Materials for Guiding Elicitation of Self-Reported Formulas Describing Knowledge in a Multi-Object Domain

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Materials for Guiding Elicitation of Self-Reported Formulas Describing Knowledge in a Multi-Object Domain.

This report presents a procedure for assisting someone in expressing knowledge about a domain in mathematical terms. It is particularly intended to be used with domains in which one entity is influenced by a number of entities, and the influence relations are one-directional and continuous. As a result, the relations are continuous rather than discontinuous and the mathematical expression of the relation involves functions rather than rules.

The purpose of the procedure is to provide a rough description of the dependency of the target entity on the other entities, in which appropriate functional expressions are used consistent with principles of meaningful measurement (cf. Krantz, Luce, Suppes, and Tversky, 1971). As such, it can serve as a form of externalization of an expert's knowledge that is not available from other sources (Fischhoff, 1989), as might be required for example in building expert systems.

The procedure was developed for use in a study of highway engineers' judgments of several aspects of highways (Hamm, 1990; Hammond, Hamm, and Grassia, 1986; Hammond, Hamm, Grassia, and Pearson, 1983, 1987). It is modeled after the procedures of multiattribute utility and value analysis (Keeney and Raiffa, 1976; Edwards and Newman, 1982) and after a project which used expert judgment to build a model of the effects of radiation pollution from a nuclear bomb manufacturing plant (Hammond, Anderson, Sutherland, and Marvin, 1984).

The procedure could be coded in an interactive computer program which would present the information, elicit the judgments, and produce the final formula. However, there are two advantages to using it in a paper and pencil mode. The expert or or subject will have some questions about the details or the point of the procedure which can best be answered by the analyst's or researcher's explanations. And the paper procedure gives not only flexibility (for example, sketches can be made and then refined to provide a curve that the analyst can later fit with a grid and a regression program), but also the appearance of flexibility (so that the subject is not limited by the options the computer program offers).

The material that follows consists of (a) an overview of the procedure, (b) forms to be used for various purposes at each step of the procedure, and (c) information sheets which explain concepts to the expert or subject in more detail. (Addition explanations can be found on the forms.) In addition, it is advisable to specify the definitions of the dimensions or entities which are involved in the domain. When the procedure is used in multi-subject research, these should be defined beforehand. When the procedure is used to express one person's judgment, the specification is worked out in negotiation with the person and hence can not be done ahead of time. However, it is useful to record the definitions in writing so that the problem definition does not shift unnoticed during the procedure.

Overview of the Procedure.

The various steps in the procedure have two functions (see Hamm, 1990). First, they help the expert organize the complex information of the problem and keep it in mind. Second, they constrain the process so that the resulting formula makes sense. This constraining is done in two ways: by suggestion and by requirement. It is suggested, for example, that interactions among input variables be identified and that variables that interact be grouped together. And it is required that all variables be defined consistently, so that the output of one combination process be measured the same way when input into the next combination process.

The typical sequence in producing a formula is as follows:

1. Define the measurement scale of the output dimension (Form 1) and the input dimensions (Form 2-i-a).

2. Graphically express the relation between each input dimension and the output dimension (Form 2-i-c).

3. Summarize the information about all input dimension in one table (Form 2-i-b).

4. Think about interactions. Is each input dimension's impact on the output dimension moderated or influenced by any other dimensions (Form 2-i-c)? Definitions of several interaction patterns are given on Sheet 1.

5. Group the input dimensions, according to similarity or high degree of interaction, and then arrange them in a hierarchical tree (Form 4). The tree is "hung upside down", and influence flows from twigs to trunk. The output dimension is the top node in the tree.

6. For each group of variables, give it a name, define its measurement, graph its effect on the output dimension, and consider whether it has interactions with other groups or individual variables (Form 2-g; parallel to forms 2-i-a and 2-i-c for the individual input variables).

7. For each branch of the tree, specify how the (local) input variables are to be combined into the (local) output variable. This involves choosing whether the variables are to be combined by averaging (Form 5), by multiplying (Form 6), or by defining a table that names the output for every possible combination of values of the input variables (Form 7). Sheet 2 gives guidance for choosing an organizing principle.

8. Each input into an organizing principle needs to be mapped onto the organizing principle's required input scale. This is done on Form 8 (for averaging) and Form 9 (for multiplying).

9. The top node is itself a branch in the tree representing the combination of several inputs -- individual variables or groups of variables. When the mathematical form of every branch in the tree as been specified, as well as the functions transforming individual variables for input into the organizing principles at the branches, or the outputs of low level branches for input into the organizing principles at higher level branches, then the expert's contribution is over: these specifications jointly define the model.

Experience has shown that experts with whom this procedure is used do not choose to use the multiplying method of combining variables. They feel that it is easier to express their sense of the variables' relative importance using the averaging method, and to express interactions among variables using tables. Indeed, it is difficult to grasp how to express relative importance or interactions using this multiplying method. In designing this procedure, Keeney and Raiffa's multilinear model was considered but rejected for being so complicated that people would not be able to understand how to use it. The present procedure seems to suffer from the same problem.

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Form 1: Answer Dimension Form.

Name ________________ Task ____________.

Answer dimension's units ______________.

Range of possible answers: Low ______ High ______.

A "natural 0" on a scale means that when it is called "0" there really is NONE of the quality being measured. If you had a natural 0, then it would make sense to say that an "8" is twice as much of the thing as a "4"; but if the 0 was arbitrary, it wouldn't have that sort of meaning.

For example, if you have savings of $10,000, you have twice as much money as if you had $5000, because 10000 is twice 5000. Here the $0 is a natural 0. But 32 degrees F is not twice as warm as 16 degrees F, because the 0 on the temperature scale is picked arbitrarily. In other words, it does not have a natural 0.

Does the answer dimension have a natural 0? Yes ____ No ____.

It is useful, when considering numbers that measure a dimension, to ask whether the intervals between the numbers have consistent meaning, or whether the numbers simply express order. For example, is the difference between a 1 and a 2 the same as the difference between an 11 and a 12? In the above measures of money or temperature, the intervals do have consistent meaning. However, if we were to assign numbers to grades on a test, where A = 1, B = 2, C = 3, D = 4, E = 5, and F = 6, the interval between 2 and 4 would be different in meaning from the interval between 4 and 6. All the numbers convey is that A is better than B, etc.

Do the intervals in the answer scale have a consistent meaning?

Yes _______ No _______.

Name ____________
Formula Presentation
Maximal Guidance Approach

Task ________________.
Date ________.
Judgment Analysis Presentation.

Form 2-i-a: Assessment of Individual Dimension.
List of questions about each dimension.

This sheet has a set of questions for you to answer about each of the dimensions given for use in the formula. The answers will be entered in the table on Form 2-i-b. There is also a graph for you to draw on Form 2-i-c.

1. Write the abbreviation for the name of the dimension.

2. Is this dimension important to the problem? In other words, does it make a difference in determining the answer? If not, just go on to the next dimension.

3. According to the instructions, what units is this dimension measured in? If no commonly understood units are given, write "ARBIT" for arbitrary.

4. What range does the dimension vary over? Say the highest value.

5. And the lowest value.

6. Is there a natural 0 on this dimension? (Natural 0 was defined on Form 1.)

7. If so, is it inside the range we have to consider?

8. Do the intervals on the dimension have consistent meaning (see Form 1)? In other words, do equal numerical differences mean the same thing?

9. Now think about the impact of this dimension on the answer. "Impact" means how much a particular value on this dimension influences the answer. What point (or points) on this dimension has the highest impact on the answer?

10. What point (or points) has the lowest impact on the answer?

11. (NOW DRAW GRAPH ON FORM 2-i-c.)

Name ________________ Task ______________.

Form 2-i-b. Assessment of Individual Dimensions.
Table for recording answers.

Use this table for recording answers to questions on Form 2-i-a and 2-i-c, for each of the individual input dimensions.

1. Abbreviation for dimension.						
2. Does dimension make a difference? (Yes or No)						
3. Units dimension is measured in.						
4. Range: Lowest Value:						
5. Highest Value:						
6. Is there a natural 0? (Yes or No)						
7. If so, is it in the above range? (Yes or No)						
8. Do the intervals have consistent meaning? (Yes or No)						
9. Point(s) with highest impact on answer.						
10. Point(s) with lowest impact on answer.						
11. (DRAW GRAPH NOW)						
12. Do any other dimensions influence the impact of this dimension on the answer? (Yes or No)						

Name ____________ Task ____________________.

Form 2-i-c: Assessment of Individual Dimension.
Graph of impact and list of interacting dimensions.

Name of Dimension ____________________. Abbreviation ______.

To help express the relation between this dimension and the answer, graph it here roughly. The x axis represents the dimension. Label the high and low ends of its range (from Form 2-i-b). The y axis is the impact of the dimension on the answer. It ranges from the lowest impact to the highest impact. Now sketch the form of the impact function.

Highest impact |

I
M
P
A
C
T

Lowest impact |____.

________ Low end of range ________. High end of range

Dimension ______________.

Question #12 for Form 2-i-b: Do any of the other dimensions influence the impact of this dimension on the answer? (See Sheet 1, Definition of Interaction.) If yes, list them here:

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

Think about other dimensions whose impacts on the answer might be influenced interactively by <u>this</u> dimension. Add this dimension to those dimensions' 2-i-c Forms.

Name ____________ Task __________________.
Formula Presentation Date ________.
Maximal Guidance Approach Judgment Analysis Presentation.

Form 2-g: Assessment of Intermediate Products.

This form will be used for thinking about the relation between an intermediate product (representing the combination of a group of dimensions) and the other independently standing dimensions or groups of dimensions.

1. Name (or identifying code) of this group of dimensions____________.

   Abbreviation ________.

2. List the dimensions (or groups of dimensions) that are members of this group.

3. You will later specify a procedure for combining all these dimensions (and/or groups of dimensions) into one numerical measure which represents the impact of this whole group. Although you will specify this measure later, you probably already have a sense of its characteristics, and of the relation between it and the answer. For example, you can say whether the measure has a natural 0, and whether, as it increases, its impact on the answer increases or decreases.

   Is there a natural 0 on this dimension? Yes ____ No ______. What point or points on this dimension have the highest impact on the answer? ________. What point or points have the lowest impact on the answer? ________.

4. To help express the relation between this group of dimensions and the answer, graph it here roughly. The x axis is the numerical measure representing the group of dimensions. The y axis is the impact of this group of dimensions on the answer. It ranges from the lowest impact to the highest impact.

Now sketch the form of the impact function.

Highest impact

IMPACT

Lowest impact

Low end of range — High end of range

Group of Dimensions____________.

5. Interactions. Let us now focus on the question of whether this group of dimensions' impact on the answer might be influenced by other dimensions.

   Do any of the other dimensions or groups of dimensions influence the impact of this dimension on the answer, i.e., interact with it (see Sheet 2)? If so, list them:

   Influencing Dimension/Group

   1. ________________
   2. ________________
   3. ________________
   4. ________________
   5. ________________
   6. ________________

6. Think about other dimensions or groups of dimensions whose impacts on the answer might be influenced by <u>this</u> group of dimensions. Add this group of dimensions to their forms (Form 2-i or 2-g).

Form 3: Choice of Organizing principle.

This form is for use in deciding what organizing principle to use for producing either the final answer or an intermediate product to be plugged in at a higher level in the hierarchy.

Output.

Is this the top level, producing the final answer? Yes____ No____. If so, what are the units of the final answer? ________________. What is its range? Low _____ High_____.

If this is not the top level, then the output of this organizing principle will be input for an organizing principle at a higher level.

What organizing principle is used at the next higher level? __________.

What kind of input does it require? Units_________;

Range: Lowest point______, Highest point _____;

Does it need to have a natural 0?_____

Input.

List the input dimensions:

Organizing Principle.

What organizing principle do you want to use here? (Refer to Sheet 2 for guidance in your choice, and to the Forms 2-i and 2-g that you used to describe these dimensions to see what kinds of interaction they have with each other.)

Check one: Averaging___. Multiplying___. Table (Configural)___.

Other___.

Form 4: For recording group structure and justifications.

Researcher will note on this sheet the groups that the engineer makes and the justifications for each. This process will result in a hierarchy with the answer at the top and the levels arranged below it.

Once this hierarchy is recorded it can be used as an orientation aid for showing the engineer where a particular task fits in the overall process.

Groups.

Justifications

Hierarchy.

Form 5: How to do an Averaging Organizing Principle, and Form for Recording It.

Instructions.

In an averaging organizing principle, the n different input dimensions (Sdim's) are

1. Each transformed into a common scale, **Fdim(Sdim)**.

2. Each multiplied by a weight, Wdim X Fdim(Sdim).

3. Added up, with the weights normalized:

$$\frac{\text{Sum(dim=1,n) of Wdim X Fdim(Sdim)}}{\text{Sum(dim=1,n) of Wdim}}$$

4. Thus producing an output.

This method allows a change in each input to have an impact on the output, independently of the levels of the other inputs.

The common scales Fdim(Sdim) are in the same units and have the same range as the output scale.

For convenience, we will name weights Wdim that sum up to 1, across all the dimensions involved in this averaging principle.

We will use Form 8, later, for the specification of the impact functions of our input dimensions, Fdim(Sdim).

Procedure.

Let us now go through the procedure step by step.

Input.

List the dimensions whose impacts are the input for this averaging procedure (some may be individual dimensions, others may be groups of dimensions). You will specify the impact functions later; for now just write the variables' names.

Output.

Check where output of this organizing principle will be used: Final answer ___. Input into another organizing principle ___.

If the latter, what kind of principle will the output be input into?

___________________.

Specify the output dimension. It is usually convenient to use the units and range of the final answer. If the output is to be input into an averaging organizing principle, then it must be in the answer's units and range. Other times you may want it in different units because it is to be input into a different organizing principle.

Output dimension's units _______.

Range: Lowest point _____. Highest point ______.

The inputs into this averaging process, Fdim(Sdim), must also be in these same units and have the same range.

Weights.

The next process is to name weights for each input dimension that express the relative importance of these dimensions.

In order to name these relative weights, you need to remember that they correspond to the importance of a change from the level that has the highest impact, to the level that has the lowest impact.

Refer to the Form 2 for each dimension we are combining here, and review the highest and lowest impact points of each dimension. You must think about how important a change from the highest to lowest point on each dimension is, relative to changes from highest to lowest on the other dimensions.

Fill in this table, except for the first 2 columns. "Point with Lowest Impact" refers to the point on the original dimension (x axis value) that has the lowest impact (y axis value).

Weight	Normalized Weight	Dimension	Dimens. Abbrev.	Point with Lowest Impact	Point with Highest Impact
______	______	________	______	__________	__________
______	______	________	______	__________	__________
______	______	________	______	__________	__________
______	______	________	______	__________	__________
______	______	________		__________	__________
______	______	________		__________	__________
______	______	________	______	__________	__________
______	______	________	______	__________	__________
______	______	________	______	__________	__________

Now give weights to the dimensions that reflect their relative importance, and enter these in the first column. You can use any numbers you like.

When satisfied that your weights in the first column are correct, sum the column, divide each weight by the sum and enter the quotient in the second column. These are the weights you will use in your formula.

Now write your formula. The output equals the sum of "weight Wdim times impact function Fdim(Sdim)", for each dimension "dim". You know the numerical weights now. You need write only the correct variable name for each Fdim(Sdim).

Output =

Form 6: How to do a Multiplying Organizing Principle, and Form for Recording It.

Instructions.

In a multiplying organizing principle, the n different input dimensions (Sdim's) are

1. If necessary, transformed into a scale, Fdim(Sdim), that has a natural 0.

2. Multiplied together

   Product(dim=i,n) of Fdim(Sdim)

3. Thus producing an output.

In this method, each input dimension influences the impact on the output of a change in any of the other input dimensions. Thus, the impact of a change in one dimension is not independent of the other dimensions.

The output of the multiplying principle is a number that can potentially go from 0 to Product(dim = 1,n) of Max(Fdim(Sdim)), where n is the number of dimensions involved in this multiplying principle. You will later use Form 8 to transform this output into whatever scale is needed at the next higher level.

Procedure.

Let us now go through the procedure step by step.

Inputs.

List the dimensions whose impacts are the input for this multiplying procedure (some may be individual dimensions, others may be groups of dimensions). You will specify the impact functions later; for now just write the dimensions' names, and the names of their impact functions.

The output of this organizing principle is the product of functions **Fdim** of each of the dimensions Sdim, i.e.,

Output = F1(S1) times F2(S2) times ....

Write the formula, using the correct impact function names:

Output =

Form 7: How to use a Table to do a Configural Organizing Principle, and Form for Recording It.

Instructions.

In a configural organizing principle, the n different input dimensions are

1. Each divided into subranges,

2. Which become the categories on the margin of an n-dimensional table,

3. Into the cells of which you put a value indicating the output for the levels of input that define the cell.

This method allows expression of any form of interaction among the dimensions in their impacts upon the output.

There is no requirement to transform the input.

The output (cell values) can be assigned on any scale you choose.

Procedure.

Let us now go through the procedure step by step.

Input.

List the input (individual dimensions and groups) for this configural procedure. Consult their respective Form 2's to find their ranges. Divide these ranges into subranges or categories that seem natural, whose boundaries capture distinctions. The more categories, the more work you will have later.

Output.

Check where the output of this organizing principle will be used: Final answer ____. Input into another organizing principle ______.

If the latter, what kind of principle will the output be input into?

__________________.

Often it is good to specify the output dimension in the units and range of the final answer itself. When the output will be fed into an averaging organizing principle, it has to be in the answer's units and range. Other times you may want it in different units because it is to be input into a different organizing principle.

Decide on the output dimension's unit and range:

Output dimension's units __________. Range: Lowest point ______.

Highest point ______.

Construction of Table.

On blank paper, construct a table, with n dimensions, and divide its margins into the categories you specified above. When this is done, draw all the cells. Now, assign a number from the output dimension to each cell, representing the impact on the answer of the unique confluence of all the dimensions, as specified by their marginal values. In assigning output numbers to cells, be sure to use the whole range of the output dimension somewhere in the table, including its lowest and highest numbers.

Form 8: Impact Function Forms for Averaging.

On this sheet you will record a function for expressing the impact of either an individual given dimension, or a group of dimensions, to be used in an averaging organizing principle at the next higher level. (This form is also to be used if you need to transform the output of a table or a multiplying principle into the answer scale, at the highest level of the hierarchy.) Refer to Form 5, where you recorded your organizing principle, and Form 2-i or 2-g where you roughly graphed this relation.

This function will be recorded first on a graph. Then you will write a mathematical expression for it, if you know one.

Input.

Name the dimension, or the group of dimensions, which is the input to this impact function ____________________.

If the input for this impact function is a single dimension, its scale has the units given on its Dimension Assessment Form, Form 2. If the input is a group of dimensions, its scale is a numerical measure representing the impact of those dimensions, and you should have already worked out the organizing principle for creating that measure before you specify this impact function. (With such a group measure, the units depend on the kind of organizing principle you used.)

The input dimension's units: ________.

Its range: lowest ____ highest _____.

Output.

This impact function is to be used with an averaging organizing principle (or else at the top level). Therefore, we know that the impact F(S) it produces from the dimension scale S has got to

a) be in the units of the answer dimension, and

b) range from the lowest to the highest values of the answer dimension.

Graphing the Impact Function.

On a sheet of graph paper, graph the impact function of the dimension.

1. Label the x axis with the full range of the input dimension, Sdim.

2. Label the y axis with the full range of the output dimension, Fdim(Sdim).

3. Sketch, then draw carefully, a curve or line expressing the impact Fdim(Sdim) of each value of the input dimension Sdim. This must reach both the bottom (lowest impact) and the top (highest impact); other than that it can have any form you want. Some examples are: positively sloped (increasing) and negatively sloped (decreasing) straight lines, positively and negatively sloped nonlinear curves, and curves that are U-shaped or inverted-U-shaped. You need not restrict yourself to curves that you can describe mathematically; accurate expression of the relation, with the graph, is what is important here.

Describing the Impact Function Mathematically.

When you have completed graphing your function, give a mathematical formula for it if you know one. This can be an exact formula, with numerical parameters, or just a general formula using variables. Do not spend much time on this if the answer does not come to mind quickly.

Form 9: Impact Function Forms for Multiplying.

On this sheet you will record a function for expressing the impact of either an individual given dimension, or a group of dimensions, to be used in a multiplying organizing principle at the next higher level. (Refer to Form 6, where you recorded your organizing principle, and Form 2-i-c or 2-g, where you roughly graphed this relation.)

This function will be recorded first on a graph. Then you will write a mathematical expression for it, if you know one.

Input.

Name the dimension, or the group of dimensions, which is the

input to this impact function ________________.

If the input for this impact function is a single dimension, its scale has the units given on its Dimension Assessment Form, Form 2-i-b. If the input is a group of dimensions, its scale is a numerical measure representing the impact of those dimensions, and you should have already worked out the organizing principle for that before you specify this impact function. (With such a group measure, the units depend on the kind of organizing principle you used.)

The input dimension's units: ________________.

Its range: lowest ____ highest _____.

Output.

This impact function is to be used with a multiplying organizing principle. Therefore, we know that the impact F(S) it produces from the dimension scale S must have these features:

1. Fdim(Sdim) has a meaningful natural 0, although the range of Sdim actually given need not include the Fdim(Sdim) = 0 point.

2. The range over which the impact function Fdim(Sdim) varies reflects its importance, relative to the ranges over which the impact functions of the other dimensions involved in this multiplying organizing principle vary.

If the input dimension already has these features, you do not have to specify a special impact function, just write at the bottom of this form that Fdim(Sdim) = Sdim and go on to the next form.

Graphing the Impact Function.

On a sheet of graph paper, graph the impact function of the dimension.

1. Label the x axis with the full range of the input dimension, Sdim.

2. Label the y axis with the full range of the impact, Fdim(Sdim).

3. Sketch, then draw carefully, a curve or line expressing the impact Fdim(Sdim) of each value of the input dimension Sdim. You need not restrict yourself to curves that you can describe mathematically; accurate expression of the relationship with the graph is what is important here.

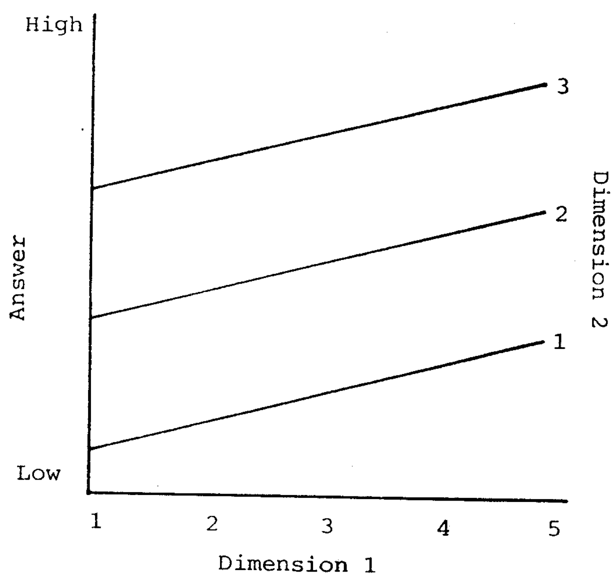
Describing the Impact Function Mathematically.

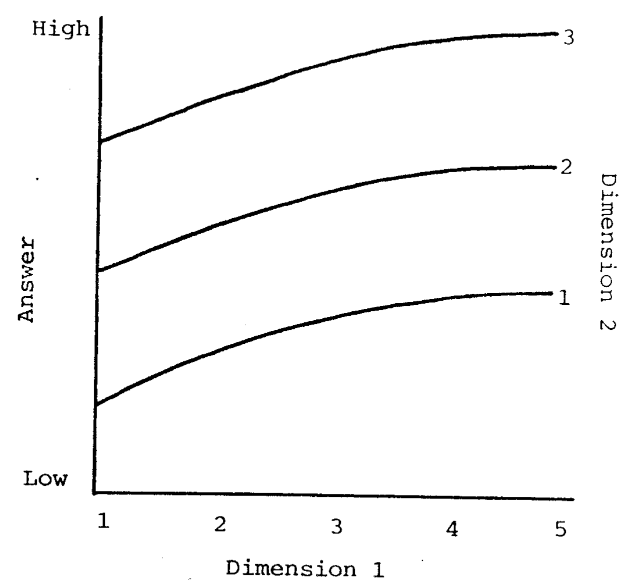
When you have completed graphing your function, give a mathematical formula for it if you know one. This can be an exact formula, with numerical parameters, or just a general formula using variables. Do not spend much time on this if the answer does not come to mind quickly.

Information Sheet 1. Definitions of Interaction Types.

This sheet defines and gives examples of interactions between dimensions, their effects on each others' influence on the answer.

No Interactions. When there are no interactions, a change in one dimension has the same impact on the asnwer, no matter at what levels the other dimensions appear. These figures illustrate the idea.

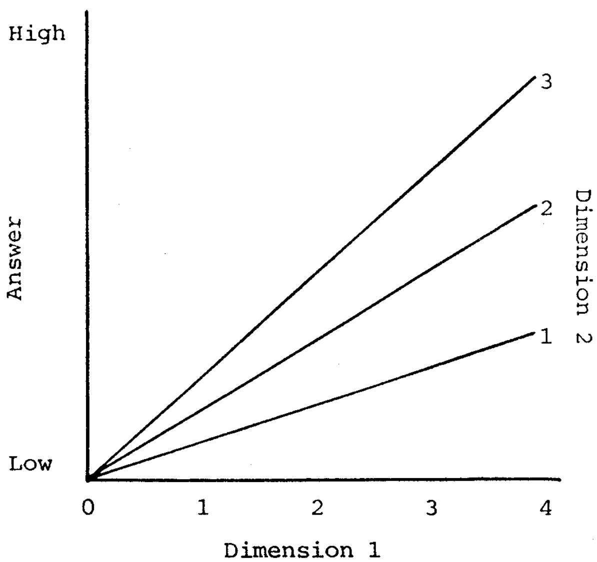


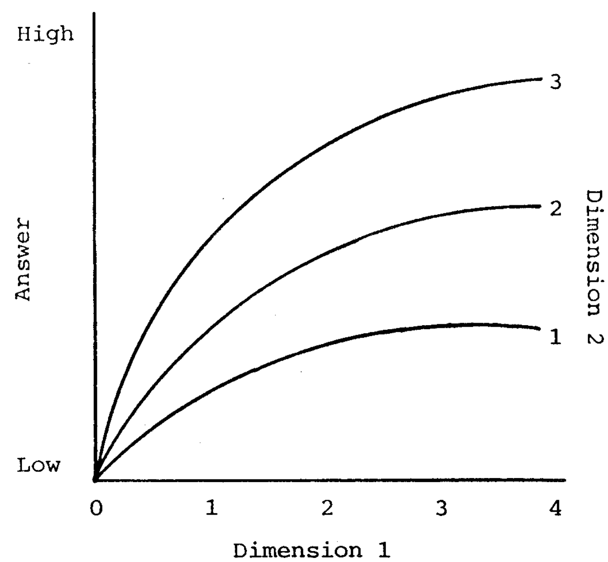


For each of the 3 levels of Dimension 2, Dimension 1 has the same impact function on the Answer. The only difference between the curves is that the height of the impact function has changed, expressing Dimension 2's impact on the answer. This is true for both the left and right hand figures, although the form of the impact is a straight line on the left, a curve on the right.

Interactions. If a change in the level of one dimension can affect the impact of a change in another dimension on the answer, we say they interact.

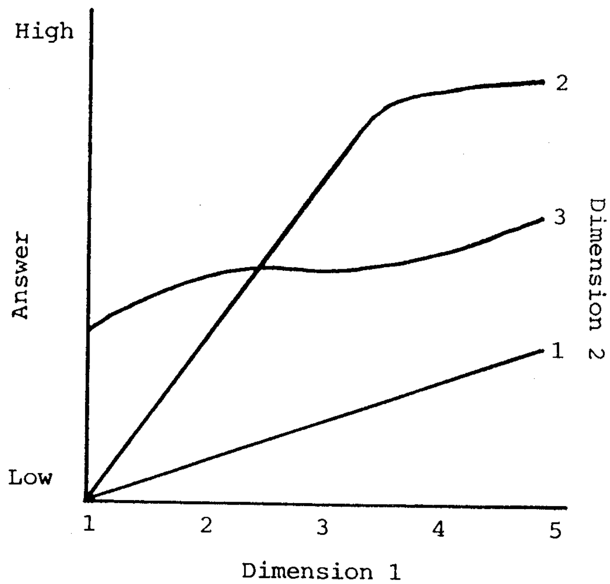
Simple Interactions. In simple interactions, a change in the level of one dimension will affect the size of the impact of a change in another dimension, but will not affect the basic form of the other dimension's impact function. These figures illustrate the idea.





For each of the 3 levels of Dimension 2, the function expressing the impact of Dimension 1 on the Answer has the same basic form, but its slope or strength has changed.

<u>Complicated Interactions</u>. In complicated interactions, a change in the level of one dimension will affect the basic form of the function expressing another dimension's impact on the answer. These figures illustrate the idea.



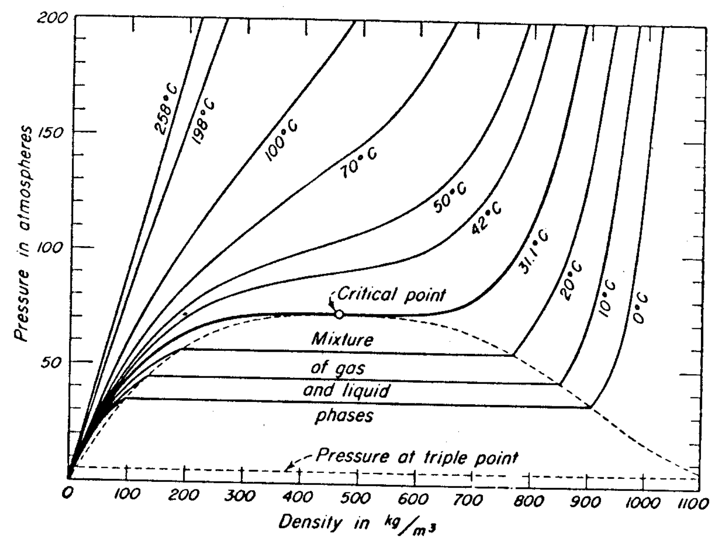


Fig. 13. Isothermals for CO_2, including the critical isothermal at 31.1° C, and showing the region in which vapor and liquid coexist. At the bottom of this region, below the 5-atm pressure of the triple point indicated by the horizontal broken line, is a region in which vapor and solid coexist.

In the left hand figure, for each of the 3 levels of Dimension 2, the function expressing the impact of Dimension 1 on the Answer has a different basic form. The right hand figure is an illustration from a physics textbook. The curves express pressure of carbon dioxide as a function of density (i.e., mass/volume), and there is a different curve for each temperature. Density and temperature have a complicated interaction.

Information Sheet 2: The Choice of Organizing Principle.

This sheet has information about three forms of organizing principle you may want to use -- averaging, multiplying, and tabular (configural). It tells about how they can be expressed -- in particular kinds of mathematical expressions, or in a table. It discusses constraints on the kinds of numbers these formulas can take as inputs and the kinds of numbers they can produce. It gives guidance on selecting which organizing principle is appropriate for your task, based on formal considerations.

Averaging.

Description. In averaging, you add the impacts of the dimensions together. The impacts are functions of the scale the dimension was originally measured in. The impact functions can be expressed with graphs or with mathematical expressions. To express the relative importance of the different dimensions, you multiply each impact by a weight. The weights all add up to 1.0. The output of an averaging organizing principle will be a number on the answer scale. However, it can be combined with other numbers before the final answer is produced.

Constraints. The input and output dimensions need not have natural 0's. We will make all the impact functions of the input dimensions have the same units as the output dimension, e.g., the answer. Then it is easy to figure out the appropriate weights.

When is averaging the appropriate organizing principle? It works well when the dimensions' impacts are independent of each other.

Multiplying.

Description. In multiplying, you multiply the impacts of the dimensions together. If you wish to (although this may not be necessary), you may use a function to transform a dimension's original scale into its impact. If you do this, the impact functions would be expressed with graphs and/or with mathematical expressions. The relative importance of the different dimensions is expressed by the range of their possible impacts. The output of a multiplying organizing principle can range from 0 to an arbitrarily large number, so a further step will be required to relate it to the answer.

Constraints. The input and output dimensions need to have natural 0's. If there is no natural 0, you should not use a multiplying function. If the natural 0 is there, but it is not expressed accurately by the units the dimension was originally measured in, then you will need to transform the scale into one that reflects the natural 0.

When is multiplying the appropriate organizing principle? It works well when the dimensions interact with each other in a simple way (see Sheet 1), such as when they affect the extent, but not the basic form, of each other's impacts on the answer. Multiplying is particularly useful if there are levels of some dimensions that can veto the impacts of other dimensions.

Tabular (Configural).

Description. To handle configural interactions, you construct a table whose input dimensions are the margins, i.e., the labels on the rows and columns, and whose output is the contents of the cells, expressing the output for that particular combination of inputs. There is no need to develop mathematical descriptions of impact functions for an individual input dimension, because you can label the rows and columns using the original units the dimensions were given in. To express that one dimension is relatively more important than another, you just make the cell values change more as this dimension varies than as the other one does. The output (cell values) of a table can be on any scale; for simplicity we recommend making it be on the answer scale, although it can be combined with other numbers before the final answer is produced.

Constraints. A table can take input dimensions that are measured in any kind of scale, e.g., one with or without a natural 0. Its outputs also can be in any scale. However, if the next higher level of the hierarchy requires input of a particular scale quality, you either have to put numbers in the cells that have that quality, or plan on transforming your cell numbers into a scale that has the qualities needed later.

When is a table the appropriate organizing principle? It is needed particularly when you have interactions that you can not express with a multiplying principle, e.g., complicated ones where the dimensions affect the form of each other's impacts. Tables can also be used in cases of simple interactions or no interactions, but they have the disadvantages that they have discontinuities, that it is hard to keep control of scale type, and that they are tedious for you to specify.

Alternative Organizing Principles.

If you have another organizing principle that you would like to use, explain it to the researcher and if he can understand it and thinks he can guarantee that the numbers will come out right, you can use it.