Solar spectrum and solar energy harvesting

As we discussed in class, the solar spectrum at the Earth's surface originates from the solar black-body radiation and is modified by absorption from various gases in the atmosphere. In this problem, your task is to estimate how much of the solar spectrum a given material can absorb.

For simplicity, let's ignore the atmospheric absorption, and treat sunlight as black-body radiation from a T=5800 K source. You can use the provided *Mathematica* notebook to get started.

The radiant energy density between λ and λ + d λ can be described by Planck distribution law for blackbody radiation:

$$\rho_{\lambda}(T)d\lambda = \frac{8\pi hc}{\lambda^{5}} * \frac{d\lambda}{Exp\left(\frac{hc}{\lambda k_{B}T}\right) - 1}$$

The constants h, c, and k_B have their usual meanings.

- (a) What are the units of $\rho_{\lambda}(T)d\lambda$?
- (b) Plot $\rho_{\lambda}(T)$ for 5800 K black body radiation. Which wavelength corresponds to the maximum radiant energy density? There are multiple ways to get the answer and *Mathematica* can help.
- (c) Single-crystalline silicon, the most commonly used light-absorbing material in photovoltaics, can absorb photons with energies above 1.1 eV. What *fraction of solar energy* can be absorbed by single-crystalline silicon? You will likely need to use a *Mathematica* trick and we can help you with this.
- (d) Sometimes we are interested in what *fraction of solar photons* can be absorbed by a material. Write down the expression for solar photon density (units of photons/m³) (Hint: remember that each photon has the energy of hc/λ). What wavelength corresponds to maximum photon density for the 5800 K black-body spectrum? How does this value compare to your answer in (a)? Does this make sense?
- (e) What is the fraction of solar photons that can be absorbed by single-crystalline silicon?