces. For this work, we use this simplified framework to understand how critical sheath physics at the vehicle boundary can impact the ETC process and to discuss trends on how it influences ETC performance. A two-dimensional extension of the model is presented in Section 3.4 that accounts for variations in surface properties.

Secondly, the present chapter assumes a uniform plasma in the inter-electrode space such that  $\Delta\phi = 0$ . For a hypersonic leading edge, the bulk plasma expands and cools as it travels downstream from the cathode to the anode and a non-zero  $\Delta\phi$  is expected. Importantly, the magnitude of  $\Delta\phi$ , which depends on the specific leading edge geometry and electrode placement, reduces the effectiveness of the voltage source on the ETC system. However, to keep the results of the following chapter independent of a specific geometry, we assume a uniform bulk plasma. An approach to model  $\Delta\phi$  in a hypersonic environment, as well as a detailed discussion on its role in an ETC system is found in Chapter 6.

### 3.2 Current-Voltage Traces of the ETC System

We begin our analysis with a base case for the ETC system and show the corresponding parameters in Table 3.1. Parameters of interest include the cathode surface area,  $A^c$ , the anode to cathode area ratio,  $A$ , the cathode and anode work functions,  $WF^c$  and  $WF^a$ , Richardson's

constant,  $A_R$ , material emissivity,  $\epsilon$ , and electrode wall temperatures,  $T_w^c$  and  $T_w^a$ . Plasma and wall temperatures roughly correspond to conditions experienced by a hypersonic leading edge flying at 6 km/s and 60 km altitude. Given the high wall temperature and low work function value at the cathode, we expect that the cathode is space-charge-limited in the base case.

**Table 3.1:** Base simulation parameters for the ETC circuit analysis.

| ETC System Parameters                      | Value                |
|--------------------------------------------|----------------------|
| $A$                                        | 50                   |
| $A^c$ , cm <sup>2</sup>                    | 4.0                  |
| WF <sup>c</sup> , eV                       | 2.0                  |
| WF <sup>a</sup> , eV                       | 2.0                  |
| $A_R$ , A/(m <sup>2</sup> K <sup>2</sup> ) | $120 \times 10^4$    |
| $\epsilon$                                 | 1.0                  |
| $T_w^c$ , K                                | 2000                 |
| $T_w^a$ , K                                | 1000                 |
| $T_i$ , K                                  | 2000                 |
| $T_e$ , K                                  | 2000                 |
| $n_e$ , 1/m <sup>3</sup>                   | $1.0 \times 10^{19}$ |
| $E_i$ , eV                                 | 15.76                |

To quantify the performance of the ETC system, we define a total cooling metric,  $\tilde{q}$ , and a cooling performance metric,  $\eta$  in Eqns. 3.2 and 3.3. The value of  $\tilde{q}$  is a normalization of the total heat flux traveling through the sheath to radiative cooling, such that  $\tilde{q} < 0$  and  $\tilde{q} > 0$  indicate net cooling or heating of the surface, respectively, due to the combination of thermionic emission and charged particles fluxes through the sheath. The value of  $\eta$  represents the ratio of useful cooling at the cathode to all power entering the system (i.e. sheath heating and power source). This can be interpreted as a cooling efficiency metric for the ETC system, where a value of  $\eta \approx 0$  indicates that the level of cooling achieved is small compared to power entering the system. Both metrics are defined in terms of the flowfield ion heating  $q_{i,f}$ , electron heating,  $q_{e,f}$ , thermionic cooling  $q_{etc}$ , radiative cooling via the Stefan-Boltzmann law,  $q_{rad}$ , as well as the internal power of the ETC system,  $P_s$ . We note that, under certain conditions, the internal power of the system is negative,

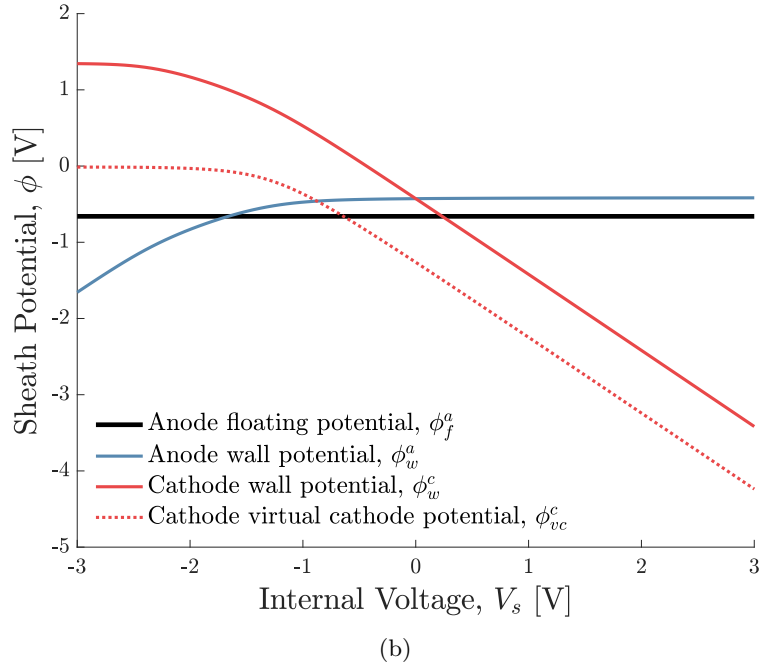
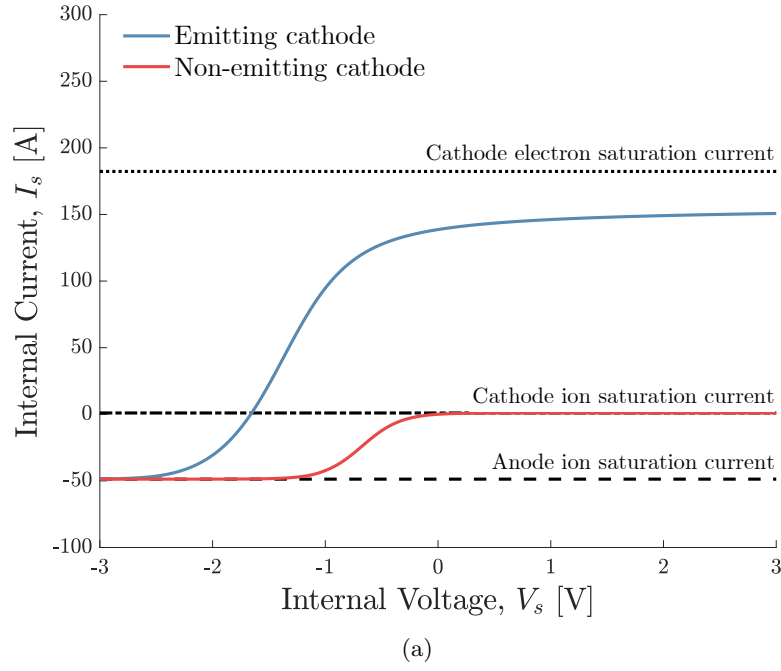
implying that electrical power is being generated rather than consumed. To focus on the cooling aspects of ETC systems, we disregard negative values of internal power in the formulation of  $\eta$ .

$$\tilde{q} = \frac{q_{i,f}^c + q_{e,f}^c - q_{etc}^c}{q_{rad}^c} \quad (3.2)$$

$$\eta = \frac{A^c q_{etc}^c}{A^c (q_{i,f}^c + q_{e,f}^c) + A^a (q_{i,f}^a + q_{e,f}^a) + \max([P_s, 0])} \quad (3.3)$$

We compute current-voltage (IV) traces to characterize the ETC system. These traces provide insight into the dynamics of the system (e.g. internal power requirements, limiting behavior) over a range of configurations, and have been captured experimentally [91, 14]. To compute the IV trace, we solve Eq. 3.1 over a range of internal biases,  $V_s \in [-3, 3]$  V and obtain the current in the system  $I_s$ . As a comparison, we also compute the internal current of the system assuming no thermionic emission at the cathode. For this non-thermionic case, the system is akin to an asymmetric dual Langmuir probe, where we can expect internal currents to be bounded at the ion saturation current at high and low voltages.

Figure 3.2(a) shows the internal current of the system for both cases. As expected, the non-thermionic curve is bounded at low and high potentials by the anode and cathode ion saturation currents, respectively. It should be noted that the asymmetry in saturation currents for the non-thermionic case is due to the area ratio,  $A$ , being non-unity. Conversely, we see that the blue curve, denoting a thermionic cathode, is capable of pushing a total current through the circuit that is much greater than the non-thermionic surface. We show sheath potentials for the thermionic system in Fig. 3.2(b). As the internal voltage becomes more positive and the internal current increases, the anode wall potential rises above the anode floating potential to collect a net electron current. Additionally, we see that the difference between the cathode wall potential,  $\phi_w^c$  and virtual cathode potential,  $\phi_{vc}^c$ , is nonzero. This confirms that a virtual cathode exists and the cathode is space-charge limited where the sheath potential takes a structure similar to that of Fig. 2.4(c).



**Figure 3.2:** (a) IV trace of the ETC system described in Table 3.1 for a non-emitting and emitting cathode. The presence of thermionic emission at the cathode drives the total current in the system to higher values compared to a non-ETC case. (b) Sheath potentials of the ETC system, including cathode and anode wall potentials,  $\phi_w$ , anode floating potential,  $\phi_f^a$ , and the virtual cathode potential  $\phi_{vc}^c$ .



For this reason, the internal current of the thermionic cathode approaches the electron saturation current at the cathode, instead of Richardson’s Law.

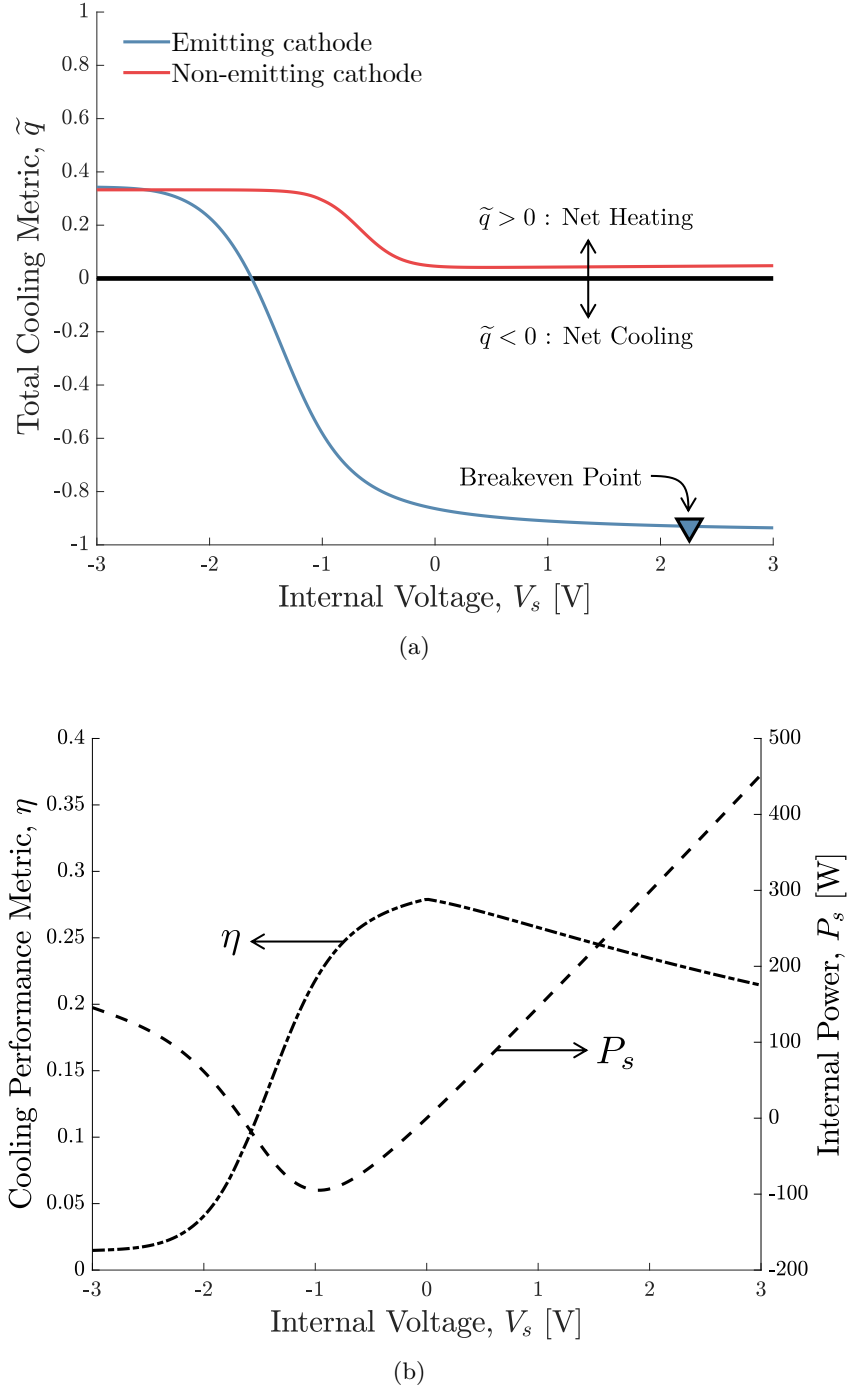
We turn to computations of the total cooling metric,  $\tilde{q}$ , in Fig. 3.3(a) to understand the role of thermionic emission from a system perspective. For large, negative internal voltages, both the non-thermionic and thermionic cases predict net heating as the system is being biased to push current in a direction opposite to that of thermionic emission, resulting in a large flowfield electron heat flux,  $q_{e,f}$ , at the cathode. For large, positive internal voltages, the non-thermionic surface (red curve) reflects the majority of flowfield electrons, but still experiences net heating due to flowfield ions,  $q_{i,f}$ , being accelerated into the surface. However, for the emitting cathode (blue-curve), thermionic emission allows for the introduction of the thermionic cooling flux,  $q_{etc}$ , at the surface. The thermionic cooling flux is so much greater than flowfield ion heating that cooling on the order of  $\mathcal{O}(1 \times q_{rad})$  is predicted for this configuration. Finally, we illustrate a “breakeven” point on curves of  $\tilde{q}$  where the total cooling power is equal to the power draw of the circuit (Eq. 3.4). As power scales with the internal voltage, values of  $\tilde{q}$  to the right of the breakeven point indicate the power draw of the ETC circuit is greater than the total cooling flux the ETC system is providing.

$$P_s^{breakeven} = q_{etc} - q_{i,f} - q_{e,f} \quad (3.4)$$

Figure 3.3(b) shows a measure of cooling efficiency via the cooling performance metric,  $\eta$ . Interestingly,  $\eta$  reaches its maximum value at 0 V, where net thermionic cooling is still predicted. The reason for this behavior is that thermionic emission naturally raises the wall potential compared to a non-thermionic surface, even without an applied bias. As the anode is non-thermionic and we have assumed a constant plasma between the electrodes, the rise in the cathode wall potential,  $\phi_w^c$ , creates a potential difference between the electrodes that pushes electrons from the anode to the cathode. Thus, we get a “free” voltage contribution with the addition of the anode. As a 0 V configuration implies the absence of a voltage source (electrodes are shorted), the contribution of

the internal power,  $P_s$ , to  $\eta$  becomes zero, driving up the value of the cooling performance metric. As a result, the model predicts an efficient operating condition at 0 V, where significant thermionic emission is achieved without the need for external power.

Finally, it should be noted that in viewing the internal power,  $P_s$  in Fig. 3.3(b), there exists a small regime near  $V_s \approx -1$  V where  $P_s < 0$ . In this regime, power must be leaving the system for the circuit to be balanced. This suggests that it may be possible for the ETC system to power a resistive load or external apparatus by taking advantage of the hypersonic plasma around the vehicle. This arises from the thermodynamic characteristics of the ETC system. Thermal energy from a hot surface (i.e. cathode) is being converted into electrical energy before reaching a cooler surface (i.e. anode), resembling a Carnot cycle.



**Figure 3.3:** (a) Total cooling metric  $\tilde{q}$  (Eq. 3.2) for the ETC system described in Table 3.1. Symbol ( $\nabla$ ) corresponds to the breakeven point for the ETC system where the total cooling power equals the power draw of the circuit. (b) Cooling performance metric  $\eta$  for the ETC system (Eq. 3.3) and power consumption  $P_s$  for the voltage source.

### 3.3 Parameter Study of the ETC System

We extend the characterization of the ETC circuit by performing a parametric analysis of system parameters. Specifically, we focus on three parameters, namely the electron number density,  $n_e$  (section 3.3.1), electrode area ratio,  $A = A^a/A^c$  (section 3.3.2), and the change in work function between the cathode and anode,  $\Delta WF = WF^a - WF^c$  (section 3.3.3). For each parameter study, we compute curves  $\tilde{q}$  and  $\eta$  for a sweep of internal voltages  $V_s \in [-3, 3]$  V. Parameters not being varied are held constant according to Table 3.1.

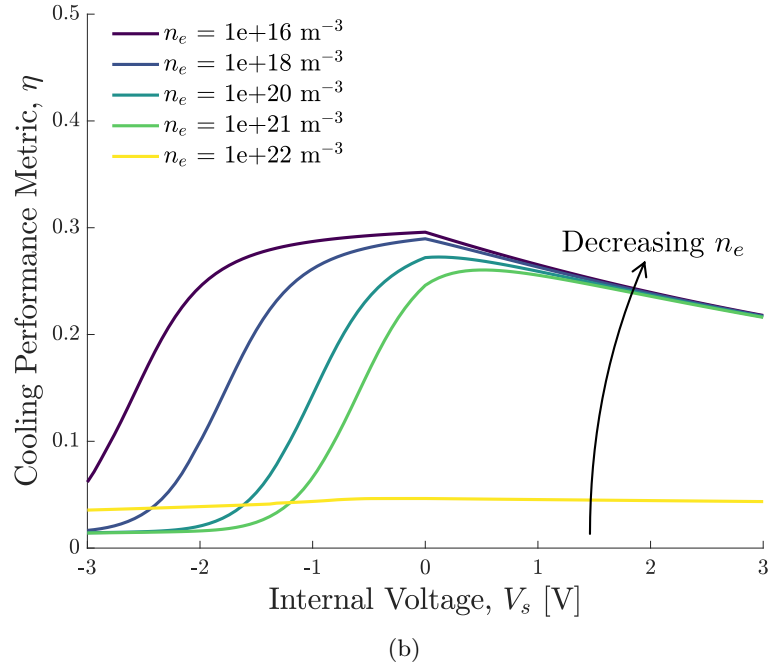
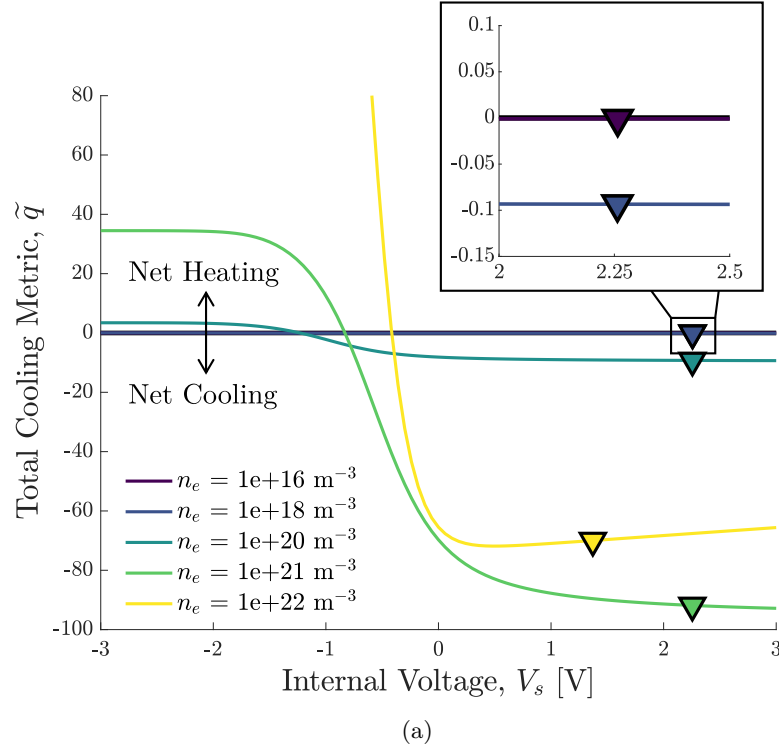
#### 3.3.1 Impact of Varying the Plasma Density, $n_e$

We first look at the role of the electron number density,  $n_e$ . For low values of  $n_e$ , we expect space-charge-limits to hinder thermionic emission such that thermionic cooling provides nearly no benefit. Conversely, as plasma density increases, space-charge-limits relax and thermionic cooling increases until the cathode transitions to a Richardson-limited mode. However, ion and electron heating from the plasma scale with the plasma density, making it unclear if either a space-charge-limited or Richardson limited regime is more favorable for net cooling and power requirements of the ETC system. To explore this relationship, we vary the plasma density to include both space-charge-limited and Richardson-limited modes.

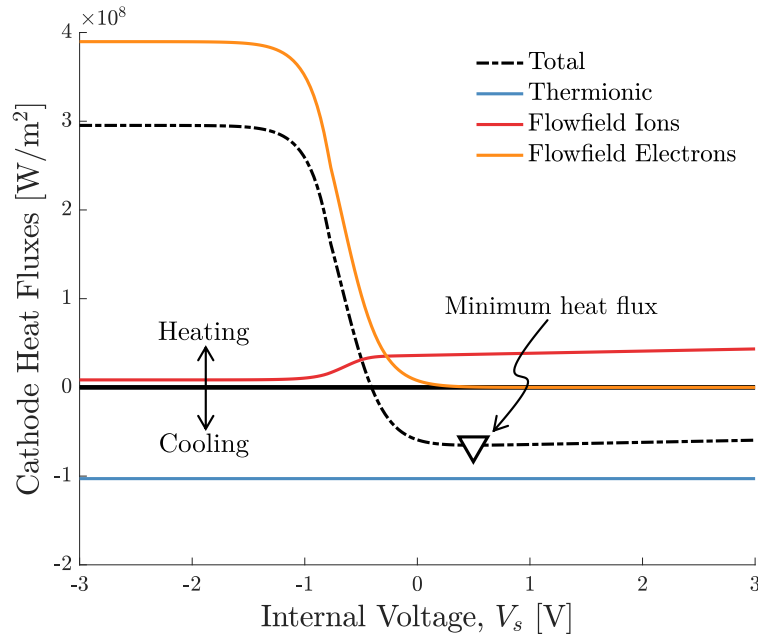
We show curves of the total cooling metric for plasma densities  $n_e \in \{10^{16}, 10^{18}, 10^{20}, 10^{21}, 10^{22}\}$   $1/\text{m}^3$  in Fig. 3.4(a). We note that the ETC system is space-charge-limited for all cases below  $n_e = 10^{22}$   $1/\text{m}^3$ . For the SCL cases, the magnitude of the total cooling metric increases with  $n_e$  as the higher plasma density is capable of accepting a larger thermionic current. This behavior continues until the system becomes Richardson limited (which occurs between  $10^{21}$  and  $10^{22}$   $1/\text{m}^3$  cases). Specifically, the Richardson-limited  $n_e = 10^{22}$   $1/\text{m}^3$  case is seen to predict higher heating compared to the SCL  $n_e = 10^{21}$   $1/\text{m}^3$  case. This is expected, as Richardson-limited operation suggests thermionic cooling is near-constant at its maximum value. However, as  $V_s$  increases, the cathode wall potential  $\phi_w^c$  increases in magnitude. This enhances ion heating  $q_{i,f}$  via the sheath

potential contribution in Eq. 2.23. To emphasize the role of ion heating, we show heat flux components for the  $n_e = 10^{22} \text{ 1/m}^3$  case in Fig. 3.5, where ion heating drives the total heat flux at high values of the internal voltage (and eventually overpowers thermionic cooling). This leads to the observed minimum in the total heat flux at  $V_s \approx 0.5 \text{ V}$ , where the system has taken a cathode sheath potential that balances heating from flowfield electrons and ions. In fact, due to ion heating, a minimum value of  $\tilde{q}$  is expected for both SCL and Richardson-limited cases. However, in the SCL cases presented, the locations of minimum  $\tilde{q}$  are at high values of  $V_s$  off to the right of the page. As the system approaches the Richardson limit, the location of the minimum moves to the left. Finally, we note that the location of the breakeven point (Eq. 3.4) for all SCL cases occurs at the same value of  $V_s \approx 2.25 \text{ V}$ . This is because, at SCL conditions, both the thermionic and flowfield ion + electron currents are proportional to the local plasma density, as opposed to a Richardson limited case where thermionic currents follow Eq. 2.1. As a result, the plasma density cancels from Eq. 3.4.

Figure 3.4(b) shows the cooling performance metric over varied  $n_e$ . The results mirror those from Fig. 3.3(b), where a maximum in cooling efficiency is predicted at 0 V for all cases. For the SCL cases, curves of  $\eta$  collapse together as the internal voltage increases. This behavior is expected for large values of  $V_s$  as, under space-charge-limits, the ratio of thermionic current to flowfield currents is nearly constant. However, the power draw increases with the internal voltage, driving the cooling performance metric down as the internal voltage is increased. For the Richardson-limited cases, the value of  $\eta$  is observed to be significantly lower, as the thermionic current is lower than what is allowed by the bulk plasma. Thus, to maximize cooling efficiency, it is suggested to operate at the space-charge limit.



**Figure 3.4:** Parameter study of the ETC system with varied plasma density,  $n_e$ , showing curves of (a) total cooling metric  $\tilde{q}$  (symbol  $\nabla$  corresponds to the breakeven point for the ETC system (Eq. 3.4), where the total cooling power equals the power draw of the circuit) and (b) cooling performance metric  $\eta$ .



**Figure 3.5:** Heat flux components at the cathode for the Richardson limited  $n_e = 10^{22} \text{ 1/m}^3$  case. The total heat flux reaches a minimum value at  $V_s = 0.5 \text{ V}$ . As  $V_s$  continues to rise, ion heating increases. Symbol ( $\nabla$ ) indicates the location of minimum total heat flux.

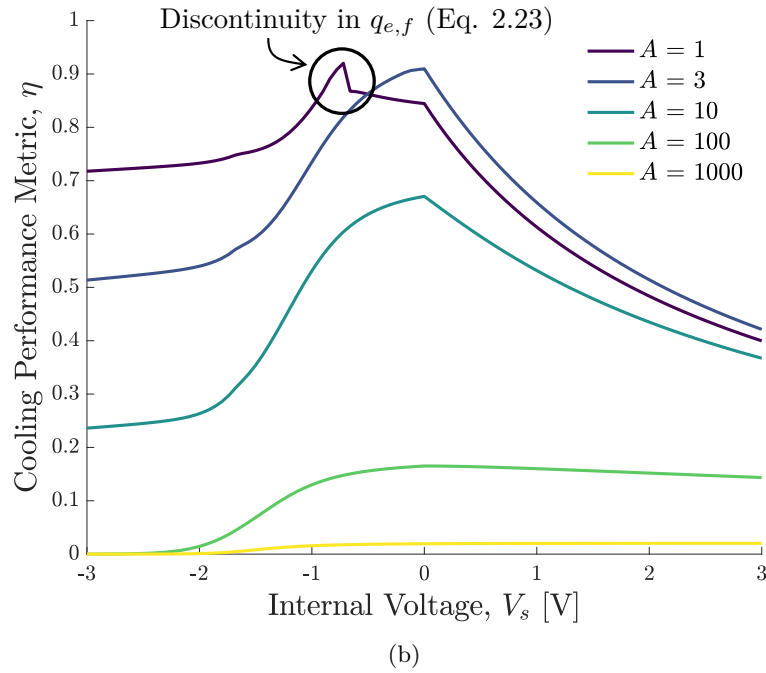
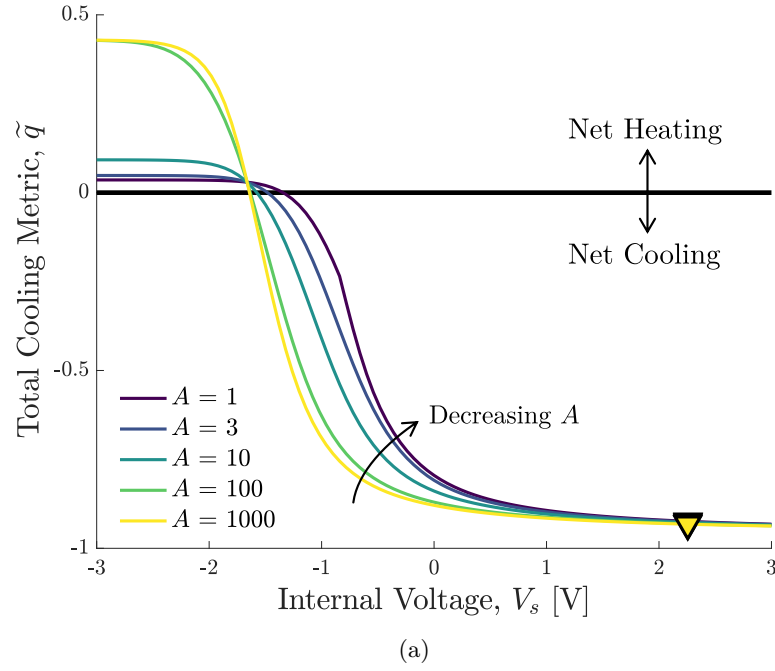
### 3.3.2 Impact of Varying the Electrode Area Ratio, $A$

Next, we consider the role of the electrode area ratio,  $A$ . Likely we can expect a lower limit on the value of  $A$  where the anode surface is small and unable to collect sufficient current to supply the emissive cathode. Additionally, it is likely that, at very large values of  $A$ , the anode can easily collect sufficient current and any further increase in  $A$  does not significantly change collection capability. To identify the role of the anode size on ETC performance, we vary  $A$  from small to large values.

The total cooling metric for electrode area ratios  $A \in \{1, 3, 10, 100, 1000\}$  is plotted in Fig. 3.6(a). Curves of  $\tilde{q}$  suggest that cooling is improved as the anode area is increased relative to the cathode. To understand this behavior, it is important to envision that as the anode increases in size, it experiences a larger flux from flowfield electrons and does not need to be biased far from the floating potential,  $\phi_f$ , in order to collect enough electrons to complete the circuit. Thus, for  $A \gg 1$ ,  $\phi_w^a$  approaches the plasma floating potential  $\phi_f$ . However, the anode is electrically coupled to the cathode via  $V_s$ . As a net collection of electrons at the anode requires  $\phi_w^a > \phi_f$ , increasing  $A$  and driving the anode wall potential towards  $\phi_f$  drives the wall potential at the cathode down. A lower value of  $\phi_w^c$  decreases the expected flux of flowfield electrons to the surface, and via Eq. 2.24, lowers the electron heat flux. This process has diminishing returns however, as the expected electron flux at the cathode decays exponentially with  $\phi_w^c$ . This is why the curves of  $\tilde{q}$  coalesce at high  $V_s$ .

We show curves of  $\eta$  for varied  $A$  in Fig. 3.6(b). We note that the cooling performance metric scales inversely with  $A$ , suggesting that the anode size should be small relative to the cathode in order to maximize cooling efficiency. For  $A \gg 1$ , the large anode experiences a large energy flux from the plasma without a commensurate increase in ETC current (as shown by asymptote in the total cooling metric), driving the value of  $\eta$  to zero. As  $A$  decreases, the anode wall potential,  $\phi_w^a$ , must rise above the floating potential to increase the collected current density over the smaller surface and complete the circuit. This causes  $\phi_w^a$  to weaken in magnitude and approach 0 V. The





**Figure 3.6:** Parameter study of the ETC system with varied electrode area ratio,  $A$ , showing curves of (a) total cooling metric  $\tilde{q}$  (symbol ( $\nabla$ ) corresponds to the breakeven point for the ETC system (Eq. 3.4), where the total cooling power equals the power draw of the circuit) and (b) cooling performance metric  $\eta$ .

smaller magnitude of  $\phi_w^a$  reduces ion heating at the anode via Eq. 2.23 and increases the cooling performance metric. However, as  $A$  becomes increasingly small, the anode wall potential eventually becomes positive, forming an electron sheath. An ion sheath is present at the anode for all cases except for the  $A = 1$  case where the sheath transitions to an electron sheath for  $V_s > -0.75$  V. The presence of the electron sheath leads to significantly increased electron heating at the anode. As a result, curves of the cooling performance metric are lower for the  $A = 1$  case compared to the  $A = 3$  case. This suggests that there exists an ideal value of  $A$  that maximizes the cooling efficiency of the ETC system. We note that the sharp kink at  $V_s = -0.75$  for the  $A = 1$  case is due to a discontinuity in sheath heating expressions (Eqs. 2.23 and 2.24).

### 3.3.3 Impact of Varying the Change in the Work Function, $\Delta WF$

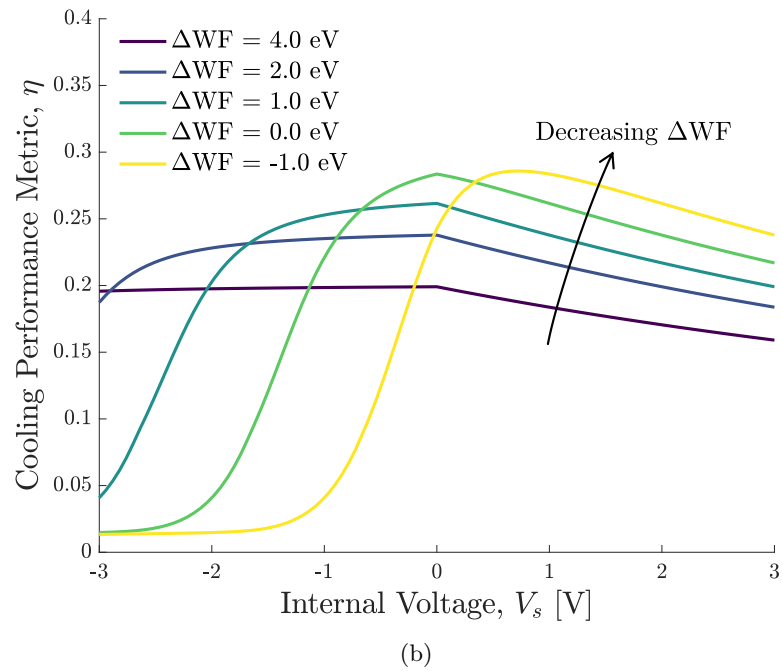
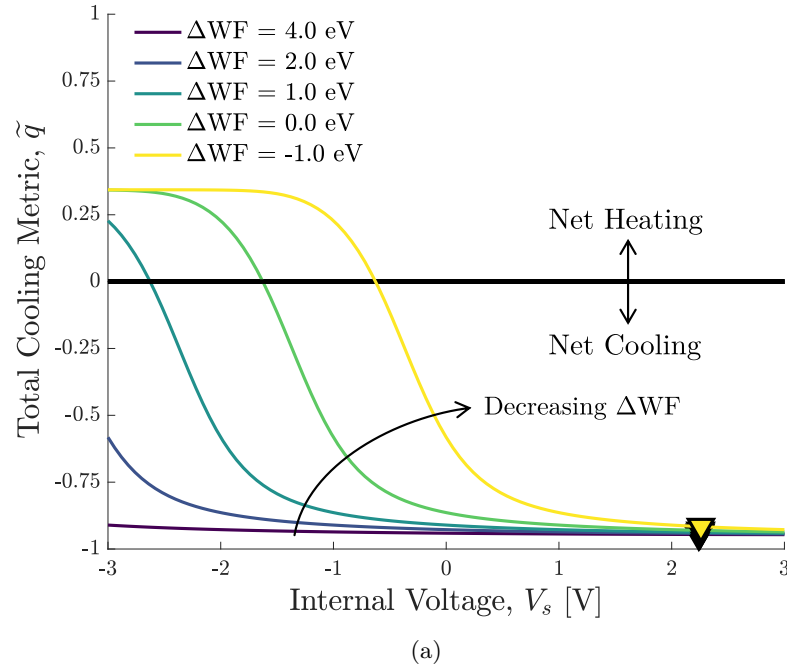
Finally, we consider the change in work function between the cathode and anode,  $\Delta WF$ . As the value of the anode work function is a component of the total potential in the circuit, it is expected that the role of  $\Delta WF$  is similar to that of the internal voltage source where a high value of  $\Delta W$  behaves similarly to a high value of  $V_s$ . However, unlike the internal voltage source which can be controlled, once the anode material is chosen the anode work function remains constant. To identify favorable materials to choose for the anode, we vary  $\Delta WF$  to include both negative and positive values.

Plotted in Fig. 3.7(a) are curves of  $\tilde{q}$  for the change in work function  $\Delta WF \in \{4.0, 2.0, 1.0, 0.0, -1.0\}$  eV. We note that the changing value of  $\Delta WF$  shifts curves of the total cooling metric to the left and right in a linear fashion, and that cooling is enhanced when the anode work function is larger than that of the cathode. This supports our expectation of  $\Delta WF$  behaving similar to that of the internal voltage source. This behavior can be understood by viewing the potential diagram in Fig. 3.1. If the value of  $WF^a$  is much higher relative to  $WF^c$ , the total potential on the anode side is large and the cathode sheath potential,  $\phi_w^c$  must increase in magnitude. The higher magnitude of  $\phi_w^c$  reflects more flowfield electrons and allows for more thermionic electrons to escape the space-charge-limited system, at that operating point. However, the cathode sheath

potential also increases in strength as  $V_s$  is increased. This has the same effect as a large value of  $\Delta WF$ , where flowfield electron flux decays to zero and increased thermionic emission is impossible. As a result, the curves of  $\tilde{q}$  coalesce at high  $V_s$ .

Finally, curves of  $\eta$  are presented for the varied values of  $\Delta WF$  in Fig. 3.7(b). Similarly, we note that a configuration at 0 V typically corresponds to the highest value of the cooling performance metric. The only exception to this behavior is the  $\Delta WF = -1.0$  eV case, where via the total cooling metric in Fig. 3.7(a), the negative value of  $\Delta WF$  has shifted the curve of  $\tilde{q}$  to the right such that it is more efficient to put energy into the circuit rather than operate at a 0 V configuration. As the internal voltage becomes large, so does the input power to the ETC circuit, and curves of  $\eta$  begin to decrease. Additionally, it is observed that anodes with a work function equal (or even lower) than the cathode work function increase the cooling efficiency of the system. This behavior is primarily due to electron heating at the anode, where a portion of the deposited heat flux is proportional to the anode work function (Eq. 2.24). However, while a lower value of  $\Delta WF$  aids in efficiency, curves of  $\tilde{q}$  predict that this comes at the cost of lower net cooling at the cathode.

The trends of the preceding ETC system parameter study suggest that total cooling at the cathode is enhanced when the both anode area and material work function are large relative to the cathode. In addition, ETC systems that are at the cusp of being space-charge-limited have the ability to generate the largest magnitude of total cooling at the cathode. Regarding power, the parameter study suggests that a configuration with no internal voltage source (0 V) can provide significant cooling without the need for external power. The cooling efficiency of the system can be improved with an anode work function that is close to (or negative) relative to the cathode, along with an optimally chosen anode electrode area.

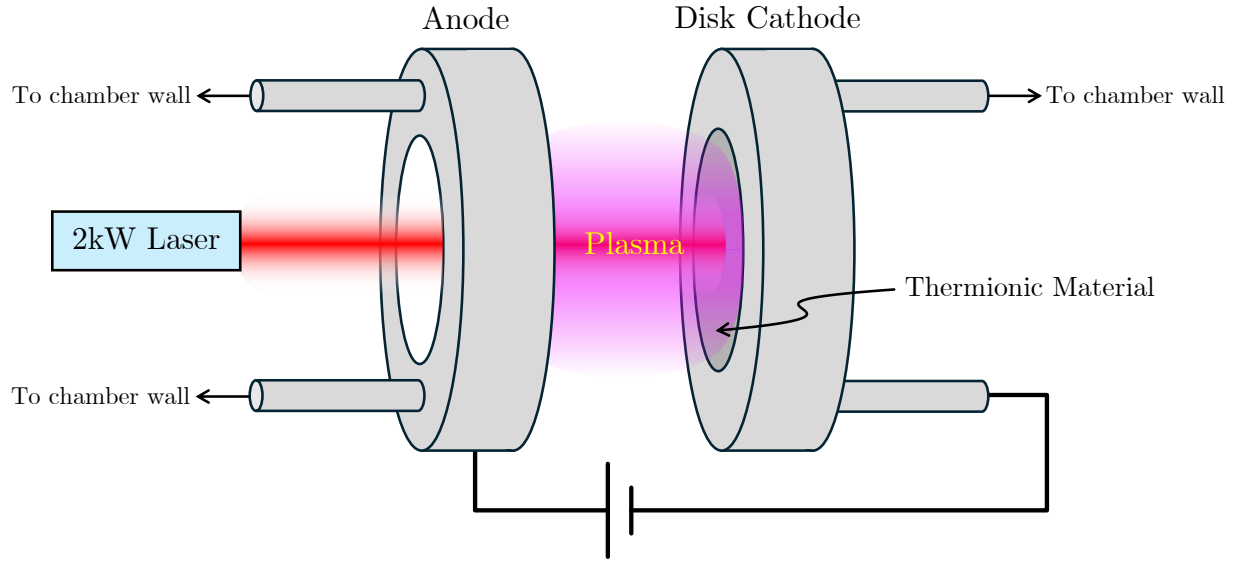


**Figure 3.7:** Parameter study of the ETC system with varied changes in work function,  $\Delta WF$ , showing curves of (a) total cooling metric  $\tilde{q}$  (Symbol ( $\nabla$ ) corresponds to the breakeven point for the ETC system (Eq. 3.4), where the total cooling power equals the power draw of the circuit) and (b) cooling performance metric  $\eta$ .

### 3.4 Experimental Validation

The final section covers a comparison of the ETC model to experimental measurements, starting with a description of the experimental setup (Section 3.4.1). Section 3.4.2 describes the coupling of the ETC model with a two-dimensional heat conduction model to accurately model the experimental setup. Several assumptions regarding the experimental measurements are described in Section 3.4.3, followed by validation efforts and discussion in Section 3.4.4.

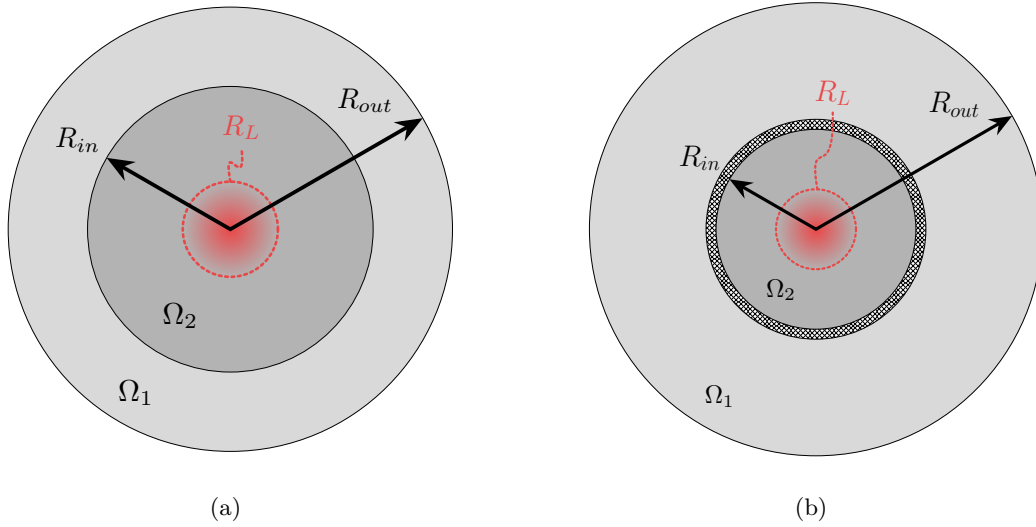
#### 3.4.1 Description of Experimental Facility



**Figure 3.8:** An illustrative sketch of the experimental setup by Bak et al. [14] Gapped electrodes are immersed in Argon at 1 Torr. Electrodes are supported by rods connected to the chamber wall. A 2 kW laser heats the cathode surface, which is made of a thermionic material, igniting a plasma discharge. A voltage source between the electrodes controls the system bias. Detailed views of the setup can be found in Bak et al. [14].

Recently, work by Bak et al. [14] presented an experimental setup to resolve issues of isolating the thermionic current and explicitly quantify thermionic cooling fluxes. Figure 3.8 shows an illustrative sketch of the experiment. The experimental setup was comprised of gapped electrodes immersed in Argon at 1 Torr. Cathode surface temperatures were controlled via a 2 kW laser with a spot radius of  $R_L = 3.0$  mm that is directed at the center of the thermionic material.

Simultaneously, a two-color pyrometer, pointed at the same location, collected measurements of surface temperature. A voltage source and current probe were connected between the cathode and the anode. To initiate thermionic emission, the laser was activated and the cathode surface became heated. A constant 15 V bias was then applied between the electrodes via the voltage source. When sufficient thermionic electrons are present, the bias accelerates electrons into the bulk gas. Accelerated electrons impact and ionize neutral argon atoms, igniting a steady state plasma discharge between the electrodes and completing the circuit. With this approach, the authors were able to obtain measurements of both steady-state and transient surface temperatures at the cathode center, as well as the internal current of an ETC system over a range of incident heat fluxes characterizing a hypersonic leading edge.



**Figure 3.9:** Geometries for the (a) disk and (b) segmented cathode configurations in Bak et al. [14]. Outer and inner surfaces are denoted by  $\Omega_1$  and  $\Omega_2$ , respectively. Hatch marks indicate a gap between materials.

Figure 3.9 shows two cathode configurations explored by Bak et al. [14], with geometrical descriptions in Table 3.2. The disk cathode is comprised of an emissive inner surface of lanthanated tungsten which is stated to have a work function of 2.5-2.7 eV. The inner surface of the segmented cathode is pure tungsten which is reported to have a work function of 4.54 eV. It is noted that the

**Table 3.2:** Parameters for the disk and segmented cathode configurations in Bak et al. [14]

| Cathode Configuration | Outer Radius, $R_{out}$ , mm | Inner Radius, $R_{in}$ , mm | Outer Material, $\Omega_1$ | Inner Material $\Omega_2$ |
|-----------------------|------------------------------|-----------------------------|----------------------------|---------------------------|
| Disk                  | 14                           | 9.0                         | Molybdenum                 | Lanthanated Tungsten      |
| Segmented             | 17                           | 4.8                         | 304 Stainless Steel        | Tungsten                  |

inner surface of this cathode is not in contact with the outer ring. Presently, we focus on the disk geometry as the majority of the ETC measurements were taken with the disk cathode. However the segmented cathode is discussed in detail in Sec. 3.4.3. The anode geometry is not described in this work, but details can be found in Bak et al. [14].

### 3.4.2 A 2D Coupled ETC + Heat Conduction Model

As the electrode surface in Bak et al. [14] is two-dimensional, it is necessary to extend the 1D ETC model in Sec. 3.1. Instead of a singular cathode and anode surface, the cathode and anode surfaces are discretized into  $N_c$ ,  $N_a$  cells, respectively. This results in a  $N_c \times N_a$  system, which upon solving, computes the sheath potentials at each discretized cell. Applying conservation of voltage and current, we show the form of the residual in Eq. 3.5, where  $\mathbf{w} = [\phi_w^{c,1}, \dots, \phi_w^{c,N_c}, \phi_w^{a,1}, \dots, \phi_w^{a,N_a}]^T$  is the extended vector of sheath potentials at each cell.

$$\mathbf{R}(\mathbf{w}) = \begin{bmatrix} \text{WF}^c - \phi_w^{c,1} + \Delta\phi_p + \phi_w^{a,1} - \text{WF}^a - V_s \\ \vdots \\ \text{WF}^c - \phi_w^{c,1} + \Delta\phi_p + \phi_w^{a,N_a} - \text{WF}^a - V_s \\ \text{WF}^c - \phi_w^{c,2} + \Delta\phi_p + \phi_w^{a,1} - \text{WF}^a - V_s \\ \vdots \\ \text{WF}^c - \phi_w^{c,N_c} + \Delta\phi_p + \phi_w^{a,1} - \text{WF}^a - V_s \\ \sum_{k=1}^{N_c} \left[ A^{c,k} \left( J_{i,f}^{c,k} + J_{etc}^{c,k} - J_{e,f}^{c,k} \right) \right] + \sum_{k=1}^{N_a} \left[ A^{a,k} \left( J_{i,f}^{a,k} - J_{e,f}^{a,k} \right) \right] \end{bmatrix} \quad (3.5)$$

To account for thermal conduction into the material, the ETC system model is coupled

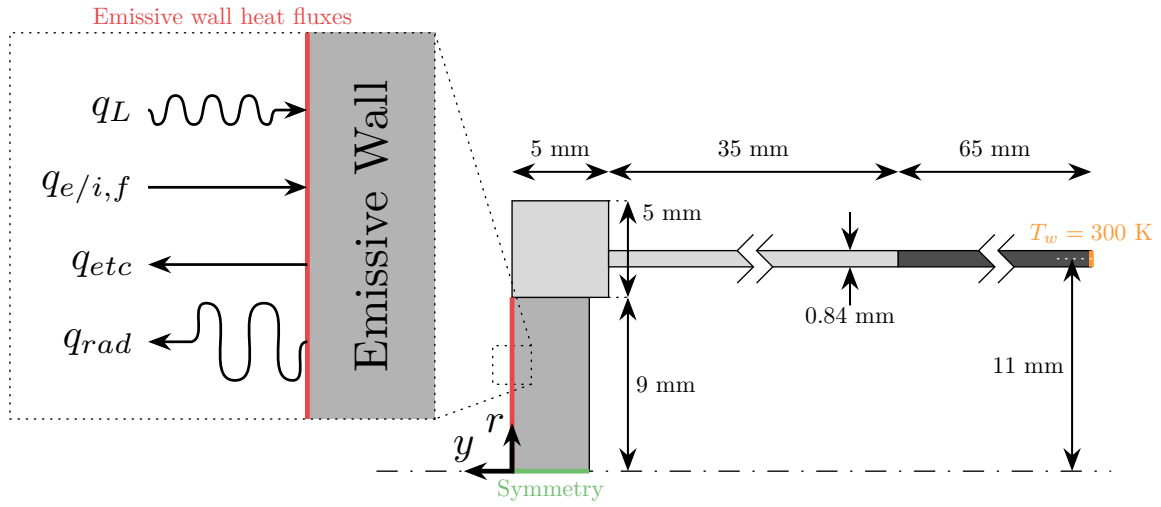
to a two-dimensional heat conduction analysis via a finite element framework in MATLAB [68]. Figure 3.10 shows the computational domain of interest, over which the steady state heat equation,  $\nabla \cdot (\kappa \nabla T) = 0$ , is solved. The disk cathode is supported by a molybdenum rod which transitions to a 304 stainless steel rod connected to the chamber wall. Values for the thermal conductivity and emissivity of the cathode (treated as tungsten) and molybdenum rod are specified as a function of temperature, details of which can be found in Appendix C of Bak et al. [14]. For the 304 stainless steel rod, a thermal conductivity of 16.2 W/(mK) and emissivity of 0.12 are specified [24].

We note two geometrical changes from the true experimental setup. Firstly, the stainless steel rod is given a length of 65 mm which is shorter than the reported length of 300 mm. A similar heat conduction analysis was performed in Bak et al. [14] using the 300 mm dimension, and it was found that the internal temperature had nearly reached the isothermal boundary value at 65 mm in length. Additionally, the width of the molybdenum rod is taken to be 0.84 mm (the same dimension as the 304 stainless steel rod) instead of the stated 0.21 mm dimension. Surface temperatures over the cathode are computed using both dimensions and no observable differences are found, and thus, the 0.84 mm dimension is selected to simplify the geometry and reduce the complexity of the heat conduction problem.

We illustrate the boundary conditions for the coupled ETC + heat conduction problem in Fig. 3.10. Notably, the heat fluxes at the emissive cathode surface ( $0 \text{ mm} \leq r \leq 9 \text{ mm}$ ) are specified as laser, electron, and ion heating,  $q_L, q_{e,f}, q_{i,f}$  as well as thermionic and radiative cooling,  $q_{etc}, q_{rad}$ . The laser heating profile is approximated as a Gaussian over the emissive surface,  $q_L = I_0 \exp(-r^2/R_L^2)$ , where  $I_0$  is the laser heating intensity at the center of the cathode. Potential differences due to the inter-electrode space are assumed to be zero, i.e.  $\Delta\phi_p = 0$ .

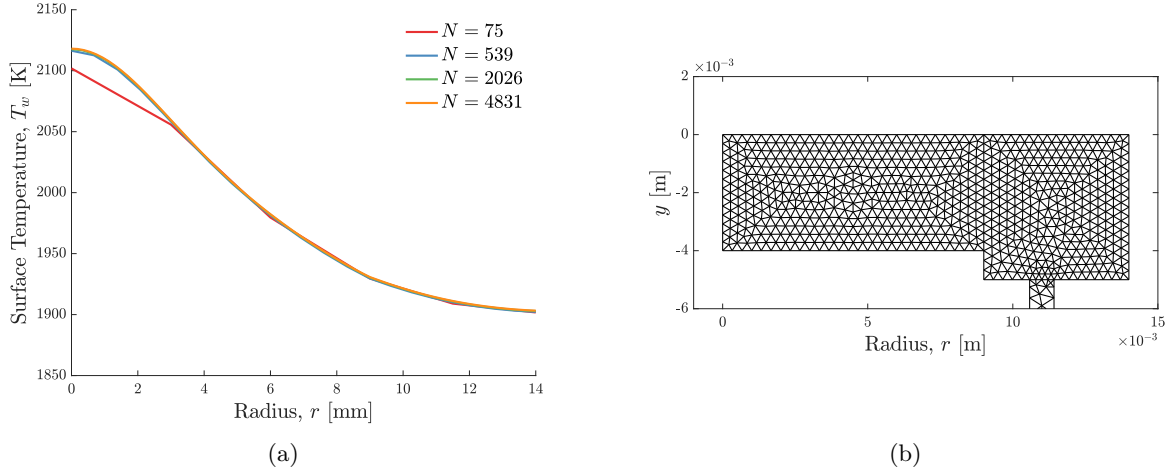
Finally, a suitable mesh size for the coupled ETC analysis is obtained via a grid convergence study. Figure 3.11(a) shows surface temperatures along the emissive cathode for four levels of grid refinement and a constant laser profile. The third refinement level, corresponding to a global node count of  $N = 2026$  and average mesh size of  $\Delta r = 0.31 \text{ mm}$ , is found to be suitable, with further refinement not significantly changing surface temperatures. The mesh corresponding to the third





**Figure 3.10:** Geometric dimensions and boundary conditions for the coupled ETC + heat conduction problem. The emissive wall experiences laser, ion, and electron heating,  $q_L, q_{i,f}, q_{e,f}$ , as well as thermionic and radiative cooling,  $q_{etc}, q_{rad}$ . All boundaries not labeled are set to a radiative boundary condition.

refinement level is shown in Fig. 3.11(a).

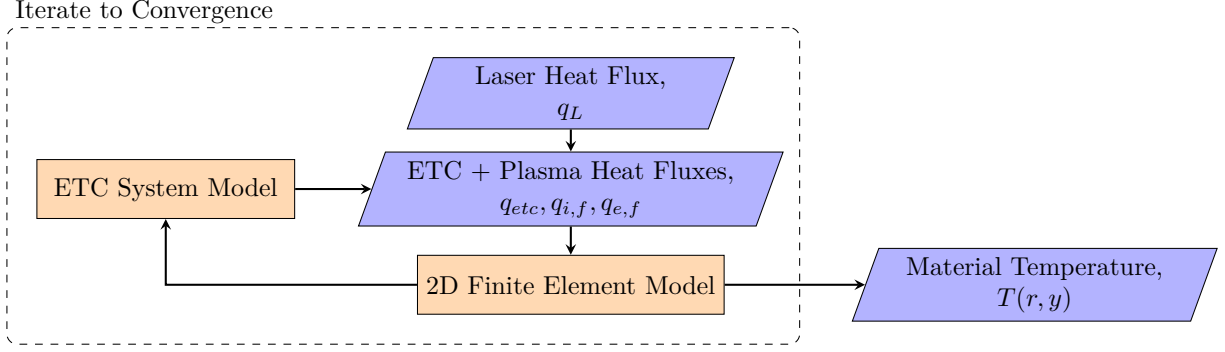


**Figure 3.11:** (a) Surface temperatures along the emissive cathode for several mesh refinement levels,  $N$ . (b) Computational domain for the  $N = 2026$  mesh.

We show a final depiction of the coupled ETC + heat conduction model in Fig. 3.12. The ETC system model and finite element model share relevant heat flux information along the emitting cathode boundary, notably  $q_{etc}$ ,  $q_{i,f}$ ,  $q_{e,f}$  and  $q_L$ . The finite element model solves the heat conduction problem over the domain, computing the equivalent conductive heat flux,  $q_{cond}$ , along the cathode boundary. This process is repeated until the change in wall temperatures over the emissive boundary is below a specified tolerance. This results in a spatial material temperature distribution as a function of  $r$ , and  $y$ , that includes the influence of thermionic cooling.

### 3.4.3 Estimating the Experimental Environment

To apply the ETC model to the experimental setup, a notion of the plasma composition, notably the electron and ion temperatures,  $T_e$  and  $T_i$ , as well as electron density,  $n_e$ , is required as a function of the cathode radius,  $r$ , and the internal current,  $I_s$ . However, Bak et al. [14] only provided a single description of the composition as “ $n_e \sim 1 \times 10^{19} \text{ 1/m}^3$  and  $T_e \sim 1 \text{ eV}$ ”. As a result, this work relies on first order approximations to estimate the plasma properties. It is recognized that the approximations are not well suited for all experimental conditions, but they are expected



**Figure 3.12:** Flow chart of the coupled ETC + heat conduction model.

to scale reasonably when thermionic current is large and provide insight into the operation of the experimental setup.

As thermionic emission increases in strength it is expected that the plasma density between the electrodes rises correspondingly. To estimate the plasma density at the center of the cathode,  $n_e^0$ , we construct a scaling argument via a nondimensional parameter. We assume that  $n_e^0$  is a function of the gas ionization potential,  $E_i$ , cathode surface area,  $A^c$ , electron thermal velocity,  $v_{th,e}$ , total current,  $I_s$ , and the applied electrode voltage,  $V_s$ . Thus, we assemble a Pi parameter as

$$\Pi_1 = (n_e^0)^{\alpha_1} (I_s)^{\alpha_2} (V_s)^{\alpha_3} (E_i)^{\alpha_4} (A^c)^{\alpha_5} (v_{th,e})^{\alpha_6} \quad (3.6)$$

We then solve for  $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6]^T$ , by finding the nullspace,  $N$ , of the dimensional matrix,  $\mathbf{M}$ , where

$$\boldsymbol{\alpha} = N(\mathbf{M}), \quad \text{where } \mathbf{M} = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ -3 & 0 & 2 & 2 & 2 & 1 \\ 0 & 0 & -3 & -2 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \quad (3.7)$$

The resulting scaling argument is shown in Eq. 3.8. The physical trends implied by the expression are consistent with expectations. Increasing input power (i.e. the product  $I_s V_s$ ), raises the available energy for ionization processes and therefore increases the plasma density. Conversely, larger ionization potentials require more energy per ionization event, reducing the achievable plasma density. A larger cathode area similarly lowers the density by expanding the volume occupied by the plasma. Finally, higher electron velocities enhance diffusive transport, which also acts to reduce the plasma density.

$$n_e^0 \propto \frac{I_s V_s}{e E_i A^c v_{th,e}} \quad (3.8)$$

To simplify the analysis, the electron and ion temperatures are assumed to be spatially uniform and constant as a function of  $I_s$ . This assumption is based from the operation of hollow cathodes for electric propulsion which, similar to the setup of Bak et al. [14], ignite a plasma via thermionic emission and have typical operating conditions consisting of  $\mathcal{O}(1)$  Torr neutral pressures, and  $\mathcal{O}(1 - 10)$  A discharge currents. Importantly, detailed analysis of hollow cathode plasmas have shown that electron temperatures are mostly constant as a function of both spatial location and cathode discharge current [50, 92, 139, 138]. Heavy particle temperatures, which are coupled to the plasma pressure, are observed to vary with location and discharge current [50, 92], however the variation in  $T_i$  is not extremely large. Additionally, for plasma densities of  $n_e \sim 1 \times 10^{19} \text{ 1/m}^3$  where  $T_e \gg T_i$ , both the ion current and role of  $T_i$  in determining thermionic emission strength is expected to be small.

As a result, for a fixed geometry and a single species background gas, the scaling relationship can be simplified to  $n_e^0 \propto I_s V_s = P_s$ . This relationship is physically reasonable as it simply suggests that the plasma number density scales with the input power to the plasma, and in fact, has been observed in recent work on thermionic energy converters and hollow cathode operation [72, 139]. We note that the measurements by Bak et al. [14] were taken at a constant voltage and the relationship can be further simplified to  $n_e^0 \propto I_s$ . For the present work, electron and ion temperatures are taken

to be  $T_e = 1$  eV and  $T_i = 2000$  K, from the plasma description in Bak et al. [14] and a representative hollow cathode discharge at 0.75 Torr, 10 A [50], respectively.

While the scaling relationship in Eq. 3.8 is helpful, it does not provide a *spatial* estimate of the plasma over the cathode surface, which is required for the 2D ETC system model. To this end, we approximate the radial distribution of the plasma density as a function of the cathode radius,  $r$ , via a Bessel function, shown in Eq. 3.9,

$$n_e(r) = \begin{cases} n_e^0 B_0(2.4r/r_{eff}) & r \leq r_{eff} \\ 0 & r > r_{eff} \end{cases} \quad (3.9)$$

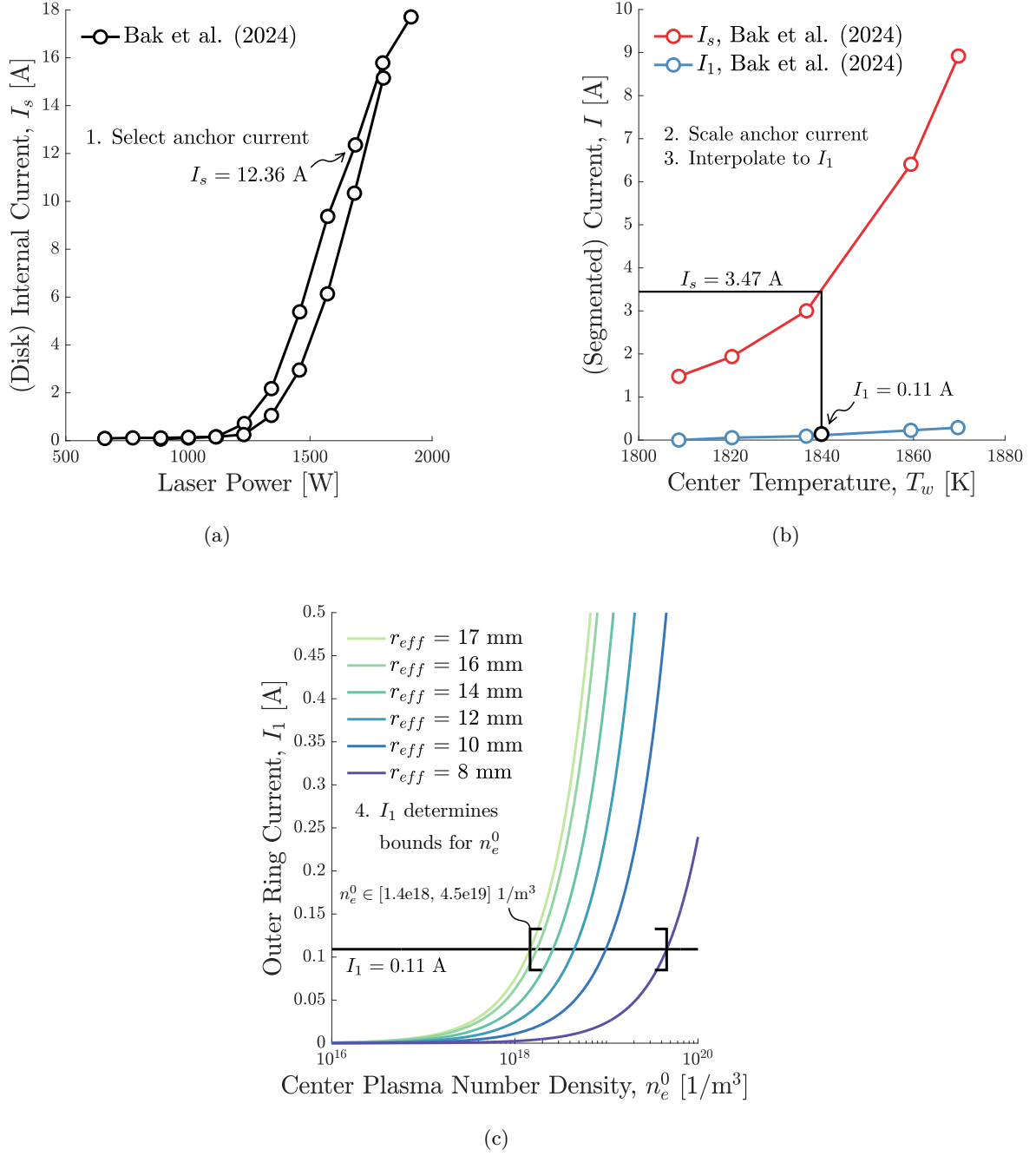
where  $B_0$  is the zero-order Bessel function of the first kind and  $r_{eff}$  is a specified effective radius of the plasma. Bessel functions are often found in solutions to diffusion-governed systems in cylindrical coordinates and a similar approach is taken in Bak et al. [14]. Given experimentally measured values of  $n_e^0$ ,  $I_s$ , and  $r_{eff}$ , the scaling relationship of Eq. 3.8 coupled with the spatial density relationship in Eq. 3.9 can be used to determine  $n_e$  as a general function of  $I_s$  and  $r$ . Values of  $I_s$  are provided in Bak et al. [14], but no explicit measurements of  $n_e^0$  or  $r_{eff}$  were taken.

To obtain estimates of  $n_e^0$  as a function of the experimentally measured values of  $I_s$  we now consider the segmented cathode configuration from Fig. 3.9. Owing to the electrically isolated regions  $\Omega_1$  and  $\Omega_2$ , the segmented cathode measures a current from each region, i.e.  $I_1 = \int_{\Omega_1} J dA$  and  $I_2 = \int_{\Omega_2} J dA$ , which are reported in Bak et al. [14]. Importantly, as  $\Omega_1$  in the segmented cathode is made of a high work function material and not directly heated by the laser,  $I_1$  can be assumed to be solely due to flowfield electrons and ions. Furthermore, since the cathode is at a strong negative bias via the voltage source, we can further neglect the electron current. Using these assumptions we construct an analytical expression for the outer ring current in Eq. 3.9 as

$$\begin{aligned}
I_1 &= \int_{\Omega_1} J_{i,f} - J_{e,f} dA \\
&\approx \int_{\Omega_1} J_{i,f} dA \\
&= \int_0^{2\pi} \int_{R_{in}}^{r_{eff}} en_e(r) V_{i,se} r dr d\theta
\end{aligned} \tag{3.10}$$

where  $V_{i,se}$  is computed from Eq. 2.13a assuming a strong potential ( $\eta \rightarrow \infty$ ). Thus, Eq. 3.10 relates an experimentally measured current from the segmented cathode in Bak et al. [14] to the center plasma density  $n_e^0$ .

Using Eq. 3.10, we now outline a process for determining  $n_e^0$  as a function of the total current in the disk cathode,  $I_s$  in Fig. 3.13. Starting in Fig. 3.13(a), experimentally measured values of the disk cathode current,  $I_s$ , are provided as a function of the laser power. We select an arbitrary anchor current value from this data, taken as  $I_s = 12.36$  A. Next, in Fig. 3.13(b), we show experimentally measured values of the segmented cathode currents, where  $I_s$  in this case is the sum of each segmented area, i.e.  $I_s = I_1 + I_2$ . The anchor current from the disk cathode,  $I_s = 12.36$  A, is modified using the scaling relationship of Eq. 3.8 to account for the change in cathode area. Holding the center plasma density,  $n_e^0$ , constant between the disk and segmented cathodes, the smaller cathode area,  $A^c$ , in the segmented configuration requires a smaller equivalent current, yielding  $I_s = 3.47$  A in the segmented configuration. We then interpolate the measurements in Fig. 3.13(b) to obtain the corresponding value of the outer ring current,  $I_1 = 0.11$  A, from the total current value. Finally, we turn to Fig. 3.13(c), where we leverage the analytical expression for  $I_1$  as a function of  $n_e^0$  (Eq. 3.10). Note that curves of  $I_1$  for varied values of the effective plasma radius,  $r_{eff}$  have been computed. The highest value of  $r_{eff} = 17$  mm corresponds to the outer ring radius  $R_{out}$ , while the lowest value of 8 mm is estimated from visual photos of the ignited plasma in Bak et al. [14]. The true value of  $r_{eff}$  is unknown, but it is likely contained within this range. Overlayed on Fig. 3.13(c) is the value of the outer ring current,  $I_1 = 0.11$  A, which through the previous steps, is related to the original anchor current taken at the disk cathode. The intersection of the



**Figure 3.13:** (a) Measurements of the internal current,  $I_s$  for the disk cathode, reproduced from Bak et al. [14]. (b) Values of  $I_1$  and  $I_3 = I_s$  for the segmented cathode, reproduced from Bak et al. [14]. (c) Guard ring ion current,  $I_1$  for the segmented cathode computed with Eq. 3.10 for varied center plasma density,  $n_e^0$ , and effective plasma radius,  $r_{eff}$ .

$I_1 = 0.11$  line and the curves of Eq. 3.10 correspond to the range  $n_e^0 \in [1.4 \times 10^{18}, 4.5 \times 10^{19}] \text{ 1/m}^3$ , representing possible center plasma densities above the disk cathode for the original anchor current value of  $I_s = 12.36 \text{ A}$ . Though the computed range covers an order of magnitude in  $n_e^0$ , there is qualitative agreement with the author's description of the plasma composition as " $n_e \sim 1 \times 10^{19} \text{ 1/m}^3$ ".

Best agreement to the experimental results of Bak et al. [14] is found with a value of  $n_e^0 = 2.3 \times 10^{19} \text{ 1/m}^3$ . Thus, using the simplified scaling law of  $n_e^0 \propto I_s$  by recognizing that voltage is held constant in the experiment, as well as the chosen anchor point of  $(I_s, n_e^0) = (12.36 \text{ A}, 2.3 \times 10^{19} \text{ 1/m}^3)$ , the final linear scaling relationship for the center plasma density above the disk cathode as

$$n_e^0 = (1.861 \times 10^{18}) I_s \quad (3.11)$$

Equations 3.9 and 3.11 model the spatial distribution of plasma density over the cathode as a function of the internal current. We note that several values spanning the range of feasible values of  $n_e^0$  are also investigated when constructing the scaling relationship. Scaling relationships using values taken at the lower range of  $n_e^0$  reduce the coefficient in Eq. 3.11, resulting in a lower plasma density for a given current,  $I_s$ . This introduced SCL behavior into the computed results that were not reported in Bak et al. [14]. In addition, we note that the selected value of  $n_e^0 = 2.3 \times 10^{19} \text{ 1/m}^3$  corresponds to an effective plasma radius of  $r_{eff} \approx 8.5 \text{ mm}$ , which corresponds well to the visual images of the plasma discharge [14].

#### 3.4.4 Comparison to Experiment

Returning to the disk cathode geometry, the coupled ETC + heat conduction model is evaluated over a range of incident laser heating intensities for three values of the cathode material work function  $WF^c \in \{2.5, 2.65, 2.7\} \text{ eV}$ . These values are chosen to cover the reported value of the work function for lanthanated tungsten (2.5 - 2.7 eV). All cases use the plasma density scaling parameter from Eq. 3.11, except for a fourth case which uses  $WF^c = 2.65 \text{ eV}$  and a constant value



**Table 3.3:** Constant simulation parameters for the 2D ETC + heat conduction problem.

| ETC System Parameters                      | Value            |
|--------------------------------------------|------------------|
| $A$                                        | 3.46             |
| $A^c$ , cm <sup>2</sup>                    | 2.54             |
| WF <sup>a</sup> , eV                       | 4.22             |
| $A_R$ , A/(m <sup>2</sup> K <sup>2</sup> ) | $70 \times 10^4$ |
| $T_i$ , K                                  | 2000             |
| $T_e$ , K                                  | 11604            |
| $E_i$ , eV                                 | 15.76            |

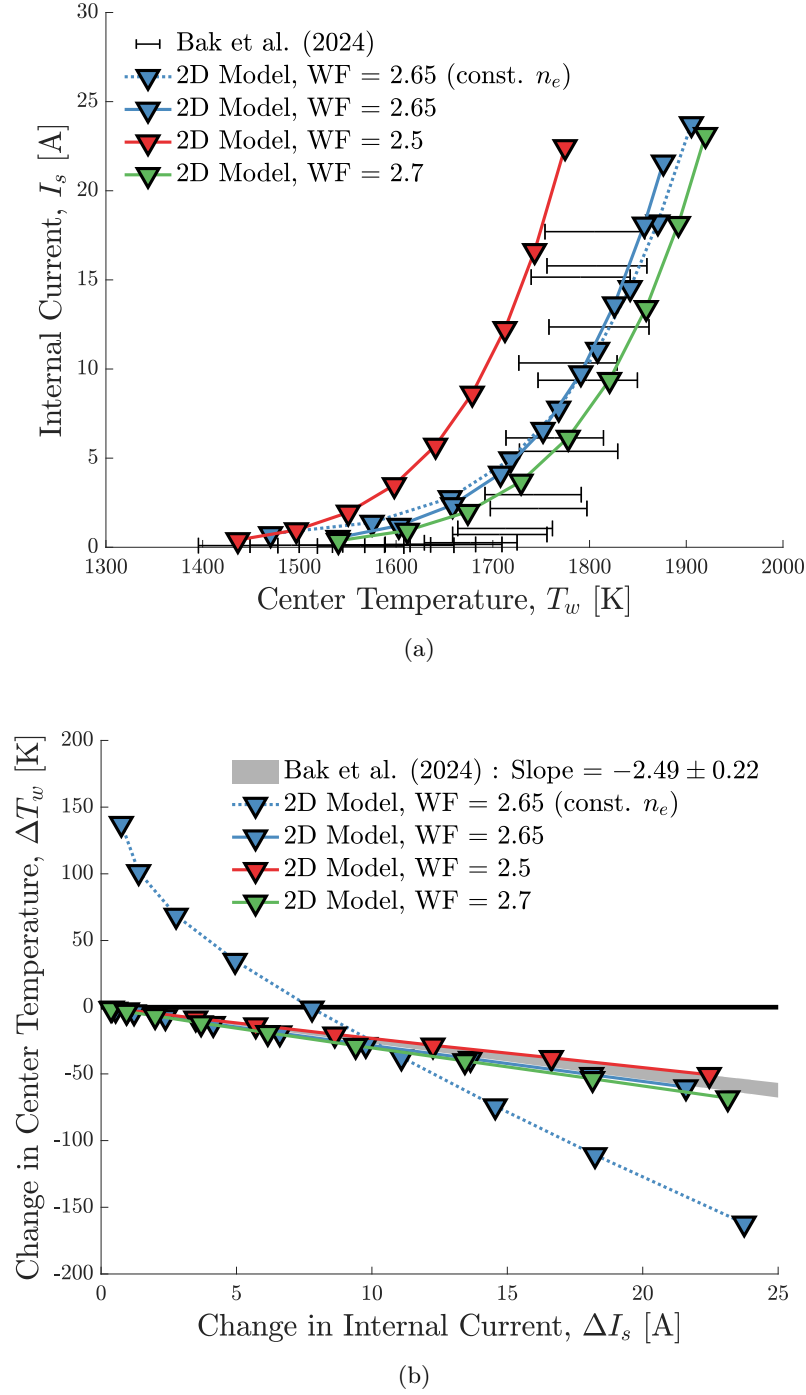
of  $n_e^0 = 1 \times 10^{19}$  1/m<sup>3</sup> to highlight the role of the scaling relationship. It is noted in Bak et al. [14] that there exists a skewed uncertainty of +2% to +8% in surface temperature measurements due to the calibration procedure of the pyrometer. The probe used to measure the current is reported to have a +1% to -1% uncertainty bound. As total current values are  $\mathcal{O}(1)$ , the uncertainty magnitudes are small and not considered in the present work. Constant parameters for the ETC model are listed in Table 3.3.

Figure 3.14(a) shows the total current,  $I_s$  as a function of the center surface temperature,  $T_w$ , from the 2D model overlayed with experimental measurements from Bak et al. [14]. Both the model and experiment show an exponential relationship in measured current,  $I_s$  with the center wall temperature,  $T_w$ , and reasonable agreement exists at high  $T_w$  for the WF<sup>c</sup> = 2.65 and 2.7 eV cases. However, at low surface temperatures, it is evident that the model is overpredicting the system current. Physically in this regime, thermionic emission is expected to be nearly inactive and current values are dominated by the composition of the bulk plasma. As the model is overpredicting  $I_s$ , this indicates the approximations to compute the plasma composition are overestimating the plasma density. Notably, a constant electron temperature is assumed when it is expected that  $T_e$  lowers with the discharge current, and thus, reduces the ion current to the surface. Additionally, when thermionic emission is low, the ignition mode of the plasma may not be the same as when thermionic emission is large, and plasma density may no longer scale linearly with  $I_s$ . Finally, while it is expected that plasma losses from the center of the experimental setup are dominated by

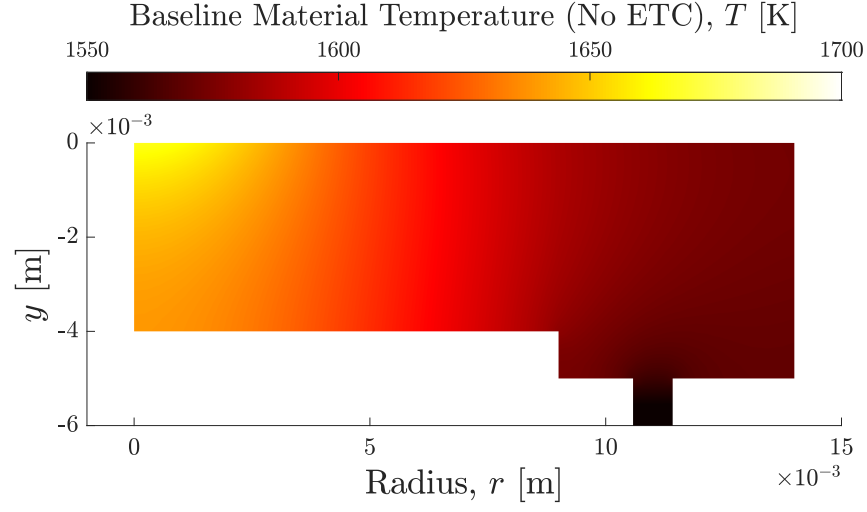
diffusion when density (and current) is high, this may not be true when the plasma density and current values are low, making the Bessel function in Eq. 3.9 a potentially poor approximation of the radial plasma density.

An important observation from Bak et al. [14] was a linear relationship between the change in total current,  $\Delta I_s$ , and  $\Delta T_w$ , the change in center wall temperature, when compared to the non-ETC case. The non-ETC measurements were obtained by activating the laser but removing the applied bias between the electrodes such that no plasma ignition occurs. We present values of  $\Delta I_s$ , and  $\Delta T_w$  from the 2D model in Fig. 3.14(b) in addition to the reported slope of  $-2.49 \pm 0.22$  K/A. In this case, good agreement is observed over the entire range of  $\Delta I_s$ , with all cases using the scaling parameter showing a linear relationship with  $\Delta T_w$ . Best agreement is found with a work function between 2.5 and 2.65 eV, confirming that the model's behavior aligns with the stated material properties. This agreement also gives confidence that both the particle and heat fluxes at the emissive surface are scaling appropriately with increased thermionic emission. It is noted that without the scaling of the plasma density, the coupled model does not show a linear relationship and strongly diverges from the measured slope. We compute this case and illustrate the divergence in slope in Fig. 3.14(b) via the dashed line. This divergence is expected, as since  $n_e^0$  is constant, ion heating dominates at low values of  $\Delta I_s$  where temperatures are low and thermionic current is near zero.

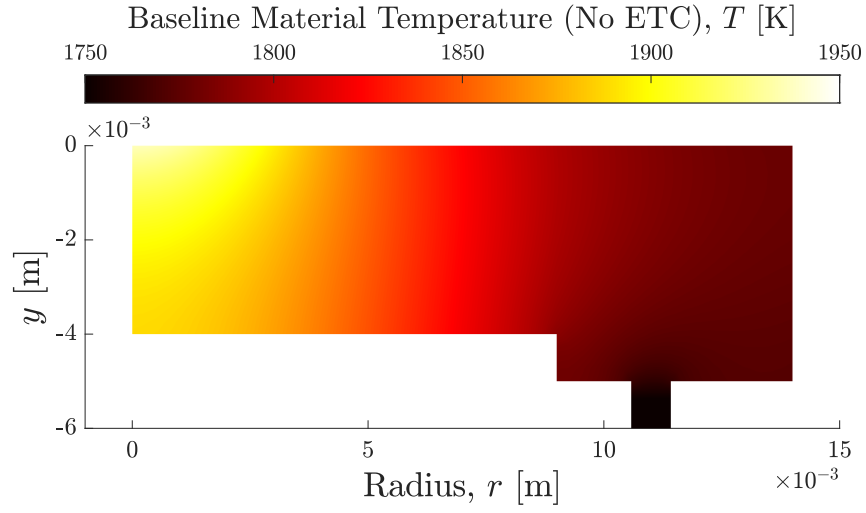
The influence of ETC on the material temperature,  $T$ , is now considered. Although thermionic cooling occurs at the cathode surface, the resulting heat flux propagates through the material via conduction and can reduce temperatures throughout the geometry. Based on the experimental comparison in Fig. 3.14(b), two representative operating conditions are examined with total currents of  $I_s = 2.4$  A and  $I_s = 21.6$  A. These cases correspond to low and high thermionic current levels, allowing the influence of ETC to be isolated. As a baseline, contours of the material temperature distribution without ETC are shown in Figs. 3.15(a) and 3.15(b). In both cases, the peak temperature occurs near the center of the emissive cathode ( $r = 0, y = 0$ ), where the incident laser produces the largest heat flux.



**Figure 3.14:** (a) Internal current,  $I_s$ , as a function of the surface temperature at the center of the cathode,  $T_w$ . Error bars represent a +2% to +8% uncertainty in the measured wall temperature. (b) Change in surface temperature compared to a non-ETC case,  $\Delta T_w$ , as a function of the change in total current  $\Delta I_s$ . Shaded region indicates the reported uncertainty in the measured linear slope.



(a)



(b)

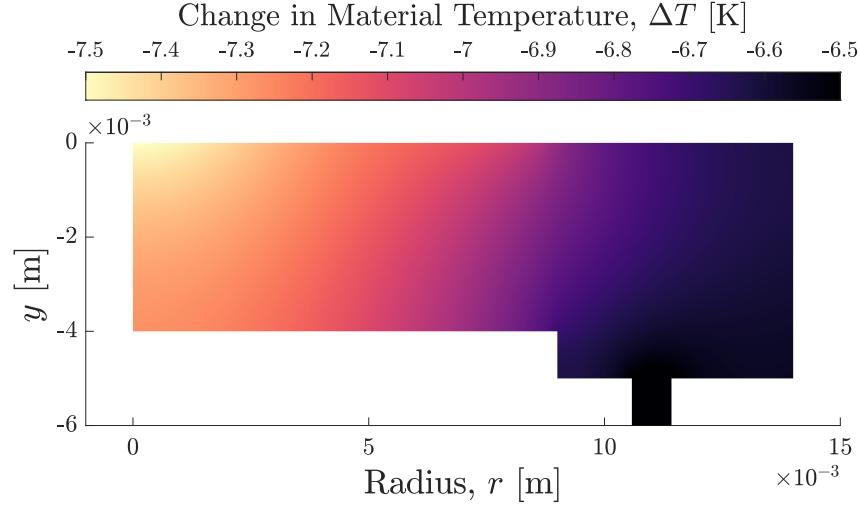
**Figure 3.15:** Contours of the baseline material temperature without ETC,  $T$ , for  $WF^c = 2.65$  eV at (a)  $I_s = 2.4$  A and (b)  $I_s = 21.6$  A. Both cases show peak temperatures at the center of the emissive cathode, where laser heating is the strongest.

ETC is now introduced, and the resulting change in material temperature due to the inclusion of ETC,  $\Delta T$ , is shown in Fig. 3.16 for the same operating conditions. For the low-current case (Fig. 3.16(a)), thermionic emission produces only a small temperature reduction across the domain, with  $\Delta T$  generally less than 8 K. This behavior is expected because the total thermionic current is small, resulting in a limited cooling flux. In contrast, the high-current case (Fig. 3.16(b)) exhibits much stronger cooling. The largest temperature reductions occur near the center of the cathode, where the wall temperature, and therefore the thermionic emission, is greatest, producing cooling of up to 60 K. Through thermal conduction, this cooling effect spreads throughout the material, reducing temperatures across the entire geometry.

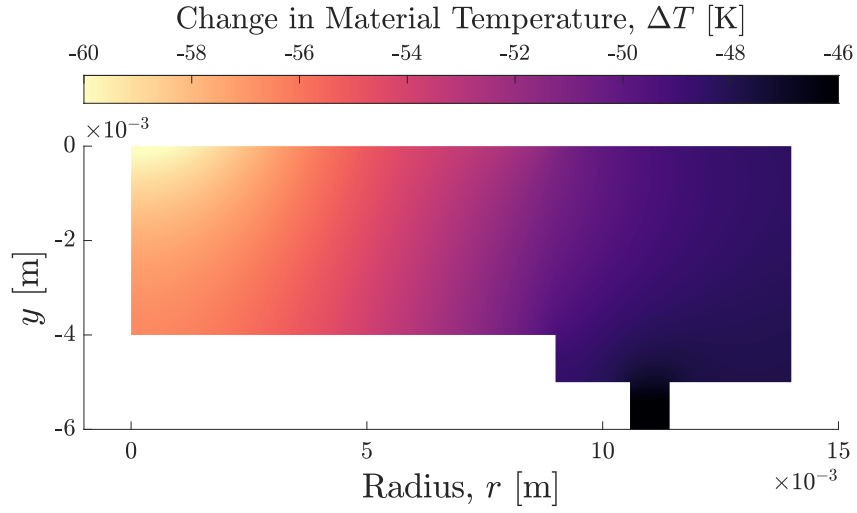
Wall temperatures,  $T_w$ , along the emissive surface at  $y = 0$  m are shown in Figs. 3.17(a) and 3.17(b) for the two current cases. Without ETC, the  $I_s = 2.4$  A case exhibits a peak temperature of 1666 K, while the  $I_s = 21.6$  A case reaches a peak temperature of approximately 1937 K. When ETC is introduced, cooling is observed in both cases, reducing the peak temperatures to 1658 K for the low current case and 1876 K for the high current case. Importantly, although the thermionic emission flux varies along the surface, the resulting reduction in wall temperature is relatively uniform across the domain. This behavior is likely due to thermal conduction within the material, which spreads the influence of thermionic cooling throughout the geometry.

The normalized heat-flux balance along the emissive surface is shown in Fig. 3.18, where  $q_{total}$  is defined as the sum of the absolute values of all heat flux contributions. Positive values indicate heat transfer into the surface, while negative values represent cooling. The heat-flux distribution for the low-current case,  $I_s = 2.4$  A, is shown in Fig. 3.16(a). Near the center of the geometry, heating is dominated by the incident laser flux, while cooling occurs primarily through thermal conduction into the material. Thermionic cooling is minimal in this configuration, contributing less than 2% of the total cooling. Near the edge of the geometry, laser heating has decayed and material conduction becomes the dominant source of heating. This is balanced mostly by radiative cooling.

The high-current case,  $I_s = 21.6$  A, is shown in Fig. 3.16(b). In this configuration, thermionic

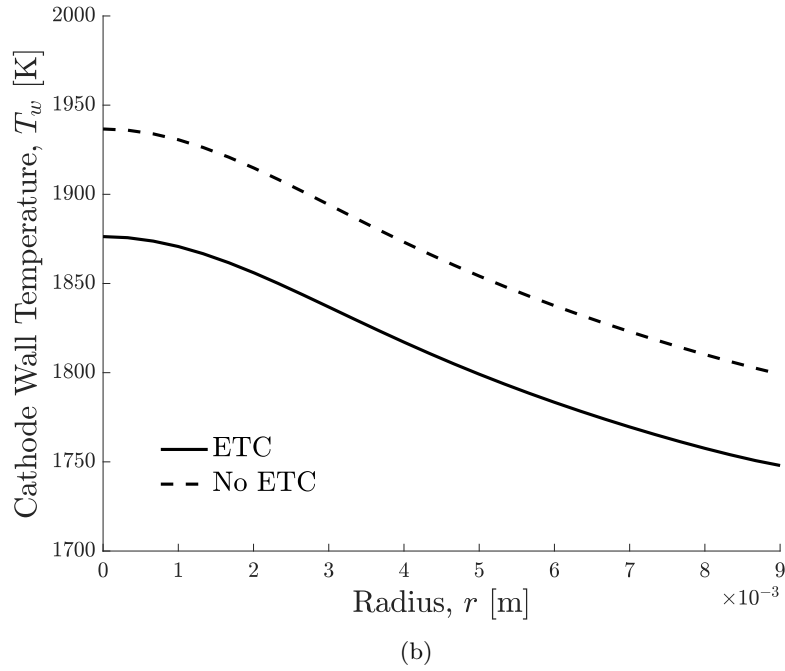
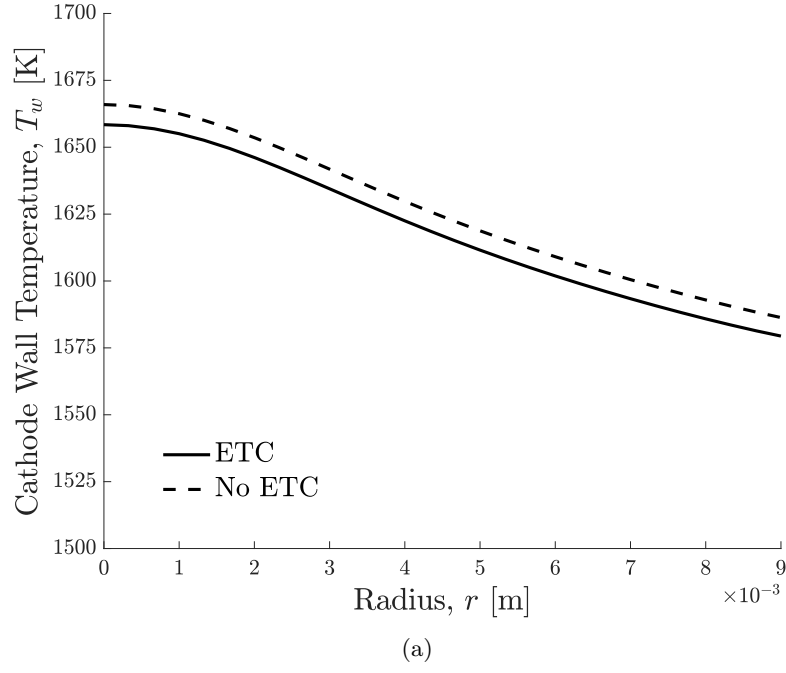


(a)



(b)

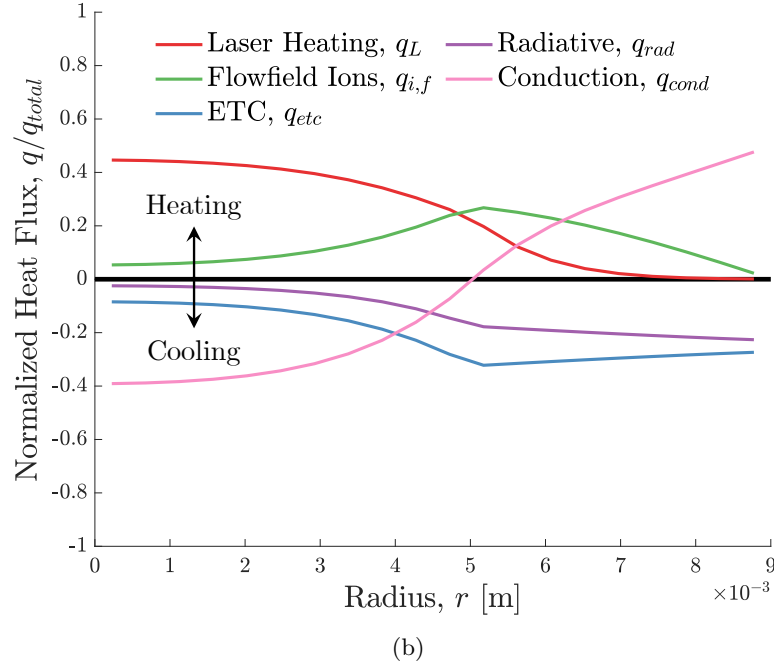
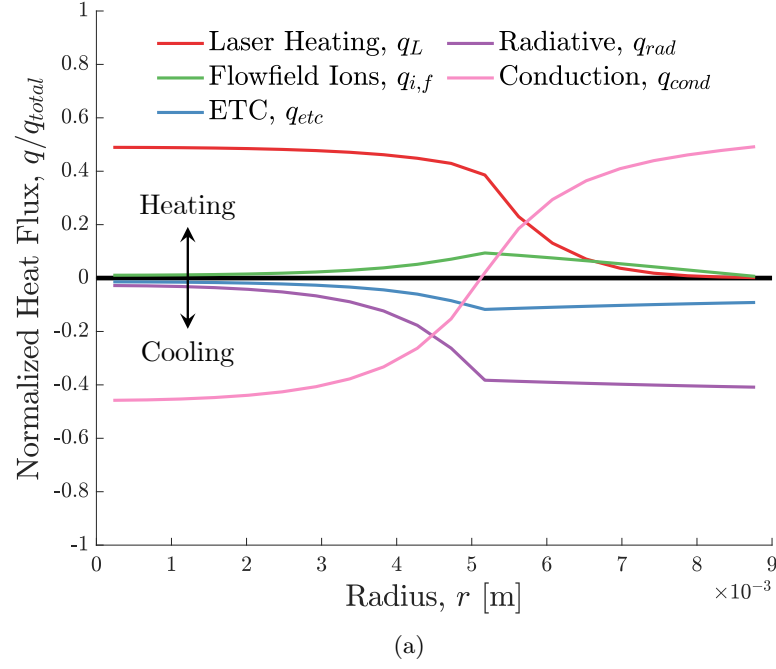
**Figure 3.16:** Contours of the change in the material temperature compared to a non-ETC case,  $\Delta T$ , for  $WF^c = 2.65$  eV at (a)  $I_s = 2.4$  A and (b)  $I_s = 21.6$  A. Both cases show a cooling of the entire geometry due to ETC. The cooling effect is substantially stronger in the  $I_s = 21.6$  A case as the total thermionic current and corresponding thermionic cooling flux is larger in magnitude compared to the low current case.



**Figure 3.17:** Wall temperatures,  $T_w$ , along the emissive cathode for  $WF^c = 2.65$  eV at (a)  $I_s = 2.4$  A and (b)  $I_s = 21.6$  A. Solid (—) and dashed (---) lines denote cases with ETC active and inactive, respectively.

cooling becomes a significant component of the surface heat balance at close to 10% and even exceeding radiative cooling near the cathode center. Ion heating also becomes more pronounced for this case. This behavior results from the increased thermionic emission at higher current, which produces a denser plasma in the inter-electrode region and consequently enhances ion fluxes to the surface. Similar to the low current case, conduction dominates heating at the edge of the cathode where laser heating has decayed. However, wall temperatures are still high enough to support thermionic emission, which becomes the dominant source of cooling in this area at close to 30% of the total cooling flux.





**Figure 3.18:** Heat fluxes along the emissive cathode surface normalized to the total heat flux,  $q_{total}$  for  $WF^c = 2.65$  eV at (a)  $I_s = 2.4$  A and (b)  $I_s = 21.6$  A. Positive values indicate surface heating while negative values indicate surface cooling. Electron heat fluxes are near zero and are omitted.

### 3.5 Conclusion

In this chapter, a novel model for an ETC system was presented. The model integrates kinetically-informed plasma sheath expressions with current and voltage conservation within the ETC system to enable the computation of current-voltage traces and power requirements. By examining variations in key ETC design parameters, this chapter identified configurations of ETC systems that enhance thermionic cooling as well as system efficiency. Finally, the ETC system model was validated against experimental measurements, where qualitative and quantitative agreement was observed.

A parameter study was performed over several ETC system parameters, such as electrode area ratios, plasma densities, and electrode materials. Results indicated that cooling of an ETC system is enhanced when both the anode surface area and work function are large relative to the equivalent values at the cathode. Additionally, ETC systems that are at the transition point from being space-charge-limited to Richardson-limited are expected to generate the largest magnitude of total cooling at the cathode. Across all cases, there exists an optimal internal voltage that maximizes the net cooling performance of the system.

The parameter study also computed a metric of cooling efficiency, which relates the total cooling of the ETC system to the required input power. Results indicated that cooling efficiency is enhanced when the system cathode is operating under space-charge-limited conditions, the anode material work function is close to, or lower than, that of the cathode, and an optimal value of the anode-to-cathode area ratio is chosen. Across nearly all cases, the most efficient configuration was predicted when the anode and cathode are internally shorted as significant cooling is still achieved without the need for an external power source. This configuration does not require an internal voltage source, potentially reducing both the complexity and cost of implementing an ETC system. We also note that in certain configurations the ETC system is predicted to generate power, indicating the potential for an ETC system to act as a power source on board a hypersonic vehicle.

Validation of the model was attempted with an explicit comparison to experimental measure-

ments of an ETC system. A coupled ETC + material conduction problem was solved and, for the material properties specified, the model was found to qualitatively agree with reported values of changes in surface temperature and current compared to a non-ETC case. This suggests that the sheath current and heat fluxes incorporated in the ETC model are physically reasonable and scale well with increasing thermionic emission. However, validation of the model was limited in that the plasma density and temperature were not measured, requiring the use of functional approximations for the radial plasma density as well as a density scaling parameter. This was apparent in that individual values of surface and temperature and current predicted by the model lie outside the uncertainty intervals of the experiment at low values of the internal current. It is expected that this disagreement is due to physical aspects of the experiment not captured by the approximations to the plasma composition. Notably, a constant electron temperature was assumed for the bulk plasma in the model, whereas is expected that electron temperature depends on the plasma density. Additionally, while it is expected that plasma losses from the center of the experimental setup are dominated by diffusion when density (and current) is high, this may not be true when the plasma density and current values are low, making the Bessel function a potentially poor approximation of the radial plasma density. Finally, the absorption coefficient of the material with regards to the incident laser was taken to be a constant in the model, where physically it may be a function of the material temperature.

It is desirable for future experimental studies to validate the trends presented in this work. To this end, it is suggested that future experimental studies have explicit control over both plasma density and cathode surface temperature to control the transition of the ETC system from a space-charge-limited to a Richardson-limited mode. To aid in the observation of cooling at a 0 V internal bias, it is recommended to construct an anode with a larger surface area and higher work function relative to the cathode. Further validation of the model can be achieved through variations in anode size and work function to test their influence on thermionic cooling. In addition to measurements of surface temperature and internal current, we recommend obtaining local measurements of plasma temperature and electron density near electrode surfaces to improve the fidelity of plasma property

inputs in computational models. Lastly, it is suggested that clear estimates of the cathode work function are made as the work function value can vary strongly as a function of the material purity, manufacturing process, grain orientation, and oxidation levels.

## Chapter 4

### Hypersonic Operating Regimes of ETC Systems

As discussed in Chapter 2, ETC systems can enter a space-charge-limited (SCL) mode when the thermionic emission current reaches a critical threshold and no longer can be increased. As thermionic current is proportional to the cooling flux, SCL modes prevent the ideal cooling flux from being realized and can be undesirable for a TPS application. The critical threshold for SCL modes depends on the local electron density near the emitting surface with higher plasma densities delaying the onset of SCL behavior. Consequently, identifying the hypersonic environments in which SCL operation is expected is essential for assessing the viability of ETC as a TPS technology.

The following chapter leverages the ETC system model presented in Chapter 3 to ascertain flight regimes where thermionic cooling is achievable, identify SCL regimes, and quantify power requirements of an ETC system. The low computational cost of the model allows rapid evaluation across a wide range of material properties, geometries, applied voltages, and bulk plasma conditions. This is in contrast to prior ETC studies that have mostly focused on fixed geometries, prescribed materials, and specific freestream conditions, all of which obscure the identification of broader hypersonic regimes where ETC is viable. Moreover, prior studies have largely modeled thermionic emission in isolation rather than the complete ETC electric circuit and have not quantified system-level considerations (e.g. power requirements) making the practicality of ETC an open question.

This chapter is structured as follows. First, in Section 4.1, a model for estimating the plasma properties of a post-shock hypersonic environment is described. The model is then coupled with the one-dimensional ETC model in Section 4.2 and the performance of the ETC system is analyzed for

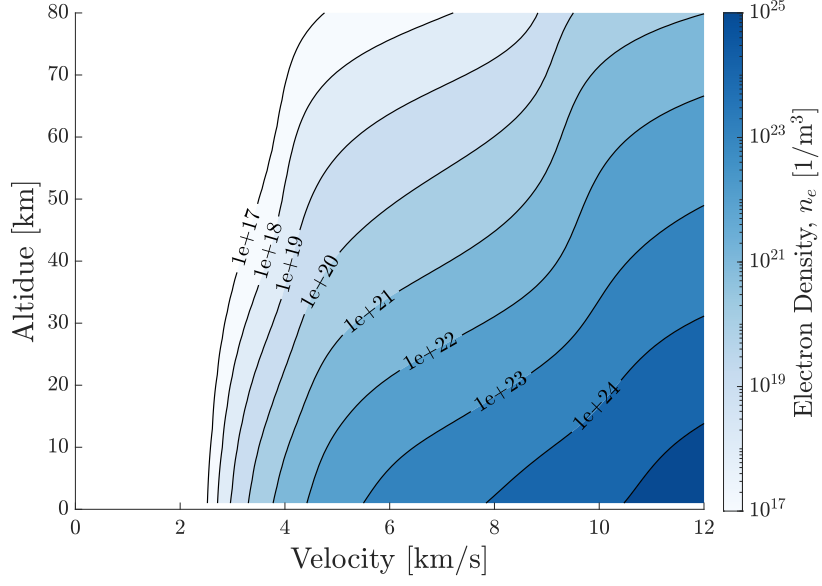
a wide range of velocities and altitudes. Specially, surface temperatures, with and without ETC, thermionic cooling fluxes, locations of SCL operation, and power requirements are quantified for varied thermionic materials and vehicle geometries. General flight regimes that are ideal for an ETC system are identified and discussed in detail. Finally, conclusions and recommendations for likely use cases for ETC as a cooling technology are presented in Sec 4.3.

#### 4.1 A Model for a Post-Shock Plasma

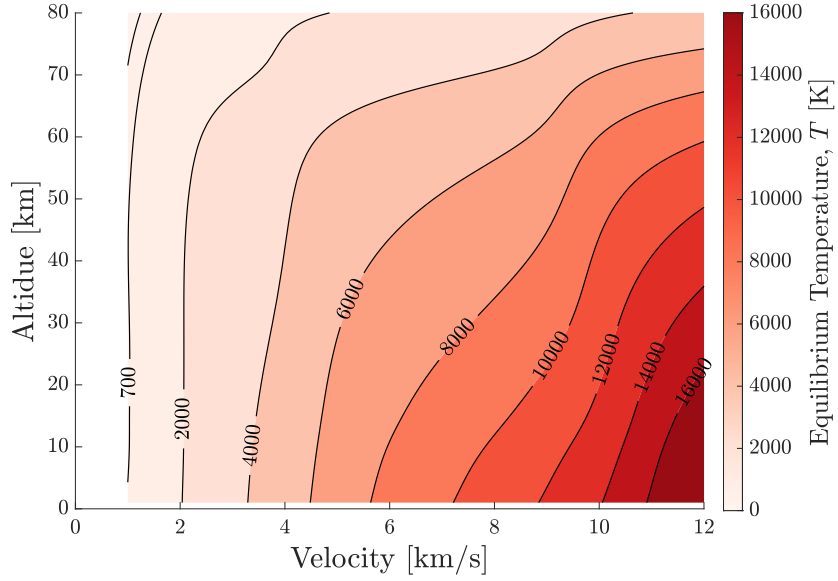
We present a model implemented in MATLAB to estimate the composition of a post-shock hypersonic environment, following similar approaches used in studies of hypersonic plasma composition [6, 5, 64]. The freestream chemical composition, density, and temperature as a function of altitude are taken from the U.S. Naval Research Laboratory’s NRLMSIS-00 atmospheric model [109], and the total enthalpy of the flow is computed as a function of the freestream velocity,  $v_\infty$ . Cantera [53], an open-source chemical kinetics software, is used to compute the thermochemical composition of the post-shock flow assuming local thermochemical equilibrium (LTE) and an 11-species air composition where the set of chemical species is denoted as  $\mathbf{S} = \{\text{N}_2, \text{O}_2, \text{N}, \text{O}, \text{NO}, \text{N}_2^+, \text{O}_2^+, \text{N}^+, \text{O}^+, \text{NO}^+, \text{e}\}$ .

The outputs of the MATLAB model are values of the post-shock plasma density,  $n_e$ , and equilibrium temperature,  $T$  as a function of the freestream velocity and altitude. We show contours of the post-shock  $n_e$  and  $T$  in Figs. 4.1(a) and 4.1(b) respectively, for a range of hypersonic flight conditions. Each pair of values define a plasma composition that is input directly into the ETC model, where  $T$  is used for the electron temperature,  $T_e$ . This allows for rapid interrogation of the ETC system across a wide range of hypersonic environments. While the assumption of LTE likely overpredicts values of  $n_e$  and  $T_e$ , it provides a suitable estimate of the post-shock condition for the purposes of identifying flight regimes where ETC may be useful. Further discussion on this assumption is found in Section 4.2.3.

To model the stagnation point heat flux,  $q_{conv}$ , This work employs a correlation by Brandis and Johnston [25] (Eq. 4.1), which is a function of the freestream velocity, density,  $\rho_\infty$ , and nose



(a)



(b)

**Figure 4.1:** Contours of (a) post-shock electron density,  $n_e$ , and (b) equilibrium temperature  $T$ , predicted by the post-shock plasma model. Both values define a plasma state as an input to the 1D ETC model.

radius,  $R_n$ . The functional form of Eq. 4.1 was found by fitting 648 CFD simulations, covering altitudes between 38.5 km and 83.5 km, velocities between 3 km/s and 17 km/s, and nose radii

between 0.2 m and 10 m. For the purpose of quantifying the scaling behaviors of ETC systems, the spread in hypersonic flight conditions covered by this correlation is expected to be sufficient.

$$q_{conv} = \begin{cases} (7.445 \times 10^{-5}) \rho_{\infty}^{0.4705} v_{\infty}^{3.089} R_n^{-0.52} & 3000 \leq v_{\infty} < 9500 \text{ m/s} \\ (1.270 \times 10^{-1}) \rho_{\infty}^{0.4678} v_{\infty}^{2.524} R_n^{-0.52} & 9500 \leq v_{\infty} \leq 17000 \text{ m/s} \end{cases} \quad [\text{W/m}^2] \quad (4.1)$$

In addition to convective heating, electromagnetic radiation from the shock-layer is expected to play a role in surface heating at high velocities [22, 73]. This phenomenon stems from electronic relaxation processes within the post-shock plasma, where electronically excited atoms and molecules emit photons as electrons return to lower energy states. Photons that impact the vehicle surface are absorbed and introduce a heat flux, denoted as  $q_{r,f}$ . A correlation for this heat flux was developed by Brandis and Johnston [25] with the final expression shown in Eq. 4.2 through Eq. 4.7. Stated by the authors, Eq. 4.2 is valid for velocities between 9.5 and 17 km/s. In the present work,  $q_{r,f}$  is taken as zero outside of these boundaries.

$$q_{r,f} = C R_n^a \rho_{\infty}^b f(v_{\infty}) \quad [\text{W/m}^2] \quad (4.2)$$

$$C = 3.416 \times 10^8 \quad (4.3)$$

$$a = \min \left\{ (3.175 \times 10^6) v_{\infty}^{-1.8} \rho_{\infty}^{-0.1575}, a_{max} \right\} \quad (4.4)$$

$$a_{max} = \begin{cases} 0.61 & 0 \leq R_n \leq 0.5 \text{ m} \\ 1.23 & 0.5 < R_n \leq 2 \text{ m} \\ 0.49 & 2 < R_n \leq 10 \text{ m} \end{cases} \quad (4.5)$$

$$b = 1.261 \quad (4.6)$$

$$f(v_{\infty}) = -53.26 + \frac{6555}{1 + \left( \frac{16000}{v_{\infty}} \right)^{8.25}} \quad (4.7)$$



At high altitudes, the freestream environment becomes increasingly rarefied, leading to a decrease in molecular collision frequencies and, eventually, a breakdown of continuum-based models. This breakdown is particularly important in regards to estimating both chemical composition in the post-shock region and convective heat fluxes at the vehicle surface. Equation 4.8 defines the Knudsen number, which captures the degree of rarefaction in an environment by taking the ratio of the fluid mean free path,  $\lambda$ , to a characteristic length,  $L$ . For  $Kn > 10$ , the fluid is typically treated as free-molecular, while for  $Kn < 0.01$ , continuum models are well suited. Thus, to account for rarefied effects, the present work utilizes bridging relations that leverage local values of  $Kn$  to interpolate between continuum and free-molecular limits.

$$Kn = \frac{\lambda}{L} \quad (4.8)$$

To smoothly model the post-shock chemical composition (i.e. the species density,  $n_s$ , where  $s \in \mathcal{S}$ , as well as the equilibrium temperature,  $T$ ) in the transitional and rarefied regimes, the present work uses a bridging function proposed by Wilmoth et al. [152]. Expressions for the bridged value,  $x$ , is shown in Eq. 4.9 where  $x_{fm}$  and  $x_{ct}$  are the freestream and continuum values, respectively. The switching parameter,  $F$ , described in Eq. 4.10, is a function of the freestream Knudsen number,  $Kn_\infty$  as well as two free parameters,  $\Delta Kn$  and  $Kn_m$ . The value of  $\Delta Kn$  represents the logarithmic width of the transitional regime with center at  $Kn_m$ . Values of  $Kn_m = 0.25$  and  $\Delta Kn = 6.21$  are chosen based on recommended values from Direct Simulation Monte Carlo (DSMC) simulations of blunt body craft [152].

$$x = x_{fm}F + x_{ct}(1 - F) \quad (4.9)$$

$$F = \frac{1}{2} \left( 1 + \operatorname{erf} \left[ \frac{\sqrt{\pi}}{\Delta Kn} \log \left\{ \frac{Kn_\infty}{Kn_m} \right\} \right] \right) \quad (4.10)$$

To compute accurate values of  $q_{conv}$  at high altitudes, the present work utilizes a similar bridging function by Singh and Schwartzentruber [131]. The bridged heat flux coefficient,  $K$ , is

shown in Eq. 4.11 as a function of the continuum and free-molecular heat flux coefficients,  $K_{ct}$  and  $K_{fm}$ , as well as a rarefaction parameter,  $W_r = M_\infty^{2\omega}/Re_\infty$ , where  $M_\infty$  and  $Re_\infty$  are the freestream Mach and Reynolds number, respectively, and  $\omega$  is the viscosity-temperature index, taken to be 0.75 for air [131]. The constants  $C_2$  and  $C_3$  control the location at which the bridging function becomes active and the speed at which the function switches to the free-molecular limit. Equation 4.1 is used to compute the continuum heat flux,  $q_{ct}$ , while the free-molecular heat flux,  $q_{fm}$  is computed via Eq. 4.14.

$$K = K_{ct} + (K_{fm} - K_{ct}) \max \left[ \frac{W_r - C_2}{W_r + C_3}, 0 \right] \quad (4.11)$$

$$K_{ct} = \frac{q_{ct}}{\frac{1}{2}\rho_\infty V_\infty^3 (1 - \alpha W_r)} \quad (4.12)$$

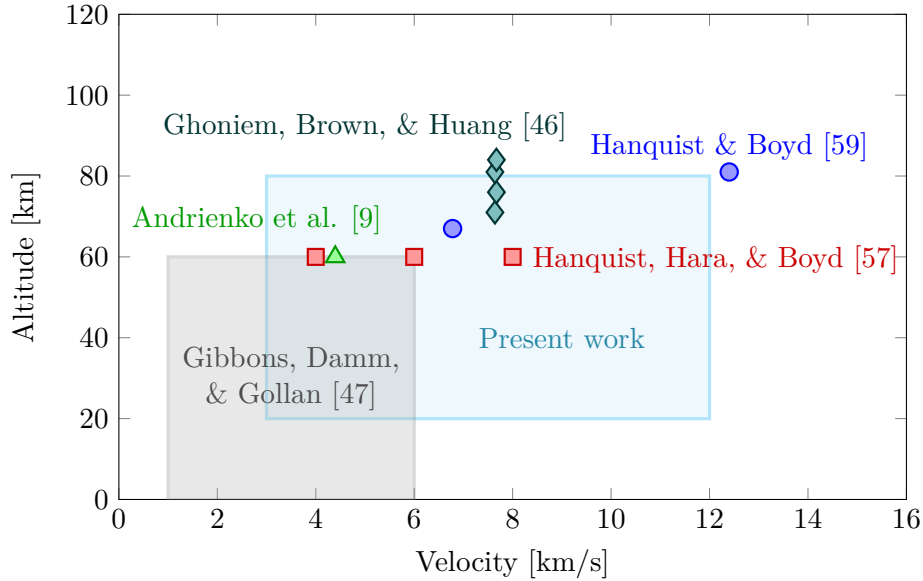
$$K_{fm} = \frac{q_{fm}}{\frac{1}{2}\rho_\infty V_\infty^3} \quad (4.13)$$

$$q_{fm} = \rho_\infty \left( \sqrt{\frac{2k_B T_\infty}{m_i}} \right)^3 \frac{1}{4\sqrt{\pi}} \left\{ \left[ \frac{\gamma}{2} M_\infty^2 + \frac{\gamma}{\gamma - 1} - \frac{\gamma + 1}{2(\gamma - 1)} \frac{T_w}{T_\infty} \right] \right. \\ \times \left[ \exp \left( -\frac{\gamma}{2} M_\infty^2 \right) + \sqrt{\frac{\pi\gamma}{2}} M_\infty \left[ 1 + \operatorname{erf} \left( \sqrt{\frac{\pi\gamma}{2}} M_\infty \right) \right] \right] \\ \left. - \frac{1}{2} \exp \left( -\frac{\gamma}{2} M_\infty^2 \right) \right\} \quad (4.14)$$

The values of constants  $C_2$ ,  $C_3$ , and the fitting parameter,  $\alpha$ , are found through comparison to DSMC simulations for a given geometry. The present work takes provided values of  $C_2 = 1.10$ ,  $C_3 = 1.8$ , and  $\alpha = -0.476$  from Singh and Schwartzentruber [131] corresponding to a 6 in. radius sphere. While this geometry does not match the nose radius chosen in the ETC study, it is sufficiently similar for the purpose of assessing ETC system scaling across the chosen hypersonic environments and large deviations from the discussed behavior are not expected.

## 4.2 Operational Envelopes of ETC Systems

The following section leverages the ETC system model in conjunction with the post-shock plasma model to investigate ETC performance across a range of hypersonic flight regimes. To limit the scope of the analysis, the present work focuses on altitudes between 20–80 km and velocities between 3–12 km/s. These conditions span a broad range of hypersonic environments relevant to ETC operation. Figure 4.2 summarizes the freestream conditions examined in prior CFD studies of ETC and contrasts them with the expanded range of regimes considered in this work.



**Figure 4.2:** Flight conditions of past investigations into ETC that have incorporated space-charge-limited physics.

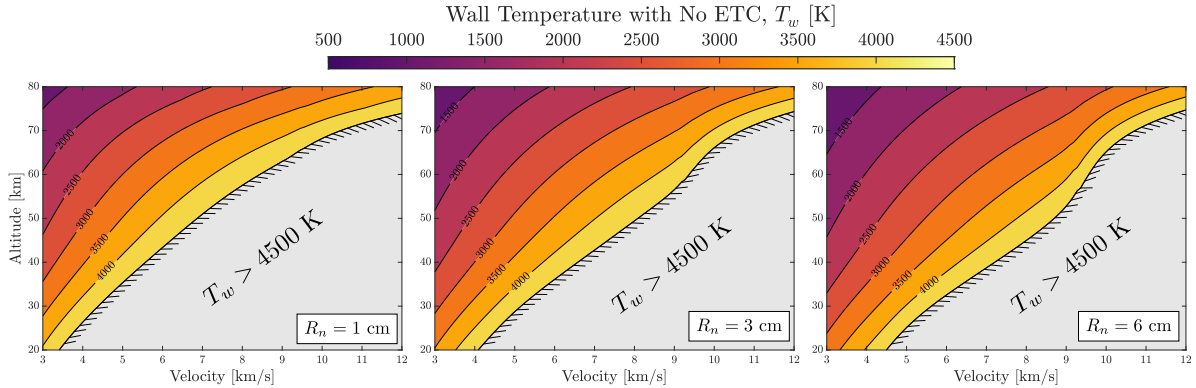
First, Section 4.2.1 presents a baseline case without ETC. In Section 4.2.2, ETC is introduced as a cooling mechanism and surface temperatures are compared to the baseline case. Next, in Section 4.2.3, locations of space-charge-limits are identified and their implications towards ETC performance are discussed. Finally, in Section 4.2.4, power requirements of the ETC system are computed and their implications on the feasibility of ETC systems is discussed.

#### 4.2.1 Surface Temperatures without ETC Systems

To establish a baseline for expected surface temperatures, we first consider a non-emitting surface. The surface is assumed to be subjected to convective heating,  $q_{conv}$ , shock-layer radiative heating,  $q_{r,f}$ , electron and ion sheath heating,  $q_{e,f}$  and  $q_{i,f}$ , and radiative cooling,  $q_{rad}$ . Radiative cooling is determined via the Stefan-Boltzmann Law where  $\epsilon = 0.89$  in accordance with the convective correlation [25]. Although there is no thermionic emission,  $q_{i,f}$  and  $q_{e,f}$ , are still present due to the floating sheath forms over the surface. All heat fluxes are shown in Eq. 4.15 below.

$$q_{rad} = q_{conv} + q_{r,f} + q_{i,f} + q_{e,f} \quad (4.15)$$

We solve Eq. 4.15 over a range of altitudes and velocities in order to obtain the steady state surface temperature without ETC effects denoted as  $T_w$ . To identify the influence of geometry on ETC performance, we repeat this computation for three chosen nose radii:  $R_n \in \{1, 3, 6\}$  cm. surface temperatures above 4500 K are omitted as this is above the melting point of even the most high-temperature materials.



**Figure 4.3:** Contours of baseline surface temperatures at radiative equilibrium without ETC,  $T_w$  corresponding to a nose radius,  $R_n$  of 1 cm (left), 3 cm (middle), and 6 cm (right). Grayed-out regions indicate surface temperatures above 4500 K.

Figure 4.3 show contours of  $T_w$  for the three chosen nose radii. Immediately, we observe that material temperature limits restrict a significant portion of the investigated flight regimes, particularly for low altitudes and high velocities. As expected, for a given altitude and velocity, a

larger nose radius experiences lower surface temperatures. This is illustrated by the gray region of  $T_w > 4500$  K shrinking as  $R_n$  increases.

#### 4.2.2 Surface Temperatures with ETC Systems

We now consider the effect of an ETC system on the surface temperature. The ETC system introduces a thermionic cooling flux,  $q_{etc}$ , to the surface in addition to modifying values of the electron and ion sheath heating contributions. Using the electron density and equilibrium temperature predicted by the post-shock model, we solve the ETC system model for  $q_{etc}$ ,  $q_{i,f}$ , and  $q_{e,f}$ . Incorporating these heat fluxes into the energy balance at the surface yields a final cathode surface temperature that includes the effect of the ETC system, denoted as  $T_w^{etc}$ .

Table 4.1 shows relevant parameters of the ETC system. We point out that the anode to cathode area ratio,  $A$ , and internal voltage,  $V_s$ , are set to 50 and 1.0 V, respectively. In Chapter 3, it was found that increasing the values of  $A$  and  $V_s$  in an ETC system enhances the cooling at the cathode. However, this process has diminishing returns. The chosen values for  $A$  and  $V_s$  represent intermediate values that are reasonable for an ETC system. In addition to three values of the nose radius,  $R_n \in \{1, 3, 6\}$  cm, we consider four values of the cathode work function:  $WF^c \in \{2, 3, 4, 5\}$  eV. Thus, the 12 combinations of  $R_n$  and  $WF^c$  span a wide range of sharp leading edge geometries and thermionic materials.

**Table 4.1:** Base simulation parameters for the ETC circuit analysis.

| ETC System Parameters                      | Value             |
|--------------------------------------------|-------------------|
| $A$                                        | 50                |
| $A^c$ , cm <sup>2</sup>                    | 6.0               |
| $A_R$ , A/(m <sup>2</sup> K <sup>2</sup> ) | $120 \times 10^4$ |
| $V_s$ , V                                  | 1.0               |

Figure 4.4 shows contours of the expected surface temperature with ETC, denoted as  $T_w^{etc}$ . Each column corresponds to the same nose radii from the previous section, while each row corre-

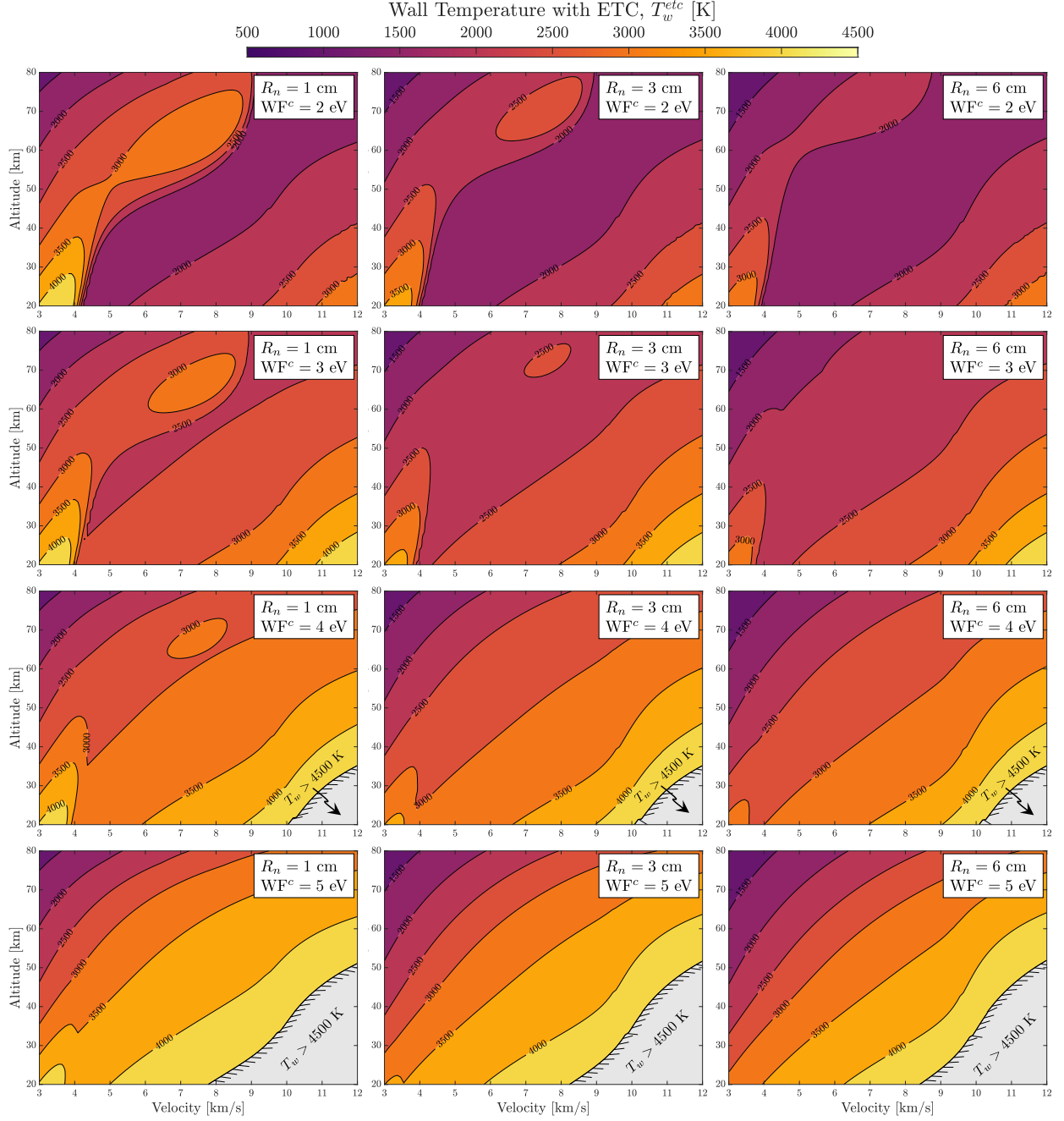
sponds to a different cathode work function. The most emissive material and sharpest geometry is in the top left of Fig. 4.4. Similarly to Fig. 4.3, surface temperatures above 4500 K are grayed-out.

We observe in Fig. 4.4 that the introduction of the ETC system significantly reduces surface temperatures over the investigated hypersonic environments and nose radii. This is best illustrated in the first and second rows in Fig. 4.4, where surface temperatures never exceed 4500 K over the entire hypersonic flight regime (3-12 km/s and 20-80 km). For higher cathode work functions that are less emissive (third and fourth rows), material temperature limits begin to appear at high speeds and low altitudes. However, even for the highest work function of 5 eV (bottom row), the extent of feasible environments with the ETC system is larger compared to the purely radiative surface.

Additionally, we note that surface temperatures in Fig. 4.4 are occasionally predicted to be non-linear with altitude and velocity. This is particularly evident for the  $WF^c = 2$  eV and  $R_n = 1$  cm case (top left) at 20 km altitude, where high temperatures are expected until  $\sim 4$  km/s where  $T_w^{etc}$  rapidly drops for higher velocities. The sudden change in surface temperature arises from the transition between a SCL to Richardson-limited operational mode and a sudden increase in thermionic cooling. Further discussion on SCL regimes, which inhibit thermionic cooling, is provided in the following section. Importantly, this behavior suggests that, for the chosen materials and geometries, ETC systems are not expected to provide significant cooling below 3-4 km/s.

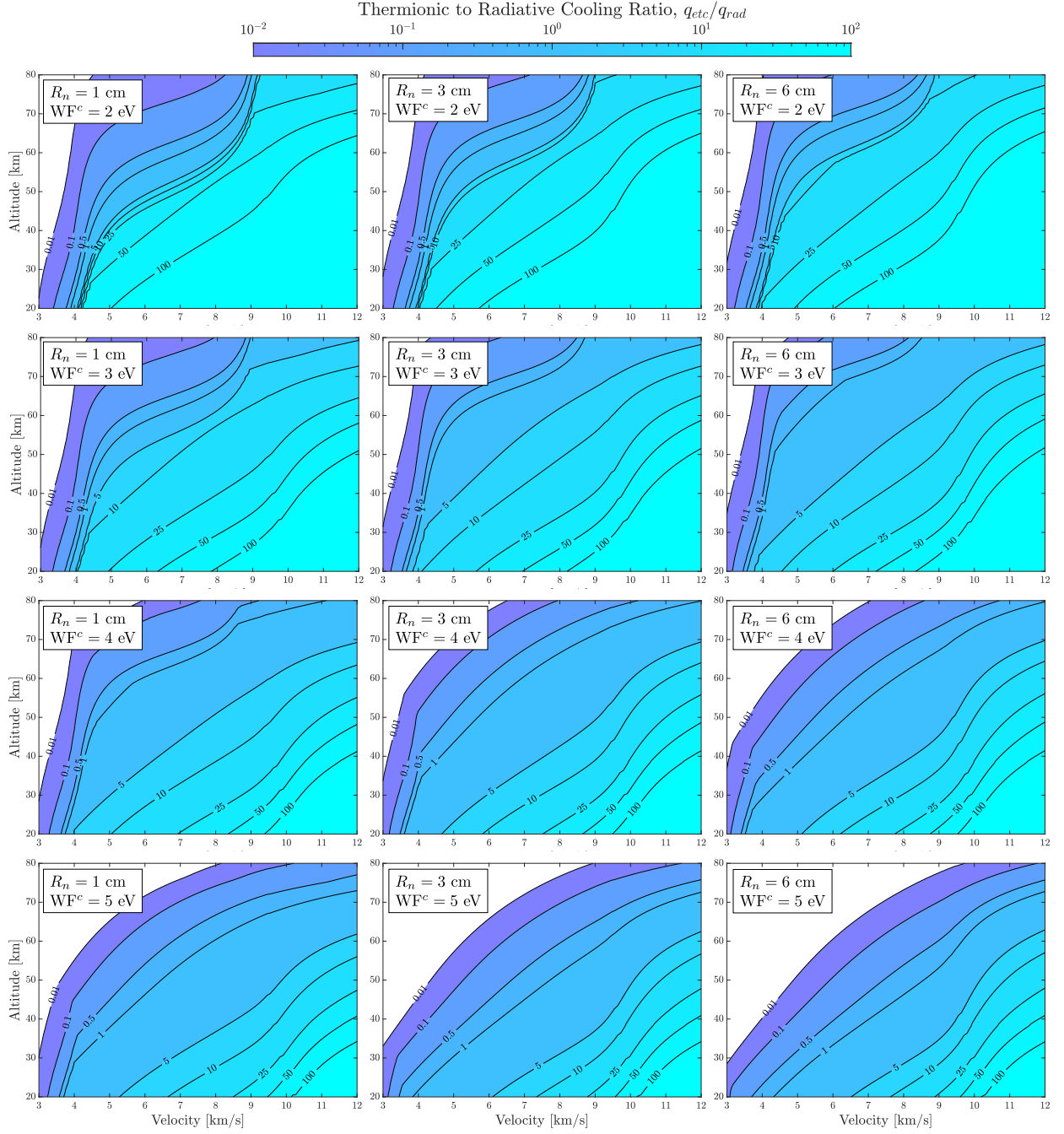
Finally, contours of the ratio of thermionic to radiative cooling,  $q_{etc}/q_{rad}$ , are presented in Fig. 4.5. Values of  $q_{etc}/q_{rad} > 1$  are desirable, as they indicate that the thermionic cooling flux exceeds the radiative cooling flux. At lower altitudes and higher velocities, the surface temperature and plasma density increases, both of which enable increased thermionic emission. As a result,  $q_{etc}/q_{rad}$  increases from values near zero to order unity and, in some cases, to values exceeding 100 over the investigated flight regimes. This trend highlights a key advantage of ETC systems: for increasingly severe hypersonic environments, thermionic cooling can scale more strongly than radiative cooling.

Considering the  $R_n = 1$  cm and  $WF^c = 2$  eV case (top left), an interesting behavior is ob-



**Figure 4.4:** Contours of surface temperature at radiative equilibrium with ETC,  $T_w^{etc}$ , for varied cathode materials and nose radii. Rows correspond to the cathode work functions of 2, 3, 4, and 5 eV (top to bottom). Columns correspond to nose radii of 1, 3, and 6 cm (left to right). Grayed-out regions indicate surface temperatures above 4500 K

served near the flight condition of 6 km/s and 60 km. As discussed in the following section, this condition lies within the SCL regime. Despite SCL operation, the ratio of thermionic to radiative



**Figure 4.5:** Contours of the ratio of thermionic to radiative cooling,  $q_{etc}/q_{rad}$  for varied cathode materials and nose radii. Rows correspond to the cathode work functions of 2, 3, 4, and 5 eV (top to bottom). Columns correspond to nose radii of 1, 3, and 6 cm (left to right).

cooling is still non-negligible, with  $q_{etc}/q_{rad}$  approaching 0.5. This indicates that thermionic cooling can remain a substantial cooling method even when the ETC system is in a SCL mode. Conse-



quently, SCL operation, while reducing the achievable thermionic current, does not completely eliminate the usefulness of ETC systems which can still provide significant cooling for a hypersonic vehicle.

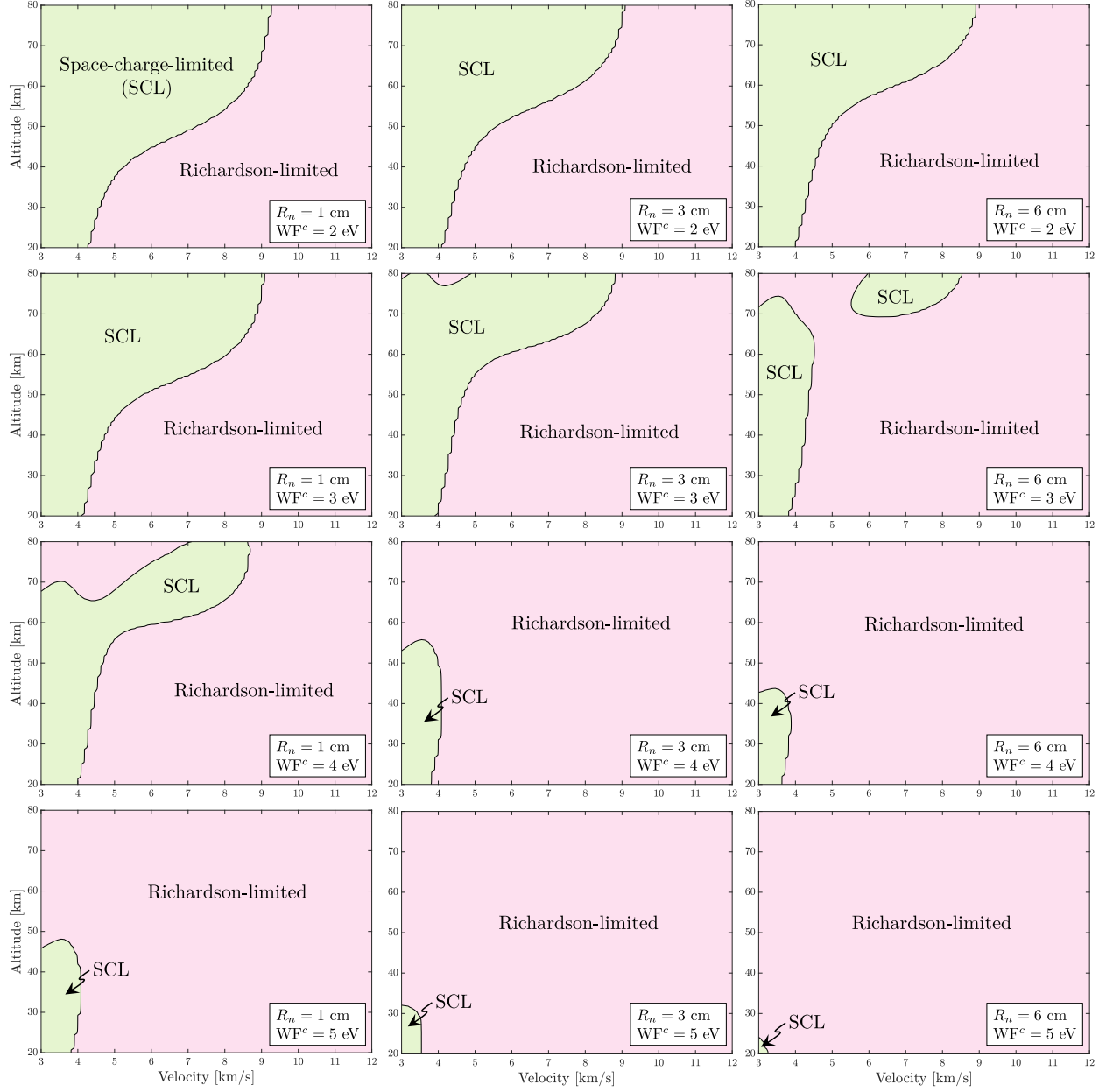
#### 4.2.3 Locations of Space-Charge-Limits

We now consider the effect of space-charge-limited modes in the operation of ETC systems. Space-charge effects prevent the emissive cathode from realizing the ideal Richardson emission current, and subsequently, the maximum thermionic cooling flux of an ETC system.

Figure 4.6 shows locations in altitude-velocity space where SCL and Richardson-limited operation are predicted. Columns and rows follow the same structure as shown in Fig. 4.4. As expected, the largest SCL regime corresponds to the  $WF^c = 2$  eV and  $R_n = 1$  cm case (top left). This is because the low work function (high thermionic emissivity) and small nose radius (high convective heating) result in large thermionic currents that overwhelm the bulk plasma's ability to conduct current away from the surface. Conversely, the SCL regime is nearly absent for the  $WF^c = 5$  eV and  $R_n = 6$  cm case (bottom right), as the high work function (low thermionic emissivity) and large nose radius (low convective heating) reduce emission currents into the plasma for a given environment.

Over all combinations of nose radius and work function, no SCL regimes are expected beyond 9 km/s. Above this velocity sufficient plasma forms to prevent the rise of space-charge effects. Below 9 km/s, two regions of SCL operation exist where the freestream plasma density is insufficient for the emission strength of the cathode. This is best illustrated by the two local maxima present in the  $WF^c = 3$  eV and  $R_n = 6$  cm case (second row, right column). A high altitude regime, centered at approximately 7.5 km/s and 75 km, arises from the rarefied environment where molecular collisions are too infrequent to produce a sufficient plasma. A low velocity regime, occurring at velocities between 3 and 4 km/s, arises from insufficient freestream enthalpy to ionize the post-shock region.

For TPS applications, it is desirable to operate an ETC system in a Richardson-limited regime as this allows for the maximum thermionic cooling flux to leave the surface. Thus, we



**Figure 4.6:** Locations of space-charge-limited (green) and Richardson-limited (pink) operation for varied cathode materials and nose radii. Rows correspond to the cathode work functions of 2, 3, 4, and 5 eV (top to bottom). Columns correspond to nose radii of 1, 3, and 6 cm (left to right).

identify two promising regimes for ETC operation where SCL operation is not expected. The first is a super-orbital regime above 8 km/s. This regime may be well suited for lunar/Martian reentry vehicles or aerocapture maneuvers. The second is a sub-orbital regime between 4 - 8 km/s and below approximately 50 km. This regime may be promising for a boost-glide platform.

The post-shock plasma model used in this work assumes LTE when computing plasma properties,  $n_e, T_e$ . Because the onset of SCL emission depends strongly on the plasma density and electron temperature near the surface, it is useful to assess how this assumption influences the predicted location of SCL regimes. To evaluate this, the present results are compared with those of Gollan et al. [47], who investigated ETC behavior using a nonequilibrium hypersonic flow solver. As shown in Fig. 4.2, the flight regimes examined in the two studies overlap, enabling a direct comparison of the predicted boundaries between Richardson-limited and SCL operation.

A key distinction between the analyses lies in how the plasma properties  $n_e, T_e$  are obtained. Gollan et al. [47] compute these quantities using a hypersonic flow solver [49], which resolves a two-dimensional flowfield while incorporating thermochemical nonequilibrium effects. Plasma properties used for the ETC analysis are extracted along the stagnation line for each simulated flight condition. In total, the study performs 66 CFD simulations spanning a range of altitudes and velocities, with each case providing detailed plasma conditions at the vehicle leading edge.

In contrast, the post-shock plasma model presented in Section 4.1 evaluates plasma properties assuming LTE. This approach is computationally inexpensive and enables a much denser exploration of the flight envelope. In the present work, the ETC system is evaluated over a grid of 100 altitudes and 100 velocities, corresponding to 10,000 model evaluations. While the equilibrium assumption neglects thermochemical nonequilibrium effects, which may result in differences between electron and heavy-species temperatures or altered ionization levels, it allows for the rapid interrogation of the parameter space and a more finely resolved map of ETC performance.

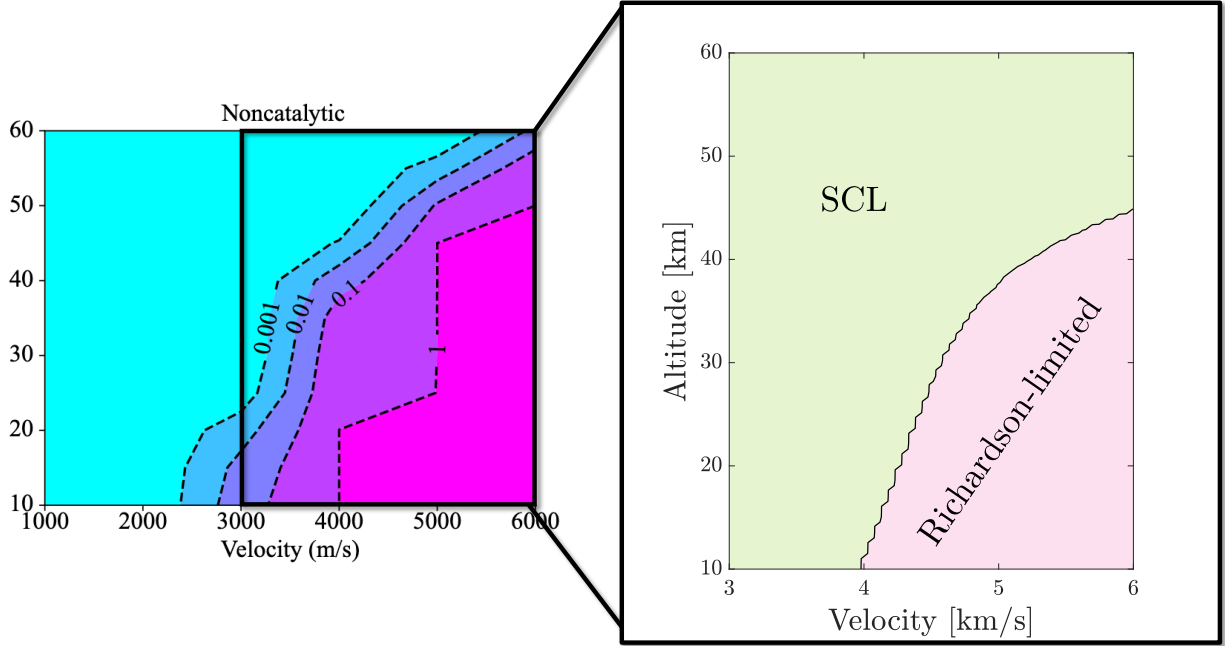
The predicted locations of SCL operation from the two analyses are compared. Both cases assume a nose radius of  $R_n = 1$  cm and a cathode work function of  $WF^c = 2$  eV. Gollan et al. [47] identify SCL behavior using a space-charge damping factor,  $\Psi$ , defined as

$$\Psi = \min(J_{etc}/J_R, 1) \quad (4.16)$$

Using this metric, the emissive cathode is in a Richardson-limited mode when  $\Psi = 1$ . When

$\Psi < 1$ , the emissive cathode is in a SCL mode as the thermionic current is less than that predicted by Richardson’s Law.

Figure 4.7 compares the two analyses over altitudes of 10–60 km and velocities of 3–6 km/s. When examining the location of the Richardson-limited regime, qualitative and quantitative agreement with Gollan et al. [47] is observed. In particular, both analyses indicate that SCL modes occur below 4 km/s and above 50 km, defining a boundary of applicable regimes for ETC systems. This agreement builds confidence in the results of this section and suggests that the LTE assumption provides a reasonable approximation for estimating regimes of SCL operation.



**Figure 4.7:** Comparison of SCL regimes between the (left) Gollan et al. [47] and (right) present work for  $R_n = 1$  cm and  $WF^c = 2$  eV. Gollan et al. shows contours of a “space-charge damping factor”,  $\Psi$  (Eq. 4.16), where values of  $\Psi = 1$  indicate Richardson-limited operation.

#### 4.2.4 Power Requirements of the ETC System

While high thermionic currents are advantageous in that they correspond to high thermionic cooling fluxes, they also increase the power draw of the ETC system. The required power draw directly influences the size and mass of the internal voltage source and represents a potential

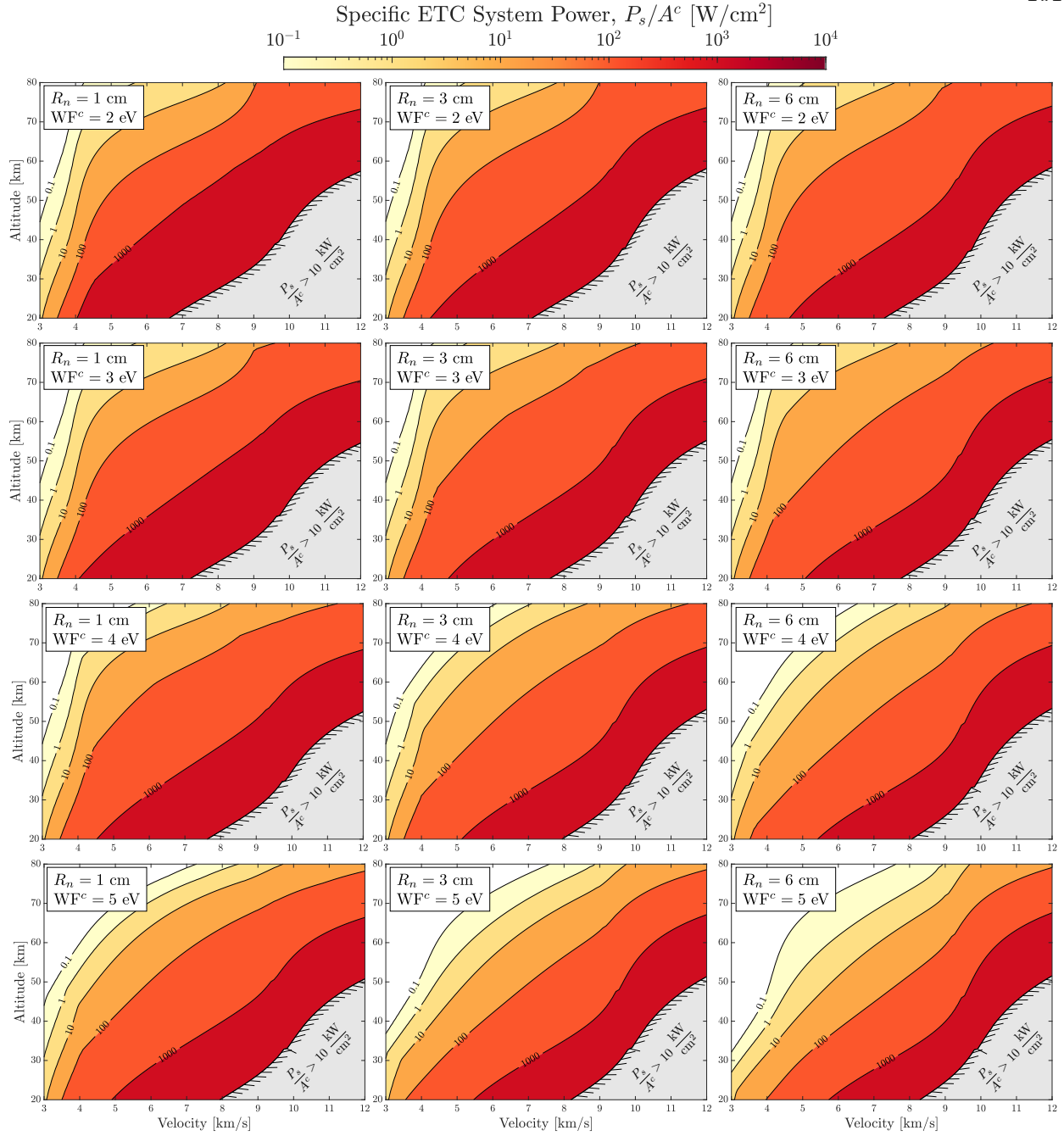
constraint on the practicality of ETC systems. Thus, we compute the power required by the ETC system as a function of the hypersonic flight environment to identify regimes where ETC may be practical.

We define the area specific power required by the ETC system in Eq. 4.17 where  $V_s$  is the applied voltage of 1 V,  $A^c$  is the cathode surface area, and  $J_s = J_{i,f} + J_{etc} - J_{e,f}$  is the total current density in the system. Thus, the specific power represents the power per unit area of the cathode required by the ETC system. We show contours of the specific power in Fig. 4.8 over the investigated hypersonic environments. Power requirements above 10 kW/cm<sup>2</sup> are grayed out as this value is determined to be infeasible for a hypersonic vehicle to provide.

$$\frac{P_s}{A^c} = V_s J_s \quad (4.17)$$

Figure 4.8 shows that, for all materials and geometries, the power draw of the ETC system is small at high altitudes and low velocities (i.e., the upper-left corner of each plot). In this regime, convective heating is weak, resulting in low surface temperatures and negligible thermionic emission. Additionally, the previous section showed that this regime typically falls within the SCL mode due to either insufficient freestream enthalpy or molecular collisions to generate a dense plasma.

Conversely, all cases show that power consumption increases rapidly at higher velocities and lower altitudes (i.e., the lower-right corner of each plot). The large freestream enthalpies in these conditions produce dense, highly ionized plasmas. As discussed in the previous section, such environments are favorable because they permit Richardson-limited operation. However, the power requirements in these regimes, which can quickly exceed 10 kW/cm<sup>2</sup>, are substantial and difficult to supply with a practically sized power source. This challenge is compounded by realistic vehicle geometries, which may contain hundreds of square centimeters of thermal protection system area, further increasing the total power demand. Flight duration presents an additional constraint: short activation periods may be feasible in these regimes, but sustained operation is unlikely. Based on these considerations, ETC systems are likely to become power-limited when velocities exceed



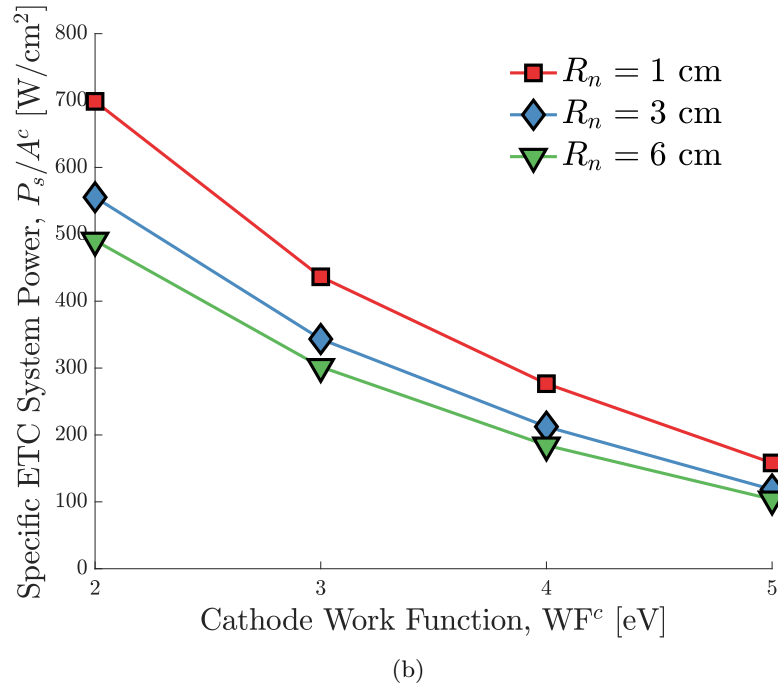
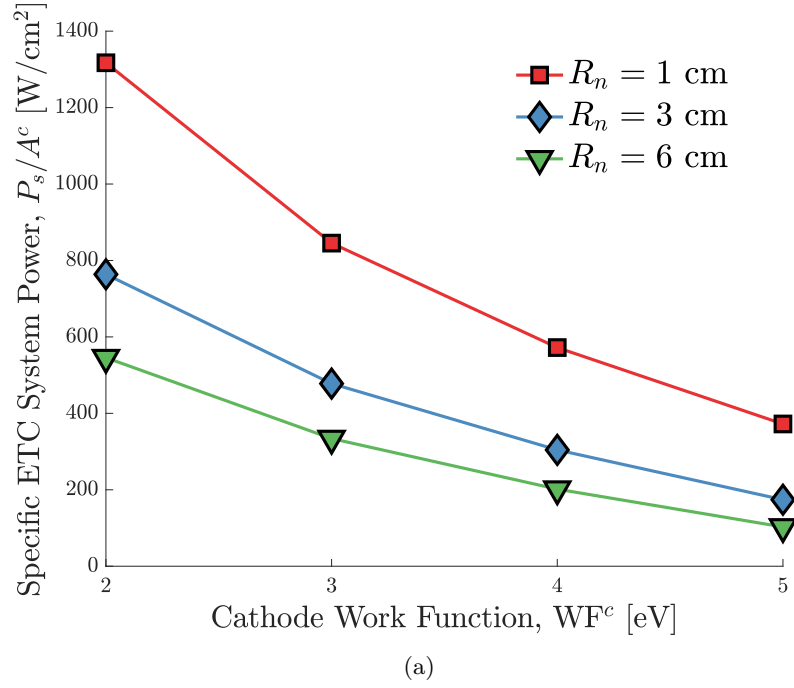
**Figure 4.8:** Contours of the specific power (power per unit area of cathode surface) required by the ETC system,  $P_s/A^c$ , for work functions of (left) 2 eV and (right) 5 eV. Grayed-out regions indicate power requirements above 10 kW/cm<sup>2</sup>, which is determined to be infeasible for a practical TPS.

approximately 8 km/s and altitudes are below 50 km.

Representative conditions of 5 km/s at 30 km as well as 10 km/s at 70 km are selected for the

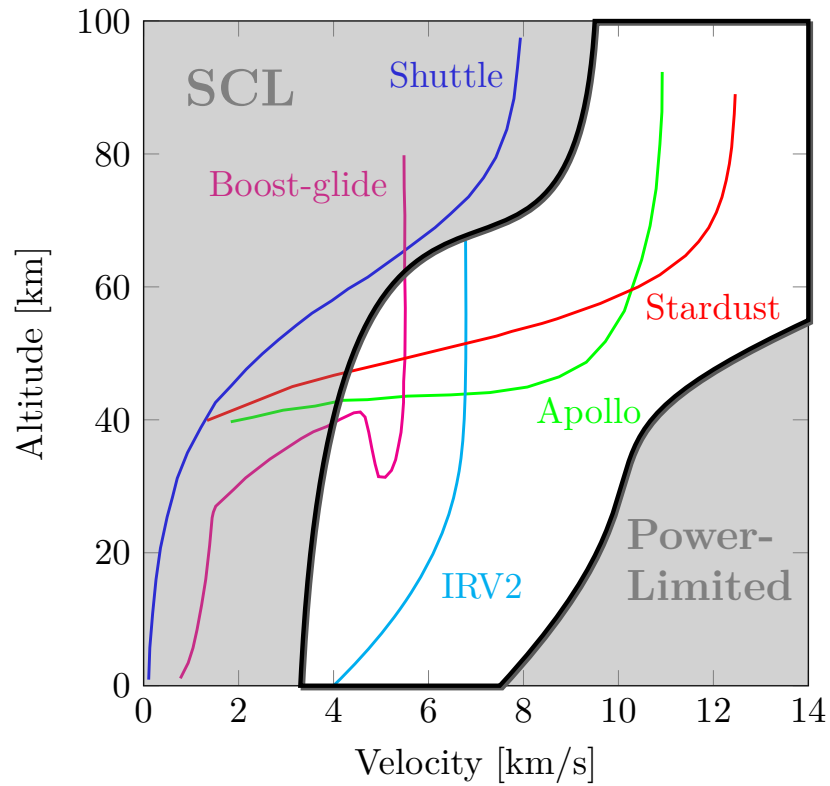
sub-orbital and super-orbital regimes, respectively. ETC specific power requirements are shown for each condition in Figs. 4.9(a) and 4.9(b). In both the sub-orbital and super-orbital environments, increasing the cathode work function or nose radius reduces the required power. Comparing the two regimes, the average power density is approximately  $600 \text{ W/cm}^2$  in the suborbital case and  $300 \text{ W/cm}^2$  in the super-orbital case. However, because orbital reentry vehicles typically have large surface areas, the total ETC system power required for orbital vehicles may be significantly higher despite the reduced power density.

With the computation of ETC power requirements, combined with the previously identified SCL regimes, sufficient information is now available to describe the overall operating regimes of ETC systems. Section 4.2.3 identified two regions of SCL behavior. The first occurs at high altitudes, where rarefied air results in infrequent molecular collisions and insufficient plasma density. The second occurs at low velocities, where the freestream enthalpy is too small to generate a sufficiently dense plasma. The present section quantified the power requirements of ETC systems and showed that, at high velocities and low altitudes, the required power becomes extremely large and sustained ETC operation is unlikely. Figure 4.10 overlays these constraints with historical hypersonic flight trajectories, including orbital reentry vehicles and boost–glide platforms. Importantly, a region emerges where power requirements remain reasonable and SCL behavior is not expected. This regime includes both super-orbital vehicles and boost–glide vehicles.



**Figure 4.9:** Curves of specific power (power per unit area of cathode surface) required by the ETC system,  $P_s/A^c$ , as a function of cathode work function,  $WF^c$ , for varying nose radii,  $R_n$ . Results correspond to (a) a sub-orbital environment at 5 km/s and 30 km and (b) a super-orbital environment at 10 km/s and 70 km.





**Figure 4.10:** Flight trajectories of historic hypersonic vehicles. Qualitative depictions of flight regimes where ETC systems are expected to encounter SCL operation or excessive power limitations are shaded in gray. An ideal region for ETC systems is outlined in black, which includes low altitude boost-glide vehicles, as well as high altitude super-orbital re-entry vehicles.

### 4.3 Conclusion

The objective of this chapter was to identify flight regimes where ETC systems are most effective. A predictive model for a post-shock hypersonic environment was paired with an ETC system model and evaluated over a range of altitudes and velocities. This approach addresses a gap in the assessment of ETC as a cooling technology by providing evaluations of cooling performance, space-charge-limited operation, and power requirements over broad hypersonic flight regimes as opposed to specifically chosen flight conditions.

surface temperatures of a radiative surface with and without ETC were compared over velocities of 3–12 km/s and altitudes of 20–80 km. Various thermionic materials and cathode geometries were considered. Across all configurations, ETC significantly reduced surface temperatures and expanded the range of environments in which hypersonic vehicles could operate. In particular, thermionic cooling was found to scale with the plasma density of the post-shock environment. In contrast to conventional TPSs, this characteristic allows ETC systems to increase their cooling ability as hypersonic conditions become more severe. This makes ETC systems uniquely suited for high-hypersonic flight environments.

Two regimes of flight were identified to be free of space-charge effects and well suited for ETC operation. The first is a sub-orbital regime from between 4–8 km/s and below 50 km, potentially well suited for a boost-glide platform. The second is a super-orbital regime above 8 km/s and above 60 km, potentially well suited for reentry vehicles or aerocapture maneuvers. ETC operation at both high velocities ( $>8$  km/s) and low altitudes ( $<60$  km) is unlikely due to large power requirements. Additionally, no meaningful ETC operation is expected at velocities below 3–4 km/s due to space-charge effects. The results of this analysis build confidence that ETC systems can be a practical approach for hypersonic thermal protection and hopefully motivate continued study in these regimes

## Chapter 5

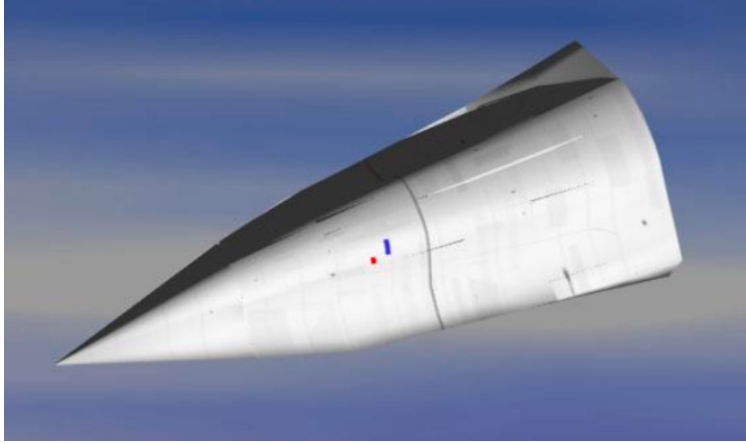
### Performance Expansion of Hypersonic Vehicles Enabled by Electron Transpiration Cooling

Shown in Chapter 4, ETC systems are predicted to perform well at high velocities and are potentially well suited for boost–glide hypersonic vehicles. A key characteristic of ETC is that thermionic cooling scales with the bulk plasma density, enabling stronger cooling in high-enthalpy ionized hypersonic environments. For boost–glide vehicles, the most severe heating typically occurs immediately after the boost phase ends, when the vehicle detaches from the launch vehicle and enters the glide phase at high velocity. The flight conditions upon entering the glide phase largely determine overall mission performance as the vehicle is unpowered and must glide using the energy imparted during the boost phase. As thermionic cooling is most effective under these extreme conditions, it has the potential to mitigate peak heating while enabling more aggressive initial states. To evaluate this potential, the following chapter applies the ETC model to a representative boost–glide vehicle to assess both the thermal benefits and the associated electrical power requirements.

This chapter is organized as follows. First, Section 5.1 introduces a canonical boost–glide vehicle, including its aerodynamic parameters and material properties. Section 5.2 presents the trajectory optimization framework and its coupling with the ETC system model, enabling the computation of hypersonic trajectories subject to specified constraints. The vehicle performance is then evaluated for two prototypical mission objectives in Sections 5.3 and 5.4, where the influence of the ETC system on boost–glide performance is quantified. Finally, Section 5.5 examines the

power requirements associated with ETC operation.

## 5.1 Vehicle Description



**Figure 5.1:** Image of the Common Aero Vehicle (CAV-H) from Phillips [108]

The Common Aero Vehicle (CAV-H) is chosen to represent a hypersonic boost-glide platform. A depiction of the CAV-H is shown in Fig. 5.1. The CAV-H is a high-lift hypersonic reentry vehicle with extended range and crossrange capabilities as well as a sharp leading edge [108]. As a glide platform, the CAV-H has no ability to generate thrust and must rely on control inputs to maneuver. Table 5.1 shows the relevant vehicle parameters where  $L/D$  is the lift-to-drag ratio,  $a$  is the aerodynamic loading,  $m$  is the vehicle mass,  $A_{ref}$  is the wing reference area, and  $R_n$  is the nose radius.

The ETC system applied to the CAV-H is the same as shown in Table 4.1. The cathode material is chosen to be hafnium carbide (HfC) which has a melting point of 4173 K, emissivity of 0.9, and work function of 3.2 eV [1, 2]. HfC is potentially well suited as a cathode material due to its high melting point and emissivity in conjunction with a relatively low work function.

**Table 5.1:** CAV-H vehicle parameters from Phillips [108].

| Vehicle Parameters           | Value |
|------------------------------|-------|
| $L/D_{max}$                  | 3.50  |
| $a_{max}$ , m/s <sup>2</sup> | 1.80  |
| $m$ , kg                     | 907   |
| $A_{ref}$ , m <sup>2</sup>   | 0.48  |
| $R_n$ , m                    | 0.03  |

## 5.2 A Hypersonic Trajectory Optimization Framework

Hypersonic trajectory optimization is handled via the in-house solver TOPHAT [116]. For a given objective (e.g. maximizing vehicle range), TOPHAT determines feasible hypersonic trajectories by solving a constrained nonlinear optimization problem. The vector of parameters over which the optimization problem is solved is defined as  $\mathbf{X} = \{h_0, v_0, \gamma_0, \theta_j\}$ , where  $h_0$  is the initial altitude,  $v_0$  is the initial velocity, and  $\gamma_0$  is the initial flight path angle. In addition,  $\theta_j$  is a vector of bank angle values where  $j$  ranges from 1 to 5. Thus,  $\theta_j$  defines a fourth-order polynomial which controls the bank angle control profile over the trajectory, ensuring that the control profile remains smooth. Table 5.2 shows the trajectory constraints considered for the CAV-H vehicle. Values of the lower and upper bounds are selected to provide a broad parameter space for exploration by TOPHAT.

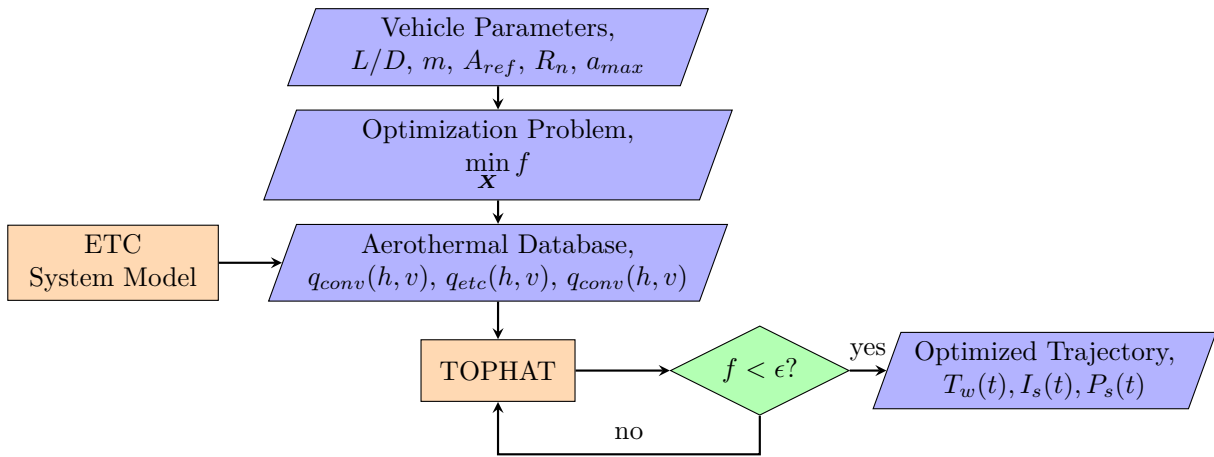
The 1D ETC model from Chapter 3 is coupled to TOPHAT to create a hypersonic trajectory optimization framework that includes ETC physics. A program flow chart in Fig. 5.2 illustrates this coupling. ETC heat flux data (i.e.  $q_{i,f}, q_{e,f}, q_{etc}$ ) is computed by the 1D model as a function of altitude,  $h$ , and velocity,  $v$ , alongside convective heat fluxes computed via the Brandis and Johnston correlation (Eq. 4.1). Heat flux data is then assembled into an aerothermal database and passed into TOPHAT, in addition to the vehicle parameters of the CAV-H, and a user-defined description of the constrained optimization problem. Using this information, TOPHAT solves the constrained optimization problem for the CAV-H to a specified tolerance. This results in an optimal trajectory of the CAV-H, where in addition to the trajectory itself, TOPHAT computes values of

**Table 5.2:** Trajectory constraints for the CAV-H vehicle within TOPHAT.

| Optimization<br>Constraints | Lower<br>Bound | Upper<br>Bound |
|-----------------------------|----------------|----------------|
| $h_0$ , km                  | 20             | 70             |
| $v_0$ , km/s                | 0.5            | 6.0            |
| $\gamma_0$ , deg.           | -0.5           | 0.5            |
| $\theta$ , deg.             | -180           | 180            |

the wall temperature,  $T_w$ , system current,  $I_s$ , and system power,  $P_s$ , as a function of time. Wall temperatures are evaluated via TOPHAT's radiative equilibrium solver.

The following sections (Sections 5.3 and 5.4) provide details of two specific optimization problems and corresponding constraints that are solved via this framework. The optimization problems are chosen to both reflect realistic performance goals of boost-glide vehicles as well as highlight the potential usefulness of an ETC system.



**Figure 5.2:** Flow chart of the coupled ETC + TOPHAT framework.

### 5.3 Optimization Problem 1: Maximizing Vehicle Range

The first objective is to maximize the downrange reach of the CAV-H vehicle,  $D_f$ . For the present objective we use a constant angle of attack of 11 degrees corresponding to the the maximum lift-to-drag ratio for the CAV-H vehicle of  $L/D_{max} = 3.5$ . Expanding the analysis to account for time-varying aerodynamics is beyond the scope of the present work, but should be considered in future investigations. We describe the objective through an optimization problem in Eq. 5.1. In addition to the trajectory constraints (Table 5.2), we enforce the maximum aerodynamic loading constraint of the CAV-H,  $a_{max} = 1.8g$ , as well as a specified maximum allowable wall temperature constraint,  $T_w^{max} = 4173$  K equal to the melting point of HfC, at all points along the trajectory.

$$\begin{aligned}
 & \underset{\mathbf{X}}{\text{minimize}} && -D_f^2 \\
 & \text{subject to} && T_{w,max} - T_w(t) \geq 0 \quad \forall t, \\
 & && a_{max} - a(t) \geq 0 \quad \forall t
 \end{aligned} \tag{5.1}$$

We note that TOPHAT employs a modified particle swarm optimization routine to determine optimal bank-angle control profiles, and is paired with a penalty function approach to handle path constraints [75, 66, 110]. This approach incorporates constraints from Eq. 5.1 into the objective function and modifies the true optimization problem being solved. Equation 5.2 shows the modified optimization problem for the maximum range study, where a vector of temperature and acceleration constraints are defined as  $\mathbf{h}(\mathbf{X}(t)) = [T_{w,max} - T_w(t), a_{max} - a(t)]^T$ , respectively, and  $C_s$  is a constant. The penalty weight,  $C_s$ , is chosen such that the product  $C_s |h_s(\mathbf{X})|$  is large compared to  $D_f^2$  to discourage constraint violation. Further details on the use of penalty functions with particle swarm methods can be found in Pontani and Conway [110].

$$\underset{\mathbf{X}}{\text{minimize}} \quad -D_f^2 + \sum_{s=1}^2 C_s |h_s(\mathbf{X})| \tag{5.2}$$



As wall temperatures are a strong driver of thermionic current, we consider three specified values of the maximum wall temperature constraint,  $T_w^{max} \in \{2500, 2750, 3000\}$  K. For each value of  $T_w^{max}$  we solve the optimization problem twice, where the CAV-H experiences (1) radiative cooling only and (2) radiative cooling and thermionic cooling from the ETC system. Thus, differences in the trajectory between the two solutions can be attributed to the ETC system.

We present a summary of values of  $D_f$  for all cases of  $T_w^{max}$  in Table 5.3. For the  $T_w^{max} = 3000$  K case, ETC increases the downrange reach of the CAV-H from 3345 km to 10100 km, an improvement of approximately 202%. Details of the optimized solutions and the role of ETC in extending vehicle range are presented in the following sections. Section 5.3.1 describes the optimized vehicle trajectories, Section 5.3.2 examines profiles of wall temperature and thermionic cooling performance, and Section 5.3.3 discusses the power requirements of the ETC system.

**Table 5.3:** Summarized values of downrange reach and flight time for maximizing vehicle range.

| Case            | Max. Allowable Wall Temperature<br>$T_w^{max}$ , K | Downrange Reach<br>$D_f$ , km | Flight Time<br>$t$ , s. |
|-----------------|----------------------------------------------------|-------------------------------|-------------------------|
| Radiation       | 2500                                               | 1520                          | 1110                    |
|                 | 2750                                               | 2300                          | 1370                    |
|                 | 3000                                               | 3350                          | 1620                    |
| Radiation & ETC | 2500                                               | 1530                          | 1120                    |
|                 | 2750                                               | 2360                          | 1390                    |
|                 | 3000                                               | 10100                         | 3010                    |

### 5.3.1 Vehicle Trajectories

Figure 5.3(a) shows the optimized trajectories of the CAV-H in altitude-velocity space. Values of the imposed constraints on the maximum initial velocity (6 km/s) and altitude (70 km) are denoted by hatched lines. It is observed that a phugoid motion (small amplitude oscillations in altitude) exists for the investigated trajectories as a consequence of the high value of  $L/D$ . The oscillations can be mitigated through the use of control surfaces and would have implications

regarding the thermal performance of the CAV-H (e.g. higher integrated heat loads). However, this is beyond the scope of the present work. As acceleration is constrained to be under 1.8 g's, the resulting trajectories are still feasible for the vehicle. In addition to Fig. 5.3(a), Table 5.4 shows the optimal values for the initial conditions of all cases, including the initial altitude, velocity, and flight path angle.

**Table 5.4:** Optimal values for initial altitude, velocity, and flight path angle for maximizing vehicle range.

| Case            | Max. Allowable Wall Temperature<br>$T_w^{max}$ , K | Initial Altitude<br>$h_0$ , km | Initial Velocity<br>$v_0$ , km/s | Initial Flight Path Angle<br>$\gamma_0$ , deg. |
|-----------------|----------------------------------------------------|--------------------------------|----------------------------------|------------------------------------------------|
| Radiation       | 2500                                               | 34.1                           | 2.7                              | 0.49                                           |
|                 | 2750                                               | 36.8                           | 3.3                              | 0.23                                           |
|                 | 3000                                               | 39.7                           | 4.0                              | 0.50                                           |
| Radiation & ETC | 2500                                               | 34.1                           | 2.7                              | 0.47                                           |
|                 | 2750                                               | 36.9                           | 3.3                              | 0.19                                           |
|                 | 3000                                               | 67.6                           | 6.0                              | 0.49                                           |

Table 5.4 shows that, for the  $T_w^{max} = 2500$  K and 2750 K cases, the presence of the ETC system does not significantly change the initial condition selected by the optimizer. However, for the  $T_w^{max} = 3000$  K case, the ETC system results in a considerably different initial condition. Specifically, the initial velocity and altitude has increased from approximately 4 km/s and 40 km with radiative cooling only, to 6 km/s and 68 km with ETC. Importantly, the optimized initial velocity is equal to the imposed constraint of 6 km/s, indicating that this constraint is active in the solution. Visually, Fig. 5.3(a) illustrates this with the launch condition for the  $T_w^{max} = 3000$  K case starting on top of the hatched line at 6 km/s. We note that the change in initial condition for this case after ETC is introduced shows that the ETC system has expanded the feasible insertion envelope of the CAV-H. In fact, if a larger maximum initial velocity were permitted, the optimizer would select an even higher initial velocity.

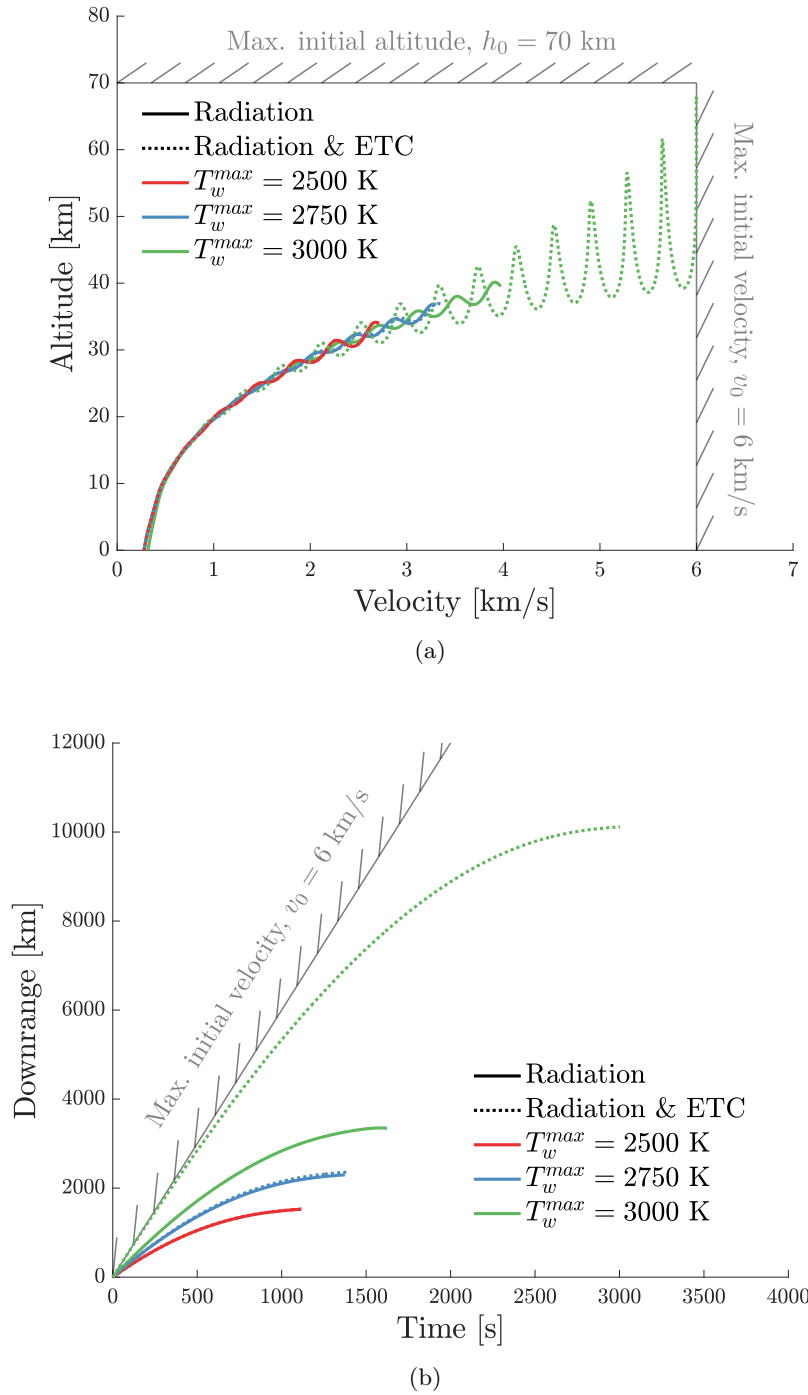
Profiles of the downrange distance of the CAV-H as a function of time are shown in Fig. 5.3(b). The hatched line denotes the downrange distance corresponding to a constant velocity of 6 km/s, representing the maximum achievable range imposed by the initial velocity constraint. All

trajectories diverge from this line due to aerodynamic drag, which decelerates the vehicle over time.

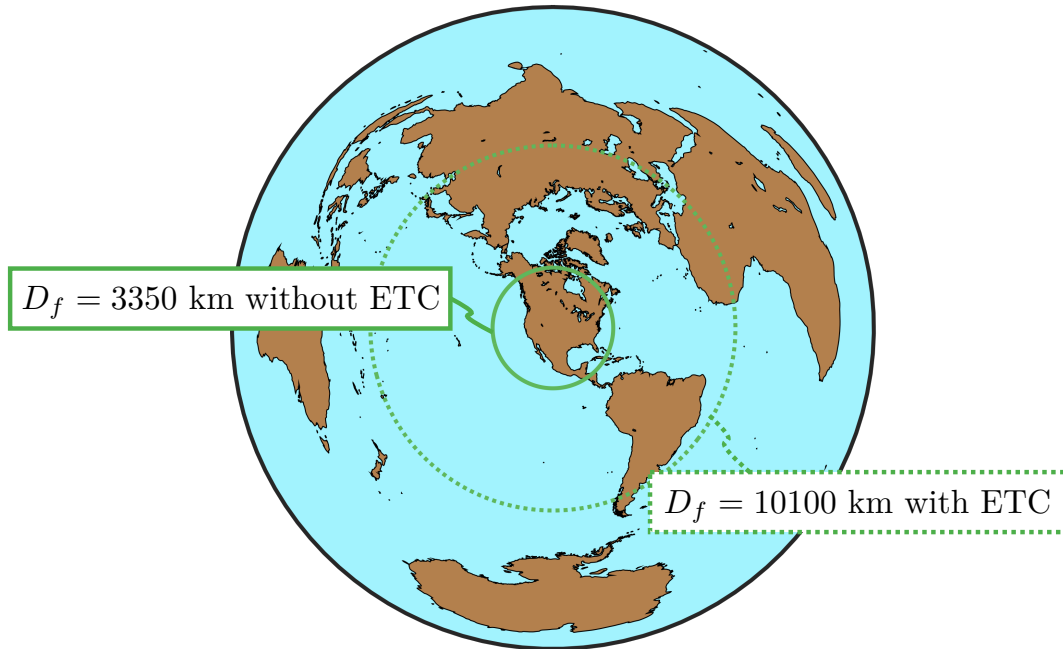
When considering radiative cooling only, Fig. 5.3(b) shows that the downrange reach of the CAV-H increases with increasing  $T_w^{max}$ . This trend is driven by the higher initial velocities observed with larger values of  $T_w^{max}$  and visualized by trajectories that lie progressively closer to the hatched line. Higher initial velocities impart increased kinetic energy into the glide vehicle that allows for farther flights. This comes at the cost of increased convective heating, but can be permitted at larger values of  $T_w^{max}$ .

The  $T_w^{max} = 2500$  K and 2750 K cases show no significant differences with the inclusion of ETC. However, the  $T_w^{max} = 3000$  K case exhibits a substantial increase in downrange reach, exceeding 10,000 km, corresponding to a 202% increase relative to the radiation only case. Again, this improvement is primarily driven by the high initial velocity of 6 km/s enabled by ETC. Visually, Fig. 5.3(b) shows this high initial velocity where the downrange distance initially follows the hatched line before diverging as aerodynamic drag reduces the CAV-H velocity.

Figure 5.4 illustrates the influence the ETC system has on vehicle performance by drawing the expected range of the CAV-H with and without ETC for the  $T_w^{max} = 3000$  K case. If launched from Boulder, CO, the CAV-H with radiative cooling is limited in range to most of North America. However, with ETC, the CAV-H's range is extended to 10118 km, which allows it to reach much of northern Europe, South America, and parts of Africa.



**Figure 5.3:** Optimized trajectories for maximizing vehicle range showing (a) profiles of altitude and velocity and (b) downrange reach as a function of flight time. Colors correspond to values of the maximum allowable wall temperature:  $T_w^{max} = 2500$  K (red),  $T_w^{max} = 2750$  K (blue),  $T_w^{max} = 3000$  K (green). Linestyles denote the presence of radiative cooling only (solid) or radiative and thermionic cooling (dotted).



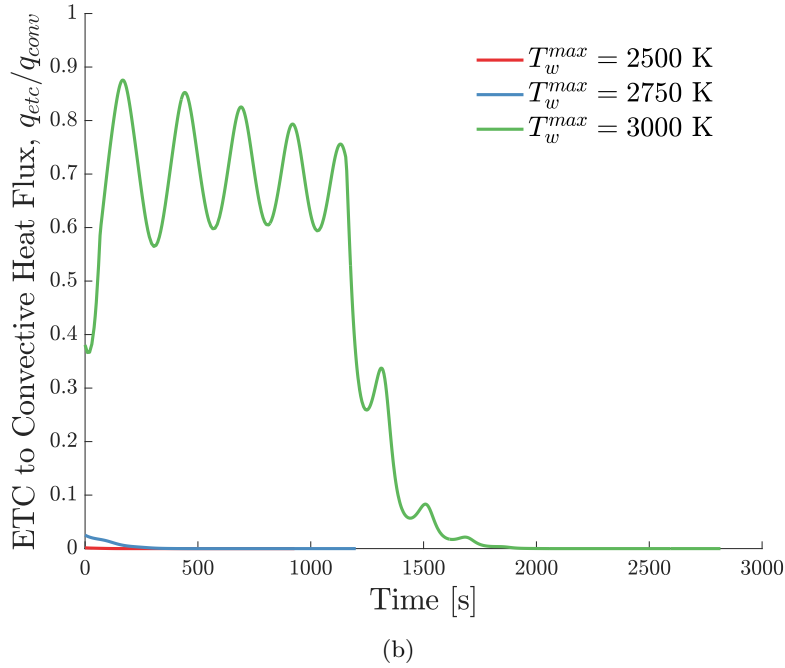
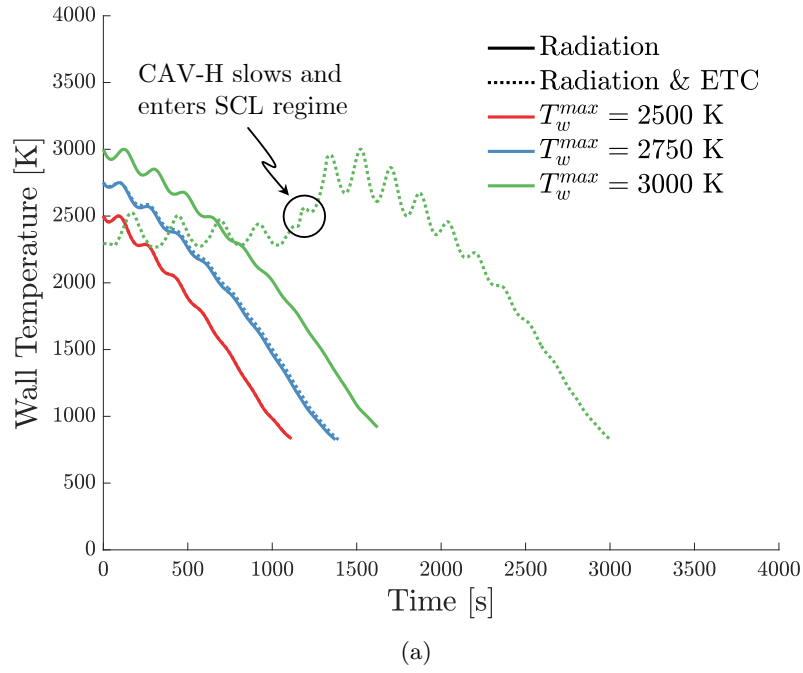
**Figure 5.4:** Downrange reach of the CAV-H for the  $T_w^{max} = 3000 \text{ K}$  case centered at Boulder CO ( $40.0190^\circ \text{ N}$ ,  $105.2747^\circ \text{ W}$ ). Linestyles denote (solid) radiative cooling only and (dotted) radiative cooling and ETC.

### 5.3.2 Wall Temperatures and Heat Fluxes

Wall temperatures and thermal fluxes experienced by the CAV-H are now considered. Figure 5.5(a) shows profiles of the steady-state wall temperature as a function of time. From the values of downrange reach in the previous section (Fig. 5.3(b)), higher values of  $T_w^{max}$  were shown to correspond to higher initial velocities. The present results expand on this behavior. For cases with radiative cooling only, the optimizer selects initial conditions such that the wall temperature begins at its maximum allowable value (i.e., 2500 K, 2750 K, and 3000 K). Because the CAV-H is an unpowered glide vehicle, maximizing range is correlated to the initial kinetic energy. Higher allowable wall temperatures permit greater convective heating at the start of the trajectory, enabling higher initial velocities.

When ETC is introduced, the  $T_w^{max} = 2500$  K and 2750 K cases show no significant change in wall temperature. This is perhaps confusing as the wall temperatures are certainly high enough to support thermionic emission. Recall from Section 4.2.2, ETC is expected to be ineffective at velocities below approximately 3-4 km/s due to SCL operation. The  $T_w^{max} = 2500$  and 2750 K cases, which showed no change when ETC was introduced, have initial velocities of 2.7 and 3.3 km/s respectively (Table 5.4) and are too low for ETC to be effective.

In contrast, the  $T_w^{max} = 3000$  K case exhibits cooling of over 700 K when ETC is included with cooling exceeding 700 K. The high allowable wall temperature in this case enables the vehicle to begin in a velocity regime where ETC is active. The optimizer identifies the added thermionic cooling and selects an even more aggressive initial velocity of 6 km/s. Following  $t = 0$ , the wall temperature remains relatively constant at approximately 2400 K before increasing sharply at  $t \approx 1250$  s. At this point, aerodynamic deceleration causes the vehicle to enter an SCL regime, leading to a rapid reduction in thermionic cooling and a corresponding rise in wall temperature. We note that the rise in wall temperature is an observed characteristic of operating ETC systems during the entire phase of hypersonic flight. Notably, the highest wall temperatures occur after entering the SCL regime, as evidenced by the wall temperature reaching its maximum value shortly



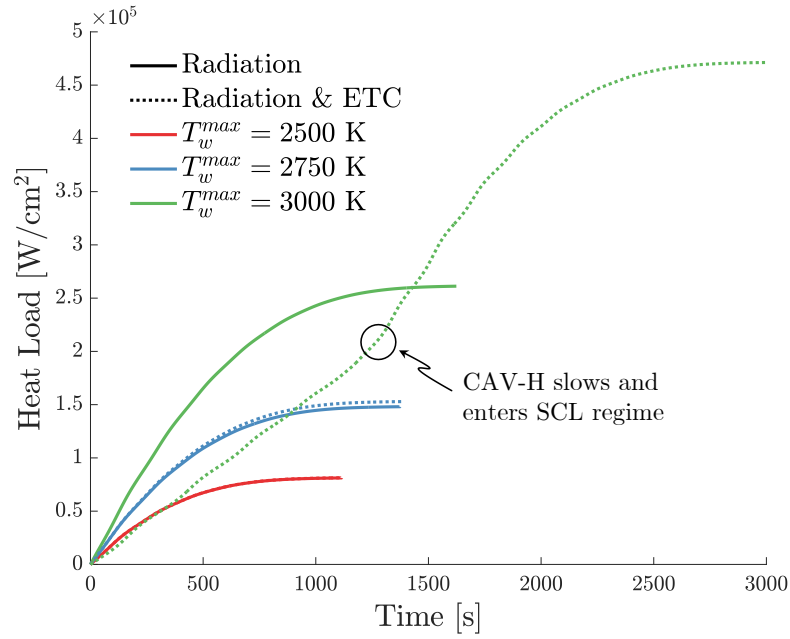
**Figure 5.5:** Optimized trajectories for maximizing vehicle range showing (a) wall temperature and (b) ratio of thermionic cooling to convective heat flux. Colors correspond to specified values of the maximum allowable wall temperature:  $T_w^{max} = 2500$  K (red),  $T_w^{max} = 2750$  K (blue),  $T_w^{max} = 3000$  K (green). Linestyles denote the presence of radiative cooling only (solid) or radiative and thermionic cooling (dotted).

after  $t \approx 1250$  s.

The effectiveness of ETC is further illustrated in Fig. 5.5(b), which shows the ratio of thermionic to convective heat flux,  $q_{etc}/q_{conv}$ . Consistent with previous observations, the  $T_w^{max} = 2500$  K and 2750 K cases exhibit values near zero for most of the trajectory due to the SCL mode. In contrast, the  $T_w^{max} = 3000$  K case shows that ETC rejects more than 80% of the initial convective heat load at the initial phase of flight before transitioning into the SCL regime.

Finally, Fig. 5.6 presents the integrated heat load,  $Q = \int_0^t (q_{conv} + q_{etc}) dt$ , for each case. For radiative cooling only (i.e.,  $q_{etc} = 0$ ), all cases exhibit similar behavior: the heat load increases rapidly during the initial phase of flight, when velocity and convective heating are highest, and then levels off as the vehicle decelerates. For the  $T_w^{max} = 3000$  K case, the inclusion of ETC modifies this behavior. The heat load grows more gradually during the early phase of flight. This behavior reflects the added cooling effect thermionic emission has on the vehicle surface. However, the longer flight duration enabled by ETC results in a final integrated heat load that is nearly double that of the case with only radiation. This may impose additional constraints on vehicle design.





**Figure 5.6:** Optimized trajectories for maximizing vehicle range showing the integrated heat load,  $Q$ , into the vehicle surface. Colors correspond to specified values of the maximum allowable wall temperature:  $T_w^{\text{max}} = 2500 \text{ K}$  (red),  $T_w^{\text{max}} = 2750 \text{ K}$  (blue),  $T_w^{\text{max}} = 3000 \text{ K}$  (green). Linestyles denote the presence of radiative cooling only (solid) or radiative and thermionic cooling (dotted).

### 5.3.3 Power Requirements

Power and energy requirements of the ETC system for the optimization problem of maximizing vehicle range are now examined. The instantaneous power supplied by the internal circuit is described in Eq. 5.3, where  $V_s$  is the internal voltage and  $I_s$  is the total current in the circuit. Additionally, we define the total energy consumed by the ETC system,  $E_s$ , in Eq. 5.4 as the time integral of the instantaneous power.

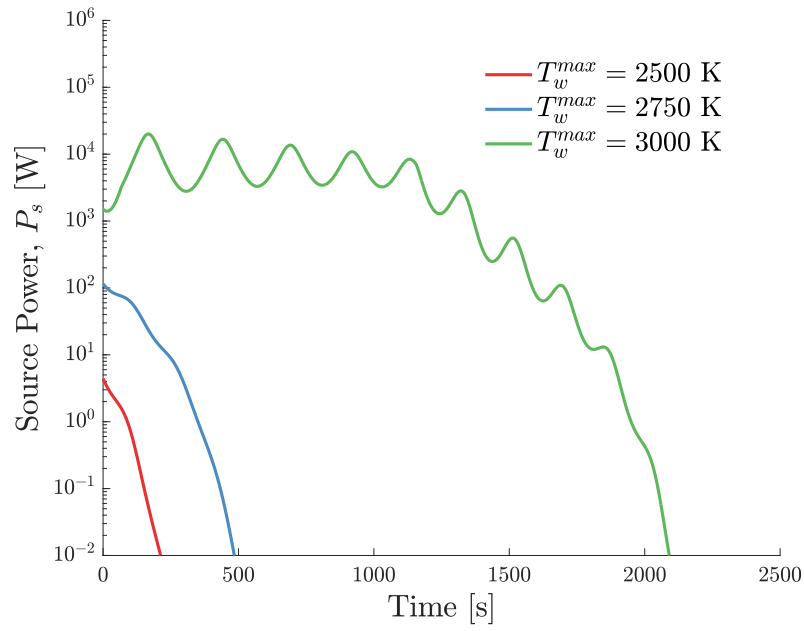
$$P_s = I_s V_s \quad (5.3)$$

$$E_s(t) = \int_0^t P_s dt \quad (5.4)$$

Figure 5.7 shows profiles of the instantaneous power,  $P_s$ , for the maximizing range optimization study with curves for each value of the maximum allowable temperature  $T_w^{max}$ . The power draw is largest for the  $T_w^{max} = 3000$  K case, where it initially oscillates between  $10^3 - 10^4$  W due to the phugoid motion of the CAV-H. At  $t \approx 1250$  s, the CAV-H enters a SCL regime, causing ETC power to decay as thermionic emission becomes restricted. The remaining cases ( $T_w^{max} = 2500$  and 2750 K) show a low power draw of below 100 W due to the vehicle continually operating within a SCL regime where thermionic cooling is inconsequential to the performance of the vehicle.

Table 5.5 summarizes select electrical characteristics of the ETC system for all values of  $T_w^{max}$ , including peak power draw, total energy, time active, and average power. The time active is defined as the interval during which the integrated energy  $E_s(t)$ , rises from 1% to 99% of its final value. The average power is computed as the total energy divided by the time active.

We focus on the  $T_w^{max} = 3000$  K case, where ETC is effective in increasing vehicle range. From Table 5.5, the peak power consumption is 20 kW, with an average power draw of 6.6 kW over a duration of 1330 s. This is a large power requirement for a hypersonic vehicle to provide, however it is not infeasible. As a reference, the 25 kg battery system of a 2025 Formula 1 car is capable of storing 4-6 kWh of energy and produces up to 120 kW of power over a lap [134]. These characteristics make it more than sufficient for powering the ETC system in this case. However,



**Figure 5.7:** The required power of the CAV-H ETC system,  $P_s$ , for the optimization problem of maximizing the vehicle range. Colors correspond to specified values of the maximum allowable wall temperature:  $T_w^{max} = 2500$  K (red),  $T_w^{max} = 2750$  K (blue),  $T_w^{max} = 3000$  K (green).

given electrical constraints of a vehicle, it may be suggested to activate the ETC system for only short time periods. Continued discussion on voltage source sizing is found in Section 5.5.

**Table 5.5:** Electrical characteristics of the ETC system for the optimization studies of maximizing vehicle range.

| Max. Allowable Wall Temperature<br>$T_w^{max}$ , K | Peak Power<br>kW     | Total Energy<br>$E_s$ , kWh | Time Active<br>s | Average Power<br>kW  |
|----------------------------------------------------|----------------------|-----------------------------|------------------|----------------------|
| 2500                                               | $4.4 \times 10^{-3}$ | $6.6 \times 10^{-5}$        | 155              | $1.5 \times 10^{-3}$ |
| 2750                                               | $1.1 \times 10^{-1}$ | $3.6 \times 10^{-3}$        | 307              | $4.2 \times 10^{-3}$ |
| 3000                                               | 20.                  | 2.4                         | 1330             | 6.6                  |

#### 5.4 Optimization Problem 2: Minimizing Vehicle Heat Load

The second objective is to minimize the integrated heat load,  $Q$ , of the CAV-H vehicle while reaching a specified downrange target,  $D_{f,target}$ , set at 4000 km. Values of  $Q$  can be related to the weight and thickness of the vehicle TPS, making the heat load a favorable objective to minimize [96]. We show the governing optimization problem in Eq. 5.5. Vehicle and ETC system parameters along with trajectory and load factor constraints are identical to those in the previous section. For this objective, a constant maximum wall temperature of  $T_w^{max} = 4173$  K is specified, corresponding to the melting point of HfC.

$$\begin{aligned}
& \underset{\mathbf{X}}{\text{minimize}} && Q \\
& \text{subject to} && D_{f,target} - D_f = 0, \\
& && T_{w,max} - T_w(t) \geq 0 \quad \forall t, \\
& && a_{max} - a(t) \geq 0 \quad \forall t
\end{aligned} \tag{5.5}$$

Similar to Section 5.3, penalty functions are used to incorporate path constraints into the objective function. Equation 5.6 shows the modified optimization problem for the minimum heat

load study. Penalty weights  $C_d$  and  $C_s$  are constants chosen such that their respective products in Eq. 5.6 are large compared to  $Q$  to discourage constraint violation.

$$\underset{\mathbf{X}}{\text{minimize}} \quad Q + C_d (D_{f,target} - D_f)^2 + \sum_{s=1}^2 C_s |h_s(\mathbf{X})| \quad (5.6)$$

As aerodynamic properties, such as the lift-to-drag ratio, are strong drivers of the vehicle integrated heat load, we consider three specified values of  $L/D \in \{1.5, 2.5, 3.5\}$ . Again, we solve Eq. 5.5 with (1) radiative cooling and (2) with radiative and thermionic cooling. Differences in the integrated heat load can then be attributed to the ETC system. We note that the value of  $D_{f,target}$  is chosen such that all cases are capable of reaching the target, regardless of whether ETC is active or not.

We present a summary of values of  $Q$  for all cases of  $L/D$  in Table 5.6. For all cases, ETC decreases the integrated heat load of the CAV-H. Details of the optimized solutions and the role of ETC in reducing heat loads are presented in the following sections. Section 5.4.1 describes the optimized vehicle trajectories, Section 5.4.2 examines profiles of wall temperature and thermionic cooling performance, and Section 5.4.3 discusses the power requirements of the ETC system.

**Table 5.6:** Summarized values of integrated heat load and flight time for minimizing vehicle integrated heat load.

| Case            | Lift to Drag Ratio<br>$L/D$ | Integrated Heat Load<br>$Q$ , J/cm <sup>2</sup> | Flight Time<br>$t$ , s |
|-----------------|-----------------------------|-------------------------------------------------|------------------------|
| Radiation       | 1.5                         | $5.89 \times 10^5$                              | 1190                   |
|                 | 2.5                         | $4.06 \times 10^5$                              | 1510                   |
|                 | 3.5                         | $3.10 \times 10^5$                              | 1810                   |
| Radiation & ETC | 1.5                         | $2.40 \times 10^5$                              | 1170                   |
|                 | 2.5                         | $2.67 \times 10^5$                              | 1520                   |
|                 | 3.5                         | $2.75 \times 10^5$                              | 1820                   |

### 5.4.1 Vehicle Trajectories

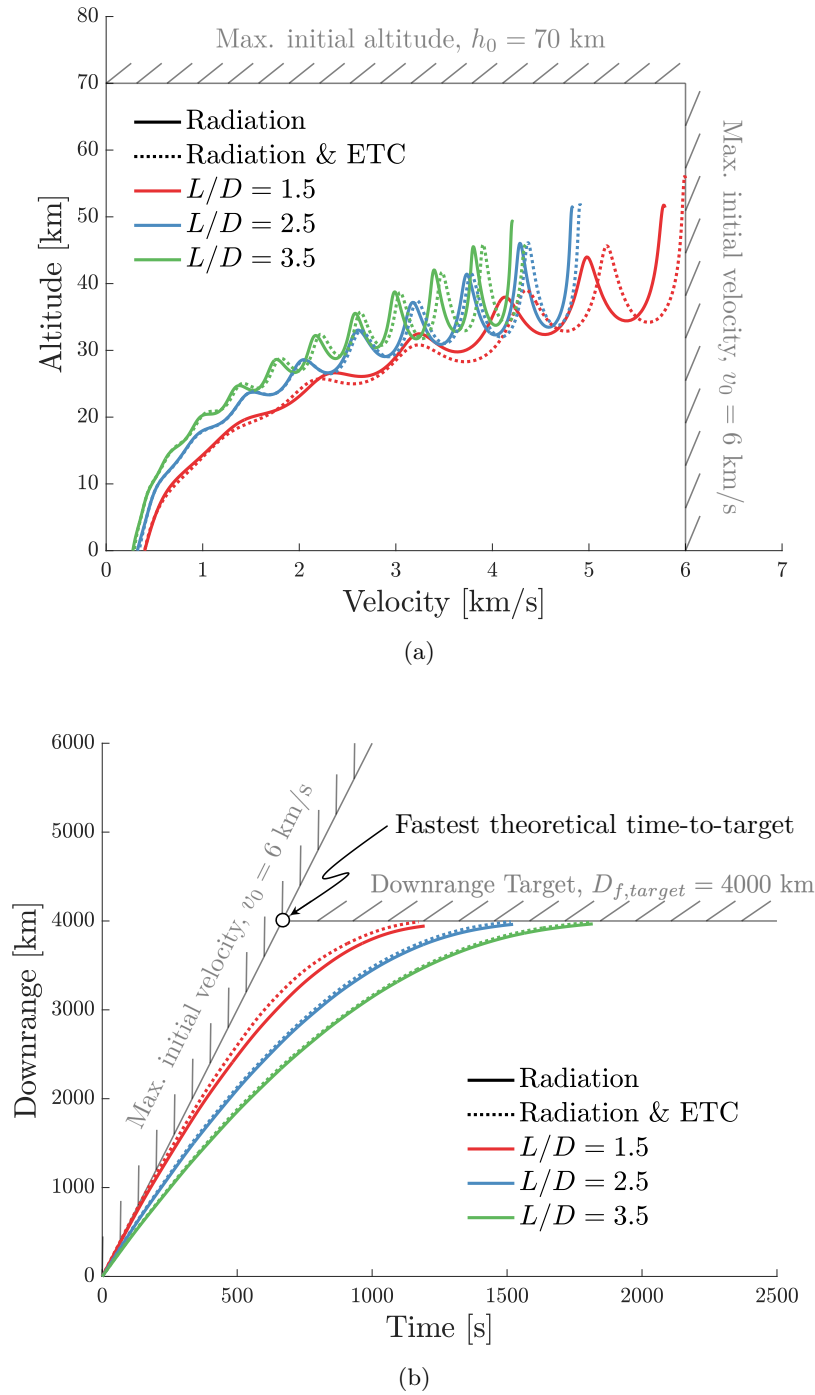
Figure 5.8(a) shows profiles of the optimized trajectories of the CAV-H in altitude-velocity space. Values of the imposed constraints on the maximum initial velocity (6 km/s) and altitude (70 km) are denoted by hatched lines. Similar to Section 5.3, all cases exhibit phugoid motion, with oscillation frequency increasing with  $L/D$ . Optimal values for the initial conditions of all cases, namely the initial altitude, velocity, and flight path angle, are summarized in Table 5.7.

**Table 5.7:** Optimal values for initial altitude, velocity, and flight path angle for minimizing the integrated heat load while reaching a specified target.

| Case            | Lift to Drag Ratio<br>$L/D$ | Initial Altitude<br>$h_0$ , km | Initial Velocity<br>$v_0$ , km/s | Initial Flight Path Angle<br>$\gamma_0$ , deg. |
|-----------------|-----------------------------|--------------------------------|----------------------------------|------------------------------------------------|
| Radiation       | 1.5                         | 51.4                           | 5.8                              | 0.48                                           |
|                 | 2.5                         | 51.3                           | 4.8                              | 0.47                                           |
|                 | 3.5                         | 49.3                           | 4.2                              | 0.46                                           |
| Radiation & ETC | 1.5                         | 55.8                           | 6.0                              | 0.48                                           |
|                 | 2.5                         | 51.8                           | 4.9                              | 0.46                                           |
|                 | 3.5                         | 45.8                           | 4.3                              | 0.11                                           |

Table 5.7 shows that, with radiative cooling only, the optimizer selects higher initial velocities for lower  $L/D$  configurations. Visually, trajectory profiles in Fig. 5.8(a) illustrate this trend with initial conditions moving to the right for lower values of  $L/D$ . This trend is expected, as reduced lift requires a higher initial velocity to counter the increased ballistic aerodynamics and reach the prescribed downrange target. When ETC is introduced, the altitude-velocity profiles closely follow those of the corresponding radiative-only cases. Small increases in initial velocities exist, but the overall vehicle trajectories remain largely unchanged, likely a consequence of enforcing a downrange target for the CAV-H. This suggests that, for the optimization problem of minimizing the integrated heat load, the ETC system does not appreciably expand the launch envelope.

Shown in Fig 5.8(b) are profiles of the downrange reach of the CAV-H as a function of time. Similar to the Fig. 5.3(b), the sloped hatched line denotes the maximum achievable range imposed by the initial velocity constraint of 6 km/s. A second horizontal hatched line denotes the



**Figure 5.8:** Optimized trajectories for minimizing vehicle integrated heat load showing (a) profiles of altitude and velocity and (b) downrange reach as a function of flight time. Colors correspond to specified values of the vehicle lift-to-drag ratio:  $L/D = 1.5$  (red),  $L/D = 2.5$  (blue),  $L/D = 3.5$  (green). Linestyles denote the presence of radiative cooling only (solid) or radiative and thermionic cooling (dotted).

target,  $D_{f,target}$ , at 4000 km. Thus, the intersection of the two hatched lines represents the fastest theoretical time-to-target of the CAV-H.

All cases satisfy the mission objective by reaching the target distance. Lower  $L/D$  configurations, which are seen to have higher higher initial velocities, reach the target in a shorter time. The inclusion of ETC results in a slight reduction in time-to-target. However, differences in time-to-target relative to the cases with radiative cooling only are small.

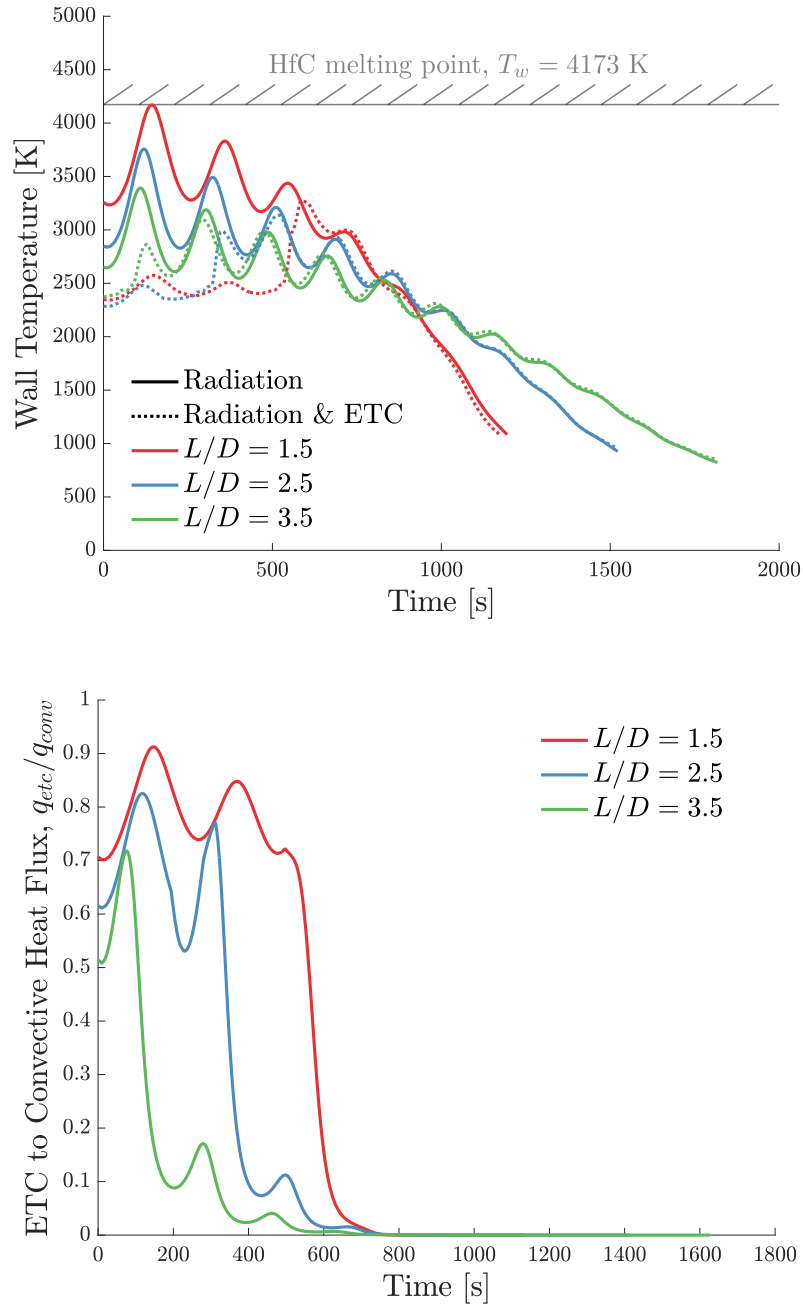
#### 5.4.2 Wall Temperatures and Heat Fluxes

Wall temperatures and thermal fluxes experiences by the CAV-H in the minimizing heat load problem are now considered. Figure 5.9(a) shows profiles of wall temperature for all values of the lift-to-drag ratio,  $L/D$ . For cases with radiative cooling only, the role of  $L/D$  is to trade flight time for peak wall temperatures. Higher  $L/D$  values reduce peak wall temperatures by requiring lower initial velocities to reach the target, thereby reducing convective heating in the initial phase of flight. Conversely, lower  $L/D$  values allow the CAV-H to reach the target in less time, but result in higher wall temperatures due to their increased initial velocity. For the  $L/D = 1.5$  case, wall temperatures approach the melting point of HfC, indicating that further reductions in  $L/D$  are likely infeasible for the given temperature limit.

When ETC is included, Fig. 5.9(a) shows that surface temperatures are substantially reduced during the initial phase of flight. The largest reduction occurs for the  $L/D = 1.5$  case, where initial temperatures decrease by over 800 K. Between  $t = 0$  s and  $t \approx 600$  s, wall temperatures increase and converge toward the solution without ETC. This behavior corresponds to the CAV-H decelerating due to aerodynamic drag and entering an SCL regime, where thermionic cooling becomes negligible.

Importantly, the time required to fully transition into SCL operation increases with  $L/D$ . For the  $L/D = 1.5$  case this occurs over  $\sim 20$  s, while for the  $L/D = 3.5$  case this takes nearly  $\sim 250$  s. This is because higher  $L/D$  trajectories have lower initial velocities and longer flight durations, both of which expand the transition time. The change in temperature a vehicle experiences during this transition may be important for vehicle design, as abrupt thermal excursions can potentially



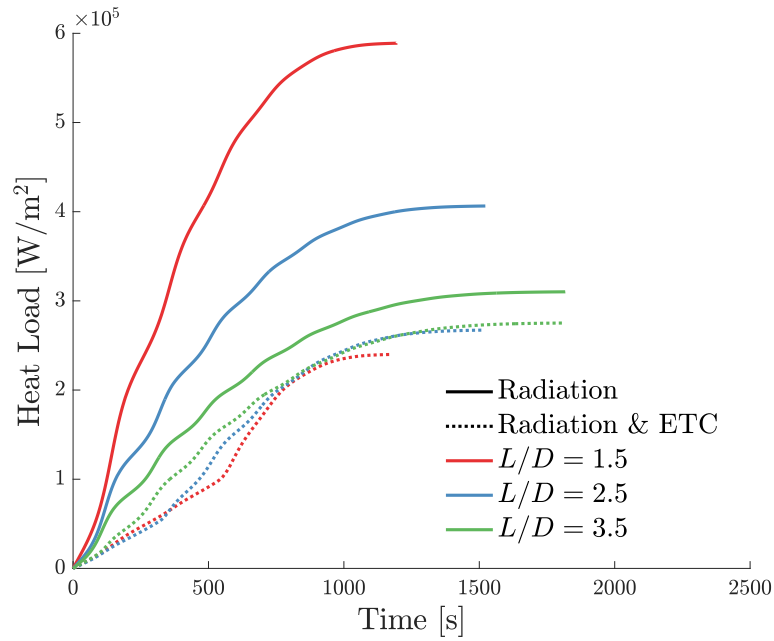


**Figure 5.9:** Optimized trajectories for minimizing the vehicle integrated heat load showing (a) wall temperature and (b) ratio of thermionic cooling to convective heat flux. Colors correspond to specified values of the vehicle lift-to-drag ratio:  $L/D = 1.5$  (red),  $L/D = 2.5$  (blue),  $L/D = 3.5$  (green). Linestyles denote the presence of radiative cooling only (solid) or radiative and thermionic cooling (dotted).

impact material integrity and internal component performance.

The ratio of thermionic cooling to convective heating,  $q_{etc}/q_{conv}$ , is shown in Fig. 5.9(b). Lower  $L/D$  configurations correspond to higher fractions of convective heating being rejected by ETC, with the  $L/D = 1.5$  case rejecting up to 90% of  $q_{conv}$ . Notably, this case experiences the highest convective heating rates (due to its higher initial velocity) while also achieving the greatest relative cooling. All cases show  $q_{etc}/q_{conv}$  decaying to zero by approximately  $t = 800$  s as the vehicle decelerates and enters the SCL regime. Consistent with Fig. 5.9(a), this decay occurs over longer time scales for higher  $L/D$  trajectories.

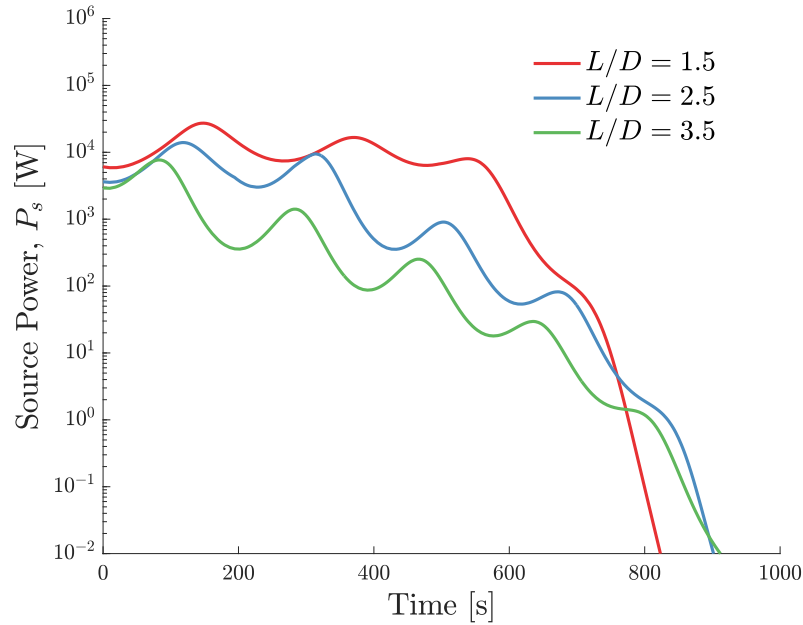
Finally, Fig. 5.10 shows the integrated heat load,  $Q$  for each case. For radiative cooling only, it is clear that the highest value of  $L/D = 3.5$  achieves the lowest value of  $Q$ . In contrast, when ETC is enabled, it is the lowest value of  $L/D = 1.5$  that achieves the minimum heat load. Lower  $L/D$  trajectories are more ballistic, resulting in shorter flight times but increased peak convective heating. However, the ETC system works to mitigate the convective heating during the initial phase of flight (where convective heating is the largest), allowing the vehicle to benefit from the shorter flight time. As a result, the ETC system removes the necessity of relying on high  $L/D$  to reduce heat loads, providing a new approach towards designing an effective hypersonic vehicle.



**Figure 5.10:** Optimized trajectories for minimizing the vehicle integrated heat load showing the integrated heat load,  $Q$ , into the vehicle surface. Colors correspond to specified values of the vehicle lift-to-drag ratio:  $L/D = 1.5$  (red),  $L/D = 2.5$  (blue),  $L/D = 3.5$  (green). Linestyles denote the presence of radiative cooling only (solid) or radiative and thermionic cooling (dotted)

### 5.4.3 Power Requirements

Power and energy requirements of the ETC system in the CAV-H are examined in the context of the minimum heat load optimization problem. Figure 5.11 shows profiles of the instantaneous power,  $P_s$ , with curves for each value of the lift-to-drag ratio. All cases are characterized by fluctuations in power draw, owing to the oscillatory nature of the trajectory. The high velocity  $L/D = 1.5$  case shows a moderately constant power draw over the first 600 seconds before  $P_s$  decays rapidly. As  $L/D$  increases, the decay of  $P_s$  begins to broaden in time. Akin to profiles of the ratio of ETC cooling to convective heating (Fig. 5.9(b)), this broadening is due to the decreasing speed at which the CAV-H enters a SCL regime. From an electrical standpoint, a gentle decay in  $P_s$  may be easier to maintain as the voltage source discharges.



**Figure 5.11:** The required power of the CAV-H ETC system,  $P_s$ , for the optimization problem of minimizing integrated heat load. Colors correspond to specified values of the lift-to-drag ratio:  $L/D = 1.5$  (red),  $L/D = 2.5$  (blue),  $L/D = 3.5$  (green).

Table 5.8 summarizes the electrical characteristics of the ETC system, including peak power, total energy consumption, activation time, and average power. Peak power, average power, and total energy all scale inversely with  $L/D$ . The average power draw ranges from approximately

1.7 kW to 11 kW. Similar to the maximum-range study (Table 5.5), these power requirements are substantial but not infeasible to provide on a hypersonic vehicle. However, depending on system-level electrical constraints, ETC operation may need to be limited to short activation periods.

**Table 5.8:** Electrical characteristics of the ETC system for the optimization studies minimizing vehicle heat load while reaching a downrange target.

| Lift to Drag Ratio<br>$L/D$ | Peak Power<br>kW | Total Energy<br>$E_s$ , kWh | Time Active<br>s | Average Power<br>kW |
|-----------------------------|------------------|-----------------------------|------------------|---------------------|
| 1.5                         | 27               | 1.8                         | 582              | 11                  |
| 2.5                         | 14               | $6.9 \times 10^{-1}$        | 527              | 4.7                 |
| 3.5                         | 7.6              | $2.3 \times 10^{-1}$        | 487              | 1.7                 |

## 5.5 Sizing of the ETC System Power Source

Estimated values of the mass and size of the ETC power source are now computed. Among the primary components of an ETC system (i.e., cathode, anode, and voltage source), the power source is expected to dominate both mass and volume. Consequently, its size and weight are integral to quantifying the feasibility of ETC systems, as their scaling must be compatible with the available space and mass constraints of a hypersonic vehicle.

In both optimization problems, ETC systems are characterized by high peak power and activation times on the order of minutes. To this end, we consider three candidate energy storage technologies: lithium-ion batteries, lithium-polymer batteries, and supercapacitors. These systems are selected for their high specific power, discharge characteristics, as well as commercial availability [129]. Table 5.9 summarizes the performance of these systems, including values of the average specific energy, specific power, and volumetric energy and power density.

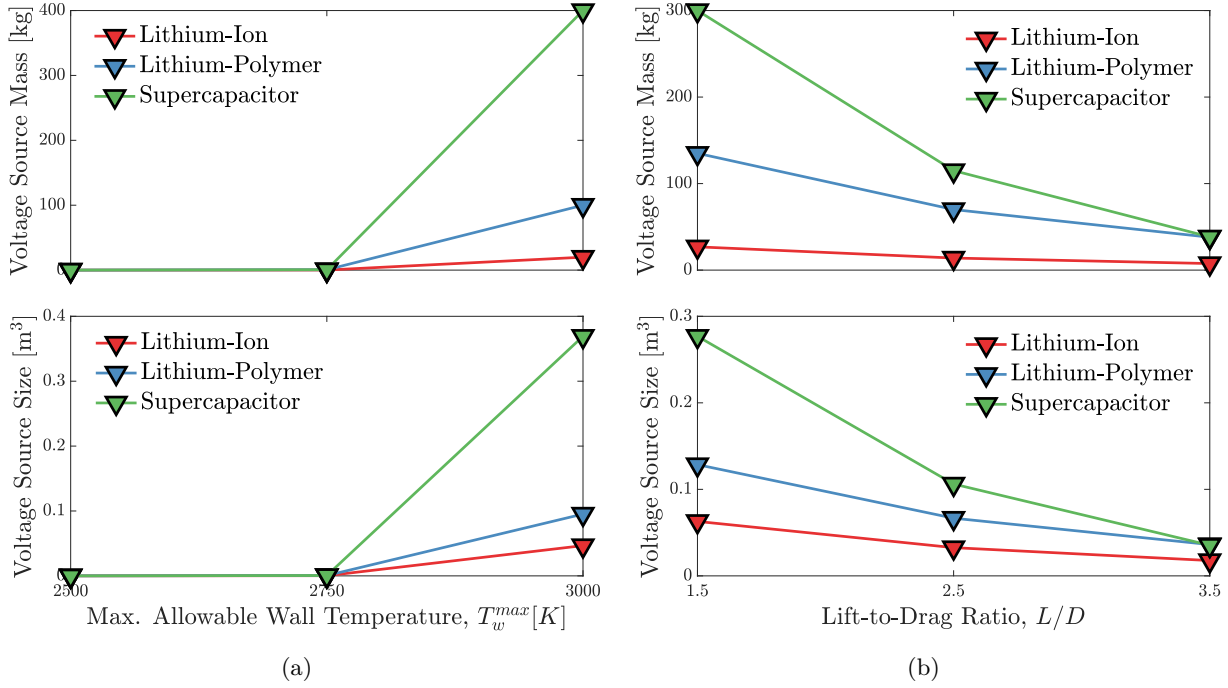
Using the electrical characteristics of the maximum range and minimum heat flux problems (Tables 5.5 and 5.8) in conjunction with the performance data of the candidate energy storage systems (Table 5.9), values of the expected size and mass for each storage system are computed. The mass is taken as the maximum of two constraints: the ratio of total energy to average specific

**Table 5.9:** Average performance data of three candidate energy storage systems.

| Energy Storage System | Avg. Specific Energy<br>Wh/kg | Avg. Specific Power<br>W/kg | Avg. Energy Density<br>kWh/m <sup>3</sup> | Avg. Power Density<br>kW/m <sup>3</sup> | Ref           |
|-----------------------|-------------------------------|-----------------------------|-------------------------------------------|-----------------------------------------|---------------|
| Lithium-Ion           | 165                           | 1004                        | 297                                       | 429                                     | [95]          |
| Lithium-Polymer       | 547                           | 200                         | 260                                       | 210                                     | [95]          |
| Supercapacitor        | 6.0                           | 5250                        | 6.5                                       | 50200                                   | [37, 112, 38] |

energy, and the ratio of peak power to average specific power. An analogous procedure is used to estimate the required volume, using the corresponding average energy density and power density values.

We present both mass and size values for the maximum range study and minimum heat flux study in Figs. 5.12(a) and 5.12(b), respectively. For both studies and all cases, the lithium-ion battery predicts the smallest mass and size of the ETC system voltage source. We note that despite the high power draw observed in the optimization studies, the predicted mass of the power source is reasonable, on the order of 10 kg. This is likely due to the moderate flight times, which do not require the ETC system to be active for a long duration. The largest power source is shown in Fig. 5.12(b) with a value of 26.9 kg. This value is not prohibitively large for use in a realistic vehicle and supports the viability of ETC as a feasible technology for thermal management.



**Figure 5.12:** Estimated size and mass of the CAV-H ETC system voltage source using lithium-ion (red) lithium-polymer (blue) and supercapacitor (green) storage systems. Plots correspond to (a) the maximum range optimization problem and (b) the minimum heat load optimization problem.

## 5.6 Conclusion

The objective of this chapter was to evaluate the performance and practicality of an ETC system as a hypersonic thermal protection system. The ETC system model was coupled with a hypersonic trajectory optimization framework and applied to a representative boost–glide vehicle. The coupled framework enabled hypersonic mission planning and the computation of trajectories with and without ETC. This approach addresses a gap in the assessment of ETC as a cooling technology by providing informed estimates of system size, weight, and power requirements, while also quantifying the influence of ETC on vehicle performance.

First, a mission objective of maximizing vehicle range was introduced. The introduction of thermionic cooling allowed for faster and more aggressive initial conditions to be explored by the optimizer. This translated into an extended range, with one case showing an improvement of

over 200%, and represents an expansion of the feasible launch envelope of the boost-glide vehicle. Interestingly, when operating an ETC system the highest wall temperatures are seen to occur when the hypersonic vehicle enters an SCL regime and thermionic cooling deactivates. This suggests that quantifying the location of SCL regimes is of utmost importance to successfully operating an ETC system.

A second mission objective of minimizing the integrated heat load was considered. In this mission, the presence of the ETC system led the optimizer to identify low lift-to-drag configurations to be optimal in achieving the lowest heat load. This was in contrast to cases without ETC, where high lift-to-drag ratios resulted in the lowest heat load. This suggests ETC may enable a fundamentally different thermal management strategy in which low lift-to-drag, short-duration trajectories may be considered to be optimal through active suppression of peak convective heating.

Finally, considering energy requirements, ETC systems were observed to be characterized by high power consumption when active. This suggests that ETC may potentially be best suited for short-duration operation during periods of high thermal loading, such as the initial pull-up maneuver of a boost-glide platform. Estimates of the size and mass of a power source were computed for the two optimization problems involving the boost-glide vehicle. Between multiple energy sources, lithium-ion batteries were shown to be best suited for ETC systems, yielding values of predicted mass and size that are feasible to carry on flight hardware.



## Chapter 6

### The Role of Bulk Plasma Dynamics in Electron Transpiration Cooling Systems

In the preceding chapters, ETC systems have been shown to have potential as a TPS for hypersonic vehicles. In particular, the model developed in Chapter 3 showed that ETC systems are capable of significant thermionic cooling at levels even greater than radiative cooling. In Chapters 4 and 5, ETC was shown to be well suited for a wide range of hypersonic conditions, and was capable of increasing vehicle performance for reasonable size and power requirements.

However, the preceding results were obtained through the assumption of an ideal bulk plasma. For a realistic ETC system, current leaving the cathode will experience a change in potential as it travels to the anode due to interactions with the bulk plasma. Changes in potential are likely to form owing to chemical recombination, thermal relaxation, and fluid expansion between the electrodes.

Critically, this potential drop must be overcome by the internal voltage source of the ETC system. Unfortunately, directly increasing the operating voltage will increase size and mass of the power system. For sufficiently large potential differences, the resulting voltage source may become impractically large, limiting the feasibility of ETC as a TPS technology. Thus, to evaluate the effect of the bulk plasma on ETC system performance, the following chapter describes a method of modeling the interaction between ETC electrodes and the bulk plasma in a hypersonic flow solver.

This chapter is organized as follows. First, Section 6.2 introduces a hypersonic flow solver and discusses the details and assumptions of the modeling of thermochemical nonequilibrium and bulk plasma properties. Next, Section 6.3 describes the formulation of boundary conditions in the

hypersonic solver that are created with the purpose of capturing the effects of thermionic emission and the plasma sheath. Section 6.4 describes the chosen geometry, computational domain, and numerical setup for the hypersonic solver. Finally, Section 6.6 shows the results of the analysis, including estimates of bulk plasma potentials and absorbed/emitted currents along the electrodes.

## 6.1 The Navier-Stokes Equations

The hypersonic flowfields considered in the present work are chemically reacting and under thermodynamic nonequilibrium. These physics are governed by the compressible Navier-Stokes equations coupled with conservation of energy:

Conservation of Mass:

$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u} + \mathbf{J}_s) = \dot{w}_s \quad (6.1)$$

Conservation of Momentum:

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} + p \bar{\bar{\mathbf{I}}} - \bar{\bar{\tau}}) = 0 \quad (6.2)$$

Conservation of Total Energy:

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot \left( (\mathcal{E} + p) \mathbf{u} - \bar{\bar{\tau}} \cdot \mathbf{u} + \mathbf{q}_{tr} + \mathbf{q}_{vee} + \sum_s (\mathbf{J}_s h_s) \right) = 0 \quad (6.3)$$

Conservation of Vibrational, Electronic, and Electron-Translational Energy:

$$\frac{\partial \mathcal{E}_{vee}}{\partial t} + \nabla \cdot \left( \mathcal{E}_{vee} \mathbf{u} + \mathbf{q}_{vee} + \sum_s (\mathbf{J}_s e_{vee,s}) \right) = \dot{w}_{vee} \quad (6.4)$$

Starting with mass conservation (Eq. 6.1),  $\rho_s$  is the density of chemical species  $s$ ,  $\mathbf{u}$  is the average bulk velocity vector for all species,  $\mathbf{J}_s$  is the species diffusive flux, and  $\dot{w}_s$  is the species net source term from chemical reactions. Within momentum conservation (Eq. 6.2) is the total mixture density for all species,  $\rho = \sum_s \rho_s$ , and thermodynamic pressure,  $p$ , computed using Dalton's law of partial pressures for an ideal gas,

$$p = \sum_s^{\text{NS}} \rho_s \frac{R_u}{M_s} \quad (6.5)$$

where NS is the number of chemical species,  $R_u$  is the universal gas constant, and  $M_s$  is the species molar mass. The identity tensor is given by  $\bar{\bar{I}}$  and the viscous stress tensor is given by  $\bar{\bar{\tau}}$ , which assumes a Newtonian fluid and Stokes hypothesis. Two energy conservation equations are included, namely conservation of total energy  $\mathcal{E}$  (Eq. 6.3) and vibrational-electronic-electron energy  $\mathcal{E}_{vee}$  (Eq. 6.4) per unit volume, where  $\mathbf{q}_{tr}$  and  $\mathbf{q}_{vee}$  are translational-rotational and vibrational-electron-electron heat fluxes, respectively,  $h_s$  is the species enthalpy,  $e_{vee,s}$  is the species internal energy per unit mass, and  $\dot{w}_{vee}$  is the vibrational-electronic-electron energy source term. Further discussion on the energy equations is found in Section 6.2.2.

Integral to the derivation of Eqs. 6.1-6.4 is the assumption that the fluid behaves as a continuum. Under this assumption, fluid properties are continuous, differentiable, and well-defined throughout the domain. As introduced in Section 4.1, this approximation is generally valid for Knudsen numbers  $Kn < 0.01$ . At higher values of  $Kn$ , however, Eqs. 6.1–6.4 break down, and higher-order effects associated with the discrete interactions between fluid particles must be considered. In such regimes, the flow is more appropriately modeled using direct kinetic (DK) solutions of the Boltzmann equation or particle-based simulations, including Direct Simulation Monte Carlo (DSMC) and Particle-in-Cell (PIC) methods.

The results of Chapter 4 indicate that ETC can provide useful cooling at high velocities (above  $\sim 9$  km/s) and at high altitudes. However, that analysis was limited to altitudes up to 80 km, where continuum assumptions are reasonable. At higher altitudes, rarefaction effects become significant and the bulk flow is no longer a continuum. High-fidelity analysis of ETC in these regimes would therefore require methods such as DK or PIC, which is beyond the scope of the present work. The following sections in this chapter analyze the flowfield surrounding an ETC system within the continuum regime, where care has been taken to ensure Eqs. 6.1-6.4 are applicable.

## 6.2 LeMANS: A Hypersonic Flow Solver

The following section introduces the (“Le”) Michigan Aerothermodynamic Navier-Stokes Solver (LeMANS). LeMANS is a parallel, three-dimensional unstructured finite volume CFD code that solves the compressible Navier-Stokes equations [124]. LeMANS has been previously benchmarked with similar hypersonic solvers (US3D, DPLR, LAURA), and validated with comparisons to flight data (RAM-C and FIRE-II) as well as test facility data (HYPULSE and CUBRC) [56, 76, 17, 36, 123, 124].

In section 6.2.1, the governing equations of LeMANS are introduced and a description of the numerical implementation, including transport relations, is given. Next, in Section 6.2.2, treatments for thermochemical nonequilibrium in LeMANS are described. Section 6.2.3 derives the EHD source terms in the governing equations, and discusses the assumptions being made. Finally, Section 6.2.4 introduces two empirical models for the plasma conductivity.

### 6.2.1 Numerical Implementation

Equations 6.1-6.4 are reorganized into flux components to form the set of governing equations within LeMANS:

$$\frac{\partial \mathbf{Q}}{\partial t} + \nabla \cdot (\mathbf{F} + \mathbf{F}_v) = \mathbf{S}_{flow} + \mathbf{S}_{EHD} \quad (6.6)$$

comprised of vectors of conserved variables,  $\mathbf{Q}$ , inviscid fluxes,  $\mathbf{F}$ , viscous fluxes  $\mathbf{F}_v$ , thermochemical source terms from the flow,  $\mathbf{S}_{flow}$ , and electrohydrodynamic (EHD) source terms,  $\mathbf{S}_{EHD}$ , shown below as

$$\begin{aligned}
\mathbf{Q} &= \begin{bmatrix} \rho_1 \\ \vdots \\ \rho_{n_s} \\ \rho \mathbf{u} \\ \mathcal{E} \\ \mathcal{E}_{vee} \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \rho_1 \mathbf{u} \\ \vdots \\ \rho_{n_s} \mathbf{u} \\ \rho \mathbf{u} \otimes \mathbf{u} + p \bar{\mathbf{I}} \\ (\mathcal{E} + p) \mathbf{u} \\ \mathcal{E}_{vee} \mathbf{u} \end{bmatrix}, \quad \mathbf{F}_v = \begin{bmatrix} \mathbf{J}_1 \\ \vdots \\ \mathbf{J}_{n_s} \\ -\bar{\bar{\tau}} \\ -\bar{\bar{\tau}} \cdot \mathbf{u} + \mathbf{q}_{tr} + \mathbf{q}_{vee} + \sum_s (\mathbf{J}_s h_s) \\ \mathbf{q}_{vee} + \sum_s (\mathbf{J}_s \epsilon_{vee,s}) \end{bmatrix} \\
\mathbf{S}_{flow} &= \begin{bmatrix} \dot{w}_s \\ \vdots \\ \dot{w}_{n_s} \\ 0 \\ 0 \\ \dot{w}_{vee} \end{bmatrix}, \quad \mathbf{S}_{EHD} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \mathbf{j} \cdot \mathbf{E} \\ \eta (\mathbf{j} \cdot \mathbf{E}) \end{bmatrix}
\end{aligned}$$

LeMANS uses a second-order discretization to solve Eq. 6.6 in the spatial domain. First-order temporal accuracy is achieved via a parallel line-implicit method. As gradients in the wall-normal direction are typically large, the governing system of equations is solved in the wall-normal direction using a tridiagonal solver for improved convergence behavior [155]. Inviscid fluxes are discretized using a modified Steger-Warming flux vector splitting scheme [85], while viscous fluxes are computed using a second-order accurate central scheme.

In the present work, LeMANS, models the flowfield as steady state and laminar. Mixture transport properties are calculated using Wilke's semi-empirical mixing rule [151], with species viscosities from Blottner's curve fits [19] and species thermal conductivities calculated using Eucken's relation [41]. Thermal fluxes are computed using Fourier's law for all active energy modes. Heavy species diffusion fluxes follow modified Fick's law as

$$\mathbf{J}_s = -\rho D_s \nabla Y_s + Y_s \sum_{k \neq e} \rho D_k \nabla Y_k \quad (6.7)$$

where  $D_s$  is the species diffusion coefficient and  $Y_s$  is the species mass fraction.

As ionization processes are expected in the present work, it is noted that LeMANS assumes a quasineutral plasma composition ( $n_e = n_i$ ), with electron transport governed by ambipolar diffusion. Under this assumption, ions and electrons diffuse at the same rate due to Coulombic coupling between charged species. As a result, the electron diffusion flux is computed as

$$\mathbf{J}_e = -\frac{1}{q_e} \sum_{s \neq e} q_s \mathbf{J}_s \quad (6.8)$$

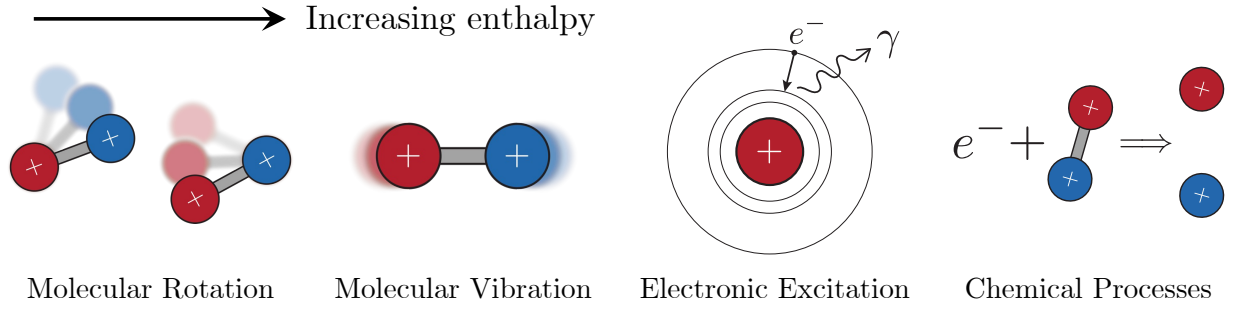
where  $q_s$  is the species charge per unit mass.

### 6.2.2 Thermochemical Nonequilibrium

When flying at hypersonic velocities, a shock forms upstream of the vehicle. Across the shock, the freestream enthalpy is redistributed into various internal energy modes. Figure 6.1 illustrates this conversion where, as a function of increasing freestream enthalpy, modes of molecular rotation, molecular vibration, excited bound electronic states, and chemical processes (e.g. disassociation and ionization) become active. Physically, the excitation of these modes occurs through collisions and is characterized by finite relaxation times,  $\tau$ . The ordering of these relaxation times, which define the timescale required for each mode to reach equilibrium, is in general given by

$$\tau_{translational} < \tau_{rotational} < \tau_{vibrational} < \tau_{electronic} < \tau_{dissociation} < \tau_{ionization} \quad (6.9)$$

where  $\tau_{dissociation}$  and  $\tau_{ionization}$  correspond to characteristic chemical reaction times for each process. Importantly, the activation of these modes in hypersonic environments leads to the breakdown of ideal gas assumptions, significantly complicating analysis.



**Figure 6.1:** Depiction of the progression of active energy modes in a hypersonic flow for increasing levels of freestream enthalpy.

Thermal nonequilibrium is incorporated into LeMANS by separately tracking active energy modes of the flow (i.e. translational, rotational, vibrational, electronic, electron translational). We note that the electron translational mode is considered separately, as electrons have significantly less mass than other particles and their respective velocity distribution function (VDF) comprises a much wider range of thermal velocities. As it requires very few collision for the rotational mode to equilibriate with the translational mode, we assume a single temperature defining the translational rotational energy mode as

$$T_{tr} = T_{translational} = T_{rotational} \quad (6.10)$$

Similarly, as the energy transfer between vibrational modes and the translational energy of free electrons is fast, and that the electronic energy mode is in general mostly excited by the translational mode of electrons [81], it can be assumed that the vibrational, electronic, and electron temperatures are described by a single temperature as

$$T_{vee} = T_{vibrational} = T_{electronic} = T_{electron} \quad (6.11)$$

The total energy,  $\mathcal{E}$ , is the sum of the kinetic energy in the flow, species enthalpy of formation,  $h_s^0$ , and the internal energy modes,

$$\mathcal{E} = \sum_{s \neq e} \rho_s C_{v_{tr},s} T_{tr} + \frac{1}{2} \rho \|\mathbf{u}\|^2 + \sum_s \rho_s h_s^0 + \mathcal{E}_{vee} \quad (6.12)$$

where  $C_{v_{tr},s}$  is the translational-rotational specific heat at constant volume. The vibrational-electronic-electron energy is obtained by summing the vibrational energy,  $e_{v,s}$ , and electronic energy,  $e_{el,s}$ , of each species with the electron-translational energy as

$$\mathcal{E}_{vee} = \sum_{s \neq e} \rho_s (e_{v,s} + e_{el,s}) + \frac{3}{2} \frac{R_u}{M_e} T_{vee} \quad (6.13)$$

Energy exchange between translational and vibrational energy modes is modeled through the Landau-Teller expression coupled with the Millikan-White curve fits for vibrational relaxation time [93]. Park's high temperature correction to the Millikan-White curves is incorporated as well [102]. Chemical nonequilibrium is also included through a finite-rate chemical model for 11-species air, defined as  $\mathbf{S} = \{\text{N}_2, \text{O}_2, \text{N}, \text{O}, \text{NO}, \text{N}_2^+, \text{O}_2^+, \text{N}^+, \text{O}^+, \text{NO}^+, \text{e}\}$ . Park's two temperature model is used to factor in the influence of thermal nonequilibrium on chemical reaction rates [101].

### 6.2.3 Electrohydrodynamic Source Terms

Electric field effects in the flowfield are included in LeMANS through the source terms  $\mathbf{S}_{EHD}$ . Specifically, the source terms contain a  $\mathbf{j} \cdot \mathbf{E}$  contribution, known as Joule heating, in the total and vibrational-electronic-electron energy equations. This heating term arises from the collisions of electrons accelerated by the electric field with ions and neutrals, and thus, represents a conversion of electric current into thermal energy. A fraction  $\eta$  of the total Joule heating goes to the vibrational-electronic-electron energy mode. For electron temperatures below 2 eV,  $\eta$  has been shown to range between 0.79 and 0.98 [100], thus the present work takes an estimated value of 0.9.

The EHD source terms arise from simplifications to the magnetohydrodynamic (MHD) equations, which couple Maxwell's equations to the Navier-Stokes equations [33]. In the MHD equations, these source terms include  $\mathbf{E}$ ,  $\mathbf{j}$ , as well as the magnetic field,  $\mathbf{B}$ . However, this coupling can be computationally expensive, and the present work takes steps to simplify the governing equations.



We define the magnetic Reynolds number,  $\text{Re}_m$ , in Eq. 6.14 as the ratio of magnetic induction to magnetic diffusion, where  $u$  is the flow velocity,  $L$  is a characteristic length,  $\mu_0 = 1.257 \times 10^{-6}$  N/A<sup>2</sup> is the magnetic permeability of free space, and  $\sigma$  is the plasma conductivity in Siemens per meter. For  $\text{Re}_m \ll 1$ , induced magnetic fields can be considered to be negligible. Importantly, this means one needs only to consider externally prescribed magnetic field components. The present work does not consider an applied magnetic field such that  $\mathbf{B}$  is taken to be zero over the domain. Previous work investigating ETC in hypersonic regimes have found this assumption to be valid [58, 57, 9].

$$\text{Re}_m = \frac{uL}{(\mu_0\sigma)^{-1}} \quad (6.14)$$

Current density through an electrically conductive fluid, such as the hypersonic environments considered in the present work, can be related to the electric and magnetic fields in the plasma by a Generalized Ohm's Law. This is shown in Eq. 6.15 as

$$\mathbf{j} = \tilde{\sigma} (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \quad (6.15)$$

where  $\tilde{\sigma}$  is the electrical conductivity tensor. In the case of no magnetic fields, Eq. 6.15 can be simplified to

$$\mathbf{j} = \sigma \mathbf{E} \quad (6.16)$$

We note that the assumption of an isotropic plasma with no external magnetic fields reduces the plasma conductivity to a scalar value. Detailed derivations of the Generalized Ohm's Law can be found in [76].

The Ampère-Maxwell Law is shown in Eq. 6.17

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (6.17)$$

where  $\epsilon_0$  is the vacuum permittivity. Taking the divergence of both sides of Eq. 6.17, identifying that the divergence of a curl is zero, and assuming steady state electric fields and zero magnetic field, we obtain a simplified version of the Ampère-Maxwell law in Eq. 6.18.

$$\nabla \cdot \mathbf{j} = 0 \quad (6.18)$$

Substituting Eq. 6.16 for  $\mathbf{j}$  we obtain

$$\nabla \cdot (\sigma \mathbf{E}) = 0 \quad (6.19)$$

We note that the electric field can be decomposed through Helmholtz's theorem into irrotational and divergence free components. For steady state fields with a low magnetic Reynolds number, this leaves the irrotational component of the electric field as

$$\mathbf{E} = -\nabla \phi \quad (6.20)$$

where  $\phi$  is the scalar electric potential over the domain. Substituting this into Eq. 6.19, we obtain a Poisson type equation of the following form

$$\nabla \cdot \mathbf{j} = \nabla \cdot (\sigma \nabla \phi) = 0 \quad (6.21)$$

Within LeMANS, the electric potential in the domain is found by applying a second order finite-volume discretization on Eq. 6.21 coupled with a Successive Over-Relaxation iterative technique. Upon computing the electric potential, Eqs. 6.20 and 6.16 are used to compute the electric field and current density, respectively, in the domain.

#### 6.2.4 Plasma Conductivity

The values of the electric potential,  $\phi$ , over the domain depend on the electrical conductivity,  $\sigma$ , of the gas. From a physical perspective, conductivity is associated with the drift of

charged particles under the acceleration of an electric field. Collisions with charged particles, such as electron-neutral or electron-ion interactions, work to redistribute momentum away from the direction of the electric field. This is akin to a drag or frictional force on the drift of charged particles that reduces the conductivity of the bulk gas. Electron-electron interactions are an exception to this idea as the interaction conserves momentum in the direction of drift. Only through indirect means (e.g. changing the energy spectrum of electrons by driving the EVDF to a Maxwellian) does this interaction modify the gas conductivity.

The present work explores two models for  $\sigma$ . First is the model by Chapman and Cowling [30], which was derived for a weakly ionized gas. For weakly ionized conditions, electron-neutral collisions are expected to be more frequent compared to Coulomb interactions (electron-ion). Using this idea, Chapman and Cowling [30] assume that, after attaining a drift velocity due to the electric field, electrons lose all momentum in the direction of the electric field upon a collision. The result is a coupling between the conductivity and the binary diffusion coefficient of the collision. The model is shown in Eq. 6.22, where  $n_n$  is the neutral number density and  $Q$  is the effective electron-neutral collision cross section of the gas. For the present work,  $Q$  is taken as  $5 \times 10^{-19} \text{ m}^2$ , which was observed to produce results consistent with other semi-analytic models in the hypersonic regime [18].

$$\sigma = 3.34 \times 10^{-10} \frac{n_e}{n_n Q \sqrt{T_{tr}}} \quad (6.22)$$

The second model, developed by Spitzer and Härm [132], was developed for fully ionized gases. Owing to the high ionization levels in this regime, Coulomb interactions may impede the movement of electrons as much as with neutrals. The model by Spitzer and Härm [132] leverages this by assuming a fully ionized Lorentz gas where ion movement is neglected and electron-electron interactions are again ignored. The expression is shown in Eq. 6.23. Importantly, dependence of the gas conductivity on an ionization fraction term disappears, leaving only a weak electron density dependence via  $\ln[n_e]$ .

$$\sigma = \frac{(1.56 \times 10^{-2}) T_{tr}^{3/2}}{\ln \left[ (1.23 \times 10^7) T_{tr}^{3/2} n_e^{-1/2} \right]} \quad (6.23)$$

The plasma conductivity will strongly influence the potential distribution in the flowfield, which by extension, is coupled to the size of the internal power source for the ETC system. As the two presented models were derived for plasmas at the limits of weakly and strongly ionized plasmas, it is expected that the corresponding results will bound the true plasma conductivity and the flowfield potential difference for an ETC system. To this end, the present analysis considers both conductivity models to assess the practicality of a complete ETC system.

### 6.3 Modeling Thermionic Emission and Electron Collection

For cathode and anode surfaces, the production rates of charged species are computed as a function of the current densities through the plasma sheath. Equation 6.24 shows the total electron production rate,  $\dot{w}_e$ , taken as the difference between the thermionic and flowfield electron current through the sheath where  $N_{Av} = 6.022 \times 10^{23}$  1/mol is Avogadro's number. Electron production rates at anode surfaces take a similar form but are considered to be non-emitting (i.e.  $J_{etc} = 0$ ).

$$\dot{w}_e = \frac{J_{etc}}{eN_{Av}} - \frac{J_{e,f}}{eN_{Av}} \quad (6.24)$$

The total ion production rate at electrode surfaces is set to be equal to that of the electron production rate to maintain quasineutrality of the bulk flowfield. The total ion production rate is then divided among the population of ions near the wall. Shown in Eq. 6.25 is the ion production rate,  $\dot{w}_i$ , for species  $i$ , where  $X_{i,cl}$  is the species mole fraction at the first wall-normal cell, and  $i \in \mathbf{I}$ , where  $\mathbf{I} = \{N_2^+, O_2^+, N^+, O^+, NO^+\}$  is denoted as the set of all positively charged species.

$$\dot{w}_i = X_{i,cl} \dot{w}_e \quad \forall i \in \mathbf{I} \quad (6.25)$$

The source of ions is assumed to come from the population of neutral species near the wall. Thus, the neutral species production rate,  $\dot{w}_n$ , is augmented in Eq. 6.26 according to the ion production rate, where ion species  $i$  recombines into neutral species  $n \in \mathbf{N}$ , where  $\mathbf{N} = \{\text{N}_2, \text{O}_2, \text{N}, \text{O}, \text{NO}\}$  is the set of all chemically neutral species.

$$\dot{w}_n += -\dot{w}_i \quad \forall i \in \mathbf{I}, n \in \mathbf{N} \quad (6.26)$$

where species  $i$  recombines to neutral species  $n$

Wall species mass fractions,  $Y_{s,w}$ , are computed using the previously determined species production rates by solving mass and momentum conservation equations at the surface, described in Martin and Boyd [90]. Species mass and momentum conservation equations are shown in Eqs. 6.27 and 6.28, where  $D_s$  is the species diffusion coefficient and subscript  $( )_{cl}$  denotes the first wall-normal cell.

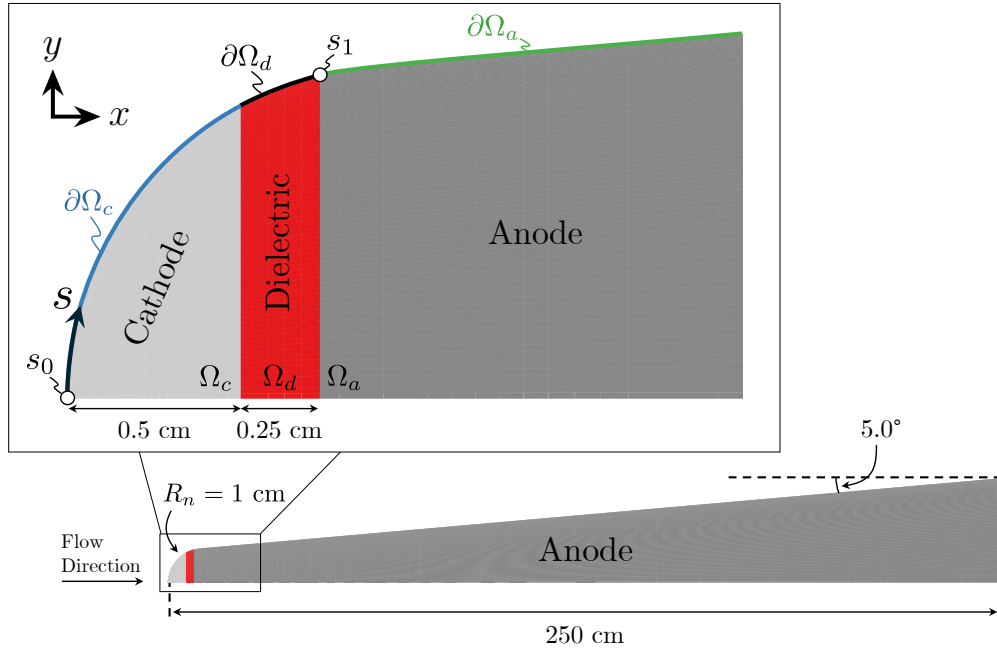
$$\rho_w D_{s,cl} \left( \frac{Y_{s,cl} - Y_{s,w}}{dx} \right) - \dot{m} Y_{s,w} + \dot{w}_s \frac{M_s}{1000} = 0 \quad \forall s \in \mathbf{S} \quad (6.27)$$

$$\rho_w R_{mix,w} T_w + \frac{\dot{m}^2}{\rho_w} - \rho_{cl} v_{cl}^2 - p_{cl} = 0 \quad (6.28)$$

$$\text{where} \quad R_{mix,w} = R_u \sum_s \frac{Y_{s,w}}{M_s} \quad (6.29)$$

## 6.4 Test Case Description

To investigate the performance of an ETC system, the present work considers a sharp leading edge geometry. Geometric dimensions are shown in Fig. 6.2, where the leading edge is noted to have a nose radius,  $R_n = 1$  cm. Defining the origin at the stagnation point, the cathode boundary,  $\Omega_c$ , is defined for  $x \in [0, 0.5]$  cm, while the anode surface,  $\Omega_a$ , begins at  $x = 0.75$  cm and extends until the end of the vehicle. The extent of the cathode surface spans the region of highest expected convective heating, while the anode is designed with a large surface area to promote electron collection. The region between the electrodes consists of an electrically insulating dielectric material,  $\Omega_d$ .



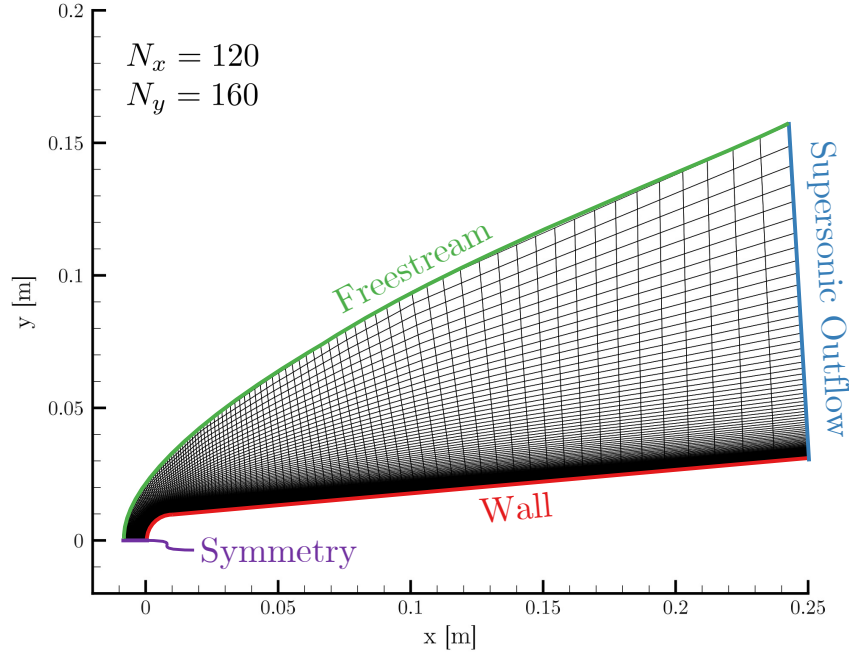
**Figure 6.2:** Illustration of the test case geometry with shaded regions denoting the cathode, anode and dielectric surfaces.

Additionally, Fig. 6.2 shows the arc length,  $s$ , defined along the outer surface of the leading edge. Over the entire surface, this includes the freestream-facing surfaces of the cathode,  $\partial\Omega_c$ , the dielectric material,  $\partial\Omega_d$ , and the anode,  $\partial\Omega_a$ . The dielectric surface is considered to be catalytic such that all charged species are neutralized upon impact with the wall. Additionally, reference locations along the arc length,  $s_0$  and  $s_1$ , are defined at the start of the cathode and anode boundaries, respectively.

## 6.5 Numerical Setup

Figure 6.3 shows the computational domain of the investigated geometry along with the chosen boundary conditions. The mesh consists of 120 cells in the  $x$ -direction and 160 in the  $y$ -direction. A wall-normal spacing of  $1 \times 10^{-6}$  m is used to properly resolve surface quantities. Care has been taken to ensure cells near the wall are orthogonal with respect to the vehicle surface.

ETC system parameters are shown in Table 6.1. Both electrodes are comprised of hafnium



**Figure 6.3:** Computational domain of the investigated geometry showing wall, symmetry, freestream, and supersonic outflow boundary conditions.

carbide (HfC), which is well suited for an ETC system owing to its moderate work function of 3.2 eV, high melting point of 4173 K, and high emissivity taken to be  $\epsilon = 0.9$ . We note that the cathode and anode wall potentials are prescribed as  $\phi_w^c = -3$  V and  $\phi_w^a = 1$  V. These values are chosen to be moderate in magnitude so that they do not require an unrealistically large internal voltage, while still providing sufficient bias for the cathode/anode to emit/collect near their maximum electron current.

**Table 6.1:** Simulation parameters for the ETC system.

| ETC Parameters                             | Value             |
|--------------------------------------------|-------------------|
| $WF^c$ , eV                                | 3.2               |
| $WF^a$ , eV                                | 3.2               |
| $A_R$ , A/(m <sup>2</sup> K <sup>2</sup> ) | $120 \times 10^4$ |
| $\phi_w^c$ , V                             | -3.0              |
| $\phi_w^a$ , V                             | 1.0               |
| $\epsilon$                                 | 0.9               |

Freestream parameters for the ETC system are shown in Table 6.2. The conditions correspond to an altitude of 60 km which was used in a previous ETC investigation [7]. For the chosen work function (3.2 eV), nose radius (1 cm) and flight environment (60 km altitude at 7 km/s), it is expected that the emissive cathode will be in an SCL mode. However, this flight condition lies near the boundary between SCL and Richardson-limited modes, and the cathode may transition between the two regimes due to thermionic emission at the cathode surface.

**Table 6.2:** Freestream conditions for the ETC test case corresponding to 60 km altitude (\*unless otherwise stated).

| Freestream Conditions            | Value                  |
|----------------------------------|------------------------|
| $\rho_{N_2}$ , kg/m <sup>3</sup> | $1.760 \times 10^{-4}$ |
| $\rho_{O_2}$ , kg/m <sup>3</sup> | $5.393 \times 10^{-5}$ |
| $u_\infty$ , m/s                 | 7000*                  |
| $T_\infty$ , K                   | 238.1                  |

Boundary conditions for the Poisson solver are shown in Table 6.3. Symmetry, outflow, and freestream boundaries satisfy a zero gradient electric field,  $\mathbf{E} \cdot \hat{\mathbf{n}} = 0$ . The wall is comprised of three boundary types: cathode, anode, and dielectric. At the cathode boundary, the total current through the sheath is computed as the sum of the thermionic, flowfield ion, and flowfield electron current densities traveling through the sheath:  $J_{total} = J_{etc} + J_{i,f} - J_{e,f}$ . The current is assumed to be in the wall-normal direction and is enforced as a Neumann condition:  $\mathbf{j} \cdot \hat{\mathbf{n}} = -J_{total}$ . The anode boundary acts as the reference potential in the domain and is set such that  $\phi = 0$  V. All other wall boundaries are treated as an electrically insulating wall:  $\mathbf{j} \cdot \hat{\mathbf{n}} = 0$ .

Finally, the wall is considered to be at radiative equilibrium where heat fluxes from convection and the plasma sheath are balanced by thermionic and radiative cooling fluxes. The surface energy balance is shown in Eq. 6.30, where radiative cooling is determined via the Stefan-Boltzmann Law. Brent's method [97] is employed to iteratively solve for the wall temperature,  $T_w$ . Thermionic emission is absent over the dielectric and anode surfaces, leaving radiative cooling to balance heating from convection and the plasma sheath.



**Table 6.3:** Boundary conditions for the electric potential in LeMANS. The normal vector,  $\hat{\mathbf{n}}$ , is defined to be positive pointing into the computational domain.

| Boundary          | Type      | Condition                                        |
|-------------------|-----------|--------------------------------------------------|
| Symmetry          | Neumann   | $\mathbf{E} \cdot \hat{\mathbf{n}} = 0$          |
| Outflow           | Neumann   | $\mathbf{E} \cdot \hat{\mathbf{n}} = 0$          |
| Freestream        | Neumann   | $\mathbf{E} \cdot \hat{\mathbf{n}} = 0$          |
| Wall (cathode)    | Neumann   | $\mathbf{j} \cdot \hat{\mathbf{n}} = -J_{total}$ |
| Wall (anode)      | Dirichlet | $\phi = 0$                                       |
| Wall (insulating) | Neumann   | $\mathbf{j} \cdot \hat{\mathbf{n}} = 0$          |

$$q_{rad} + q_{etc} = q_{conv} + q_{i,f} + q_{e,f} \quad (6.30)$$

## 6.6 Two-Dimensional Simulations of an ETC System

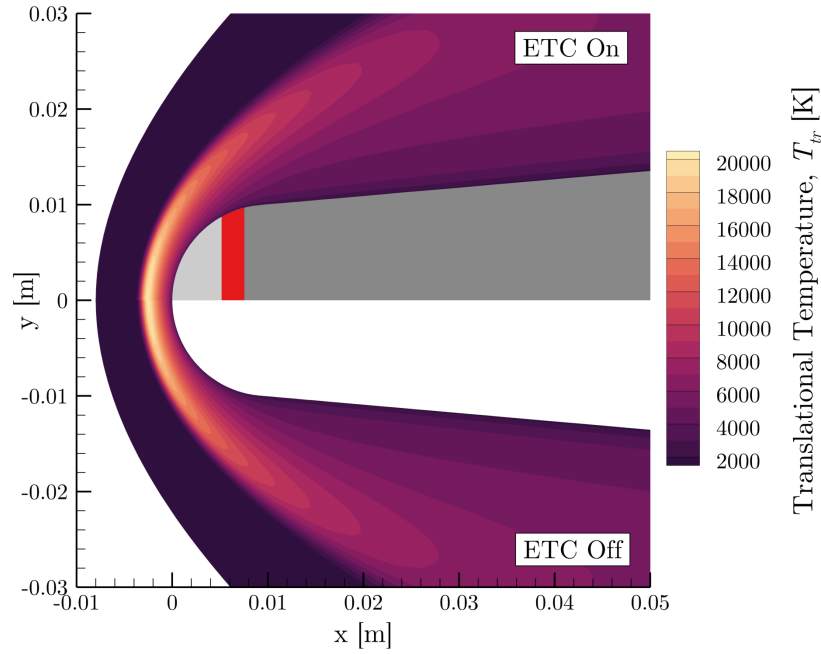
The following section investigates the hypersonic environment surrounding an ETC system. Two cases are considered: (1) an ETC off case where the entire surface is catalytic to charged species and both electrode biasing and thermionic emission are deactivated and (2) ETC on case that follows the description of Section 6.4, with an emissive cathode, dielectric spacer, and collecting anode.

First, Section 6.6.1 presents the effect of ETC on the flowfield surrounding the vehicle as well as wall temperatures. Next Section 6.6.2 quantifies the changes to the bulk plasma as well as the structure of the electric field between the electrodes. The effect of the electric field on the size of the required voltage source is also discussed. Finally, in Section 6.6.3, currents at the cathode and anode are computed and their implications on the steady state operation of ETC are considered.

### 6.6.1 Wall Temperatures and Heat Fluxes

Figure 6.4 shows contours of the translational temperature,  $T_{tr}$ , near the leading edge for an ETC On and ETC Off configuration. Both cases show that peak values of  $T_{tr}$  occur at the location

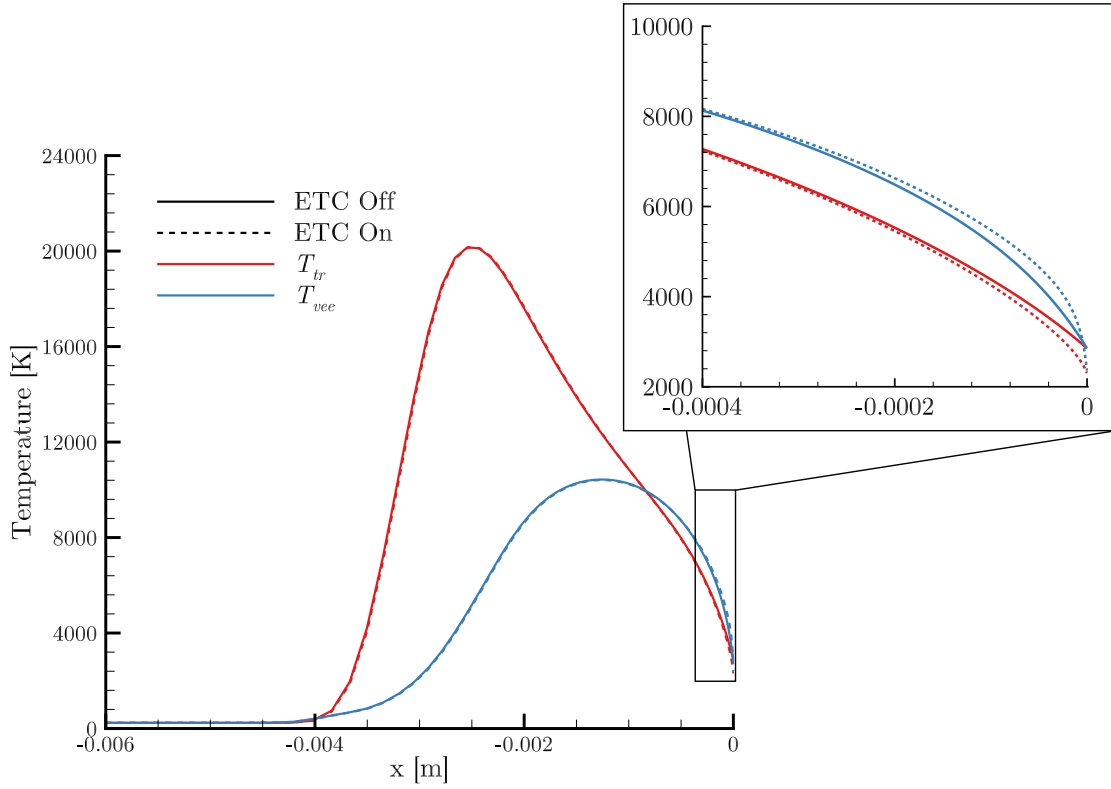
of the shock where the kinetic energy of the freestream is first redistributed into the translational energy mode. Over the domain, the presence of ETC does not result in a large change in  $T_{tr}$ .



**Figure 6.4:** Comparison of the ETC On and ETC Off cases showing flowfield contours of the translational temperature,  $T_{tr}$ . Freestream conditions correspond to those in Table 6.2

Translational and vibrational ( $T_{vee}$ ) temperatures along the stagnation line are shown in Fig. 6.5 for both cases. The largest disagreement between the ETC On and Off cases occurs near the wall. In particular, the ETC On case displays hotter translational and vibrational temperatures near the wall in addition to a larger degree of thermal nonequilibrium. Importantly, the gradients of temperature at the wall, especially that of  $T_{vee}$ , are seen to be higher for the ETC On case, indicating an increased sensitivity of conductive heating to the presence of electrons near the surface. This suggests that accurate modeling of the parameters governing vibrational–electronic–electron coupling are likely important for reliably predicting thermal behavior at the cathode.

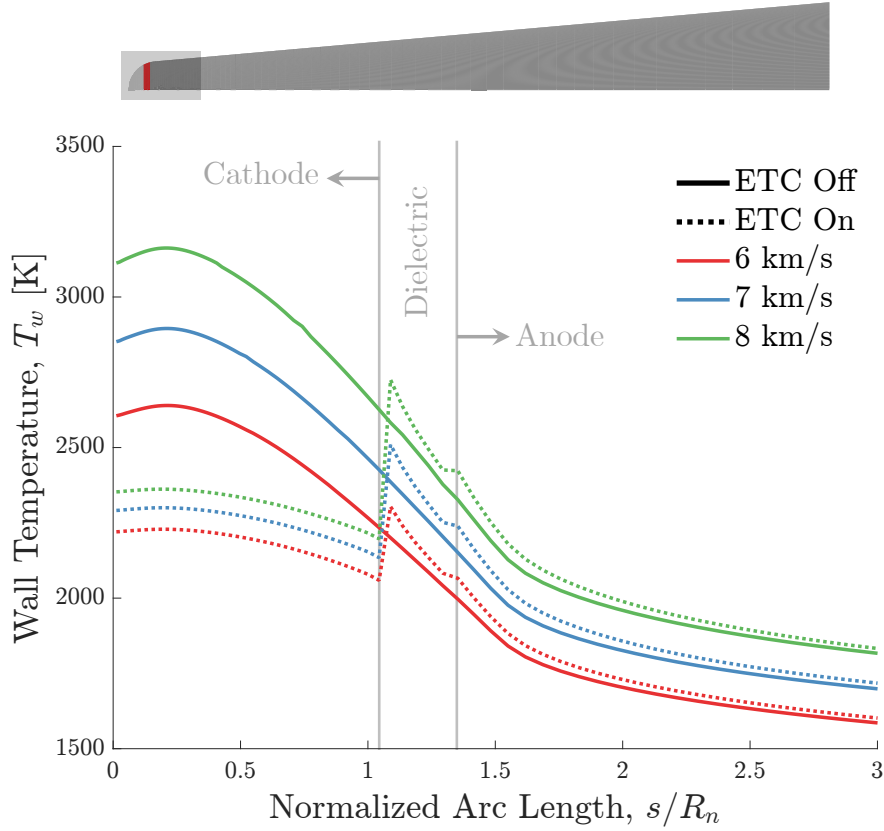
To assess the cooling performance of ETC, Fig. 6.6 presents the wall temperature,  $T_w$ , for the ETC On and Off configurations, including additional freestream velocities of 6 km/s and 8



**Figure 6.5:** Comparison of the ETC On and ETC Off cases showing values of the translational ( $T_{tr}$ ) and vibrational temperature ( $T_{vib}$ ) along the stagnation line. Freestream conditions correspond to those in Table 6.2

km/s. Thermionic emission is shown to significantly reduce wall temperatures at the leading edge. However, when ETC is active, the peak wall temperature shifts downstream to the start of the dielectric surface. This contrasts with the ETC Off case, where the maximum temperature occurs near the stagnation point. The elevated temperatures over the dielectric and anode surfaces are attributed to enhanced catalytic and sheath heating due to the increased plasma density resulting from thermionic emission at the cathode. This behavior indicates that surface temperatures downstream of the cathode, particularly at the end of the cathode surface, may become a critical design constraint for vehicles employing ETC systems.

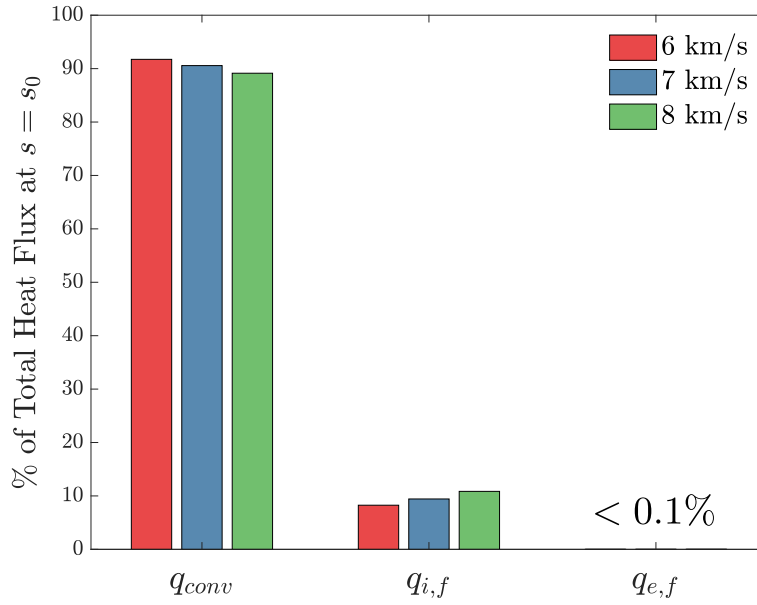
To identify the dominant heating mechanisms in the ETC On configuration, Fig. 6.7(a) presents heat flux components at the stagnation point ( $s_0$  in Fig. 6.2) expressed as a percentage of the total heat flux. The high plasma density in the stagnation region drives significant sheath



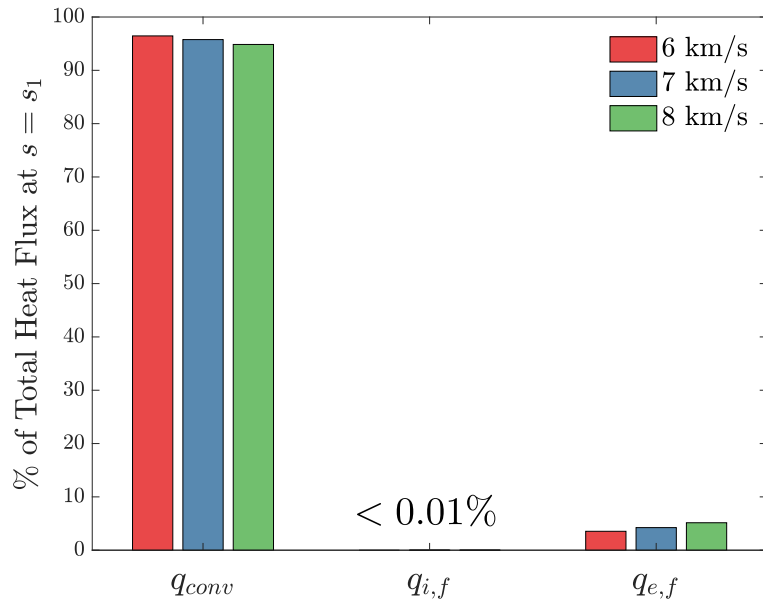
**Figure 6.6:** Comparison of the wall temperature,  $T_w$ , for the ETC On and ETC Off configurations as a function of arc length,  $s$ . Three freestream velocities of 6 km/s, 7 km/s and 8 km/s are investigated.

heating from flowfield ions,  $q_{i,f}$ , on the order of 10% of the total heat flux, while electron sheath heating,  $q_{e,f}$ , is negligible. This behavior is consistent from the dynamics of the ion sheath, which accelerates flowfield ions to the surface while reflecting the majority of flowfield electrons.

Similarly, Fig. 6.7(b) shows the heat flux components at the start of the anode surface ( $s_1$  in Fig. 6.2) where an electron sheath is present. At this location, flowfield electrons are accelerated to the surface, resulting in a small heat flux contribution on the order of a few percent, while flowfield ions are reflected and are negligible. It is noted that  $q_{i,f}$  (Eq. 2.23) does not include the ionization potential, which instead is captured in  $q_{conv}$ .



(a)



(b)

**Figure 6.7:** Percentage contributions to the total heat flux from convective heating,  $q_{conv}$ , sheath heating from flowfield ions,  $q_{i,f}$ , and sheath heating from flowfield electrons,  $q_{e,f}$ . Evaluations are taken at (a) the stagnation point on the cathode surface,  $s = s_0$ , and (b) the anode surface adjacent to the dielectric material,  $s = s_1$ .

### 6.6.2 The Effect of the Bulk Plasma on ETC Systems

Figure 6.8(a), shows contours of the electron number density,  $n_e$ . When ETC is off, the value of  $n_e$  is mostly constant over the cathode surface, with gentle gradients away from the wall. When ETC is active, the near-wall electron density increases sharply and a strong wall-normal gradient forms as electrons are emitted from the cathode surface and diffuse into the flow. Additionally, emission at the cathode is seen to strongly elevate the plasma density downstream of the leading edge and over the anode surface by one to two orders of magnitude. This is a favorable outcome for ETC operation as high plasma densities over the anode increase the amount of available current for the anode to collect.

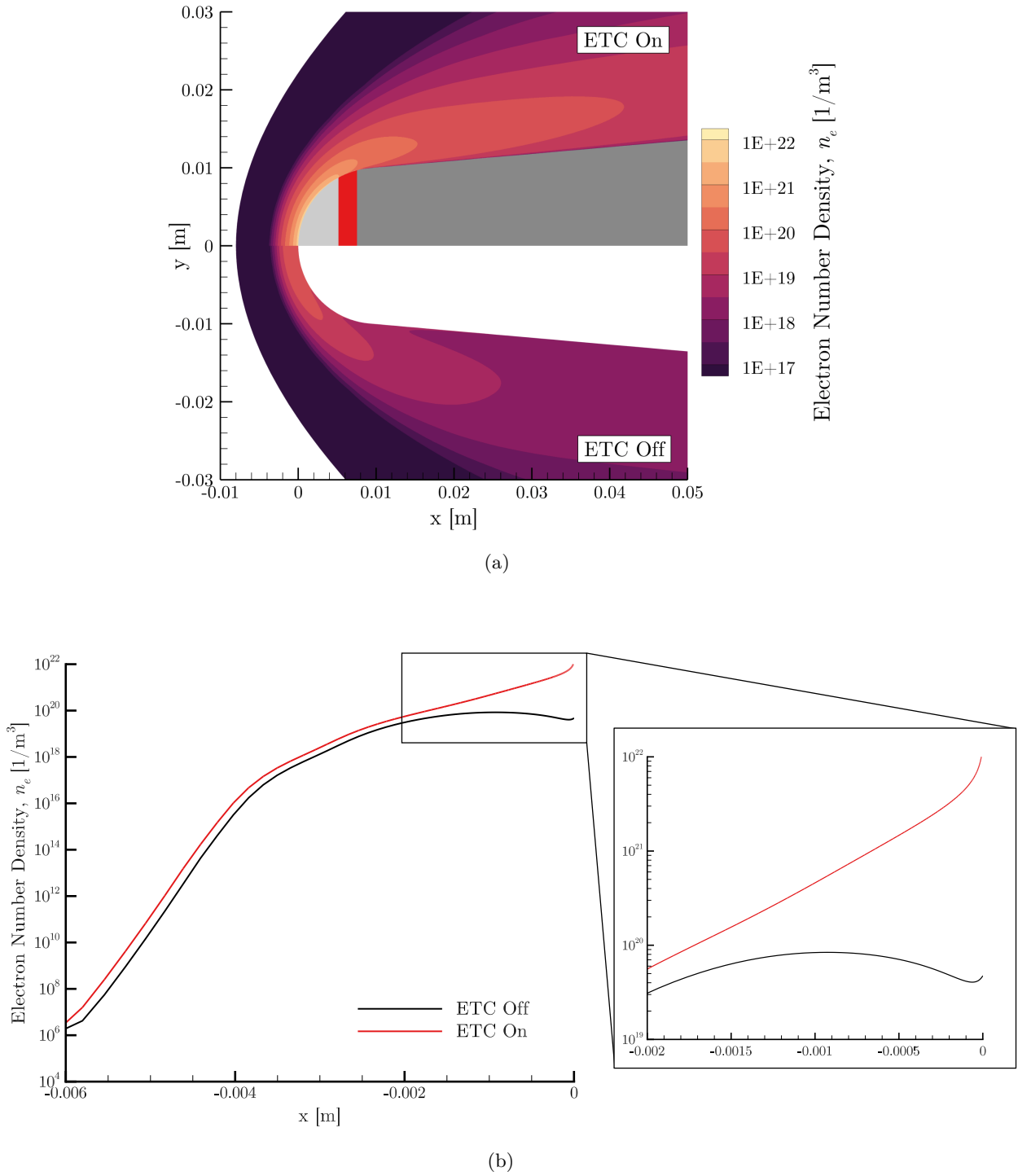
Values of  $n_e$  along the stagnation line are shown in Fig. 6.8(b) for both cases. For the ETC Off case, the near-wall plasma density is on the order of  $1 \times 10^{19} \text{ 1/m}^3$ , but when ETC is on, plasma density at the wall sharply increases to nearly  $1 \times 10^{22} \text{ 1/m}^3$ . The magnitude of ionization in the flowfield is quantified through the ionization fraction,  $\alpha$ , defined as

$$\alpha = \frac{n_e}{n_n + n_i} \quad (6.31)$$

Generally, a plasma is considered to be weakly ionized when  $\alpha < 0.01$  and fully ionized when  $\alpha \approx 1$ .

Figure 6.9(a) shows contours of  $\alpha$  in the flowfield for values above and below 0.01. When ETC is off, the entire flowfield is weakly ionized. When ETC is on, the majority of the flow is weakly ionized, however a small region above the cathode has an ionization fraction above 0.01 due to the large thermionic currents leaving the cathode surface. Figure 6.9(b), which computes curves along the stagnation line, shows the ionization fraction near the stagnation point approaches  $\alpha = 0.054$  when ETC is activated.

Since  $\alpha > 0.01$  near the cathode, the plasma in this region may no longer strictly satisfy the assumptions of a weakly ionized plasma. However, the ionization fraction is not substantially beyond the threshold of a weakly ionized plasma and is confined to a relatively small region of the flowfield. Within this context, the weakly ionized approximation is still expected to provide a



**Figure 6.8:** Comparison of the ETC On and ETC Off cases showing the electron number density,  $n_e$ , (a) in the flowfield and (b) along the stagnation line. Freestream conditions correspond to those in Table 6.2.

reasonable description for the present case.

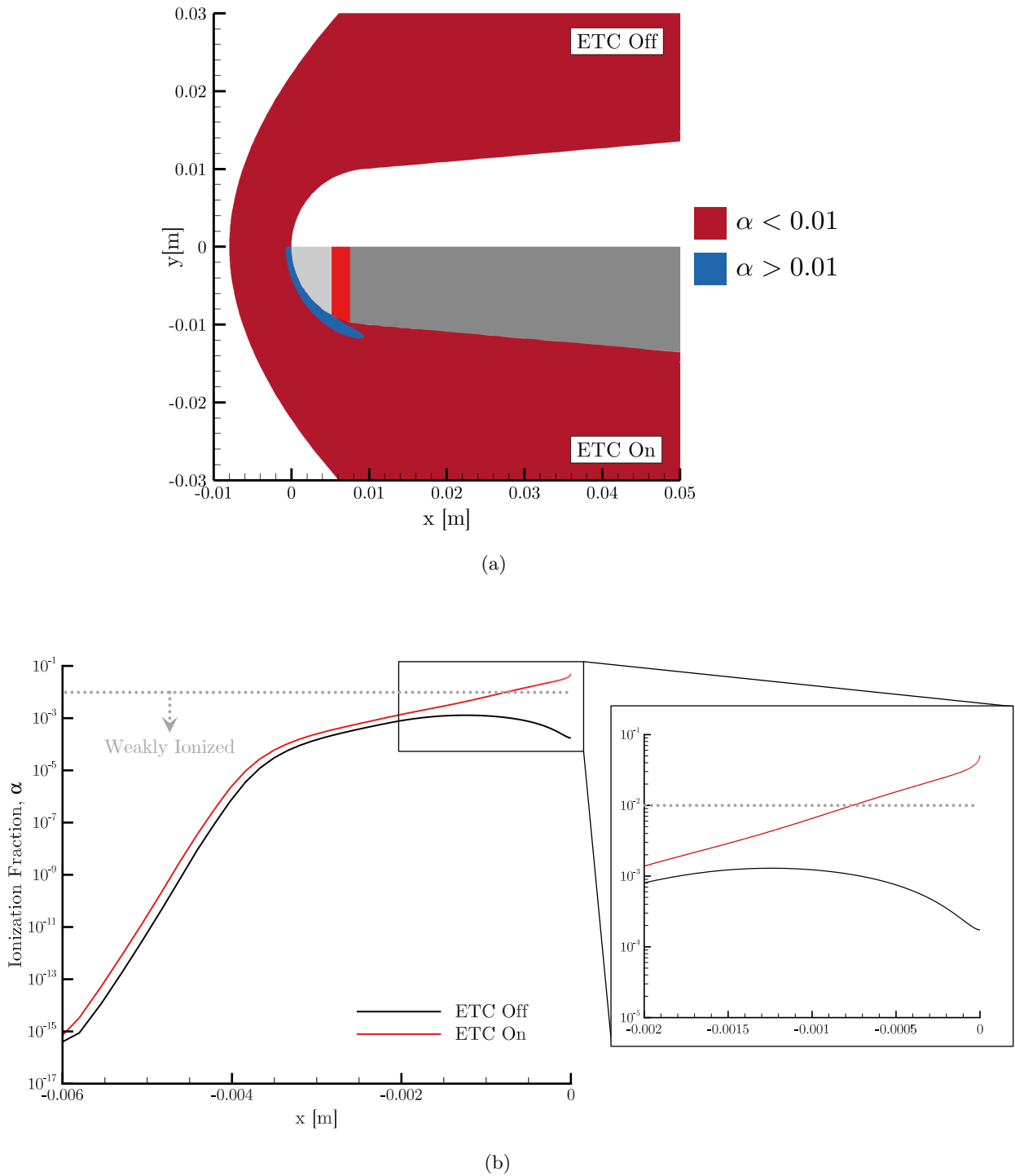
For more emissive materials or flight conditions with more plasma, the ionization fraction may be even higher. This suggests that mindful consideration should be given to the plasma dynamics when modeling thermionic emission, as weakly ionized assumptions may no longer be appropriate. Analysis of ETC systems within a MHD framework, investigations into the diffusion of charged species, similar to Woods [154], and drift-diffusion approximations, are reserved for future work and discussed in Section 7.3.

The present work considers two models for the plasma conductivity, namely those by Chapman & Cowling, and Spitzer & Härm. Figure 6.10 compares the predicted value of  $\sigma$  in the flowfield between both models. It is seen that the location and extent of high conductivity is strongly different between the models. The model by Chapman & Cowling predicts high conductivity near the surface, particularly near the emissive cathode, before dropping rapidly as one approaches the anode. Conversely, the model by Spitzer & Härm predicts highest conductivity near the shock, with regions of high conductivity extending downstream over the anode surface.

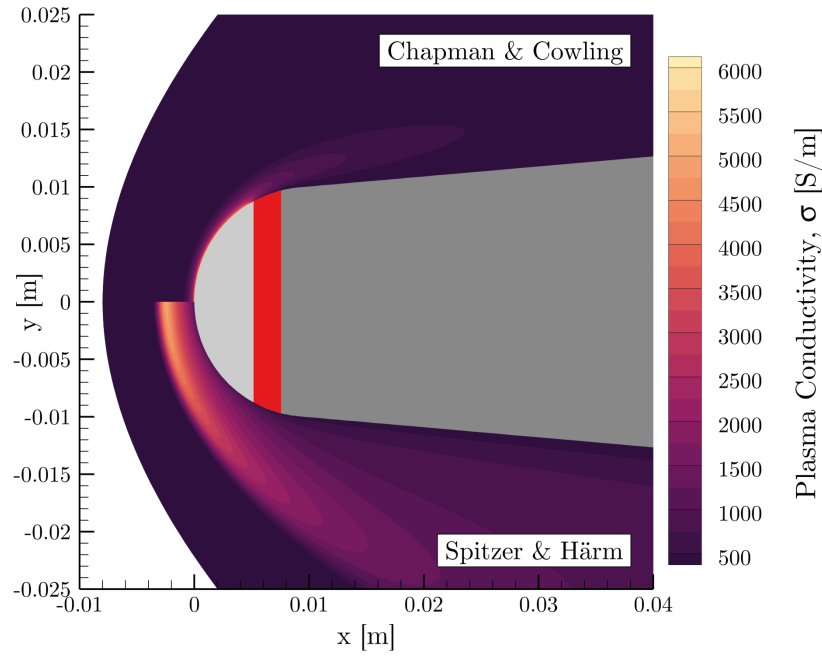
The model differences in plasma conductivity are further highlighted in Fig. 6.11, which shows values of  $\sigma$  along the stagnation line. The order of magnitude of the conductivity is seen to be in agreement with both models predicting a maximum value approximately between 5000 and 6500 S/m. However, Chapman & Cowling predicts the highest conductivity near the emissive surface while Spitzer & Härm predicts a maximum near the shock. This result aligns with prior understanding of the models, as the Chapman & Cowling model is proportional to  $n_e/n_n$  (which is highest near the emissive surface) while the Spitzer & Härm model is proportional to the translational temperature (which is highest at the shock).

The broader spatial distribution of  $\sigma$  in the model by Spitzer & Härm will likely reduce spatial gradients of electrical properties (e.g. plasma potential) in the domain relative to the model by Chapman & Cowling. In fact, Fig. 6.12 shows streamlines of the electric field emanating from the electrodes, where the electric field predicted by Spitzer & Härm is seen to extend farther from the vehicle surface. In addition, the largest potential difference occurs at the stagnation point with





**Figure 6.9:** Comparison of the ETC On and ETC Off cases showing the ionization fraction,  $\alpha$ , (a) in the flowfield and (b) along the stagnation line. Freestream conditions correspond to those in Table 6.2.

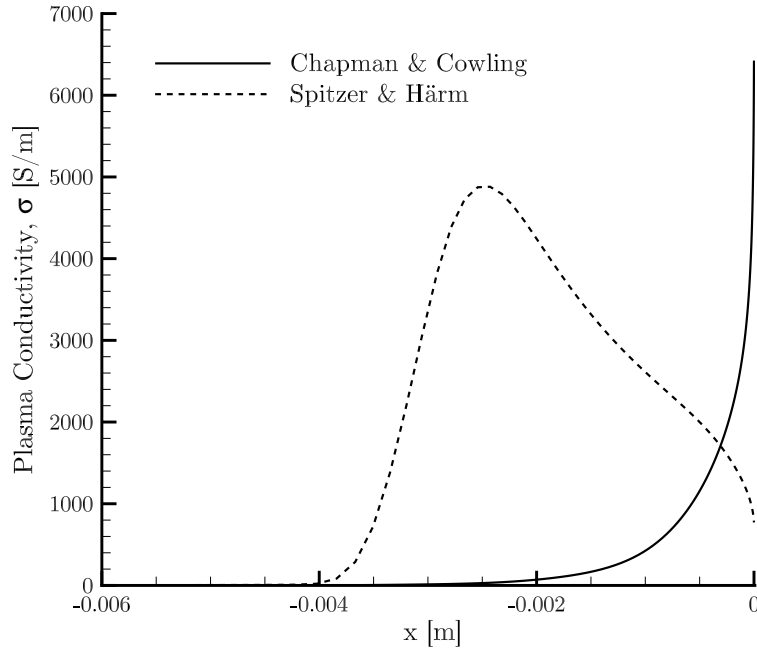


**Figure 6.10:** Comparison of the plasma conductivity,  $\sigma$ , predicted by Chapman & Cowling (Eq. 6.22) and Spitzer & Härm (Eq. 6.23) for the flowfield. Freestream conditions correspond to those in Table 6.2

predicted values of -1.75 V and -1.54 V by Chapman & Cowling and Spitzer & Härm, respectively.

A consequence of conducting current through the bulk plasma is the introduction of thermal energy via Joule heating,  $\mathbf{j} \cdot \mathbf{E}$ . Figure 6.13 shows contours of Joule heating for both conductivity models. Due to the differing distributions of  $\sigma$ , the majority of Joule heating occurs closer in the surface for the Chapman & Cowling model. For both models, the most Joule heating takes place above the cathode and the area of the anode near the dielectric surface. However, the maximum magnitude of Joule heating differs by several orders of magnitude, seen to be  $1.9 \times 10^{10} \text{ J}/(\text{m}^3\text{s})$  and  $7.7 \times 10^8 \text{ J}/(\text{m}^3\text{s})$  for Chapman & Cowling and Spitzer & Härm models, respectively.

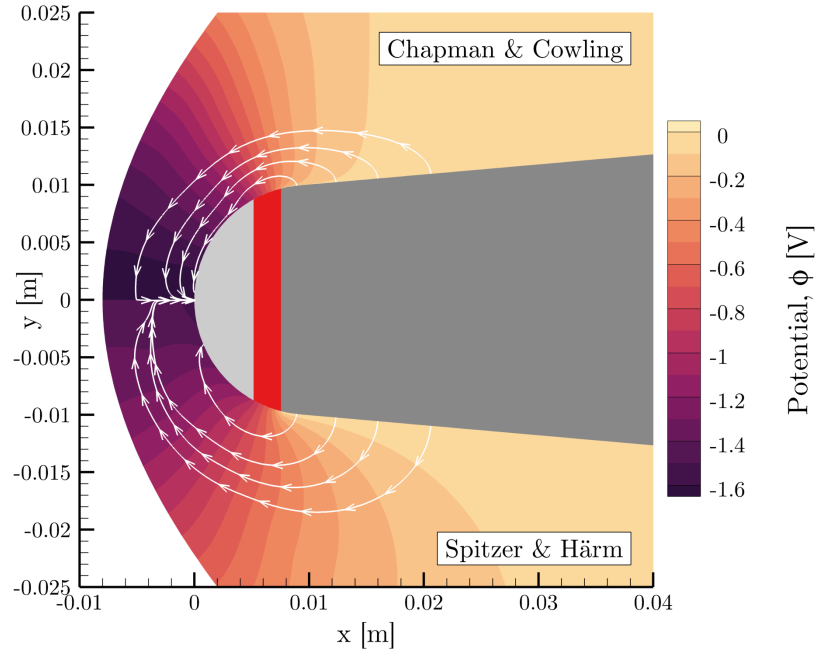
Between both models, there is agreement in the magnitude of the flowfield potential difference with values of  $\phi$  being on the order of the material work function. As the required power of the ETC system is expected to depend on the magnitude of  $\phi$ , this result implies that, for the investigated flight condition, the ETC power source will not drastically be affected by the flowfield.



**Figure 6.11:** Comparison of the plasma conductivity,  $\sigma$ , predicted by Chapman & Cowling (Eq. 6.22) and Spitzer & Härm (Eq. 6.23) along the stagnation line. Freestream conditions correspond to those in Table 6.2

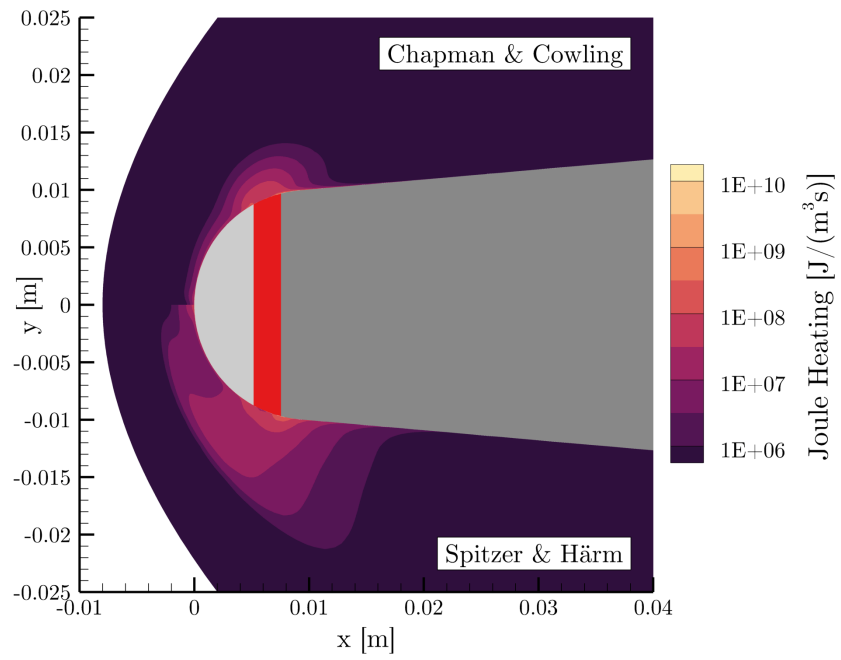
However, the large variation observed in Joule heating introduces uncertainty in the expected flowfield temperatures. This is because large values of Joule heating can lead to electrical breakdown of the plasma and arcing between the electrodes which impart extreme heat fluxes and cause material damage. Importantly, this uncertainty is largely driven by the value and distribution of the plasma conductivity. As the ETC flowfield exhibits both partially ionized and weakly ionized regions, choosing an appropriate model for  $\sigma$  that is suitable for a wide degree of ionization is a suggested focus for future work.

Additionally, it is noted that the present work invokes the assumption of a low magnetic Reynolds number ( $Re_m \ll 1$ ) following previous modeling investigations into ETC. Figure 6.14 revisits this by computing contours of  $Re_m$  for both conductivity models, using a characteristic length equal to the nose radius,  $L = 0.01$  m. Depending on the conductivity model,  $Re_m$  is seen to approach values that are not significantly less than one. This suggests that induced magnetic

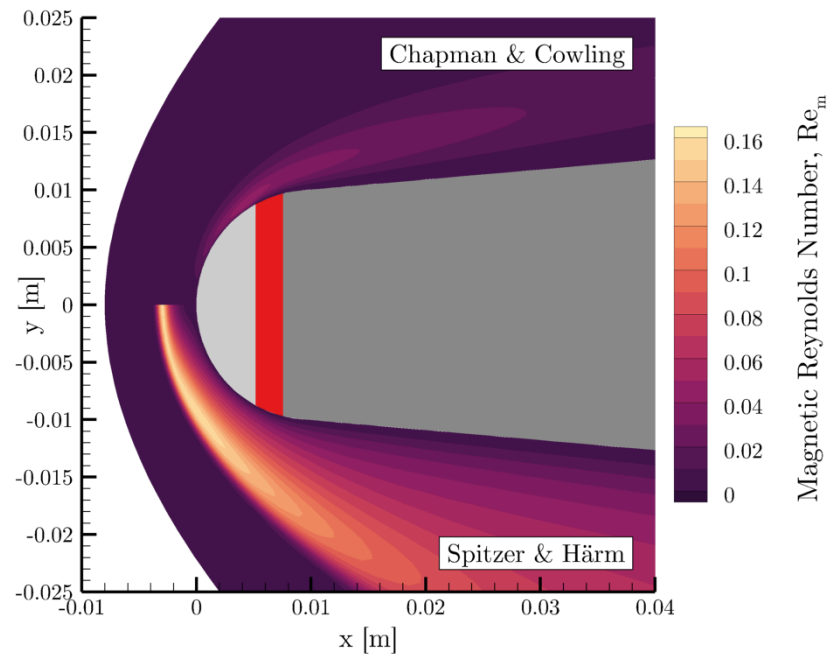


**Figure 6.12:** Comparison of the electric potential  $\phi$ , predicted by Chapman & Cowling (Eq. 6.22) and Spitzer & Härm (Eq. 6.23) for the flowfield. Selected electric field lines are shown in white. Freestream conditions correspond to those in Table 6.2

fields from the ETC system may become non-negligible for higher emissive currents, and that the modeling of ETC may be better suited to a MHD framework. However, it is not expected that the results of the present work will be significantly different with the inclusion of MHD effects.



**Figure 6.13:** Comparison of Joule heating,  $\mathbf{j} \cdot \mathbf{E}$ , predicted by Chapman & Cowling (Eq. 6.22) and Spitzer & Härm (Eq. 6.23) for the flowfield. Freestream conditions correspond to those in Table 6.2



**Figure 6.14:** Comparison of the Magnetic Reynolds Number,  $Re_m$ , predicted by Chapman & Cowling (Eq. 6.22) and Spitzer & Härm (Eq. 6.23) for the flowfield. Freestream conditions correspond to those in Table 6.2

### 6.6.3 Emitted and Collected Currents within the ETC System

To understand if the total current can be conserved within the ETC system, the present work defines the cumulative emitted and collected current,  $I_e(s)$  and  $I_c(s)$ , in Eqs. 6.32 and 6.33. These integrals represent the running total current along the electrode as a function of the arc length,  $s$ , along the electrode, and the total current  $J_{total} = J_{etc} + J_{i,f} - J_{e,f}$ . Importantly, as the geometry in the present work is 2D, this is a spanwise current, and thus, has units of Ampere per meter.

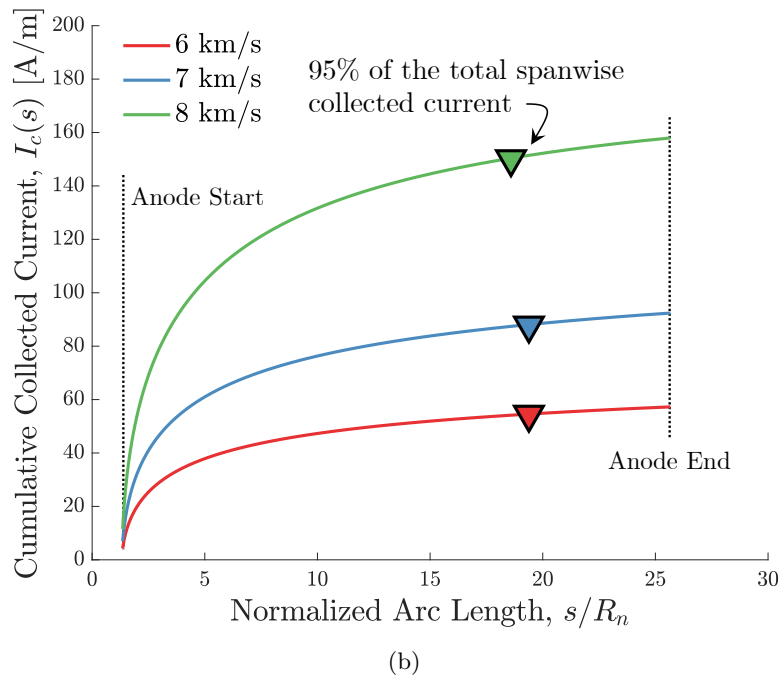
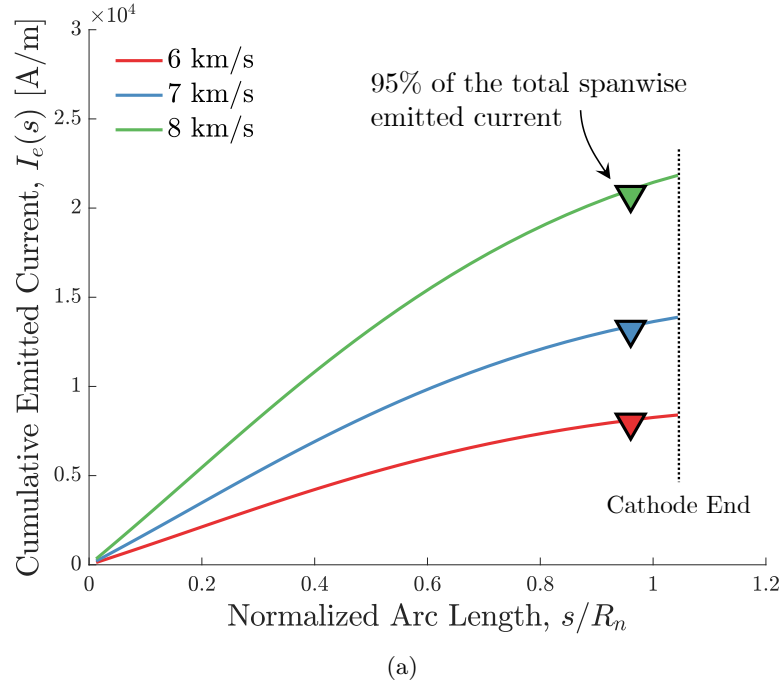
$$I_e(s) = \int_{s_0}^s J_{total} ds \quad s.t. \quad s \in \partial\Omega_c \quad (6.32)$$

$$I_c(s) = \int_{s_1}^s J_{total} ds \quad s.t. \quad s \in \partial\Omega_a \quad (6.33)$$

Figure 6.15(b) shows  $I_e(s)$  over the cathode surface for varied freestream velocities. It is noted that curves of  $I_e(s)$  are mostly linear over the arc length of the cathode. This indicates that the majority of the cathode surface is actively emitting electrons and contributing to cooling the surface. The total spanwise current emitted by the cathode is on the order of  $10^4$  A/m.

Considering the collected current, Fig. 6.15(a) shows curves of  $I_c(s)$  over the anode surface. The curves are noted to take a more logarithmic shape, with the majority of collected current captured within a short distance of the start of the anode. This suggests that it is unnecessary to have the anode surface extend a far distance downstream of the cathode. All collected currents are observed to be on the order of  $10 - 100$  A/m. As drift-diffusion processes are not included in the present work, the total anode current is largely a result of species diffusive transport to the anode sheath boundary.

Table 6.4 summarizes the total spanwise emitted and collected currents for the conditions investigated. In addition the collected percentage of emission is computed, taken as 100 times the ratio of the total collected to emitted current. For all cases, the collected percent of emission does not exceed 1% indicating that the current anode configuration is unable to fuel emission at the cathode. Interestingly, the collected percent of emission does not strongly change between



**Figure 6.15:** (a) Cumulative emitted current over the cathode surface and (b) cumulative collected current over the anode surface. Symbols indicate the arc length location where 95% of the total current has been reached.



flight conditions, suggesting that collection may be dependent on another variable, such as body geometry.

**Table 6.4:** Summarized values of the emitted and collected current for the varied freestream velocities.

| Freestream Velocity<br>[m/s] | Spanwise Emitted Current<br>[A/m] | Spanwise Collected Current<br>[A/m] | Collected Percent of Emission<br>[%] |
|------------------------------|-----------------------------------|-------------------------------------|--------------------------------------|
| 6000                         | 8401                              | 57.25                               | 0.68                                 |
| 7000                         | 13880                             | 92.39                               | 0.67                                 |
| 8000                         | 21840                             | 158.0                               | 0.72                                 |

## 6.7 Conclusion

The objective of this chapter was to quantify the effect of the bulk plasma on the performance of an ETC system. Boundary conditions for an emissive cathode and collecting anode were developed and implemented into a two-temperature hypersonic solver. The boundary conditions were applied to a hypersonic leading edge geometry and flowfields with and without an ETC system were computed. The results addressed a gap in the assessment of ETC by enabling the computation of the bulk plasma potential, locations of Joule heating, as well as emissive and collecting currents for an ETC system.

First, flowfield and wall temperatures of the ETC system were investigated. The presence of thermionic emission at the cathode was observed to increase the gradients of both the translational and vibrational temperatures near the wall. This indicates that accurate modeling of the coupling between the vibrational, electronic, and electron-translational energy modes is important for assessing thermal effects near the cathode surface. Along the surface, ETC cooled the emissive cathode while increasing temperatures over the dielectric and anode regions. In particular, peak wall temperatures shift from the stagnation point to the region immediately downstream of the cathode, suggesting this location may be where maximum allowable temperature constraints on a vehicle become active.

Next, the influence of the ETC system on the bulk plasma properties was quantified. The

emissive cathode was observed to increase the flowfield electron density above the cathode by several orders of magnitude. As electrons are advected away from the cathode, the electron density in the vicinity of the collecting anode is seen to increase. For the chosen value of the cathode work function, the bulk plasma was observed to have ionization fractions in both the weakly ionized and partially ionized regime. Additionally, two models for the plasma conductivity were investigated, namely those by Chapman and Cowling and Spitzer & Härm. In both models, the overall change in potential across the flowfield was predicted to be low, implying that the power draw of the ETC system will not drastically be affected by the flowfield potential difference. The predicted distribution of conductivity surrounding the vehicle was observed to strongly differ between the models, leading to variations in the electric field structure and strength of Joule heating. These differences are expected to increase in magnitude for cases with stronger thermionic emission or higher density plasmas. However, given the level of ionization in the flowfield, the model by Chapman & Cowling is expected to be better suited for the investigated flight condition.

Finally, currents at the cathode and anode were assessed. Emitted current from the cathode substantially exceeds that collected at the anode, indicating that steady-state operation is not viable for the explored condition. As a result, thermionic emission would be expected to decay as the system adjusts. In the present work, the electron current reaching the anode surface is governed by diffusive transport to the sheath boundary. Drift-diffusion processes are not included and may enhance anode collection, representing an important area for future refinement.

## **Chapter 7**

### **Conclusion**

The following concluding chapter summarizes the findings and impact of the research presented in this dissertation. Notable insights regarding the operation of ETC systems for hypersonic vehicles is given, along with a record of the major research contributions. The chapter concludes with recommendations for future work in modeling ETC systems.

#### **7.1 Summary of Dissertation**

Chapter 2 introduced the physics governing thermionic emission into a bulk plasma and the subsequent cooling effect on a surface. This began with a description of the plasma sheath, which naturally forms above surfaces immersed in a plasma to control the flux of charged particles that reach the surface. The dynamics of the plasma sheath were discussed, including the formation of several categories of sheath shapes that modify the total current that traverses the sheath. A particularly important sheath, namely the space-charge-limited (SCL) sheath, which limits the thermionic current and prevents an ETC system from operating at its maximum cooling potential, was described. The assumptions governing the classical expressions for modeling plasma sheaths were then shown to break down at weak surface potentials, motivating the use of modified expressions incorporating kinetic corrections and pre-sheath effects. These improved models provide a more accurate framework for predicting current and heat fluxes in ETC systems and have improved numerical characteristics. The chapter concluded with a compilation of expressions for currents and heat fluxes through the expected types of plasma sheaths surrounding an ETC system.

Chapter 3 leveraged the information presented in the previous chapter to analyze a complete ETC system, including an emissive cathode, collecting anode, and internal voltage source. Employing conservation of current and voltage through the ETC circuit, a one-dimensional model was derived. The model, while neglecting two-way coupling with the bulk plasma, allowed for rapid evaluations of surface cooling and power requirements. A parameter study over varied geometries, cathode materials, internal voltages, and bulk plasma properties, identified system configurations that maximized cooling and efficiency. As ETC systems function by spreading heat from a hot to a cool location (i.e., they are thermodynamically favorable), certain configurations were observed to operate passively without an internal voltage source, or even to generate power. To build confidence in these results, the model was extended to two-dimensions and coupled to a finite element framework for comparison with experimental data. Validation attempts showed quantitative and qualitative agreement, while also providing guidance for improved experimental design.

Chapter 4 investigated the cooling performance and scaling of ETC systems in a hypersonic context. A model for estimating a post-shock environment was created and coupled to the ETC model. Owing to the low computational cost of the ETC model, this enabled numerous evaluations of ETC performance as a function of velocity, altitude, cathode material and geometry. Results identified flight regimes where SCL operation is expected. Outside of SCL regimes, ETC was shown to significantly reduce surface temperatures and expand the feasible operating envelope of hypersonic vehicles. In particular, thermionic cooling was observed to scale strongly with the bulk plasma density, making ETC systems extremely well suited for high flight velocities. Interestingly, two regimes of flight were found to be ideal for ETC systems in that they are free of SCL effects and internal power requirements are expected to be reasonable. These included a low altitude sub-orbital regime, well suited for boost-glide vehicles, and a high altitude super-orbital regime, well suited for reentry or aerocapture missions. In contrast, ETC systems were found to provide inconsequential cooling for velocities below 3-4 km/s.

Chapter 5 showcased the changes in vehicle performance an ETC system could impart on a hypersonic vehicle. Leveraging the ETC system model, an aerothermal database was created

that included thermionic cooling effects. The aerothermal database, paired with aerodynamic parameters of a realistic hypersonic boost-glide platform, was then integrated with a hypersonic trajectory optimizer to explore two example cases. For both cases, the cooling power of the ETC system improved the performance of the boost-glide vehicle by enabling faster and higher altitude initial conditions. In the first mission, this translated to increasing the range of the vehicle by over 200% compared to a case without ETC. In the second mission, this enabled more ballistic geometries (which have larger internal volumes for payloads) to be feasible. Estimated power requirements for the missions suggested that ETC systems are feasible for integration within a realistic vehicle, with masses on the order of 10 kg and volumes below  $0.1 \text{ m}^3$ .

Finally, Chapter 6 analyzed the interactions of an ETC system with a hypersonic flowfield via a computational fluid dynamics (CFD) solver. Boundary conditions, capturing sheath effects, thermionic emission, and electron collection, were derived for a hypersonic solver. Using these boundary conditions, the resulting analysis found that the highest wall temperatures occur immediately downstream of the emissive cathode. This is due to increased catalytic heating owing to the increased plasma density from thermionic emission over the cathode, and likely will be a location where temperature constraints on a material become active. When viewing the distribution of electric potential between electrodes, variation in the potential was observed to be on the order of the material work function, indicating that resistive losses in the bulk plasma are comparable to circuit-level potentials. For the condition considered, this result implied that the bulk plasma has a small impact on overall power system sizing. However, a key performance limitation was identified in that the anode was unable to collect sufficient electrons to complete the circuit within the current modeling framework.

## 7.2 Research Contributions

The notable contributions in this dissertation to the analysis and understanding of ETC systems for hypersonic applications are summarized below:

- **Development and validation of a ETC system model:**

Although several models for emissive cathodes are present in the literature, this work presented a novel approach that captures all components of the ETC system, including the emissive cathode, collecting anode, and internal voltage source. This approach enabled new computations of system-level characteristics (i.e. internal current and power draw) that facilitated comparison to recent experimental literature that provide measurements of the internal current. As of this dissertation, there are few models in the literature that are capable of modeling system-level characteristics, such as the one presented here.

- **Identification of ideal configurations for ETC systems:**

A parameter study of the ETC system identified several configurations that are of potential use to experimentalists, modeling efforts, and applications of ETC. One configuration maximizes thermionic cooling. This configuration is well suited for hypersonic vehicles requiring maximum heat rejection, as well as for experimental studies where large, measurable temperature reductions are needed (e.g. using optical diagnostics of surface temperatures). A second configuration maximizes cooling efficiency. This may be advantageous when available power, either in a flight or experimental context, is limited. A third configuration enables power generation by the circuit, where the ETC system can operate with an external load rather than a power supply. Power generation may be desirable for hypersonic systems with auxiliary electrical demands, or for generating power in order to deploy the ETC system at a later time.

- **Suggestions for future ETC experimental campaigns:**

Validation efforts in the present work yielded several suggestions for improving the design of experimental ETC measurements. It is suggested that future experimental campaigns include a method of measuring the work function of the emissive cathode. The value of the work function strongly drives the thermionic currents in the ETC system. However, the work function can often be dependent on material grain orientation, oxide layers, purity,

and manufacturing processes. Providing a range of expected values aids in quantifying uncertainty in measurements that are being used for model validation. In addition, it is suggested that measurements of the bulk plasma density, particularly near the emissive cathode, are included if possible. The plasma composition in this region determines if the system operates in a SCL or Richardson-limited mode. Determining the correct operational mode, or transitions between modes, is integral to a successful computational model.

Several material characteristics are suggested for use as a cathode material. A high melting point ( $> 1800$  K) enables operation at temperatures where significant thermionic emission can occur. A low material work function ( $< 3.5$  eV) increases the thermionic current for a given temperature, enhancing the thermionic cooling effect. Similarly, a high emissivity ( $> 0.7$ ) improves radiative cooling of the material, which can augment thermionic cooling to enable lower surface temperatures. Finally, materials that are resistant to oxidation can improve the thermionic emission performance of the cathode in environments with oxygen (e.g. air). A leading candidate is hafnium carbide, which has a melting point of 4173 K, a work function with a lower bound of 3.2 eV, and an emissivity of 0.90, in addition to being oxidation resistant [63]. Neglecting oxidation effects, several other materials exhibit favorable thermionic properties, including Lanthanum Hexaboride, barium-oxide impregnated tungsten, and copper-doped mayenite [51, 24, 86].

The material requirements of the anode are less restrictive. The most important parameter is the work function, which will influence the performance of the entire ETC system. To maximize cooling at the cathode, the anode is suggested to have a work function that is large relative to the cathode. Potential materials include tungsten or molybdenum, which have work functions of 4.54 eV and 4.2 eV, respectively, as well as high melting points [24]. An electrical insulator is needed between the electrode materials to prevent shorting. As the insulator is in contact with the hot cathode, it should also be robust to high temperatures. Several suggestions for the insulator material include quartz, alumina ceramic, and

zirconium dioxide, which have melting points of 1918 K, 2323 K, 2963 K, respectively [24].

- **Quantification of ETC performance over broad hypersonic flight regimes:**

A contribution of the present work was to expand the analysis of ETC systems from specific freestream conditions present in previous literature to a broad range of range of hypersonic flight environments. Results from this dissertation identified three distinct ranges of altitudes and velocities integral to the operation of ETC systems, including (1) a SCL regime where thermionic cooling is limited, (2) a power-limited regime where power requirements of an ETC system will likely make the technology infeasible, and (3) a Richardson-limited regime where ideal thermionic cooling can be achieved. The demarcation of these regimes aids in identifying experimental configurations and missions where ETC may be successful.

- **Exploration of improved hypersonic vehicle performance due to ETC:**

Application of an ETC system to a representative hypersonic vehicle quantified, for the first time, the potential performance benefits of ETC. First-order sizing of the ETC system was performed, providing estimates of mass, volume, and power requirements. These results helped to establish ETC as a feasible technology by clearly illustrating both its performance benefits and integration costs, offering actionable insight for hypersonic vehicle and mission designers.

- **Expansion of in-house Computational Fluid Dynamics (CFD) solver capability for modeling ETC systems:**

The CFD solver previously used to study ETC was expanded considerably. These efforts included: (i) the incorporation of electron and ion currents from the freestream which are crucial for quantifying total currents to the surface, (ii) improved sheath-driven heat flux calculations that capture the effects of the pre-sheath, (iii) reformulated electrode boundary conditions that enable the modeling of collecting anode surfaces, and (iv) the introduction of two-way coupling between ETC boundary conditions and the flowfield as opposed to the



previous approach which utilized one-way coupling. In addition, several modifications to the Poisson-solver were implemented for increased numerical stability.

- **Quantification of bulk plasma effects on ETC systems:**

Prior studies modeling the electric field surrounding ETC systems have largely neglected the presence of a collecting anode, and consequently, anode current collection has not been previously quantified. By explicitly including the anode surface, the present work resolves electric field distributions that more closely reflect realistic flight conditions. This led to improved estimates of Joule heating and resistive losses within the bulk plasma. Additionally, the present work provides, for the first time, quantitative predictions of collected current densities at the anode, addressing a key gap in prior ETC analyses.

### 7.3 Recommendations for Future Work

There remain several challenges and unknowns regarding ETC systems that were considered to be out of scope for the present work. These challenges should be addressed in future endeavors in order to continue the advancement of ETC as a potential thermal management technique for hypersonic vehicles.

Critically, advances in modeling ETC systems have outpaced the creation of experimental data. In particular, there is a lack of experimental data regarding thermionic cooling in a hypersonic environment that prevent validation of computational ETC models. Future experimental campaigns are suggested to be performed using an inductively coupled plasma (ICP) torch. ICP torches are well suited for analyzing ETC systems owing to the relatively inexpensive operation, long testing times, and high freestream ionization fractions. Several facilities exist that are capable of modeling ETC systems, including new facilities specifically designed for researching MHD effects in hypersonic environments at The University of Colorado Boulder [4, 117]. In addition, to isolate the effect of thermionic cooling from material oxidation effects, it is recommended to first perform experiments in a oxygen-free environment, such as argon or nitrogen.

From a computational standpoint, it is suggested that future researchers consider the following topics:

- **Modeling of surface chemistry at electrode surfaces:**

The present work assumes that all thermionic materials have a constant material work function. This assumption was made in order to isolate the ideal performance of ETC systems and identify if the technology is appropriate for continued research. However, surface chemistry and material response will influence the effectiveness of ETC systems and should be considered in future modeling attempts.

Low work function materials, which are desired in that they enable large thermionic currents, are often achieved through the deposition of a thin layer of an electropositive material on the bulk material surface. However, cathode poisoning via the absorption of electronegative gases or vapors on the surface will increase the work function of the material and cause a decrease in emission [71, 45, 44]. To expand the range of materials suitable for ETC applications, continued research into poisoning-resistant cathodes is needed. Recent work has identified promising candidate materials, including cerium hexaboride, which has properties similar to  $\text{LaB}_6$  but improved oxidation resistance [153, 34].

It is recommended that future work in modeling thermionic emission and ETC systems incorporates the chemical response of electrode surfaces in the computation of the material work function. Environment-dependent values of the work function will better estimate the thermionic currents that are achievable in atmospheric conditions and will likely change the expected flight regimes where ETC may be feasible.

- **Improved modeling of plasma conductivity:**

The present work investigated two models for the plasma conductivity when modeling the bulk plasma surrounding an ETC system. Importantly, differences in the models yielded starkly different distributions of the electric potential and locations of Joule heating. These

differences are expected to grow when considering thermionic currents larger than those covered in this dissertation. For large values of Joule heating, this may lead to electrical breakdown of the plasma and arcing between the electrodes, both of which are undesirable when operating an ETC system.

To better predict the influence of Joule heating, future studies should consider alternate methods of computing the plasma conductivity. Given that the presence of both weakly and ionized plasmas were observed to surround the ETC system, future conductivity models should be accurate over a wide range of ionization levels, temperatures and densities. Increased accuracy in the value of the conductivity will directly improve the confidence in modeling ETC systems in hypersonic regimes.

- **Integration of ETC boundary condition into a Magnetohydrodynamics (MHD) framework:**

The plasma modeling presented in Chapter 6 relied on the assumption of a low magnetic Reynolds number where  $Re_m \ll 1$ . This assumption led to the removal of the magnetic field from the source terms present in the hypersonic solver. While the conditions explored in the present work saw values of  $Re_m < 1$ , depending on the value of the plasma conductivity they were not always  $Re_m \ll 1$ . This indicates that induced magnetic fields from the plasma may become non-negligible under some conditions when modeling ETC systems. An improvement future researchers should consider is the implementation of the derived ETC boundary conditions into a MHD framework. Such a framework allows for the inclusion of the induced magnetic field and may be a more robust modeling approach when considering ETC systems with higher thermionic currents.

- **Improved sheath modeling:** The sheath models in the present work were found to be sufficient for quantifying the ideal performance of ETC systems. However, several improvements to the sheath models are suggested to better understand the role of the plasma sheath in the operation of ETC systems.

Sheath thickness is expected to vary with wall potential, a phenomenon known as sheath expansion. This effect increases the effective electrode surface area and, consequently, alters the current densities experienced by the surface [144]. Additionally, within the pre-sheath, collisions with neutral species introduce a drag force on charged particles accelerating toward the wall. This interaction modifies the potential distribution across the pre-sheath and can alter its size [16]. Under certain conditions, the pre-sheath may extend significantly into the flowfield, influencing the local plasma properties. The inclusion of both of these phenomena in future research endeavors would help build confidence in sheath models for hypersonic applications.

- **Inclusion of flowfield turbulence:**

The effects of turbulence in the hypersonic flow on ETC systems were not considered in this work. Turbulent flow is expected to enhance near-wall mixing, which may alter plasma density over electrode surfaces and influence sheath behavior. This is particularly relevant for the collecting anode, where insufficient current collection was observed, in that increased turbulent transport may enhance electron collection at the anode. Additionally, it is unclear if thermionic emission at the cathode could trip the boundary layer and cause an early transition to turbulence. Incorporating turbulence modeling into future analyses would provide a more complete understanding of the physically relevant phenomena at hand.

- **Time-varying models of thermionic emission:** The ETC circuits modeled in the present work were assumed to be in a direct-current configuration (i.e. current is constant and in one direction for a given configuration). A natural extension is the introduction of time-varying voltage sources, such as pulsed or square-wave inputs. The oscillations in the voltage source may enable net cooling by the ETC system while reducing the time-averaged power consumption. In addition, given that certain configurations can generate power, it may be possible to alternate between cooling and power-generating modes within a single

system. Exploration of the benefits of these configurations could expand the efficiency of ETC systems and build motivation for continued study.

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## Appendix A

### Fluid Equations for the Plasma Pre-Sheath

To derive an expression for the ion velocity,  $V_i$ , while incorporating kinetic effects, Loizu et al. [84] considered fluid equations for steady-state conservation of mass for ions and electrons, and conservation of momentum for ions within the pre-sheath. These expressions are presented in Eqs. A.1 through A.3, where  $S_{pi}$ ,  $S_{pe}$ , and  $S_{mi}$  are source terms for ions, electrons, and ion momentum.

$$n_i \frac{\partial V_i}{\partial x} + V_i \frac{\partial n_i}{\partial x} = S_{pi} \quad (\text{A.1})$$

$$n_e \frac{\partial V_e}{\partial x} + V_e \frac{\partial n_e}{\partial x} = S_{pe} \quad (\text{A.2})$$

$$m_i n_i V_i \frac{\partial V_i}{\partial x} = -en_i \frac{\partial \phi}{\partial x} - \frac{\partial}{\partial x} (n_i k_b T_i) + S_{mi} \quad (\text{A.3})$$

Loizu et al. [84] argue that, at the pre-sheath-sheath interface, gradients are much larger compared to source terms, leading to the condition that setting the determinant of Eqs. A.1 through A.3 equal to zero for  $S_{pi} = S_{pe} = S_{mi} = 0$  corresponds to the sheath entrance. Before solving this system, we note the third term in Eq. A.3 represents an ion pressure component. If the IVDF at the sheath edge is known, this term can be extracted via moments of the IVDF. However, the IVDF can be difficult to know a priori as it is strongly dependent on the acceleration, rate of ionization, and charge-exchange collisions in the pre-sheath. To close the system, the IVDF is represented via an equation of state where ions are treated as an ideal gas and assume gradients in pressure, density, and temperature are parallel. This approach has been analyzed with particle-in-cell approaches and found to be a reasonable approximation [83].

## Appendix B

### Radiative and Thermionic Properties of Selected Materials

**Table B.1:** Thermionic and radiative characteristics of selected high-temperature materials.

| Material                                                       | Work Function, WF [eV] | Melting Point [K] | Emissivity | Ref.          |
|----------------------------------------------------------------|------------------------|-------------------|------------|---------------|
| Tungsten                                                       | 4.54                   | 3695              | 0.22       | [24]          |
| Lanthanum Hexaboride ( $\text{LaB}_6$ )                        | 2.65                   | 2480              | 0.80       | [24]          |
| Graphite                                                       | 4.3-4.7                | 3823              | 0.75       | [120, 24]     |
| Hafnium Carbide ( $\text{HfC}$ )                               | 3.2-4.9                | 4173              | 0.90       | [87, 149, 24] |
| Nitrogen-Incorporated Ultra<br>Nanocrystalline Diamond (NUNCD) | 1.34                   | 3174              | 0.75       | [77]          |