Analogical Reasoning: A New Look at an Old Problem

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Introduction

Because many of the problems we encounter in life are well within our ability to solve, we often fail to observe how it is that we are solving those problems. Let's take for example roasting a chicken. We have made the stuffing and have prepared the chicken, but what if we do not wish the stuffing to fall out? Well, if we know how to sew, then we might derive an analogous solution from our sewing experience. In order to fix a hole, we need to take a needle and thread and sew the hole shut. So, an analogous solution to the stuffing problem would be to take some string and poke that string through the skin back and forth across the opening until it is held shut. Thus, it is entirely possible that people perform analogical reasoning for problems that they encounter every day.

But, what is analogical reasoning, what are the problems people have trying to reason analogically, and what are the processes involved? This paper will clarify the available psychological research on these issues.

Motivation and logic of examining analogical reasoning

When people encounter a new problem, they can use previous knowledge to help solve that new problem. Altering a previous problem solving episode to fit a new problem solving context is often called analogical reasoning. When the new problem context is in a different domain than the familiar problem, successfully performing analogical reasoning is particularly difficult. Sometimes the level of knowledge that a person has in the known domain may affect their ability to solve the problem. In addition, since across domain analogical reasoning is often difficult, people would like to remember their solution to that problem or learn from that problem solving episode. Therefore, the ability to perform analogical reasoning can indicate the level of knowledge achieved in the familiar domain, because, not only must the solver remember an applicable previous problem but also know how to apply that familiar problem solution to the new problem (Sternberg, 1988). Thus, learning and problem solving are intimately related.

In studying learning, researchers have held that people rely on previous knowledge whenever they can (Anderson, 1983; Carbonell, 1983; Gentner & Landers, 1985; Rumelhart & Norman, 1981; Wason & Johnson-Laird, 1972). In fact, some people would argue that every situation you find yourself in is a unique one (as in producing language) and that you constantly need to produce mappings from your previous knowledge to the new task (R. Young private communication). The use of previous knowledge is often referred to as transfer of cognitive skill. When the transfer involves two domains that are quite different or different on the surface but require the same underlying problem solving solution path, that problem solving is called analogical reasoning.

In the psychological literature, analogical reasoning research across domains has typically presented subjects with a story and solution in one domain and a problem to solve in another domain. The story problem (source) that is usually used by Gick and Holyoak (1980, 1983) is about an army general who is trying to attack an evil ruler. He must send small groups of troops simultaneously down several roads which lead to the castle. The other problem (target) given later is a medical problem in which the solver is a doctor who must save a patient by destroying a tumor (Duncker, 1945). The analogous solution is that rays must be sent in small doses simultaneously at the tumor. Subjects are typically familiar with the source domain and task of memorizing and are unfamiliar with the target domain in which they are asked to solve a problem. The noticing of the analogy and the solution of the analogous problem (target) may be difficult. However, if the solution resulting from the use of analogical reasoning is correct, then the student can remember that solution for direct application at a later presentation of the problem. Here, again, analogical reasoning can lead to knowledge acquisition.

In fact, Anderson (1983) hypothesized that new knowledge is often primarily acquired through analogical reasoning. He based his ideas on the fact that most textbooks contain examples interspersed with text and that instructors often use examples when teaching. Students often will refer to those examples when trying to solve problems (Anderson, Farrell, & Sauers, 1984; Pirolli & Anderson, 1985). In addition to the use of examples, Gentner and Gentner (1983) point out that instructors often use models in known domains to explain information in new domains (such as an atom is like a solar system). Students can draw on models taught to solve problems. However, an atom and a solar system are not exactly the same and there may be unimportant aspects of the atom that should not transfer to the solar system model. These examples illustrate the importance of analogical reasoning and indicate that the strengths and weakness of analogical reasoning that should be studied in order to provide insight into how information is acquired both within a single domain and across domains.

The basic process of analogical reasoning involves finding a problem (source) that is similar to the problem you need to solve (target) and mapping the solution of that source problem onto the target problem. Certain assumptions are made about what kind of knowledge needs to exist in order for analogical reasoning to occur successfully. The person must have a source problem to retrieve from memory and that source problem must be retrieved successfully. Assuming the source is retrieved, it needs to be complete in order for the mapping to take place. For example, the person must remember that an atom is like a solar system and, in addition, that electrons revolve around the nucleus.

Although retrieval of a source analog is very important to analogical reasoning, the representation of the problem may be even more critical for success. Three types of representation are: simple sequences, general methods, and mental models (Carroll, Olson, & Anderson, 1987). An example of a simple sequence to print a document would be: Type print, document name, and return. Simple sequences are rote actions which people learn in response to some stimuli. They are often tied to a specific situation and are often performed unconsciously. On the other hand, general methods are often performed in service of a goal. Although one could argue that a goal is always involved, general methods usually involve several layers of goals and selection between or combination of possible simple sequences (Card, Moran, & Newell, 1983). The idea is that the person recognizes certain properties of the situation and rather than automatically performing something, the person selects among the sets of possible actions (combines groups of them together).

If a person has a known procedure which matches the current problem solving situation, then that procedure would apply. What happens when there are no directly matching cues? These general methods are procedurally oriented and do not have any explanation attached about the specific steps within the sequences. To respond to this problem, researchers in human computer interaction generally refer to the knowledge about the function of steps as a mental model. Even though pure procedures do not have explanations or annotations, mental models can contain this additional information. Examples of this information include "how does this work" or "to what function does this apply". Therefore, this information can allow a person to reason about procedures and this ability to reason using existing knowledge is critical to analogical reasoning.

Although in theory we can comment on analogical reasoning, processes and representations, we need to be able to support theory with experimentation. The processes of analogical reasoning have been studied extensively. Because we often use analogical reasoning to solve problems in real life, one would expect that the phenomenon could be reproduced and studied in the laboratory. However, research in the field has indicated that people often fail to use that previous knowledge across domains in laboratory experiments (Gentner & Landers, 1985; Gick & Holyoak, 1980, 1983). The literature on analogical reasoning reviewed in the next section generally shows only partial success at demonstrating the phenomenon that previous knowledge can be used across domains. This lack of success may be in part due to attempts on the part of investigators to examine one isolated part of the process; however, the parts of the process may be interactive and not completely isolatable (even though they may be identifiable).

If one focuses only on simple changes to these experiments such as using different problems or different domains, there is still some doubt that the probability of use and success of applying analogical reasoning would increase greatly. On the other hand, in studies of human and computer interaction (HCI), researchers have been able to demonstrate large amounts of positive transfer (Bovair, Kieras, & Polson, 1990). One could argue that this positive transfer has been primarily the study of common elements (similarities in surface structure) and not of deep structures. Also, transfer between surface structure elements which could be equated to common elements has been demonstrated in the literature. But, these critics would fail to note that positive transfer has been found even between two quite dissimilar text editors which indicates some deeper structure transfer (Singley & Anderson, 1989). Thus, HCI may be a good vehicle to recreate the phenomena of analogical transfer in the laboratory.

Overview of paper

In summary, far transfer across domains, can best be studied by examining analogical reasoning and the processes of analogical reasoning. This paper will begin with a definition of analogical reasoning and a description of the research performed in the field: within and across domains. Then, a discussion of the processes of analogical reasoning will be presented. Since these processes depend upon representation, mental models will be discussed at length and compared with proposed models of analogical reasoning. This paper will demonstrate that the key issues for eliciting analogical reasoning may be the types of tasks used to study analogical reasoning and the knowledge level of the subjects in the known domain.

Analogical Reasoning

Defining Analogical Reasoning

Analogical reasoning and problem solving

When confronted with a novel problem, problem solvers can choose to start afresh rather than applying their previous knowledge. There are many possible problem solving heuristics that are available and some of these heuristics are means ends analysis (Newell & Simon, 1972), breaking a problem into sub-problems (Egan & Greeno, 1974), working forward/backward (Duncker, 1945), planning (Hayes-Roth & Hayes-Roth, 1979; Larkin, 1983) and generating solutions and then testing them (Burke, 1970; Dominowski, 1981). In addition to these heuristics, problem solvers have their previous experience to guide their problem solving. The use of previous problem solutions to solve new problems is often defined as analogical reasoning. Analogical reasoning depends on the solver recognizing a similar situation in a new problem to an old one, identifying similarities between the old situation and new situation, mapping those features and then adapting the correct procedure from the old problem to solve the new one (Gick & Holyoak, 1980, 1983). Two initial problems with the literature on analogical reasoning are identifying analogical reasoning as opposed to other types of problem solving and identifying the current paradigms within analogical reasoning.

Identifying analogical reasoning

Although studies have been done on many of these problem solving heuristics individually, in real life, people may use many different combinations of these methods to solve a problem (Newell & Simon, 1972). The use of analogy, which depends so much on previous knowledge or previous exposure, would probably be distinguishable from the other types of problem solving by examining verbal protocols (Ericsson & Simon, 1984). In addition, examining streams of action data (keystroke, mouse movement) might also give an indication as to what the subject is doing. This is not to say that analogical reasoning can always be isolated; but, it is being proposed that of the heuristics mentioned previously, we should be able to identify when analogy is happening as opposed to something else. Although identifiability is a problem, another major problem in the field is that many different paradigms have been used to research analogical reasoning but they may not all actually require analogical reasoning.

Fitting the current paradigms into a definition of analogical reasoning

Analogy problems are most commonly recognized in the basic form, A:B::C:D (A is to B, as C is to D) from standardized tests. Solution to such problems usually proceeds by finding the relationships between A and B and A and C. Then, a mapping is made between these relationships to produce D. These problems have been studied extensively by Sternberg (1977) and are widely accepted as indicating that people are engaging in analogical reasoning when posed with A:B::C:? problems. However, not only do these problems fail to resemble real-life problems, they also do not even approach the complexity of other problems used in lab studies (Hayes & Simon, 1977; Reed, Ernst, & Banerji, 1974). In fact, by giving people, A:B::C:? problems, subjects are set up to solve the problem using analogy; however, in most studies, recognizing that something is analogous to something else is often the most difficult part (Gick & Holyoak, 1983).

A subset of the basic form of an analogy is the metaphor. Metaphors are used in speech to convey similarity between items. If we remove B and D, we arrive at A::C which is illustrated by the example metaphor, "A cigarette is like a time bomb" (Gentner, 1989). Thus, a subset of the basic processes for analogical reasoning should be evident in a metaphor.

Transforming the basic form of A:B::C:D, we can derive the forms of analogy used in most studies. For example, we can substitute story problems and solutions and derive the resulting formulation: story problem: solution:: medical problem: unknown solution. This form was used by Gick and Holyoak (1980, 1983) in a series of studies that are the starting point for most studies in this field. Basically, if we think about domains represented by A:B and C:D with each of these domains consisting of objects and relations, analogy is the mapping between the parts, objects, and relations of one domain onto another domain (Gentner & Landers, 1985; Quinn, 1989). Studies which use this definition of analogy will be described in detail in a following section. A:B is typically called the source or base (problem) and C:D is called the target problem.

We can also substitute examples into the equation and derive the resulting formulation: example problem: example solution:: quiz problem: unknown solution. Learning by example differs from learning by analogy because the examples used are generally in the same domain and have the same underlying structure as the target problems. People generally choose learning by examples over other methods. Although they are often more successful at learning by example than learning by analogy, learning by example is often seen as using related processes (Anderson et al., 1984; Neves, 1981; Pirolli & Anderson, 1985; VanLehn, 1983). Further, as with the basic standardized test form, examples (such as those found in textbooks) are easily recognized by students as items that would be analogous to target items. If the examples are not accessible to the students or if the student forgets that an example is relevant, the situation begins to resemble the studies of Gick and Holyoak (1980, 1983).

On the other hand, there are other forms which would require transformations farther away from the standardized test form such as discovery. We can generate discovery by requiring that A:B be very distant from C:D. This distance can be illustrated by drawing A:B from a domain using a relationship which has never been recognized before. Discovery is therefore very far from the original recognized standardized test form and may actually require more components to the analogical process, such as intermediate analogical forms.

How analogical reasoning will be covered in this paper

So, depending on whether analogical reasoning is defined in a broad sense and includes the far transforms of the standardized problem or only in a narrow sense to consider the minimal transforms (substituting story problems), very different models and experimental results may result. This section will focus on analogy in its many forms as defined above using Gick and Holyoak as a basis of comparison since their experiments are based on between (across) domain problems. In addition, for the purposes of this paper, learning by examples will be included because some useful information can be gained by looking at situations where analogical reasoning is easier to access and also because taking examples that people do not have physical access to at the time of experiment can be considered similar to across domain studies; however, the reader is asked to remember the caveats concerned with such an inclusion. The next two sections will describe first within domain analogies and then between domain analogies.

Within domain: Learning by example

Anderson: Studies in learning by examples

When students learn, they have several options open to them. They can use conceptual information found in a text book. They can use information presented in class. In addition, they can use examples. Some researchers believe that the latter is used primarily when no explicit procedure is taught (Anderson et al., 1984; Anderson, Pirolli & Farrell, 1988). For this reason, Anderson and colleagues have concentrated on learning by examples.

In their experiments, they presented three problems to a subject. A subject solution protocol (listing of actions) is presented in the paper and it is looks as if a verbal protocol was also taken, although it is not explicitly stated. An example problem in the book, Let's Talk LISP (Siklossy, 1976), was provided which was the starting point for the first problem. For the first problem, the resulting protocols indicate that the subject used the analogical reasoning process by trying to map the example problem onto the test problem. In particular, the protocol includes a section where the subject attempted to incorporate into her solution an unusual part of the example to the test problem, although that part is not needed for test problem solution. Thus, the mapping should have utilized other generalized LISP knowledge in order to avoid including this unnecessary part. The third problem provides additional evidence that other LISP knowledge mediates the analogical process when learning by examples. In that problem, the subject is required to utilize previous knowledge to generate from examples another example in order to solve the problem. However, if the LISP knowledge itself was new, as in this case, the solution to the problem would be difficult to obtain directly. For this subject in particular, solution required a hint from the tutor. This experiment demonstrated that learning from examples is mediated by the analogical reasoning process initially.

In summary, the work of Anderson and colleagues demonstrates that analogical reasoning from examples can occur and is the principle mechanism used when solving a new problem. They have also shown that learning from examples is very successful. However, learning from examples can be difficult if the student must generate an example from previous examples presented before solving the new problem. The student will often fail to realize that they need to do this and this failure is usually due to unfamiliar aspects or an incomplete understanding of the domain under study. Clearly knowing that one must generate other examples first from the given examples before one can solve the problem directly is a difficult realization in itself, though, and may be easier as one gains experience.

Ross and remindings

Ross, has studied extensively learning by example with a focus on retrieval. When solving problems, people are reminded of previous examples (Ross, 1984). In particular, they are reminded of examples that are similar to the problem on which they are currently working. Two types of reminding might occur: principle-cuing or example-analogy use. In principle-cuing, the previous example helps a student to remember the formula or concept needed to solve the current problem, but the example itself is not used. On the other hand, for example-analogy use, the learner will always try to use the previous example, even when the formula/concept is presented during problem solving. The idea is that the previous example and the formula/concept are tied together and not understood independently. The main differences between the two are found in their use. In the principle-cuing case, students only use the example once - as a cue; whereas, in the example-analogy case, students use the example throughout problem solving. Ross (1987) designed a series of experiments to test the difference between these ideas.

How examples are used (Ross,1987). His first experiment presented subjects with an example which includes both the formula and the concept. Problems were given in which similar objects either played the same role as in the examples or different roles. Two control conditions were included in which the objects had no corresponding role and in which the objects were not similar. Subjects were asked to use the formulas in the examples to work out the example problems. Then, they were given test problems for which the formula was provided. Results indicated that subjects used the examples in an example-analogy sense, but not in a principle-cue sense. That is, subjects tended to use the examples to provide information about how the objects in the test problems should be placed into the formulas and not as just a reminder to the concept.

In a second experiment, two control conditions were used. The test problems did not provide a formula. The control condition had an unrelated story line and object correspondence. This second experiment was done in two parts. The first part used problems in which the principles could be easily confused with each other (looked similar to each other) and the second part used problems identical to the first experiment in which the principles were distinct. For the first part, results indicated that subjects in the first control condition, where the story lines were the same but there was no object information, were more able to generate answers to the test problem which showed that they were accessing the example formula. When the formula was provided on the second round of tests, there was no difference between the groups. This second result replicated the first experiment.

Thus, aspects of similarity between two situations do play a role in providing principle-cuing. On the other hand, for the second part, results indicated that there was very little difference between the two groups when the principles involved were distinct from each other. It is possible therefore that with distinct principles the control condition would look more similar to the problem it is related to than other control examples and other problems. This result is similar to memory studies in which both absolute similarity and relative similarity determine what will be accessed from memory. Taken together, the two parts of this second experiment indicated that relative similarity may be the most important variable in access.

To summarize, subjects were able to draw analogies to previous examples even when those examples were not immediately available. Further, object correspondences were more important than story lines except when the principles being learned could be easily confused. What is most interesting about these experiments is that subjects in the control condition, where both story line and object correspondences were provided, produced correct responses in larger numbers than previous studies (and sometimes even larger than the other control condition). This would indicate relative similarity and/or general schemas of the situation may be important for transfer.

Ross indicated that the success of his experiment may be due to the fact that studies done by Carbonell (1983), Gentner (1983) and Holyoak (1985) all examined between-domain analogies. Further, the studies which tried to examine surface versus deep structure tended to manipulate the story lines and this was thus confounded with object similarity and use. However, the success of his study could be due to other factors, though. Part of the success may be that subjects actually interacted with the examples (worked out the formulas rather than just examining the results of someone else's worked out formulas). Further, as with Anderson's work, within domain analogies may be easier to recognize for the mere fact of their being within the same domain. There may be enough cues that can be recognized within domain. Indeed, Ross also points out that relative similarity may be responsible for some of the success here.

What affects similarity judgement (Ross, 1989a). Solving problems by analogy involves selecting the correct analog and using it to derive an analogical solution to the current problem. In terms of examples, the former analog is an example. The analog and the current problem can have certain similarities (relationships between the two items): superficial and structural (Gentner & Landers, 1985; Ross, 1989b). Superficial similarities are those that have no bearing on problem solution; whereas, structural similarities are those which are critical to problem solution. Ross (1987) indicated that superficial similarities (as defined as the story lines) played a part in the access but not in the use of a previous example. This led him to wonder what kinds of superficial similarities would affect analogical problem solving. Since some researchers believe that superficial similarities only play a part in access but not in use (Kedar-Cabelli, 1985; Anderson & Thompson, 1989), Ross also explored the use of superficial similarities.

The experiments described in Ross (1989a) used the same procedure as Ross (1987). The first set of experiments showed that object correspondences are very important and that story line correspondence may be less important. The second set of experiments showed that when the formula was not provided, object correspondences does not affect access but appears to affect how the formula is used. Finally, in the third set of experiments, Ross found that the same materials produced different results depending on whether or not one was testing access or use. Object similarities mostly affected use. These experiments together with Ross (1987) provide evidence that story line and object similarity plays a part in the analogical reasoning process and therefore different types of superficial similarity are important in the process.

Conclusions about reminding. In examining previous research, Ross (1987) indicates that lack of success in previous work may have been due to the fact that the experimenters were examining between domain issues. Also, they did not often provide the principle or underlying structure of the original story/example to the subject and, if they did, it was often not well understood or remembered by the subjects (see also Chi et al., 1981; Ross & Sofka, 1986; Silver, 1981). In Ross' experiments, even in the control conditions, well over half of the subjects were able to draw an analogy. He credits this success to providing the abstract or conceptual information along with the examples. Further, his success with distinguishing superficial similarities is credited to lack of understanding of the underlying concepts. However, it is again emphasized that his success may be due to the additional cues available when problems are solved within domain and it would be interesting to examine using his techniques between-domain analogies.

Ross (1984) defines reminding as the process whereby a person notices a previous example or part of a previous example as being related to the current situation. Relevant and irrelevant aspects are often used to produce a probe/cue which will be used to produce a reminding and novices tend to use both (although often more irrelevant aspects). Noticing or having a reminding is not enough to produce an analogical solution. Ross and Sofka (1986) indicate that use of the problem to produce cues for remindings occurred throughout the problem solving process and not just for the initial reminding. In the final analysis, Ross (1989a) indicates that reminding should probably be divided into two parts: noticing an example is applicable and retrieving that example. This breakdown is common in most other researchers' theories (Gentner, 1983; Holyoak, 1985). However, the analogical reasoning process is on-going such that access may produce use which then produces another access.

Ross (1989a) describes the difference between superficial and structural similarities and the experiments presented support that story lines appear to affect access (Ross, 1987) and object similarity affects use. Superficial similarities will play a larger part, and account for the lack of success of earlier work in this field, when there is a lack of understanding of the conceptual ideas underlying the problems (Holyoak, 1985). Thus, structural similarities are not the only determinants of use. Ross (1989b) states that the structural similarities that play the largest part in this process may be those that distinguish examples by the principle being taught. However, other structural similarities are not delineated in the work reviewed here.

Although Ross' work provides more information about how analogical reasoning occurs in a problem solving context, it fails to produce any indication about how to examine deep understanding or the understanding that subjects might have of the variables and the relationship between variables. His work thus focuses on whether or not analogy can occur and what surface features are important and is quite successful mainly for the same reasons that Anderson was successful. The question of between domain analogical reasoning still remains and the key may still be a deep understanding of the original domain(s). Again, the question is whether or not these ideas on reminding would continue to be valid across domains and what parts of these ideas are unique to the fact that the studies were done within domain.

Summary on learning by examples

The research presented in this section describes analogical reasoning for within domain problems. Anderson and colleagues have successfully demonstrated that students can and do utilize previously presented examples in solving new problems (Anderson et al., 1984, 1988). However, general knowledge about the domain can be helpful in source selection and also in generating examples from other examples.

Ross' studies were generally successful in indicating the importance of reminding in analogical reasoning. Reminding can be seen as a problem of accessing memory which may be susceptible to both distinctiveness of items and interference (Ross, 1984, 1987). Since distinctiveness and interference are commonly associated with memory issues, the representation of knowledge becomes an important issue.

Learning by example presents the easiest case of analogical reasoning. Students are often exposed to examples in the context of learning and the examples that students draw from are in the same domain as learning. So, a major problem when comparing this work to the work to be described in the next few section is that just having similar domains may be enough to trigger the use of an example (an analog) and the problem of recognition of an analog is a major problem in this literature.

Across domain: Analogical reasoning

Although analogical reasoning can be demonstrated within domain, can it occur when domains are different? Different domains pose a unique problem in that the usual cues in the environment which would tell you to use a previous problem solving episode are not necessarily available. Thus, the work described in this section uses different cover stories to provide different domains, however, the same solution procedure should be applicable between source and target.

Gick & Holyoak (1980,1983)

Gick and Holyoak (1980, 1983) examined analogical reasoning from problems in different domains. They used Duncker's radiation problem (1945) and compared performance on the problem alone with performance after a student was exposed to a story problem in another domain. The formulation was, story problem: solution:: target problem: unknown target solution. In order to perform analogical reasoning, a person must start with a representation of the problems presented. Then, a mapping must be performed between the source (given problem) and solution and between the source and the target (problem to solve). Successful analogical reasoning will generate a solution based on these mappings.

In a series of experiments, subjects were given a story problem that included a solution (see Appendix A; Gick & Holyoak, 1980). They were asked to summarize the story. Then, they were given Duncker's radiation problem. They used a version of Duncker's radiation problem as stated:

> Suppose you are a doctor faced with a patient who has a malignant tumor in his stomach. It is impossible to operate on the patient, but unless the tumor is destroyed the patient will die. There is a kind of ray that can be used to destroy the tumor. If the rays reach the tumor all at once at a sufficiently high intensity, the tumor will be destroyed. Unfortunately, at this intensity the healthy tissue that the rays pass through on the way to the tumor will also be destroyed. At lower intensities the rays are harmless to healthy tissue, but they will not affect the tumor with the rays, and at the same time avoid destroying the healthy tissue? (Duncker, 1945; Gick & Holyoak, 1980)

They used this problem because the solution rate on the problem itself is approximately 10% (2 out of 42: Duncker, 1945). Since the problem is so difficult to solve by itself, Gick and Holyoak (1980) hypothesized that it would be a good candidate for solution by analogy provided the correct analog could be found.

Gick and Holyoak (1980) found that there were differences in obtaining the dispersion solution based on story analog. Generating a solution to the story problem did produce an analogous solution to the radiation problem; however, this result does not speak to the problem of noticing an analogous situation because a hint was given to use the analog. Noticing that the analog is useful was difficult. More students could generate the dispersion solution given a hint to use the previous story analog (20% in the NoHint condition versus 92% in the Hint condition). In addition, providing the radiation problem before the story problem did not make a difference. From these results, Gick and Holyoak (1980) concluded that an analogy can be drawn even if the analog is incomplete; however, for a completely correct solution, one would probably need to have a complete analog. Strong effects of problem type (for the analog) were indicated and therefore a potential analog coded in a different context than a target may result in poorer performance. In addition, they postulated that analogical reasoning may be a conscious search process with those who are more thorough ultimately being more successful.

In a series of additional studies, Gick and Holyoak (1983) used three basic changes to the original Dispersion story: use of diagrams, use of principle, use of more source analogs. The principle was a statement added to the end of the Dispersion story:

> The general attributed his success to an important principle: If you need a large force to accomplish some purpose, but are prevented from applying such a force directly, many smaller forces applied simultaneously from different directions may work just as well. (Gick & Holyoak, p.16)

When the principle behind the problem was compared with no principle, there was no difference in performance. Also, when diagrams were compared with no diagrams, there was no difference in performance. They argued that this finding supported the idea that retrieval was semantically based. However, the use of more than one source analog had a dramatic effect on performance. First, having more than one analog was better than having just one analog even if the two analogs were dissimilar. Second, adding a diagram and more than one analog was better than no diagram. Third, adding the principle to more than one analog produced better performance than no principle. Thus, Gick and Holyoak concluded that subjects needed more analogs in order to produce a solution to the problem.

Summary of Gick and Holyoak. Gick and Holyoak's studies were designed to induce their subjects to draw an analogy in a different domain than the target domain (distant from the target domain). Taken together, Gick and Holyoak(1980, 1983) reported that 30% of the subjects spontaneously generated the correct solution with an analog and 70% generated the correct solution after a hint. Differences exist in the percentages based on the manipulations used. A major problem with these studies is that throughout their work, Gick and Holyoak failed to require subjects to solve the story problem first. A summarization task or memorization task is not the same as really using a problem in a problem solving situation and may have caused the story problem to be encoded differently. In addition, it was hard to tell what the previous exposure to the problems had been and whether or not the subjects' general knowledge level had some effect on performance in all the tasks. For example, previous knowledge could have accounted for the success of their subjects in the first experiment of Gick and Holyoak (1983) which used Maier's cord problem (1930,1931) as the target problem for a birthday party story problem.

In spite of these problems, Gick and Holyoak did show that such analogies can occur, and the sticking point seems to be the noticing of the usefulness of the story problem to the target problem. First, they argue that retrieval is semantically based. Second, due to their success with the use of more than one analog, they argue that people need more than one analog in order to produce the correct solution before a hint. Their conclusions seem very plausible when compared with other lines of research.

Tversky (1977) argues that an analog may end up more similar to a general schema than to another analog. In addition, exposure to many diverse examples can lead to superior performance (Fried & Holyoak, 1982; Posner & Keele, 1968). Chi, Feltovich and Glaser (1981) indicate that experts encode at a higher level than novices and this encoding is presumably based on exposure to many problems within their domain of expertise. People who are experts in a field are assumed to have solved many problems in that field and therefore may have generated many various connections to problem schemas. Therefore, it is plausible that with exposure to more analogs people are generating a general schema or even more cues to access particular schemas. When they are confronted with a new problem, they can encode it in such a way as to access those cues or general schemas. In other words, people could be generating some general schema from the source analog and then comparing it to their available cues from other problem solving episodes.

So, in summary, analogical reasoning in Gick and Holyoak's terms (1983) would include a transfer of knowledge from one problem to another through mapping. The mapping of two concepts are generally at the same level of abstraction; however, a relevant known analog must be available to the person. The function of an analogy is to a derive a new solution to a new problem given a previous known problem and solution. The process may be one of extracting what is needed from the known problem and solution.

Their work in general focused on example story problems which they provided to students. First, they examined the ability to draw analogies and found that students could draw analogies when given hints. Then, when studying hints and using the source analogs spontaneously, they discovered that students were unsuccessful without a hint when only one analog was given to them initially; however, students were successful given more than one analog. Their results may have been stronger had they used source problems as problems rather than stories and asked subjects to solve those source problems rather than summarize them. But, these results supply some support for far analogies across domains even for people who are experts only in the source domain.

Gentner and colleagues

The main idea driving Gentner and colleagues' work is that analogies can have a fundamental effect on a person's ideas about a domain. Not only do teachers use analogies to help students learn concepts, but scientists use analogies to drive their theoretical work (Glashow, 1980; Kepler, 1969; Koff, 1961). Given this evidence, Gentner postulates that people must be able to generate knowledge structures about a domain which can be later used to draw analogies.

The bulk of this work has been based upon the Generative Analogy hypothesis. This hypothesis states that the base domain is used as an analogical model which predicts inferences made in the target domain. These inferences rely upon structure mapping which assumes that analogies are not just a weak similarity processes but a comparison between relations.

Analogies reflect certain aspects of knowledge. In particular, they reflect the relationship between predicates of a base domain with these relationships being the most important aspect. The objects or predicates may differ between the target and base domains, but the structures or relationships between objects must be similar.

In a series of experiments, described in Gentner and Gentner (1983), designed to test this particular view of analogy, two basic analogs are studied and compared to a model of electricity: water-flow and moving-crowd.

Water-flow (hydraulics)

Electricity is often characterized as water flowing through pipes of a water system. Thus, electric current flows through wires (pipes) with voltage being equated to the pressure and milliamperes being equated to volume. A resister is similar to a narrow pipe and a battery is a pump. Using these two models, they selected problems where subjects would produce different responses based on the model used.

Moving-Crowd model

Electricity can also be characterized as a moving crowd. thus, electric current flows through wires (passageways) with voltage being equated to how hard the crowd pushes and milliamperes being equated to how many people pass a certain check point. A resistor is similar to a gate. No useful description of a battery can be found; however, Gentner and Gentner (1983) argue that this model is actually better for the use of more than one resistor in a circuit.

In the first experiment, subjects used their own model which corresponded to either the flowing-fluid model or the moving-crowd (self-generated model of electricity). Subjects who reported using both models or changing models in the middle of the experiment were not used. In addition to asking about the model used, subjects were asked specific questions about reservoirs and the flow of water in reservoirs The results on the circuit problems showed different performance depending upon the model being used. Thus, different source analogs produced different solutions in the target problems.

In the second experiment, subjects were taught one of two different models and were given a basic introduction to electricity. Simple circuit problems were given and subjects were required to solve 4 out of 5 in order to continue. Harder problems were then given along with a "thought question". The "thought question" was, "What will happen if there is no resistor in the circuit?" (Gentner & Gentner, 1983, p.121). The subjects who were given the moving crowd model performed better than subjects given the water models for parallel resistor problems; however, the subjects given the reservoir model (fluid flow) did not do better for parallel battery problems (no significant difference). Gentner and Gentner (1983) hypothesized that the failure to find the reverse performance was due to a failure in the students' understanding of the hydraulic system.

This work is interesting for several reasons. First, the source analogs for the first experiment came entirely from the students' knowledge base. Second, their experiments give evidence for the mapping of erroneous knowledge. Erroneous knowledge seems to be mapped when some problem exists in the model (source analog) that the students were using. Finally, inexperience with the source domain affected performance in the second experiment but when using their own models in the first experiment, students were able to achieve success in the problem solving. The second experiment was comparable to Gick and Holyoak's work (1980,1983) in that subjects were taught the models to be used. Clearly, the problem again arises that you may need to train a subject to a higher level in order to test analogical transfer or find subjects who are already familiar with the source domain.

Analogical reasoning in children. Examining novice skills is similar (although not the same as) examining skills of children. Children often fail in their use of analogical reasoning and tend toward producing comparisons between objects that relies on surface features (Asch & Nerlove, 1960; Dent, 1984; Gardner, Kircher, Winer & Perkins, 1975; Kogan, 1975). On the other hand, adults can produce relationships between objects and can make mappings between mutually constraining relations (Gentner, 1980; Gentner & Landers, 1985; Gentner & Stuart, 1983). Children do differ from adults in their basic knowledge level and their ability to use vocabulary (Gentner, 1977; Reynolds & Ortony, 1980; Vosniadou, 1985). Thus, for children at least, the problem may not be an inability to reason analogically so much as an inability to solve problems in general.

Gentner and Toupin (1986) examined this issue in an experiment using children of various ages. They asked children to transfer one story to another story using dolls to illustrate. Children were never asked to verbalize the higher order information. Stories were varied by using three levels of transparency: similar characters/similar roles, different characters/similar roles, and similar characters/different roles. In the last case, the objects in the target are intentionally mapped to objects in the base which have no bearing on the analogy. Their results showed that success was based on age, systematicity (logical relationships and structure) and mapping condition. Children performed better on the easier mapping conditions; children (age 8) could use higher order constraints to produce the lower order mappings.

By presenting the information for the analogies in a way that children could understand and communicate back, Gentner and Toupin (1986) were able to demonstrate that children can do analogical mapping. Assuming that the base and target are identified, the difficulty should then come in the mapping. The existence of constraining higher order relations should actually make the mapping easier. The results of their experiments support this conclusion.

This experiment did not investigate whether or not children would actually be able to retrieve the base (source problem) in a situation where they were not given the story immediately preceding the task of producing the target story. Since the two tasks matched, however, it could be argued that the children were drawing an analogy to the story that they just acted out. However, retrieval would have been easy considering they had just heard and acted out the story and there was no retention interval. In addition, even though the experiment removed the story as base and problem as target issue, story to story analogies may be easier to produce than problem to problem analogies in general.

Retrieval examined. Although, Gentner and Gentner (1983) and Gentner and Toupin (1986) examined whether or not analogical reasoning differed based on source analog, the issue of retrieval of analogs was not examined directly. In an attempt to address this issue, Gentner and Landers (1985) describes an experiment in which subjects are given 32 stories to study, 18 stories and 14 fillers. After a week retention interval, two tasks were given to subjects, a "reminding" tasks and a "soundness" rating task. The "reminding" task consisted of presenting to the subjects 18 new stories that resembled 18 original stories. If a new story reminded a subject of an old story, they were supposed to perform a recall on the old story. Stories were of three types: mere appearance (objects + first order relations), true analogy (objects + higher order relations), and false analogy (only first order relations). First order relations refers to relationships between objects that resemble the original story (analog). Higher order relations refers to connections among first order relations or other higher order relations. These connections must have the correct overall relationship between the first order relationships. After they were done with the "reminding" tasks, subjects were given a "soundness rating task: 18 story pairs and asked to rate each pair for how close they matched (inferential power).

Soundness ratings indicated that subjects could recognize analogical matches. In the reminding task, subjects were best able to access the mere-appearance matches more often than the true analogies (78% vs 44%). Even with literal similarity, subjects produce more surface matches than analogical matches (Ratterman & Gentner, 1987). However, in soundness ratings, analogical matches and literal similarity were considered more sound than false analogies or mere-appearance matches. These results of difficulty in accessing analogies have been found in many studies (Reed et al., 1974; Gick & Holyoak, 1980, 1983); however, such a clear indication as seen in these experiments that surface structure plays a large part in reminding has also has been demonstrated in other research (Novick, 1988; Reed & Ackinclose, 1986; Ross, 1984, 1987). Gentner (1989) argues that even though relational reminding is less common, it does occur (Falkenhainer, 1987; Gentner, 1982; Hesse, 1966; Johnson-Laird, 1989; Waldrop, 1987). A correct model must therefore include relational remindings with some way to indicate that they may be less common than literal similarity.

Although the results support both the fact that subjects find surface details easier to use as cues and that relational remindings can occur, this experiment suffers from some of the same problems as Gick and Holyoak's experiments. It is unclear whether or not remembering stories is the same as remember problem solving episodes or problem solving skills. In some senses, just remembering a story is not as interactive as having to solve a problem. Further, novices in solving problems also appear to focus on surface details more (Chi et al., 1981).

<u>Summary of Gentner and colleagues' work</u>. Gentner's work demonstrated that children could reason analogically when the task was presented in a visual form rather than verbal form. Adults who were using their own model were able to

solve problems in a different domain. In addition, the model being used determined what the solution to the problem would look like and also how easily a solution would be generated. Thus, they demonstrated that relational remindings can occur and that people can use models that they have generated themselves.

On the down side, they were unable to demonstrate conclusively that training subjects in using certain models could always produce the expected results. In one test, the moving-crowd model led to better performance than both versions of water-flow model used; but, in another test, the water-flow model that was predicted to lead to better performance failed to produce that performance. It may be that the training on the models is the main reason for the lack of success. Also, in Gentner and Landers (1985), when subjects were given stories to study and later returned to identify matching stories, subjects were better able to identify surface matches over analogical matches. On the other hand, the soundness ratings indicated that subjects were able to identify analogical relationships if pressed. Although this study truly resembles a memory study in which one could examine retrieval of analogs, memorization of stories may not be the same as performing problem solving tasks and therefore, memory for problem solving task episodes may differ.

Novick

Novick's research was an attempt to discover why Gick and Holyoak's research (1980, 1983) results were so poor unless a hint was given. Her hypothesis rests on the idea that the problem is represented very differently by novices and experts and this difference in problem representation is causing the difference in solution probabilities.

Novices tend to recognize problems on the basis of salient surface features whereas experts tend recognize problems with both surface and abstract, solution-relevant structural features. They notice how things are related to each other. This characterization of the difference between experts and novices is recognized by many researchers; however, note that skill is usually considered as a continuum (Adelson, 1981, 1984; Brown & French, 1979; Chi et al., 1981; Kay & Black, 1985; Schiano, Cooper, & Glaser, 1984; Gentner, 1983; Holyoak, 1985; Shoenfeld & Herrmann, 1982; Silver, 1979, 1981; VanLehn & Brown, 1980).

Novick (1988a) considered expertise in the use of analogical reasoning. The main way that people are successful in problem solving is by searching previously solved problems, although other sources may be available and may be used. Just providing similar situations may not induce successful analogical reasoning

(Bartlett, 1958). In addition, story problems may not be "sufficiently problem-like"(Bartlett, 1958,p.512). As argued previously, just asking subjects to read story problems or memorize stories may not enable subjects to use the information in the story for later problem solving.

In a series of experiments, Novick attempted to measure positive and negative transfer by examining target problem solution paths. She categorized students based on their scores for the math Scholastic Aptitude Test (SAT). The range for novice designation was 500 to 650 and the range for expert was 690 to 770.

Subjects were given four problems and a target problem - all math problems. The first, third, and fourth problems were the same for both conditions and differed from the target problem in both surface and structural relationships. The second problem was either unrelated (baseline condition) or a remote analog - differing in surface but not in structural - (remote analogue condition). The four problems were presented with their solutions procedure. Subjects used the solution procedure to solve each of the problems. Then, subjects were given 15 minutes to solve the target problem. Results of this experiment indicated that expert and novice subjects performed equally well on the baseline condition (6.3% using correct solution procedure); whereas, expert subjects used the correct solution procedure more often than the novice subjects in the remote analog condition (an increase to 56.3%).

Experiment 2 used the same procedure as experiment 1 except that a distractor problem replaced the remote analog problem. This distractor problem was designed to be superficially similar to the target problem but not use the same solution procedure. Results showed that the distractor problem produced a very strong effect for both novices and experts and therefore contradicted expectation. However, as Novick (1988a) indicated, previous researchers have also noted that people expect a problem with similar surface features to be solved in the same way (Brown, 1989; Lewis & Anderson, 1985; Medin & Ortony, 1989). Therefore, subjects probably entered the experiment with knowledge about the problem solving heuristic to use a previous problem if the current problem looks like the previous problem and in this case, that heuristic produced an incorrect problem solution procedure.

Experiment 3 attempted to address this problem by presenting both the remote analog and the distractor in the hopes that experts would be persuaded to use the remote analog rather than the distractor. In this case, another novice condition was added. Low novices scored from 460 to 540 on the SAT, high novices scored from 580 to 620, and experts scored from 700 to 770. Experts performed better than novices in that they produced more correct solution procedures and also fewer distractor condition procedures. Presentation order of remote analog or distractor did not affect solution. In addition, experts did not persist in using the distractor solution procedure whereas novices tended to persist in using it.

In further studies, Novick (in press) attempted to examine the processes of analogical reasoning. Again, SAT scores were used to access expertise level. In addition, for the source problem, real mathematic problems were used as opposed to story problems. The procedure used was the same as in Novick (1988a) except that after target problem solution, another analog to the target were given (post-target problem). If the subject did not solve the target problem in the first attempt, up to three hints were given with solutions attempts after each hint. After solving the first problem, a filler problem was given. Then, a post-target problem was given and hints provided at failed solution attempts. Hints were designed to address each of the hypothesized processes in analogical reasoning. In the second experiment, specific hints were given: what analog to use and mapping hints.

Of subjects who were initially unable to solve the target problem, 23% were able to solve the problem after a hint was given. The hint pointed to which of the initial problems was relevant. Interestingly enough, 15% more subjects were able to solve the problem after a hint on how to use the analog on the target problem was given. Solution of the target problem could be used as a predictor for the post-target problem. The results of these experiments were quite different from Novick (1988a). Spontaneous transfer was low and retrieval failure was the primary reason for problems in analogical reasoning. However, an interesting finding was that once the analog was identified, mapping was relatively easy for subjects. Schema quality was found to be a relevant factor in subsequent problem solving.

Summary of Novick work. These experiments are interesting because Novick was able to fix some of the problems of the experiments performed by Gick and Holyoak (1980,1983) and Gentner (Gentner & Gentner, 1983; Gentner & Toupin, 1986). In particular, experts were not only able to solve problems analogically with greater frequency than subjects in those previous studies but they were also able to overlook surface similarities and see the deeper structure when presented with both a distractor and a remote analog. On one sorting task, subjects who did not sort by surface structure produced more analogical problem transfer. Thus, Novick (in press) summarizes the effects of expertise as an increased probability that an analog will be retrieved and increased ability to utilize mapping skills. In particular, the two experiments together indicated that experts were more able to use hints to perform the mappings required. These experiments point to considering the target problem solution procedures as the critical data for examining analogical reasoning and not just right or wrong answers. In addition, using hints that are designed to address each of the processes of analogical reasoning can give experimenters information about the different processes involved.

Novick's work does not differ from Gick and Holyoak's work for two reasons: both show that subjects may have problems knowing to look for an analogy or they may have problems discriminating relevant from irrelevant information. In addition, since quality of schema production was a good predictor of subsequent problem solving, analogical transfer is facilitated by schema induction. Knowing mappings at various levels must therefore be necessary for transfer.

A negative aspect of these experiments could be that even experts had lower solution rates than one would desire. In a way, it is unclear that just asking a subject to solve a problem with the solution procedure available all the time would enable the subject to learn the procedure well enough to transfer. Some of the subjects may therefore have not learned the procedure during the practice solution phase. In addition, the processes of analogical reasoning were not examined and in particular, where the subjects failed was not indicated (Novick, 1988a). Also, by using SAT math scores and then using math problems, Novick did not vary domain which makes her work comparable to learning by examples more than the Gick and Holyoak studies.

Summary of across domain analogical reasoning

The research presented in this section has been across domain in the sense that cover stories for the analogs were in different domains than the target problem. For Gick and Holyoak, Gentner, and Novick, surface features of the analog and target played a large part in the recognition of analogical reasoning. However, Gentner was able to demonstrate that relational ties between objects including higher order relations also are important. Analogical reasoning can be found in children as well as adults, and performance was better for older children as well as those with more expertise in the source domain (Gentner & Gentner, 1983, Gentner & Landers, 1985; Gentner & Toupin, 1986; Novick, 1988b).

Novick's studies are interesting for their indication of greater success by experts. An argument could be made that experts have generalized schemas or better problem solving methods for their field of expertise and these generalized schemas allow them to solve the problems with greater success. In fact, in Gick and Holyoak (1983) the use and success of more than one analog was hypothesized to be due to the generation of a generalized schema. Gentner's work is also based on the idea that mental models or knowledge structures about a domain can aid in the analogical reasoning process. Greater exposure to a field would be hypothesized to produce more elaborate mental models from which to reason.

The rate of success for analogical reasoning is quite low, though (Gick & Holyoak, 1980, 1983). Examining the studies presented in this section, one can note several problems. First, in the Gick and Holyoak studies, subjects were asked to summarize rather than solve the story problems. Generation of the solution during a summarization task may result in quite different memory for that solution than if the problem had been solved by the subject himself or herself. Second, it is unclear what the criterion of "having learned" was in the Gentner and Gentner 1983 study was. One can hypothesize that if the models had been overlearned or at least learned to a very high criterion predicted outcomes would have resulted.

Components of analogical reasoning

The main components of analogical reasoning have been described as retrieval, mapping, and procedure adaptation (Holyoak,1984, 1985; Gentner, 1983; Gentner & Landers, 1985; Gentner & Toupin, 1986; Novick, 1988). Mapping is assumed to occur if an analog is retrieved. In addition, these components are interacting and not discrete (Novick, 1988b; Ross, 1989). For example, one can not think of retrieval as stopping when an analog is retrieved; however, retrieval may continue as the mapping process is started. Therefore, rather than focusing on one part of analogical reasoning, this next section will give an overview of several models for analogical reasoning that discuss the process as a whole.

Models of analogical reasoning

Models for analogical reasoning have been developed to express the process behind the results reported. The models discussed in this section have been based on human performance, although not necessarily back tested after they were created. For a review of models developed in artificial intelligence which for the most part have not been compared directly with human performance, please refer to Hall (1988).

Anderson and colleagues: A model for analogical reasoning

Giving a formula or presenting the general concepts is often not enough for people to learn new information (Anderson, et al., 1984; see also learning by doing, Anzai & Simon, 1979). People often use examples when solving problems (Pirolli &

Anderson, 1985; Reed, Dempster & Ettinger, 1985; Ross, 1984). For this reason, Anderson and his colleagues have been working on a general theory of learning which hypothesizes that analogical reasoning is the basic mechanism used when a new problem is encountered (Anderson, 1986; Anderson & Thompson, 1989). These ideas have been successfully integrated into a production system model (PUPS). The following description is based on modeling behavior of LISP and calculus students.

PUPS uses schema-like structures for the knowledge representation. A structure consists of three obligatory slots, "isa", "function", and "form". The "isa" slot defines the structure, the "function" slot defines what it does, and the "form" slot defines what is typed to LISP. Transfer generally happens between the form slots.

A typical problem would ask a student to write a LISP function for which they are told what it should do. The student has been exposed previously to material including examples. The theory assumes that any previous example will be used even if it is not applicable. If the first two elements of the function slots must match, we can put the elements into correspondence. First, the function and form slots of the given example are compared. Then, the elements of the example are variabilized. Unmapped items in the form slot are mapped onto themselves in the target domain. We can also replace something by its functional specification because what is critical is the function of an item and not the identity in making substitutions. This substitution may be critical for an analogy to take place between items that do not have an obviously similar surface structure. Further, arguments in the target function will be embellished with their functions as long as the sufficiency of functional specification can apply.

Figure 2 illustrates the basic idea. Making the initial comparison between the function of the example and the function of the goal does not provide enough information to solve the problem. So, start with the example. Notice that func1 can be substituted into it. Then, variabilize the terms. So, 6 and 2 become variabilized. We are left with "(list difference y z) → (perform subtraction y z)" (Anderson & Thompson, 1989, p.274). Note that we can again do a substitution by changing difference to "(implement subtraction)". Once it is recognized that one can variabilize subtraction, then we are done. The difficulty then becomes whether or not all the comparisons and substitutions can be carried out.

Figure 2. Example from Anderson and Thompson (1989), p.274.

```
Example
  isa: function-call
  function: (perform subtraction 6 2)
  form: (list func1 6 2)

func1
  isa: LISP function
  function: (implement subtraction)
  form: (text difference)

goal
  isa: function-call
  function: (perform division 9 3)
  form:??

func2
  isa: LISP-function
  function: (implement division)
  form: (text quotient)
```

Examples are selected using the same processes described in Anderson (1983). They are found during the process of matching productions. Activation is dependent upon the strength and number of pathways and the dominance of recency, frequency, and feature overlap.

Singley and Anderson (1989) point out that a problem with these analyses is that the underlying structure of the retrieved example must be correct for the analogical process to produce the correct result. However, for learning by example problems, the example is usually a turn of the page away. But, the problem of the correctness of the underlying analog is also a major problem in this area in general.

The key to successful analogical problem solving may be the proper function for a structure (Anderson & Thompson, 1989). Anderson and Thompson (1989) indicate that further refinements (or additions to the function slot of structures) can produce analogical transfer to another example (i.e. an example in another domain). When the analogy mechanism reaches a refined example where the function slot has an item which is not specified within the current example, a search is initiated. It is hoped that the search will provide either the specification for that additional function item or another item to use for search. Thus, one can see from this idea that analogical reasoning across domains or away from the example readily at hand could be quite difficult.

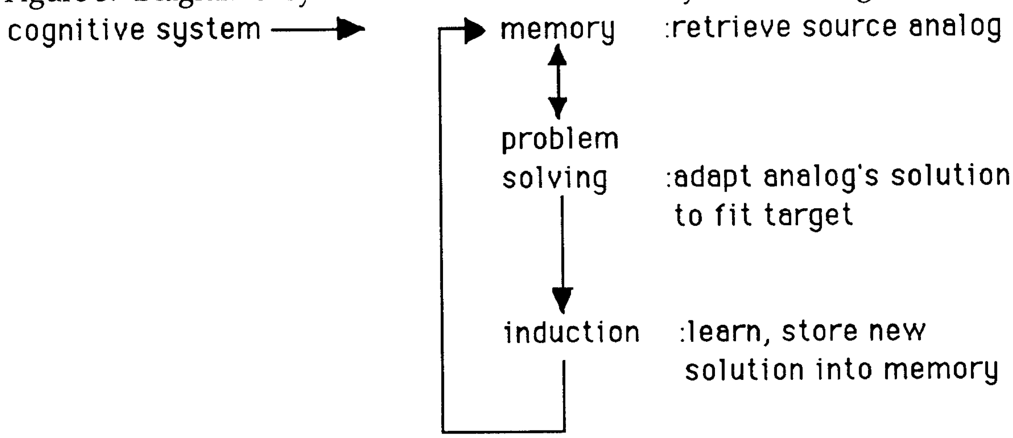
<u>Holyoak and Thagard</u>

Holyoak and Thagard propose a different model of analogical reasoning. Thagard (1988) states that the semiotics of analogy can be defined as syntax, semantics, and pragmatics. Syntax refers to the form of the symbols (elements); semantics refers to the relations of the symbols to the world; and pragmatics refers to the use of the symbols. He argues that these three components are critical to all parts of analogy but especially the mapping of an analogy.

Of the components for analogical reasoning described, mapping is generally recognized as the most important (Holyoak, 1984; Holyoak & Thagard, 1989a). Failures in analogy could be due to reminding failures as in Ross' work (1989b) but are most likely due to incorrect mappings, incomplete mapping of causal relations or a failure to notice when a mapping is either good or bad (Holyoak, 1984). Thus, their theory concentrates more on mapping mechanisms than on the other components, however, they have described and implemented two systems that contain the other components, as well (ACME, Holyoak & Thagard, 1989a; PI, Holyoak & Thagard, 1989b). For their models,the basic determinants for successful analogical transfer include the overall similarity of the analogs, the completeness of the analogy (i.e. the similarity between the goal states of the problems and relations), and the goal state of the problem solver for fostering of schema induction (single examples do not aid as much, Gick & Holyoak, 1983).

The systems start with a representation of possible analogs. When a person is given a problem (target), they retrieve a possible analog and map between the analog and the target. Then, a learning mechanism stores the result for future use. The details of this theory follow. The dimensions of analogy are: representation, retrieval, exploitation (mapping), and learning (Thagard, 1988) and are illustrated in Figure 3.

Figure 3. Diagram of system as it is described in Holyoak and Thagard (1989b).



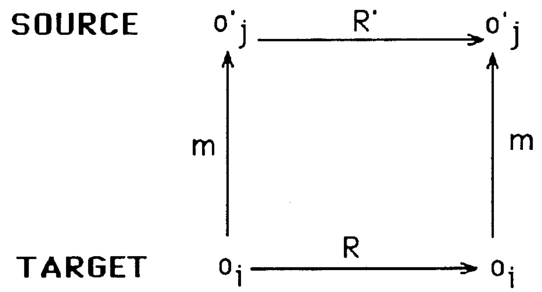
A mental representation of possible analogs must exist for the analogical processes to occur (Holyoak, 1984). This representation is most likely propositional. Therefore, it contains objects and concepts (background knowledge) in a hierarchical form (Thagard, 1988). Problem representations also include the structure of the problem, starting conditions, goals, and the past history of solution (Thagard, 1988).

Given a target, one can access a base. This retrieval, the selection of a possible analog, is critical. It depends on similar elements (Gentner & Landers, 1985), similar constraints and goals (Brown, Kane & Echols, 1986), and how many misleading factors exist (Novick, 1988a). Words in a target can trigger an analog; but, in most cases where domain is not the same between target and base, an abstract schema must be generated. This schema could be easier to apply to an analog found than the actual representation of the source (Holyoak, 1984). Several mechanisms exist to retrieve knowledge: direct, indexing, and spreading activation. Although direct access is often modeled, this direct method does not explain how people can function when not given an analog directly (Thagard, 1988). Indexing by common features, goals, and failures is also commonly used. Most often, something in the target is used as an index into memory. This will work for simple knowledge bases but for complicated knowledge bases surface features may match many things. Uses of goals can aid in these cases, though (Carbonell, 1983, 1986; Holyoak & Thagard, 1989b; Thagard, 1988). However, from these points, direct access and indexing may not be the best methods to use. Holyoak and Thagard's method of choice is spreading activation but with goals directing the spread. Subordinate to superordinate concepts get activated automatically (poodle to dog to animal). They argue that by using goals to direct the spread, they are able to retrieve all relevant possible analogs to the goal.

Once the items in an analog are activated, we can perform the mapping process. The important aspects of the source are identified, using the goals of the problem solving (Holyoak & Thagard, 1989a). Mapping between elements is sometimes situational and not always explicitly stated. Further, mapping at different levels of abstraction can occur (Holyoak, 1984). Given two problems, a general schema can be induced. This may occur through "eliminative induction" (Mackie, 1974; Winston, 1986). We can postulate a word by word abstraction or a broader context extraction. Schema induction could occur to aid in the process of mapping, because mapping the analog to a schema does not require mapping every relation but only those found in the schema. Also, novices tend to use analogy often (Pirolli & Anderson, 1985), although they may not be successful due to their lack of experience with schema induction; on the other hand, experts, who can abstract concepts in a domain or different domains, have more success with analogy in remote domains (Holyoak & Thagard, 1989b).

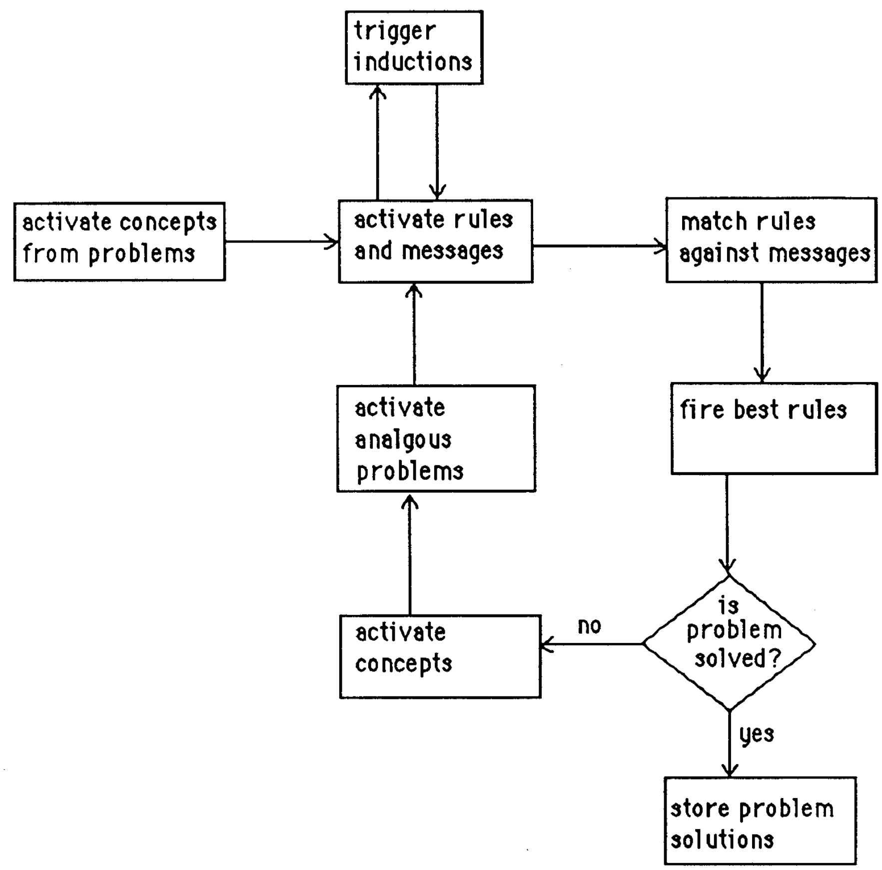
The constraints on mapping are: structural consistency (related to syntactical consistency), semantic similarity, and pragmatic centrality. An analogy is represented by the mapping between objects and relations in the target and source (Figure 4). Semantic similarity has its main affects on retrieval rather than mapping. Strict semantic similarity can not be enforced but common features are expected (Burstein, 1986; Winston, 1980). Pragmatic centrality also weighs on the initial selection of an analog. Although there is general disagreement over when goals and purposes play a part in analogical reasoning, for this theory, goals and purposes influence all levels of analogical processing. Problem solving goals are kept in mind and relevant solutions are actively examined during the mapping process (Holyoak & Thagard, 1989a; see also Brown, et al. 1986).

Figure 4. A structurally valid analogy as an isomorphism (Holyoak & Thagard, 1989a). The objects "o_i" are mapped onto objects "o_j" and relations, R to R', are also mapped.



The final item to consider is learning. Learning occurs when a successful analogical reasoning session has occurred. The general elements are extracted from the analog and the target the general elements (Thagard, 1988). New solutions, new schemas, and new rules are stored. Figure 5 shows the whole analogical process as theorized by Holyoak and Thagard (1989b).

Figure 5. Diagram of analogical problem solving in the PI model (Holyoak & Thagard, 1989b).



Structure mapping: Gentner

In a different approach, Gentner and Gentner (1983) describes how analogical reasoning can occur across domains through the use of mental models. They argue that the lack of success in Gick and Holyoak's studies (1980, 1983) was due to different models that subjects brought to the experiment. Although they do not elaborate, one can postulate that in the story problem (source) solution, when subjects are given the solution, they either fail to incorporate the solution into their model or they fail to pay attention to the solution because they already "think" they know how to solve such problems. Regardless of the possible reason, this section will describe their analysis of the analogical process.

A person has knowledge about a domain organized as a system of objects, attributes of those objects, and relationships between objects (relations). Knowledge about another domain will be organized in the same manner with analogy occurring by the systematic "structure-mapping" between systems. Objects do not have to be identical for analogy to occur; however, relationships between objects should be the same between the two domains, "preservation of relationships".

Examine Figure 6. Notice that the attributes "YELLOW", "HOT", AND "MASSIVE" are not important for the analogy to take place. The nucleus does not need to be identical to the sun. In addition, the electron does not need to be identical to a planet. These are also surface features of the model. What does matter is that the attraction between the sun and a planet resembles the attraction between a nucleus and an electron.

"Preservation of relationships" is just one of the principles upon which Gentner and colleagues base their theory of analogical reasoning. The other principle is "systematicity". Because the key to this process is "systematicity", the definition from Gentner and Toupin is provided here:

> The systematicity principle states that a base predicate that belongs to a mappable system of mutually interconnecting relations is more likely to be imported into the target than is an isolated predicate. A system of relations refers to an interconnected predicate structure in which higher-order predicates enforce constraints among lower-order predicates. The systematicity principle requires a mappable relational chain. (Gentner & Toupin, 1986)

A relation that is isolated is not used as often as one that has higher order relations or interconnecting relations. In Figure 6, for example, in comparing a solar system to an atom, "MORE MASSIVE THAN" has more interconnecting relations than "HOTTER THAN" and would be considered first even though both relations are at the same level. Figure 7 illustrates a "systematic" representation of the knowledge shown in Figure 6. Notice that there is another higher order representation shown, "CAUSE [MORE MASSIVE THAN (sun,planet), REVOLVE AROUND (planet, sun)]" (Gentner & Toupin, 1986, p.284). This higher order representation may be critical to targeting errors in mapping and for the analogical reasoning process as a whole.

Figure 6. Example from Gentner and Gentner (1983, p.103). Relations are in ovals; objects are not in ovals. S stands for subject; O stands for object; M denotes mapping to a different system.

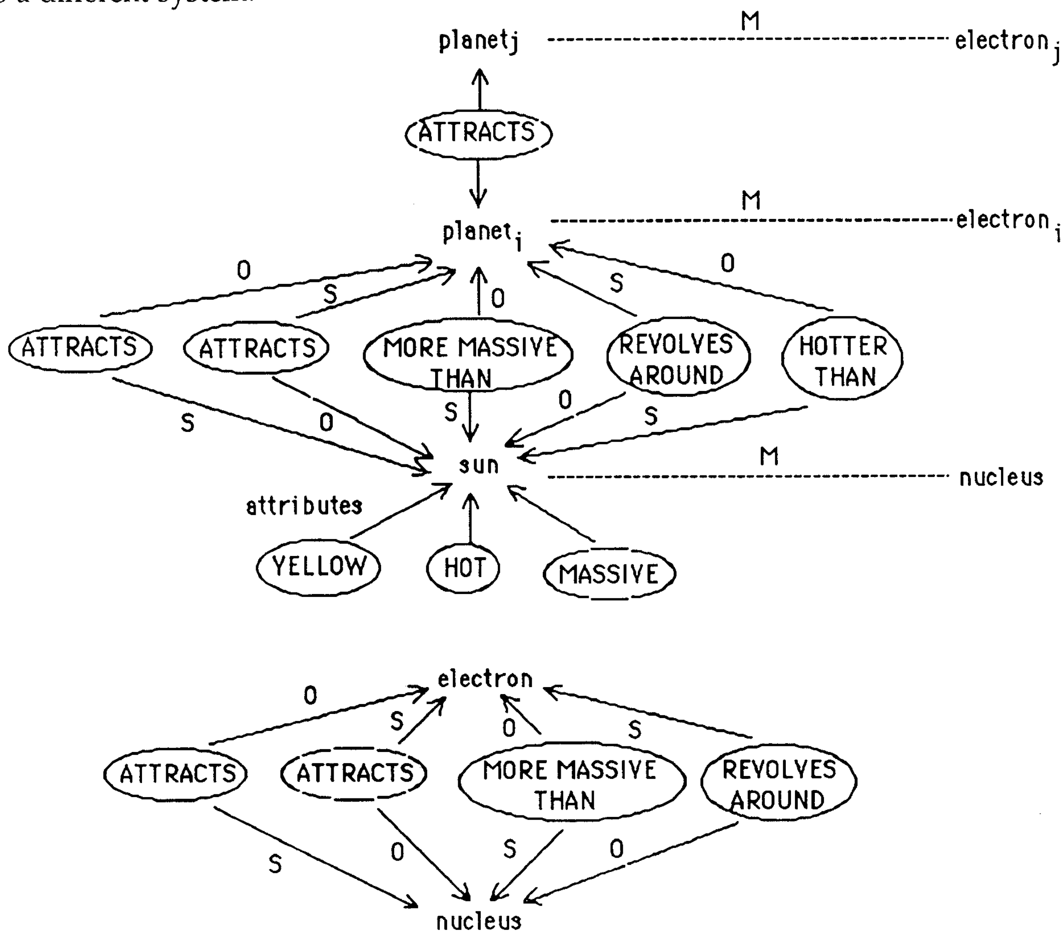
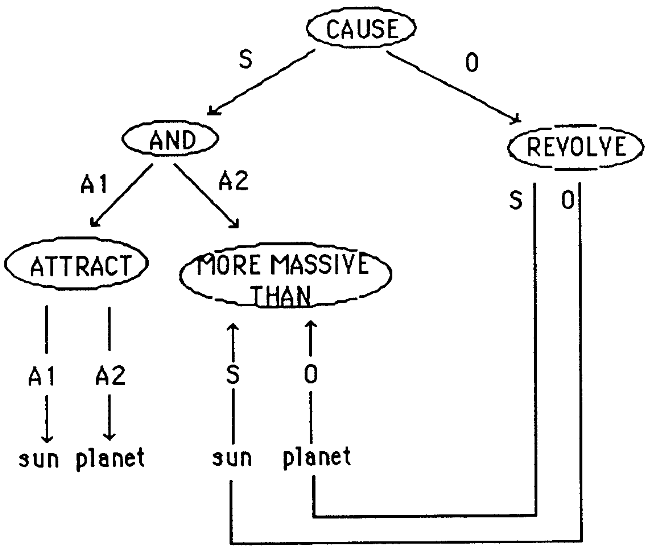


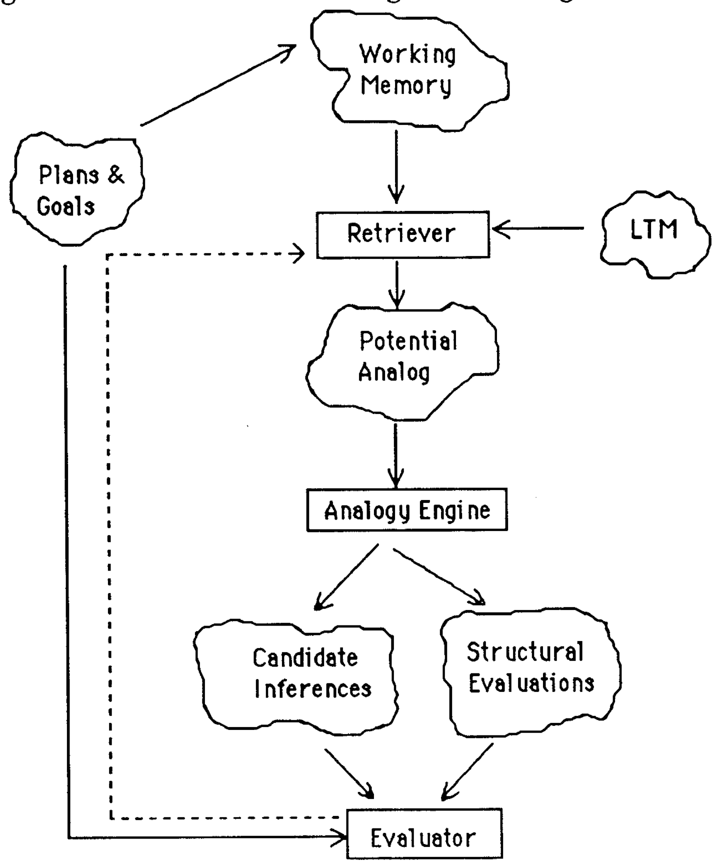
Figure 7. Diagram from Gentner and Toupin (1986, p.283).



In summary, when a person is given a source problem, a knowledge structure containing objects and relations is constructed. Another knowledge structure is produced for the target problem. At this point, the theory predicts that mapping between the structures occurs with the relations between objects being of primary importance. Although objects need not be identical, the more similar the objects are, the easier it is for subjects to perform the mappings (Gentner & Toupin, 1986). Further, the presence of higher order relations, especially constraining relations, provides for ease of mapping and ease of checking mapping once it is complete. However, since the process requires perfect relational identity, potential analogies can be missed (Gentner, 1989).

Proposed architecture. When a problem is presented, the solver creates goals for how to solve the problem, plans. These plans and goals influence the analogical process at the beginning by influencing the choice of items from memory (Gentner, 1989). The items form a set, a potential analog, which is given to the process described previously. The outcome is passed to an evaluator which uses the results and original goals and plans to determine if the process is complete. If the process is unsuccessful, then the evaluator feeds the information back. New plans and goals are formed and added to the results of the previous try and the process begins again. Thus, plans and goals affect the process in the beginning and end but the analogical processing itself takes place on the candidate analog generated from the initial search of memory. Note that since goals and plans do not influence the process directly, analogies could occur contrary to current goals (discovery situations). Also, the design of the architecture allows for separate failures to be identifiable. In particular, a failure at retrieval would feed an incorrect candidate analog to the analogy engine (Figure 8).

Figure 8. Architecture for analogical reasoning. (Gentner, 1989, p.216)



Summary of models for analogical reasoning

The models just described, including information about Carbonell (1983, 1986) are shown in Figure 9. In summary, a representation is created when a subject is exposed to a problem (target). If a person can not remember having solved the problem before, then he or she will try to solve the problem from general heuristics (Carbonell, 1983; Messamer, 1990). However, if the person can remember a previous problem solving episode, the target representation can be used to form retrieval cues in memory (Novick, in press; Ross, 1982). Assuming that this retrieval is successful, the person will then map the various aspects of the retrieved memory (analog) onto the new problem (target) (Novick, in press). Mapping is defined as the situation of finding one to one correspondences between objects and relations. After mapping, the person can be determine if the analog is useful to solving the problem (Falkenhainer, Forbus, & Gentner, 1986; Hofstadter, et al., 1987; Holyoak & Thagard, 1989a,b). Then the person must use the analog's solution to derive a solution to the target problem (Novick, in press).

Notice that even though there are many differences pointed out by these authors, the basic process of representation, retrieval, mapping, and evaluation/learning are similar. In the literature, much debate between Holyoak and Gentner has centered on how different the two models are with Gentner focusing on higher-order relations and Holyoak focusing on goal related analogical reasoning or pragmatics (Gentner, 1986; Holyoak, 1985). However, Novick (1988) points out that although there are differences in their views, both models will tend to make the same predictions because higher order relations are related to goal related issues. Thus, the straight description of the basic components will do with their combination producing an adequate model of analogical reasoning.

Figure 9. Composite view of various models for analogical reasoning.

	representation	retrieval	mapping	evaluation learning
Anderson & Thompson 1987 (within domain)	programming schema forms - syntax function - pragmatics	spreading activation; independent rule activity	use of high level plans; goals & functional knowledge to drive mapping	new rules stored
Carbonell 1983, 1986	elaborate solution traces	indexing: using goals	derivational + use of high level plans, goals and functional knowledge	new solutions stored
Gentner	mental model: interconnected system of objects, attributes and relationships between objects	direct retrieval: based on target representation, goals, and plans. More than one analog retrievable	direct mapping: higher order relations drive, mutually interconnected relations; more than individual relations will have precedence; no need for exact mappings of individual elements	Analogical reasoning is an iterative process. model stored: new solutions, new schemas
Holyoak & Thagard	data structure for problems which induce goals, starting point, solving history, and rules	spreading activation: use of target representation, goals directing; subordinate to superordinate clauses PI: parallel, rule firing, rule directed activation	activation patterns: concepts (T) to concepts (S) mapping constraints	Analogical reasoning is an iterative process. new solutions, new schemas, new rules

Representation in Analogical Reasoning

The previous section described analogical reasoning as an integrated process which included using the target problem as a way to retrieve an analog, retrieving an analog and mapping that analog to the target, generating and evaluating a solution, and learning from the generation/evaluation process. Notice that the components, retrieval, mapping, and learning, critically depend upon the representation that a person brings to the analogical process and may be critical for determining if an analogy is going to take place. For example, in the summary of Ross' work (1989b), remindings were based on cues that accessed some representation in memory. Also, in the discussions of various models for analogical reasoning, the representation of the source and target problem was different for different researchers. The possible representations presented were production system rules, semantic network, and mental models. For this paper, we will focus on mental models because production system rules, declarative structures (also propositional structures), and semantic networks can all be used to form a mental model (see also Johnson-Laird, 1983 for a similar argument).

Types of Representation

Carroll et al. (1987) characterize three different types of representation in HCI: simple sequences, general methods, and mental models. Simple sequences refer to overt actions that fit a particular situation. These sequences are often learned by rote. An example of a rote sequence is the information that a novice computer use may use when first using a computer. The novice may be shown that to print a document the command, "PRINT", must be used. A simple sequence using that command would be: Type print, document name, and return. The novice may have no choice but to memorize a few basic commands such as "PRINT" at first in order to just get started. In addition, these sequences may have little or no meaning to the novice user until he or she has had more exposure to the use of that computer or has interacted with other users.

In contrast, when a user becomes more skilled, simple sequences give way to general methods. These general methods are generally in service of a goal. They are procedurally oriented and contain no explanation of why steps are executed. General methods are captured in the GOMS model developed by Card, Moran, and Newell (1983). When a person is given a task to do using a computer, goals and subgoals are produced. There are selection rules to choose among the methods that could be performed to produce a new goal state. Thus, the GOMS model describes the organization of a task. An example is shown in Figure 10.

Figure 10. Example of a goal to modify text with two simple sequences available. (Card, Moran, & Newell, 1983, p.142)

```
GOAL: EDIT-MANUSCRIPT
    GOAL: EDIT-UNIT-TASK
    GOAL: ACQUIRE-UNIT-TASK
        GET-NEXT-PAGE
        GET-NEXT-TASK
    GOAL: EXECUTE-UNIT-TASK
        GOAL: LOCATE-LINE
            [select:   USE-QS-METHOD
                       USE-LF-METHOD]
        GOAL: MODIFY-TEXT
            [select:   USE-S-COMMAND
                       USE-M-COMMAND]
            VERIFY-EDIT.
```

Using the GOMS model as a base and modeling using production systems, Polson, Kieras and colleagues (Foltz, Davies & Polson, 1988; Lee, Polson, & Bailey, 1989; Polson & Kieras, 1984; Polson, Muncher, & Engelbeck, 1986) have had much success predicting transfer in simple tasks. The basic unit of transfer has been easily predicted through common elements, where common elements may be taken to mean exactly matching productions. From these studies, we would be able to predict how easily someone using a certain software package on a computer would be able to use another software package on the same computer based on how many common elements exist in the two situations.

Let's consider two possible scenarios different from the simple scenario of transfer between two software packages that do the same task (two text editors). One scenario could be that a person with a great deal of experience with a software package on one computer suddenly is asked to use the same type of software package on a different machine. Even though the two software packages perform the same task, because they are on different computers, the surface structures may be quite different. A second scenario could place a person with a great deal of experience with a certain computer and maybe one or two software packages, but ask the person to use a different software package which perform different tasks in general but can perform the same task as one of the previous packages that the person already knows. In both situations, there is a different domain involved but possibly the same underlying solution path. How would the person use the new software package in either scenario?

We would expect the same common elements to transfer. But, if there are very few common elements, then we would predict that the person would have great difficultly solving the new problem. However, in some cases, it may be possible for the person to have greater success than would be predicted by common elements. One case may be where the knowledge of how to use a software package on a certain machine is integrated with knowledge of how to use computers in general and in addition, specific domain knowledge of that software package (for example, knowledge of statistics combined with knowledge about statistical software packages and knowledge of how computers work). The higher order information about a system is often called a mental model. As in Gentner's work reported in the last section, these mental models could be used to perform tasks with the new software package (or the new computer).

Mental Models

The study of mental models appears to have grown out of a need to explain real human behavior and experience with real tasks. The goal has been to understand that behavior and use that knowledge to provide training. The importance of this topic can be seen in the recent proliferation of the research as described in recent publications (Gentner & Stevens, 1983; Johnson-Laird, 1989; Rouse & Morris, 1986).

This literature resembles the analogical reasoning literature in that there are as many definitions as there are researchers. However, it should be possible to isolate some commonalties. One definition is that a mental model is an internal translation of external events which allows a person to reason by manipulating the symbolic representation, a simulation of the real world (Craik, 1943). Another definition would be that a mental model represents a person's understanding of a particular content domain (Carroll et al., 1987). This definition appears to be a subset of the first definition; however, it permits us to discuss mental models in terms of particular domains. Thus, the characterization of a mental model may depend critically upon the domain being studied (Gentner & Stevens, 1983). In addition, a mental model is really based on one person's beliefs and that person's beliefs could be incorrect (Olson, 1988). This would result in a mental model which contains incorrect information and can produce superstitious or erroneous behavior. However, a simper definition could be that a mental model is a mental image or internal image of a presented problem (Johnson-Laird & Steedman, 1978). Why call the mental image a mental model rather than just a representation? Clearly a mental model in human computer interaction is more than just an image because people often have more than just an image of a computer in their minds when they are using a computer.

In which case, how does this definition differ from a representation in general? From the definitions given here, a mental model is a representation but a special type. It is a representation that represents how a person models his world in the same way as psychologists attempt to model processes (as in the analogical modeling described previously). In the case of a physical device, a mental model must do more than characterize a person's understanding. It must be able to specify what that representation should have in order for the person to be able to use the device. (Other situations may not be as clear.) Thus, a mental model of a device is a combination of the internal representation of the device and of the person interacting with the device or "how it works" knowledge.

Norman (1983) makes the distinction between a user's model and a designer's model (or a researcher's and subject's model). If more than one model of a device exists, then how many mental models does a person bring to a task? Rasmussen (1979) listed criterion for alternative mental models of a system, but Rouse and Morris (1986) point out that any one or all types of these mental models could exist for a person at any given moment. In addition, a mental model may be highly tied to the task being performed by the person (Rouse & Morris, 1986). Therefore, it is possible that a person may have more than one mental model for any particular task.

The next section will describe what a mental model means in terms of a physical device or system. Although some description and work has been done on discourse and deductive reasoning as mental models, the bulk of the research has been performed on physical devices/systems. For a review of work on mental models in discourse and deductive reasoning, see Johnson-Laird (1989).

Physical devices and systems

What a person brings to the interaction with a device is their experience, views, and conceptualization of the task they are asked to perform with the device (Norman, 1983). The combination of the internal representation of the device and of the person interacting with the device forms the mental model. A mental model of the device should provide a user with an internal representation of how that device works (Olson, 1988). In particular, the internal representation must also indicate how the person thinks the device works. Since most mental models of devices occur through exposure to the device, induction (Olson, 1988), they are therefore not always perfect. In fact, Norman (1983) points out many problems with mental models that result from this process:

1. Mental models are often incomplete.
2. The ability to "run" a mental model is often limited.
3. They are unstable.
4. One model may be confused with another.
5. They are "unscientific".
6. They are parsimonious.

In a series of studies that confirm these statements, Norman (1983) asked subjects to think aloud while using a calculator and performing simple math tasks. He found that subjects had particular beliefs about their machines and the capabilities of those machines - they were superstitious about the use. From this example, it is clear that subjects do form some representation of the machine, however, it is often not the same one that the designers intended.

Several other examples of people using physical devices are reviewed in Johnson-Laird (1989). Novices appear to have different models than experts. For example, novices tend to model surface behavior, while experts seem to model abstract relations (Larkin, 1983). Erroneous models provide the bulk of the evidence for mental models. Examples include an inability to predict trajectories of objects (McCloskey, Caramazza, & Green, 1980), what happens when coins are rolled (diSessa, 1983), and what corner of a cube is opposite the starting corner (Hinton, 1979).

Further evidence for a mental model of physical devices can be seen in the experiments performed by Kieras and Bovair (1984). These experiments provided an explicit model of a control panel device or rote instructions to subjects beforehand. Faster learning and higher retention was noted for those subjects who were given model information. However, Polson (1988) using a word processor was unable to replicate these results. These contradictory results point to methodological problems which could probably be fixed with clearer definitions of what a mental model is and what we should be teaching (Kieras, 1988). In fact, Norman (1983) had suggested that developers attempt to create a simple device model which could drive the development of an interface that is easy to learn and thus avoid the problem of teaching the model entirely. Further work has indicated that the success of direct manipulation devices may be due to this idea (Norman & Draper, 1986).

In summary, a mental model of a physical device is an internal representation of an external device that allows the user to make predictions about the device behavior. People seem to form some mental model and then adjust that model with continual exposures. Relating back to the situation where someone is using a computer to complete some task, the person would start with a model of the device. As the person is doing the task, the device is responding. With this interaction will come adjustments to the model. Although the model should be able to predict the behavior of the device, its primary function should be to aid in a person's problem solving.

How models may change with exposure to problems

When a model of a domain is created, is it general or specific to the domain? This question is still debated by many researchers, however, it is clear that people do not approach every problem as if it were a new problem (Glaser, 1984; Rouse & Morris, 1986; Wickens, 1984). The central issue is what happens to knowledge as you are exposed to different problems within a domain. One possibility is that many different models for each situation could be created. Another possible explanation could be that a single model of the domain is created and every time an exposure occurs, some change in the model is made. These first two possibilities make it impossible to explain how a person with a great deal of problem solving experience in one domain easily could find clues to another problem solving domain.

Yet another possibility is that each time you are exposed to problems in a domain, not only do changes occur to the model of the domain that you have but, changes also could occur simultaneously between that domain model and other related models. Thus, if you see many problems in statistics and you use a certain computer, each time you see a problem, your statistical knowledge could increase and your specific computer knowledge could increase. It is then possible that if you are asked to solve statistical problems on a different computer, you may have enough cues to access the original statistical knowledge. Several possible problems could keep you from utilizing this knowledge: You may not have access to that knowledge, you may never have had that knowledge, or you may not have generated generalizable computer statistics problem solving abilities in your statistical knowledge mental model.

So, in order to solve a problem either using a new software package or the same type of package on a very different computer, one could imagine a situation where you have a statistical model, a computer model, and a general model for solving problems on a computer. Each time you are presented with a statistical problem, you could increase all three models and therefore the solution on a new computer may only need to access the general "solving statistical problems on a computer" knowledge. In a sense, however, one could imagine a model for "solving statistical problems on a computer" model could just encompass the statistical model and the computer model and therefore eliminate the need for three models all together.

This combination of models could reflect the difference between expert and novice models. Again, expert and novice models may be fundamentally different (Chi & Glaser, 1984; Glaser, 1984; Greeno & Simon, 1984). A general conceptual change and a better structural organization has been documented by many researchers (Chase & Simon, 1973; Glaser, 1985; Pennington, 1985; Wiser & Carey, 1983). We could think of added conceptual knowledge or generalizations of problem schema as a consequence of more exposure to different problems in a domain. If we see expert models as transformations of novice models, then we can explain such results as diSessa (1982), McClosky (1983), and Clement (1983) which found naive ideas of certain concepts still in existence in expert models. Recall results from Gick and Holyoak (1983) in which subjects given more than one analog were more successful at solving across domain problems. They postulated that a generalized schema was being formed by those subjects. Also, Novick's experimental results showed that students with more math experience performed better on the analogical reasoning tasks than those with less ability. These results could be explained by the reflections on expert and novice models just presented. The change in mental models due to increased exposure resulting in differing levels of expertise may allow successful analogical reasoning.

<u>Relationship between mental models and analogical reasoning</u>

If we consider HCI as a form of problem solving in which people are given tasks to perform on the computer, we should be able to equate the mental models of HCI to the analogical reasoning models described in the previous section. Gentner and Gentner (1983) argue that people make analogical comparisons between systems all the time. These systems must be represented in some way and the hypothesis is that these systems are represented as mental models.

Models of domain knowledge and computer knowledge could be very specific, very general, or quite elaborated. In the case of the use of new software packages in domains different from software packages for which we are familiar or software packages on very different computers for which we are not familiar, several different models may exist: a domain knowledge model, a computer use model, and models for the use of software packages in that domain. A model or models that allow for generality or are quite elaborated may allow us to recognize that previous knowledge could be used in a new situation. If our models do not allow us to recognize the analogy, we will fail in transfer for the same reasons that students failed in the Gick and Holyoak studies (1980, 1983). On the other hand, if our models allow us to recognize that we could use analogical transfer to solve the problem but the low level similarities do not exist, we may find people who try to use an analogy to previous knowledge but nevertheless can not solve the problem. Thus, these two scenarios produce fundamentally different hypotheses about performance, whether or not analogical reasoning could be demonstrated in across domain HCI tasks.

In making a synthesis between analogical reasoning and mental models, surface similarities which often produced success could be equated with common elements. The structure and goal (pragmatics) issues could be equated to the sections of mental models that provide for explanations of how things work and why they work. In a sense, these sections of mental models can also be thought of as elaborated function slots for productions. This additional knowledge (explanations, pragmatics, function slots) can provide the material for recognizing that analogical reasoning should be used and what information should be retrieved. The mapping between the source and target problems may also depend upon the accuracy and completeness of the mental model. If a person is unclear about what a particular step in a procedure does or what it is used for, then their knowledge is incomplete and analogical reasoning can not occur. This result comes out most clearly when examining the research with mental models presented previously (refer to Norman's points on failures of mental models). Thus, the success of analogical reasoning depends upon the accuracy and completeness of the mental models for the source and target problems.

Discussion

The previous sections have examined analogical reasoning and mental models. The basic form of an analogy is found on standardized tests and is generally represented as A:B::C:D. In the literature, some source of base problem is substituted for A:B (source problem:solution) and a target problem is substituted for C:D. Subjects are given A:B and asked to solve C by finding an analogous solution. Researchers have been interested in whether or not subjects can find an analogous solution and what processes are involved.

Experimental evidence for analogical reasoning

Success and failure in experiments

In general, research on within domain analogies, learning by examples, has been quite successful in demonstrating that people can find an analogous solution in this situation; however, across domain studies have not been as successful. Several reasons may exist for this lack of success: First, subjects may need to solve the analog problem rather than just reading or summarizing it. In support of this position, Bransford, Franks, Vye, and Sherwood (1989) report on a series of studies in which "fact-oriented" study did not allow subjects to use their knowledge on new problems even though with later prompting subjects were able to repeat that "fact-oriented" information. Another argument in favor of this position would be encoding specificity. Subjects are not expecting to use the analog in anything other than another summarization task. Second, learning of a model for a domain may have to be done to a high criterion of acquisition in order to see any use of that knowledge in analogical reasoning. Inadequate training may mean that nothing can be assumed about how much knowledge the person actually started with before the target problem was presented. Associated with this criticism is the finding that experts were able to use a previous analog more easily than novices (Novick, 1988b) and those people who were given more than one analog were more successful (probably due to general schema formation, Gick & Holyoak, 1983). These findings seem to indicate that well elaborated schemas or knowledge structures aid in analogical reasoning. Either or both of these problems could be contributing to the lack of success in across domain analogical reasoning studies.

Main problem associated with failure of subjects to perform analogical reasoning

Although across domain studies have had trouble finding analogical reasoning, in general, these studies along with within domain studies have identified retrieval as the source of difficulties for subjects. Retrieval starts with cues produced by examining the target problem. The success of the retrieval appears to be based on both surface structure and relational structure (Gentner, 1989).

Research has indicated that surface similarities may be accessible to all levels of expertise but that deep structures may only be accessible by those with expertise/ability (Novick, in press). What we are then hoping for is that people with more ability (domain experience) would be able to access their domain knowledge even in the face of surface differences (as there are with two different types of computers). Assume that someone who is familiar with one domain is asked to perform a task in a very different domain. Retrieval failures could be due to a lack of knowledge in the original domain or inadequate training. Examining different levels of expertise in relation to this problem may shed some light on how all the different component processes of analogical reasoning can occur.

Analogical reasoning processes and mental models reviewed

Retrieval is only one component process of analogical reasoning. The process of analogical reasoning includes the following components: retrieval of source analog by use of some representation of the target problem, mapping of the analog to the problem to produce an analogical solution, evaluation of correctness of analogy and iteration if the process was unsuccessful. Failure of any of these components could result in failure to perform analogical reasoning. Although several analogical reasoning models were described, the basic analysis produced the idea that these models all contained the basic processes but with slightly different forms and that these models could be equated to each other analogically.

These models of analogical reasoning all depended upon some type of representation. The alternatives examined were simple sequences, general methods, and mental models. In the case where the surface structure of the source and target problem are very different, the use of knowledge about the situations in which procedures would be applicable may be necessary just to identify that analogical reasoning should be used. Simple sequences and general methods which include only sets of productions are inadequate, since pure productions do not contain this information. Annotated productions which provide function information, higher order relations between objects, and pragmatics all provide this needed additional information. These structures can be equated to each other in that incompleteness of the knowledge included in them would produce either a failure to recognize an analog or a failure in the mapping between source and target problem. These failures due to incompleteness are particularly evident when examining the literature on mental models. Thus, based on experimental evidence which indicated the importance of elaborated structures and several types of knowledge interacting for analogical reasoning, it is proposed that people have mental models which can be used to recognize the applicability of analogical reasoning in across domain problems and can affect the mapping process.

A mental model was defined as a representation which could produce a simulation of the real world, a person's understanding of a particular content domain, and specific information about how the information was attained and is to be used (such as how to use it knowledge for devices). It is possible that each time one is presented with a problem in a certain domain for which a computer program is used to compute the answer, domain knowledge, computer knowledge, and the combination are all activated. Depending on the outcome of the problem solving attempt, learning can be achieved in all three types of knowledge. Either a single mental model can represent all three types of knowledge or a mental model could exist for each type of knowledge. This would permit the detachment of any one type of knowledge for later use in a different setting. "If that which is to be transferred consists of a coherent theory of a causal explanation that is understood, it is difficult to impede a flexible application of prior knowledge" (Brown, 1989, p.370).

Conclusion

In summary, the major issues to be examined when studying analogical reasoning are: the knowledge level attained by the person on the source analog or domain and what types of tasks the person was asked to perform with that knowledge. Level of expertise, thus, plays a part in analogical reasoning in first indicating how well a subject has learned the source analog or domain and second, in indicating how well integrated the information may be in the different mental models of the source domain and related domains. Another issue is whether or not to stay with the current paradigm, analogical reasoning literature's methods, or to expand the current paradigm to incorporate HCI methods to capitalize on the success of HCI research. Staying within the current paradigm may not permit us to achieve the high amounts of transfer witnessed in HCI. In addition, it could be argued that the tasks used were too artificial (although note that Novick, Anderson, and Ross have all used real tasks and their work is distinguished by quite a bit more success). In a sense, HCI laboratory tasks resemble more closely the real world and real world tasks. This may account for the success of HCI research on transfer, since several investigators have found that in some situations people can not perform certain skills in the laboratory that they can perform in real life (Cole, 1975; Gladwin, 1970; Labov, 1979; Scribner, 1976; Rogoff, 1981). Therefore, the use of HCI tasks combined with the study of people with different levels of expertise, different amounts of exposure to different domains with computer tasks, should provide quite a bit of information about the process of analogical reasoning.

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Appendix

Story analogs used for Gick and Holyoak experiments (1980, 1983).

A small country fell under the iron rule of a dictator. The dictator ruled the country from a strong fortress. The fortress was situated in the middle of the country, surrounded by farms and villages. Many roads radiated outward from the fortress like spokes on a wheel. A great general arose who raised a large army at the border and vowed to capture the fortress and free the country of the dictator. The general knew that if his entire army could attack the fortress at once it could be captured. His troops were posed at the head of one of the roads leading to the fortress, ready to attack. However, a spy brought the general a disturbing report. The ruthless dictator had planted mines on each of the roads. The mines were set so that small bodies of men could pass over them safely, since the dictator needed to be able to move troops and workers to and from the fortress. However, any large force would detonate the mines. Not only would this blow up the road and render it impassable, but the dictator would then destroy many villages in retaliation. A full-scale direct attack on the fortress therefore appeared impossible.

Attack-Dispersion Story (Version 1)

The general, however, was undaunted. He divided his army up into small groups and dispatched each group to the head of a different road. When all was ready he gave the signal, and each group charged down a different road. All of the small groups passed safely over the mines, and the army then attacked the fortress in full strength. In this way, the general was able to capture the fortress and overthrow the dictator.