The Conceptual Structure of Word Algebra Problems

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RUNNING HEAD: Word Algebra Problems

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Abstract

It is proposed that algebra word problems can be better understood by analyzing the schema underlying each problem. An analysis of algebra word problems was developed from the work of Kintsch and Greeno (1985) in the domain of word arithmetic problems. Several experiments were conducted to determine the psychological plausibility of this model. Subjects were asked to provide similarity ratings for 8 problems, presented in pairs (28 pairs overall). The problem pairs were either identical in their conceptual structure (i.e., same underlying schema), identical in their resulting equations, identical in both, or neither. Naive subjects rated problem pairs which shared the same schematic structure as more similar than all other pairs, including problems which shared only the same equation. In Experiment 2 subjects were given a brief (25 minute) tutorial in either a traditional method or in a method developed to help students see schematic similarity across problems, then asked to perform the same rating task of Experiment 1. Subjects who were tutored in the traditional method showed similar performance as the subjects in Experiment 1. However, subjects who were tutored in the alternative method rated structurally identical problem even more similar than before. The findings support the model proposed. Furthermore, it is suggested that structural-mapping models of word algebra problems would be more successful is the mapping took place at these deeper, schematic levels.

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Footnotes

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The first author is now at the University of Colorado, Denver.

The conceptual structure of word algebra problems

Word or "story" algebra problems are hard. Even students who are good at mathematics often hate them, and neither teachers nor textbooks know how to teach them: typically they give such well-meaning advice as to "read and reread the problem until what is stated is clear" (Fuller, 1977, p.115), and otherwise are satisfied to provide students with an opportunity to practice.

Perhaps, if we understood the psychological processes that are involved in solving word problems, we could help students more effectively. Researchers in education and cognitive science have begun to lay the theoretical groundwork for such a project. In the present paper we attempt to extend a general theory of discourse comprehension (van Dijk & Kintsch, 1983) to the domain of word algebra problems. The theory has previously been applied to word arithmetic problems (Kintsch & Greeno, 1985), and the same principles can give an account of the algebra domain as well. In both cases, a conceptual problem representation must be constructed from the text according to arithmetic or algebraic schemata. To this schematic problem representation calculational operators are then applied. The difference between arithmetic and algebra is mainly one of complexity: there are more algebraic schemata than the arithmetic set schema, they may be interrelated in more ways than sets, and the calculational procedures themselves are more complex, in that both the derivation of an equation as well as its solution may require multiple steps.

After a brief discussion of the essentials of the Kintsch & Greeno model for word arithmetic, we describe the proposed generalization of the model to word algebra. Then, we report some empirical studies that suggest that the kind of conceptual structures postulated by our theory have some psychological plausibility, and, by making use of techniques emerging from other current research on word algebra problems, may perhaps make it possible to actually teach students how to solve word algebra problems.

A model for word arithmetic problems.

According to the model of Kintsch & Greeno (1985), several subprocesses need to be distinguished in word problem solving. A textbase is constructed, and is organized as a problem model, which is used to calculate the solution of the problem. The construction of the propositional textbase is much the same as with any other kind of text. What is distinctive is the kind of situation model - here called a problem model - that is formed: It is based on a set schema, and the different sets that are constructed are interrelated according to arithmetic superschemata (various kinds of superset-subset relations).

A simple example will clarify this point. Consider

Joe has 12 pencils. Then Mary gives him 5 more pencils. How many pencils does Joe have now?

This text is first analyzed into a set of propositions, such as [HAVE,JOE,[12,PENCILS]]. Then, on the basis of **arithmetic strategies**, these propositions are organized into a problem model which consists of three sets (called the **start-set,transfer-in-set**, and **result-set**), which are interrelated in a particular kind of superset-subset structure called the **transfer-in-schema**. This superschema serves to trigger a **calculation operator**, which generates the equation $12 + 5 = ?$.

The Kintsch & Greeno model has been formalized as a computer simulation by Fletcher (1985) and Dellarosa (1986). This simulation has been employed by Cummins, Kintsch, Reusser, & Weimer (in press) to determine the factors primarily responsible for the errors children make in solving word arithmetic problems. Different types of bugs were built into the simulation, and their consequences for the simulation's performance was determined. The simulation's behavior could then be compared with the errors children made in solving the same problems and recalling the problem texts.If the children's protocols exhibit the same pattern as the simulation, it could then be concluded that the particular bug that had been built into the simulation was a likely cause of the children's behavior.The results of this investigation were most instructive: it could be shown that manipulating the simulation's formal knowledge about arithmetic (that is, knowledge about sets and set relations) made it behave in a way that had no counterpart in the children's protocols. On the other hand, most of the errors the children made were generated when some simple linguistic defects were built into the simulation. Specifically, when the simulation was instructed to parse certain key words and phrases incorrectly, it exhibited the same types of errors and misunderstands as the children. The words and phrases in question had one meaning in everyday discourse, but another, more specific one in the context of the word problems, something the children had to learn specifically in school, and something they obviously still had trouble with. For instance, **some** in a word problem must be treated as a quantifier, not just as an ordinary modifier, as the children tended to do; or, as another example, **have-more-than** requires a particular, rather complicated parsing, which the children often confused with a simple **more**. It was at this linguistic level, rather than the level of formal knowledge or operations, where the reasons could be found for the extraordinary difficulty of some of these word problems for the children.

This lesson from the Cummins et al. study is very important. While it is certainly not the case that it can unhesitatingly be generalized to the domain of algebra word problems, it suggests that it might be a good bet that the translation from the problem text into a conceptual structure is a key element. Therefore, if it is possible to apply the van Dijk & Kintsch (1983) theory of discourse comprehension to word algebra problems, much as it was applied in the arithmetic domain, a better understanding of what is involved in that translation process might provide a basis for the development of instructional methods that could actually teach students how to solve such problems.

Algebraic schemata

Over 1,000 algebra story problems from standard textbooks have been compiled by Mayer (1981). Mayer classified these problems into "families". His first four families are various types of Rate problems, and comprise the bulk of the word algebra problems. Next, he considers Number, Geometry, Physics, and Statistics problems. Thus, the most widely used schema in word algebra problem is the rate schema, taking such forms as amount-per-time rate, cost-per-unit rate, portion-to-total-cost rate, and amount-to-amount rate. In addition, there is an open set of schemata that is used in geometry, physics, and statistics problems - the formulas required for the solution of these problems. Thus, it appears that a relatively small number of knowledge schemata will be needed to deal with Mayer's 1000-plus problem types, which makes the idea of extending the Kintsch & Greeno framework appear to be feasible.

The next step in this extension would be to look for the cues in the textbase of these problems to determine the conditions under which the algebraic schemata are elicited and how their interrelations are determined, eventually constructing a simulation as in the case of arithmetic processes. However, while the first part of this goal is easily achieved, the second may be impossible. It is possible, in an ad hoc manner, to point out the features in an algebra word problem that are involved in the elicitation of algebraic schemata and in the establishment of their interrelations. However, as a perusal of Mayer's problems will quickly show, just about any conceivable piece of world knowledge may play a role in this determination. Unlike with first-grade arithmetic problems, where it was possible to construct a knowledge base that supports the solution of a large portion of the typical, semantically impoverished problems found in most school books, a knowledge base for college algebra problems would have to comprise a significant segment of human world knowledge. No one today knows how to build such a knowledge base.

Thus, the path taken by Kintsch & Greeno (1985) and Cummins et al. (in press) is not open to us. Nevertheless, there are other ways to at least tentatively evaluate the extension of the model proposed here, and perhaps, put it to good use, even though a full-fledged simulation of the process of understanding these problems is currently beyond our reach.

The model makes some very definitive predictions about the nature of the conceptual structures that are supposed to be built in the process of solving these problems. The widely used technique of rating the similarity among word algebra problems may help us to determine whether our speculations about underlying conceptual structures have any degree of psychological validity. We propose to build here on the work of Reed (1987; also, Reed, Dempster & Ettinger, 1985) who has distinguished three levels of similarity to which readers may respond in solving word problems: the propositional or textbase level, where content plays the major role; the frame level where one would expect conceptual structure effects, and a level where the actual quantities are considered. In judging similarities among problems subjects rely heavily on propositional content, and are remarkably insensitive to structural analogies (see also

Dellarosa, 198x).

Structural similarities, for Reed and his coworkers, were conceived in terms of the structure of the equations used in solving the problems. In our way of thinking, the equations play a much lesser role - they are just one way-station between the conceptual structure and the solution. The dominant factor ought to be the conceptual structure - that is, the algebraic schemata used and their interrelations. We propose, therefore, to replicate Reed (1987) in such a way that the effects of conceptual similarity in terms of the algebraic schemata can be distinguished from similarity of the equations. If subjects are more sensitive to the former than they are known to be to the latter, this would be an encouraging sign that the way of thinking about word algebra problems proposed here would be worth further exploration.

Our experimental strategy can best be understood by an example. Consider the following problem:

(1) Train A leaves the station going east at 200 km/hour. Two hours later Train B leaves the same station on a parallel track also going east, but going at 250 km/hour. When will Train B overtake train A?

The distance-rate-time schema applies to both Trains A and B. Two further pieces of information are needed to relate these two schemata: that Train B leaves 2 hours later, and that the two trains travel the same distance (an inference based on the meaning of overtake - note the kind of world knowledge required here). A graphical representation of this structure is shown in Figure 1. The Distance-Rate-Time schema (DRT) is shown here as three interconnected ovals, corresponding to D, R, and T, respectively. They can be thought of as spread-sheet cells, interconnected in such a way that D = R*T. In addition, we indicate the equality of the distances both trains travel. We also indicate that the time Train A travels before it is overtaken equals the travel time for Train B plus 2 hours. The two DRT schemata, one for each train, are shown vertically, while the information interrelating them is arranged horizontally. In each case, however, ovals connected by a straight line can be read as equations, either horizontally or vertically. The numerical information given in the problem has been indicated in the appropriate ovals, and a variable *t* has been created for the time asked for in the problem. The standard equation for the problem is also shown.

Insert Figure 1 about here

Now consider another problem that is structurally equivalent:

(2) A Girl Scout troop sells the same number of peanut butter cookies as chocolate chip cookies. Peanut butter cookies come 30 to a box, while chocolate chip cookies come 25 to a box. The troop sold 28 more boxes of the chocolate chip than peanut butter. How many peanut butter cookies have been sold?

The algebraic schemata for this problem is shown in Figure 2. Note, however, that if one focuses on the construction of an equation, the equation generated from the text would be as shown in Figure 2 - not in the same form as in Figure 1: by asking for the number of cookies, a different equation is generated than by asking for the number of boxes.

Insert Figure 2 about here

On the other hand, in Problem 3 (shown in Figure 3), the conceptual structure is different from that in Problems 1 and 2, while the equation generated is of exactly the same form is in Figure 1:

(3) The area of a plot of land is the same as the area of another plot of land. The length of the first plot is 500 m, while the length of the second plot is 400 m. The width of the second plot is 25 m more than the first. What is the width of the first plot?

Problem (3) is no longer based on a rate schema, but on an "area-of rectangle" schema. Thus, the conceptual structure is different than in Figures 1 and 2. A graphical representation is shown in Figure 3. The equation derived from (3) is, however, the same as that derived from (1).

Insert Figure 3 about here

It is not simply the case that we chose to draw a different diagram in Figure 3 than in Figure 1. The difference in the diagrams reflects a fundamental conceptual difference between the problems. Mayer (1982) distinguished different types of propositions in algebra word problems. He identified "assignment propositions", which assign a numerical value to some variable, for example "the plane flew for 2 hours", and "relational propositions", which state a relation between two variables in the problem, such as "the first plane flew 2 hours longer than the second plane". The differences in the diagrams in Figures 1 and 2 on the one hand and Figure 3 on the other reflects this distinction. In the DRT schema there are assignment propositions which determine the values of D and T in the problem. The rate, however, is given as a relation between and T, 200 km per hour. The ALW schema, which has the same mathematical form, does not contain a relational component. None of the cells is expressed in terms of the others. All of the cell values are based on assignment propositions, though they are obviously related structurally. It is this difference - one of the schemata involves a relation and the other does not - that distinguishes the two problem types.

The data of Reed (1987) and others show that subjects are not very sensitive to the difference in the nature of the equations between story problems, as in (1) and (2). The reason may be that in terms of the conceptual structure upon which a solution is based, (1) and (2) do not differ according to the model proposed here. On the other hand, if that model is correct, (1) and (3) differ structurally, even though not in terms of their equations. Hence, one might predict that subjects would be more likely to detect the difference between (1) and (3) than between (1) and (2).

Experiment 1 was designed to investigate this hypothesis.

Experiment 1

The goal of this experiment is to provide some information concerning the psychological reality of the algebraic structure concept developed here. The method of Reed (1987) is employed, with some modifications. First, since the effects of story context on sorting algebra story problems are quite uncontroversial, we eliminate this effect altogether. All our problems have a different story context, so that story context does not exist as a basis for similarity judgements. Secondly, we distinguish between two types of structural similarity among problems - based on their equations, as in the original work of Reed (1987), and based on their conceptual structure in terms of the algebraic schemata and relations, as described above. Specifically, all problems in one class have relational components, while the problems in the other class have the same multiplicative relationship with no relational component. This yields a 2 x 2 table of problems, in which rows separate problems differing in their conceptual structure, and columns separate problems differing with respect to their equations. In this way subjects can compare every problem in the set with every other problem.

Method

Subjects.- Twenty subjects (10 male and 10 female) from the University of Colorado subject pool received course credit for participating in this experiment. Two subjects were excluded for failure to follow instructions.

Materials.- Eight problems were constructed. Both the form of the equation derivable from these problems and their conceptual structure was varied in a 2 x 2 design, with 2 problems in each cell. A different story context was used for each problem. The problems are shown in Table 1.

Insert Table 1 about here

Procedure.- After a brief introduction, subjects were told that they will be shown pairs of story algebra problems. They were asked to rate "how helpful would an explicit, worked-out solution to the first problem be if you were asked to solve the second problem." They were assured that they would not actually have to solve the problems. The equation for the first problem was displayed with the problem text, and subjects were told to ask for help if they did not understand it (none did ask).A rating scale from 1-20 was used, with "1" denoting "completely unhelpful." Subjects were reminded that this rating task is very similar to what they frequently do when doing homework problems: find a related example in the book that is similar to the one they are working on and mapping its solution to the present problem (Reed, 1987; Gentner & Gentner, 1983).

The eight problems were given in all possible pairs, for a total of 28 pairs.Two sets of problems were constructed. In the first set, one member of the pair, chosen randomly, was always presented first. This order was reversed in the second set. The order in which the problem pairs were presented was randomized for each subject.

Subjects were instructed to work at their own pace, but not to go back and examine or change a pair that they had already rated. Before the rating session, subjects were shown all eight problems, so that they were familiar with the rating materials.

Results

Subject Analysis.- Problem pairs were coded as having "same" or "different" structure type and "same" or "different" equation type. Each subject's mean score in these four cells was computed (4 pairs had the Same-Structure/Same Equation, while there were 8 pairs of the remaining three types). These four means were analyzed with a Within-Subjects ANOVA. The results are shown in Table 2.

Insert Table 2 about here

The ANOVA revealed a reliable effect of Structure Type, $F(1,17) = 26.6$, $p < .01$. Equation Type, in contrast, did not produce a reliable difference, $F(1,17) = 1.9$, $p > .10$. Neither Problem-Set nor Sex, which were also included in the analysis, had a reliable effect, F's < 1.

Item Analysis.- The same data were subjected to an item analysis, collapsing across subjects. The results of this Between-Items ANOVA were consistent with the Subject analysis. Structure Type was again reliable statistically, $F(1,500) = 14.1$, $p < .01$, while Equation Type was not, $F < 1$.

Another ANOVA was run to determine whether the ratings of Structure A problems differed from the ratings of the Structure B problems. Perhaps subjects could reliably tell that the ratings of the Structure A problems were similar but were unable to do that with Structure B problems, or vice versa. Pairs of problems which both have Structure A received a mean rating of 10.6, while problem pairs of Structure B were rated as 11.4. This difference was not statistically reliable, $F(1,500) = 1.5$, $p > .10$. By comparison, problem pairs composed of both Structure A and Structure B received a rating of 9.1.

Discussion

That subjects were unable to detect the similarity among the problems in terms of their Equation Type confirms the results of Reed (1987) and Dellarosa (1985). That their ratings did distinguish among the Structure Type of the problems is encouraging for the proposed theory of word algebra problem solving.

It would not have been too surprising if subjects were able to see some similarities in the Structure A problems. They all involve some sort of rate, though in different contexts and formulas. But, as the final item analysis showed, subjects were just as able to detect similarities in Structure B problems. In fact, Structure B problems were constructed to satisfy the constraint that they involve multiplicative relationships between three numbers other than rate - not an obvious relationship. Yet subjects were as reliable in detecting this basis of similarity as with rate problems.

The effect is, of course, by no means dramatic: subjects rated the structurally similar problems more similar by 2 points on a 20-point rating scale, for a 21% increase in their ratings. However, the effect was quite reliable statistically, and 17 out of the 19 subjects showed it. One might, in fact, doubt, whether there are any reason to expect larger effects. In post-experimental interviews most subjects insisted that all problems "looked alike", and that their judgments were almost random. Even if structures like the ones postulated here are being formed when word algebra problems are solved, it does by no means follow that these would be introspectively accessible. This may be an implicit part of problem solving of which subjects are neither aware, nor do they have an explicit understanding of it, just as one uses the rules of language without a knowledge of syntax. Thus, from naive subjects - even when they understand the problems involved - one could hardly expect more than glimmer of the structural principles involved.

However, if such structures are involved in understanding and solving word algebra problems, then it should be possible to make them explicit to subjects without too much trouble. Moreover, there is at least the possibility that by making these structures explicit we might find a way to <u>teach</u> how to solve word algebra problems. The goal of our next experiment is, therefore, to see whether a brief teaching session designed to introduce subjects to the spatial representation of word problem structures would be successful.

Experiment 2

We decided to give subjects a brief lesson in the graphic representation of the structure of word problems, and then have them perform the same kind of ratings as in Experiment 1. If algebraic schemata are implicitly involved in problem solving, then even a little study might make them more transparent, not to the point where subjects could actually use them effectively as problem solving tools, but at least to let people see the structural differences in our problem set more clearly.

Instruction in the traditional equation-centered method for solving algebra word problems may, of course, also improve the subjects' ability to discriminate between problems that have the same equation type and problems that do not. Hence a second group of subjects was given a brief refresher on how to derive equations from word problems, taken straight out of a standard algebra text.Our hypothesis was that making explicit the conceptual structure of these problems would sharpen the effects observed in Experiment 1, while conventional instruction probably would have very little effect (given the large amounts of previous experience subjects had with these methods in high school and college).

Method

Subjects.- 36 subjects from the University of Colorado subject pool participated in this experiment for course credit. 21 subjects were female, 15 were male.

Materials.- For the rating task, the same problems were employed as in Experiment 1. For the instruction session, 5 problems were used, two from Fuller (1977) and three from Mayer (1981). The Equation Training method was taken from Fuller (1977); it consisted in immediately writing down equations from the problem text and explained how the text was used to build up the equation. The Structure Training method consisted of an explanation of the notion of "schema", and a demonstration how such schemata can be combined to form structure graphs as in Figures 1-3, and how equations can be generated from these structures.

Procedure.- The tutoring sessions were conducted with pairs of subjects. They were told that they would be given a refresher course in solving word algebra problems. They were assured that they would not be graded in any way, nor would we even look at their attempts to solve the problems. The subjects in the Equation Training group were told that what they would be shown would very likely be similar to what they had been taught in high school or college, while the Structure Training group was warned that we would teach them a different way of looking at word algebra problems.

The first two problems were very simple, and were intended to demonstrate the basic method for setting up the problems. The other three problems were standard word problems, of a type different than those used in the rating study, The first three problems were worked out jointly on the blackboard, while the students tried to work out the next two problems on their own before they were shown the solution. Once the students arrived at the right equation, they did not have to solve it, but were told the solution.

After this 25 minute training sessions, students were taken to another room, and were there given the rating task exactly as in Experiment 1.

Results

The results from Experiment 2 are shown in Table 3. The equation Training had virtually no effect, in that the results are virtually the same as in Experiment 1. Structure Training, on the other hand, helped students to differentiate more sharply problems with a different conceptual structure.

Insert Table 3 about here

The data were analyzed in two ways. First the mean rating for each of the four cells in the table was computed for each subject. Then a 2 x 2 x 2 mixed ANOVA was performed,with Structure and Equation Type as within-subjects factors, and Instruction Type as a between-factor.Secondly, the ratings for each item pair were entered into an item analysis.

In the mixed ANOVA by subjects, the effect of Structure was significant, $F(1,34) = 98.2$, as was the Instruction x Structure interaction, $F(1,34) = 7.6$, all p's $< .01$. Equation Type, Instruction Type, and all other interactions were not significant.

The item analysis yielded substantially the same findings. There were significant effects of Structure, $F(1,1000) = 81.9$ and Instruction x Structure, $F(1,1000) = 11.5$; in addition, Instruction Type was also significant, $F(1,100) = 11.7$. No other effects were significant.

Discussion and Conclusion

A brief, informal 25 min. instruction period enabled subjects to distinguish problems of the same and different structure much better than before. The 21% effect observed in Experiment 1 increased to 50%. An equally brief traditional training period in setting up equations from word problems, on the other hand, had no discernible effects.

These results are encouraging. Subjects spontaneously perceive the structures postulated by our theory, albeit vaguely and without conscious awareness; but even a little training helps them to explicate these structures. The subjects' task was not very difficult in these experiments, of course. All they had to do is to discriminate rate problems from other three-term problems. This is, however, more than just a surface similarity effect as in Reed (1987), where subjects rated two problems involving automobiles as similar. Here, subjects could not rely on key-words or context, but had to use their conceptual problem representation. At the same time, these data are only preliminary. We don't know whether naive subjects would distinguish other features of the problem representation, nor, whether this representation would actually help students to solve word algebra problems.

We have shown here that the model of how children understand and solve word arithmetic problems, originally proposed by Kintsch & Greeno (1985), can be extended to college-level word algebra problems. The nature of this extension has been sketched, and experimental data have been reported, that, while falling short of a test of the proposed model, indicate that it may have some psychological plausibility. However, while these data provide only a weak test of our model, they hold some intrinsic interest: they are reliable and counter-intuitive, and they question current mapping models, such as Reed (1987). The relevant problem structure, the level at which analogical mappings occur, appears to be the conceptual structure as defined by our model, not the algebraic equation itself.

Further work on the present model will have to take a different track. The rating method, although useful, is insensitive and limited, and the informal, personal instruction we have used in Experiment 2 is subject to all kinds of bias and halo effects. On the other hand, as we have discussed earlier, we believe that the road to a full-fledged computer simulation of the process of algebra word problem solving is closed to us at the present time. The road we have taken, instead, is to develop a computer-based tutoring system (Nathan, Kintsch & Lewis, submitted) that works by helping students construct explicit spatial representations of the conceptual structures of word algebra problems, which normally are mental and implicit. This may provide us with at least an indirect test of the our theory, and may have some useful pedagogic fallout as well.

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Figure 1: A Distance-per-Time Rate Problem

Train A leaves the station going east at 200 km/hour. Two hours later Train B leaves the same station on a parallel track also going east, but going at 250 km/hour. When will Train B overtake Train A?

a) Equation Type:

$$250*t = 200*(t+2)$$

b) Structure Type:

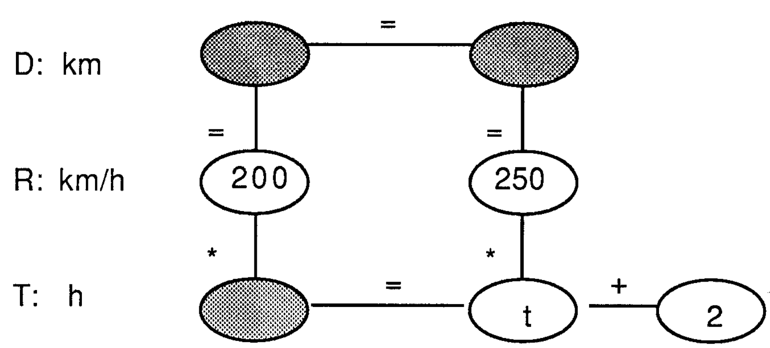

Figure 2: A Number-per-Unit Rate Problem

A Girl Scout troop sells the same number of peanut butter cookies as chocolate chip cookies. Peanut butter cookies come 30 to a box, while chocolate chip cookies come 25 to a box. The troop sold 29 more boxes of the chocolate chip than peanut butter. How many peanut butter cookies have been sold?

a) Equation Type:

$$28 + n/30 = n/25$$

b) Structure Type:

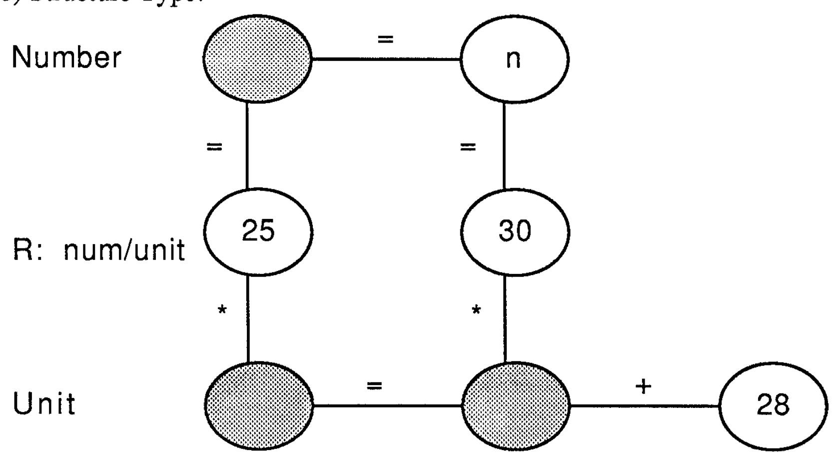

Figure 3: An Area Problem.

a) Equation Type:

$$500*w = 400*(w+25)$$

b) Structure Type:

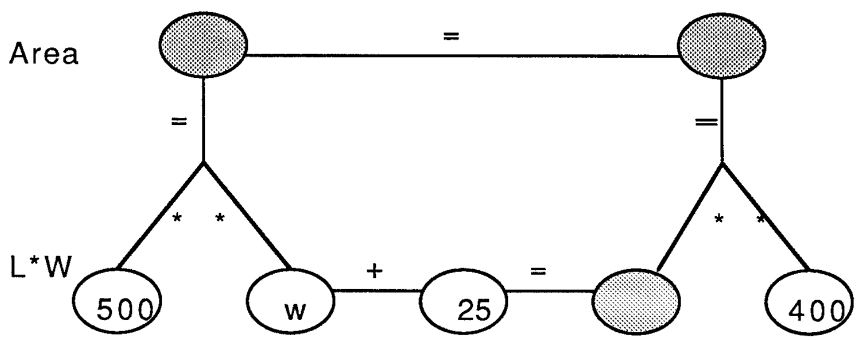

Table 1. Problems used in Experiments 1 and 2.

STRUCTURAL TYPE	EQUATION TYPE A	EQUATION TYPE B
A	Train A leaves the station going east at 200 km/hour. 2 hours later Train B leaves the same station on a parallel track also going east, but going at 250 km/hr. When will Train B overtake Train A?	Gary and Mark both have won the same number of free games playing a video game. Mark wins a free game 37% of the time he plays, while Gary wins a free game 46% of the time. Mark has played 135 more games than Gary. How many free games has each won?
A	Ernie invests some money at 10% and $350 more than that at 8%. Both earn the same amount on interest. How much was invested at each interest rate?	A Girl Scout troop sells the same number of peanut butter cookies as chocolate chip cookies. PB cookies come 30 to the box, while CC come 25 to the box. The troop sold 28 more boxes of CC than PB. How many PB cookies have been sold?
B	The area of a plot of land is the same as the area of another plot of land. The length of the first plot is 500 m, while the length of the second plot is 400 m. The width of the second plot is 25 m more than the first. What is the width of the first plot?	There are 2 electrical circuits, A and B The current in circuit A is 12 amps, while the current in circuit B is 9 amps. The resistence of B is 2 more than the resistence of A. The voltage in the 2 circuits is the same. What is the voltage in the 2 cicuits?
B	Two balls are thrown by the same person. The mass of the first ball is 150 g while the mass of the second ball is 125 g. The acceleration of the second ball is 5 units more than the acceleration of the first ball. The force of both the balls is the same. What is the acceleration of the first ball?	Two wedges of pie have crust on the outside edge which is the same length for both pieces. The radius of one pie is 5 cm more than the radius of the second, while the angle that the second wedge of pie makes is 9 degrees more than that of the first. What is the length of crust for the two pieces?

Table 2: Mean Similarity Ratings in Experiment 1

		Equation Type		
		Same	Different	
Structure	Same	11.1	11.0	11.0
Type	Different	9.2	9.0	9.1
		9.9	10.0	

Table 3: Mean Similarity Ratings in Experiment 2

TEXTBOOK METHOD:

		Equation Type: Same	Different	
Structure Type	Same	10.6	10.9	10.8
	Different	8.3	8.6	8.5
		9.5	9.8	

STRUCTURE METHOD:

		Equation Type: Same	Different	
Structure Type	Same	12.5	13.2	12.9
	Different	8.3	8.8	8.6
		10.4	11.0	