ABSTRACTION OF PROBLEM-TYPE SCHEMATA THROUGH PROBLEM COMPARISON

Denise Dellarosa
Psychology Department
University of Colorado

ABSTRACT

Several researchers have reported marked differences in the abilities of expert and novice problem-solvers to reliably sort problems according to problem structure. These differences have been attributed to the possession of schematized knowledge structures by experts that are lacking in novices. The aim of the present research was to outline and test a descriptive model of schema acquisition. The model describes the processes involved in deriving schemata from psychologically defined problem instances, and the memory structures resulting from the processes so described. Two experiments were conducted to test the model. In the first, the sufficiency of the processes comprising the model for schema abstraction was tested by requiring subjects to answer questions while reading algebra word-problem (questions that were presumed to enhance one or more hypothetical memory structures), and comparing performance on a problem sorting task following reading. Ability to sort previously read and new problems properly on the basis of problem structure was found to vary reliably with the type of question answered while reading. Questions that facilitated comparison of problems yielded better structure sorting performance than questions that focused attention within problems only. Similar results were obtained in Experiment 2, where subjects were required to match problems to symbolic representations (i.e., algebraic equations). Here subjects who answered questions that were presumed to facilitate schema abstraction according to the model performed as well as experts in matching problems to their symbolic representations.

Abstraction of Problem-type Schemata Through Problem Comparison

Expert and novice problem-solvers differ not only in their relative success at solving problems but also in qualitative aspects of their problem-solving behavior (Chi, Feltovich, & Glaser, 1981; Larkin, McDermott, Simon, & Simon, 1980a; Schoenfeld & Hermann, 1982; Simon & Simon, 1978). Novices typically attempt immediate solutions by working backward from the goal (i.e., means-ends analysis), and tend to focus on surface features of the problem statement while creating a problem representation. Experts, on the other hand, typically engage in qualitative analysis of the problem prior to attempting a solution, working forward from the results of their analysis. An important component of the qualitative stage is a classification of the problem based on an analysis of its underlying structure. Only after a problem has been identified as belonging to a certain type, and the appropriate knowledge concerning that type activated, is a solution attempted.

The implication of these facts is that acquisition of knowledge structures identifying problem-types, called problem-type schemata, is essential in the development of expertise. Indeed, models that successfully simulate the differences between expert and novice problem-solving behavior do so through the careful manipulation of schematic knowledge structures (Larkin, 1981a; 1981b; Larkin, McDermott, Simon, & Simon, 1980b). The success of such schema utilization models provides insight into the nature of schemata and the processes involved in their activation and use during problem-solving. The processes underlying the acquisition of such structures, however, are less well understood.

To describe the processes involved in schema abstraction, several researchers have appealed to traditional theories of concept formation (Anderson, 1983; Anderson, Kline, & Beasley, 1979; Elio & Anderson, 1981; Lewis & Anderson, 1983; Ross, 1982). Despite their apparent success, however, it is not entirely clear that these theories as they currently stand are appropriate for use in describing abstraction of complex problem-solving knowledge structures in formal domains. The concepts whose acquisition these theories were originally intended to describe are typically defined as collections of readily apparent surface features (e.g., red and triangle, etc.). These concepts and their respective exemplars are thus defined over surface features. Problem-types of formal domains, such as physics and mathematics, however, are defined not over surface features of problem statements but instead over underlying problem structures.

To adequately describe schema-abstraction in complex domains, it is necessary to (1) define problem-types in terms of problem structures, and (2) describe the process by which information from problem instances can be transformed into problem-type schemata. In the work presented here, a descriptive model designed to meet each of these criteria is proposed and tested. This model is depicted in Figure 1. The domain of interest here is that of word algebra problems, where problem structures are defined as quantitative relations among relevant variables (i.e., equations) cited in the problem statement. Problems that describe the same equation, or relations among variables, are considered to share a problem structure.

Insert Figure 1 about here

Since most problems are presented in verbal or textual form, a reasonable starting point involves describing problem representations derived through text comprehension processes. Accordingly, the model begins with three levels of verbal problem-statement representations that, according to van Dijk and Kintsch (1983), constitute a reader's understanding of a text. The first, called a microstructure, is a coherent conceptual representation of the text, and is constructed from the output of a linguistic parser. The second, called a macrostructure, is derived from the microstructure, and constitutes a hierarchical representation of the main ideas expressed in the text. These two structures are termed <u>text-base</u> representations in that they represent important characteristics of the text itself. The third representation, called a <u>situation model</u>, is a representation of the situation described in the text. It contains representations of the entities, relations, etc., that are referred to in the text, augmented by the reader's knowledge of the elements referred to. The text-base representations comprise memory for the text-comprehension episode as well as the text itself as an object, while the situation model constitutes memory for what the text is about.

The situation model is of particular importance to the problem-solving process in that it contains the structure of the problem. Problem texts whose cover stories differ dramatically may nonetheless describe structurally identical situations. In such a case, the problems' macrostructures would differ substantially, but their situation models would contain similar structures.

The first stage of schema abstraction involves comparing the situation models of the problem texts. This comparison process is essentially pattern-matching using the following rules:

1. Compare entities; if identical, link together with identity marker.
2. Compare relations; if identical, link together with identity marker and check arguments.
   2.1 If NOT identical, mark arguments as analogous.
3. Compare supervening structure of relations; if identical, mark whole structure as identical, and check atomic relations and arguments.
   3.1 If NOT identical, mark them as analogous.

Rule 2.1 applies to situations in which disparate entities are involved in identical relations in two texts. For example, in the phrases "fill the tub" and "fill the pool", tub and pool would be marked as analogous entities since they are both arguments to the same relational term "fill". If one were asked a typical analogies type question about these phrases such as: "Fill : tub :: Fill : ?" or "Tub (phrase 1) :: ? (phrase 2)", the correct answer would be "pool", since in tub and pool are both filled in the two phrases. Rule 3 applies to situations where two texts share a common event structure. For example, a text may describe a situation where "A is taller than B is taller than C", while a second text may describe a situation where "D is bigger than E is bigger than F". Although no entities or atomic relations are shared by either text, they do share a common structure of relations, i.e., linear orderings. In this case, the supervening structures would be marked as identical, and the atomic relations "bigger than" and "taller than" would be marked as analogous, as would the individual entities A and D, B and E, and C and F.

The memory structure resulting from this process is called a general model. It comprises the two models linked together showing identical and/or analogous entities, relations, and structures. It contains individual entities from the two models, irrelevant model-specific relations and attributes, and possible spurious matches.

From a general model, schemata can readily be formed through careful deletion of information using three principles. First, matching structures and relations are retained to form the basis for the schematic representation. Second, non-matching structures, relations, and arguments are deleted, (i.e., not carried over into the schema). Third, remaining entities are replaced with variables. For example, in the preceding case, the terms "pool" and "tub" would be replaced by variables. Thus, any text referring to items being "filled" would activate this schema.

The resulting representation is the schema. If the problem texts share nothing more than a common problem structure, than the schema will represent all and only the similarities between the problem texts that are relevant for problem classification and solution. Moreover, the resulting schematic representation feeds back on the comparison process by facilitating recognition of matches among structures. It itself can be further pruned if mismatches with subsequent problem instances occur. Thus a schema is formed through loss of particular information, and this resulting abbreviated structure facilitates recognition of like structures in the new problem instances.

Note that implicit in the pattern-matching and abstraction rules is the assumption that people notice first identities and analogies among entities and similarities among structures only after considerable work. This assumption is based on the results of several empirical studies. Novices typically fixate on commonalities among features when classifying problems (Chi et al., 1982; Silver, 1981). They can be induced to attend to problem structure, but only when attention is drawn away from surface features (Sweller, Mawer, & Ward, 1983) or after considerable instruction or practice with stimulus materials (Ross, 1982; Schoefeld & Hermann, 1982). Moreover, when memory is tested for texts, all of which share a common verbal structure, individual names and entities tend to be the most readily confused types of information (Thorndyke, 1977; Thorndyke & Hayes-Roth, 1979).

It should also be noted that schema abstraction in this model depends heavily on comparison of problem instances, and that the crucial component of this comparison process is analogical reasoning. As disparate entities and relations across problems are recognized to share common roles within the problems, common structures become apparent. This assumption is based on several studies whose results emphasize the importance of instance comparison in abstracting schemata as well as learning to use schemata provided by experimenters. Gick and Holyoak (1983) for example, found performance on an "insight" problem to be facilitated by an experimenter-provided schema only when examples were also provided. In a similar vein, Anderson, Farrell, and Sauers

(1983) found LISP function definitions (schema-like templates of LISP functions) to be of little use to students without examples of the functions so defined. A reasonable conclusion to draw from these results is that a schema alone contains too little information to fix its own interpretation. It is ambiguous enough to support any number of interpretations; schema instances serve to fix a unique interpretation among those possible.

Once correctly abstracted, the information contained in the resulting schematic representation can be used as conditions in a production system whose execution produces problem-solution. As such, the schema represents the sufficient conditions for (1) membership in a problem category, and (2) execution of a standard solution procedure attached to that category.

To summarize, it is proposed that acquisition of problem-type schemata involves abstraction from individual instances. The abstraction process itself can be decomposed into (1) a mapping of common elements across instances, (2) deletion of non-matching elements, and (3) removal of references to individual entities in favor of variables.

The experiments described here were designed to test the sufficiency of the processes contained in the model. Quite simply, if schemata can be formed in this way, then subjects who are induced to perform all of these processes should be capable of describing problem structures and using them to sort problems, while subjects who are arrested at each successive stage should possess inferior understanding of problem structures and hence produce inferior problem-type descriptions and inferior sorting performance. More specifically, three crucial hypotheses were tested in the studies to be reported here: (1) that schemata can be induced though the processes described in the model, (2) that schemata induced in this way can facilitate recognition of instances of problem types, just as those possessed by experts are purported to do, and (3) that these schemata can also facilitate recognition of symbolic representations of problem types (e.g., D=RT), as required in most formal domains. The general strategy in the experiments that follow was to control the types of memory structures produced by subjects while they processed problem texts by focusing their attention on different aspects of the texts. Some subjects were guided through all four stages of the schema acquisition process outlined in the model, while others were focused at successive stages.

Subjects' attention was directed to different aspects of the texts through the manipulation of task demands while reading, a procedure similar to a "levels of processing" task. A recognition task was employed to focus attention on the creation of <u>at most</u> a text-base and situation model. Subjects who answered this type of question were not expected to produce very rich memory structures in that task performance required attending to the wording and meaning of individual sentences, thereby discouraging analysis of the problem as a whole. A verification procedure was employed to enhance development of a situation model. Subjects who answered this type of question were required to verify the relations among entities <u>within</u> a given problem. As a result, they were expected to emerge from the orienting task with a fairly good grasp of the situation described in the text as a whole. A third question type involved analogies. Subjects who saw this type of question were required to compare entities and relations <u>across</u> texts to determine analogous entities and relations across two different problems. These subjects were expected to form general models across problems, or networks indicating analogous entities, relations and structures across problems. Finally, "schema" questions were employed, which consisted of statements describing the crucial problem structures underlying a given problem type. Subjects who saw these questions were required to match statements describing problem structures to the problems they read. These subjects were therefore expected to possess the best representation of the problem structures appropriate to the problem instances all subjects were exposed to.

Evidence for the possession of schemata was taken from classification performance on algebra word problems, written protocol information, and correlations between orienting task performance and sorting performance. Evidence for the usefulness of schemata in recognizing symbolic representations of problem types was taken from subjects' abilities to correctly match the problems to their symbolic representations (i.e., sets of equations), and analysis of errors made during solution attempts.

GENERAL METHOD

Apparatus & Materials. Text presentation and collection of responses in both experiments were controlled by a FORTRAN program executed on PDP 11/03 digital computer. Texts were presented on CRT units. Subjects made their responses on response collection boxes located in front of their CRTs.

The texts included twelve instances of each of three algebra word problem types, which were: Catch-up (CU), Dilution (DL), and Facilitation-Interference (FI). The twelve instances within each problem type were further subdivided into four topics, which were: Travel, Vat, Interest, and Work. All texts were nine lines long, and ended with a set of three asterisks. Examples of the texts and the systems of equations required for solving each of the problem types are presented in Appendix A. As is apparent, the formulae for solving the problem-types are all variations of the formula AMOUNT = RATE X TIME. However, the variations in problem structure represent non-trivial transformations for novices (cf. Bull, 1982). Moreover, the word problems were sufficiently similar in surface content to render the identification of problem-types a non-trivial task for novices. Compare, for example, these problems with the ones used by Hinsley, Hayes, and Simon (1977). The latter could easily be separated into problem types by novices simply on the basis of topical word cues that were unique to problems within a given class.

The orienting tasks included questions of one or more of four types: recognition questions, situation model questions, analogical reasoning questions and schema questions. Examples of these items are also included in Appendix A.

Recognition questions consisted of verbatim repetitions and paraphrases of phrases chosen from the texts. The paraphrases contained changes in the propositional structure of the phrases. The subjects' task was to respond 'SAME' to phrases that were presented in the text and 'DIFFERENT' to paraphrases. Situation model questions consisted of statements of the relational terms included in the text along with their respective arguments. Foils were also included wherein a relational term was paired with an incorrect argument. The subjects' task was to verify the correctness of the pairing contained in the statement by responding 'TRUE' to true pairings and 'FALSE' to foils. Analogical reasoning questions consisted of statements of the form a:b::c:? along with two alternative answers. The statement parts included entities and relations chosen from across two problem texts that were of the same problem type. Subjects were required to choose the correct answer from among the alternatives, and indicate their choices on the response box. Schema questions included two principles, only one of which described the crucial structure underlying the texts just seen (see procedure section). The subject's task was to choose the correct schematic description.

The Travel, Vat, and Interest problems were used during the acquisition stage; the Work problems served as transfer problems. Texts were chosen such that subjects saw six instances of each of the three problem-types (i.e., CU, DL, FI) and the six instances included two texts from each of the three topics (i.e., T, I, V). In addition, four warm up texts were constructed, two involving proportions and two involving two unknowns in two equations. Subjects thus saw twenty-two texts in all.

Procedure. Texts were presented a sentence at a time, and subjects used a button press to request the next sentence. Subjects were tested after each block of two texts. The word "Read" signalled reading blocks, and the word "Ready" signalled testing blocks. Texts were paired such that every possible combination of topics was tested in each problem type. The same texts were always paired together, but order of text pair presentation was randomly determined for each subject. The practice texts were always presented first. Responses and reaction times to the questions were collected, and responses were followed by feedback indicating their correctness.

In both experiments, subjects were randomly assigned to one of four groups. In Experiment 1, question assignment was as follows: The first group (Textbase) received eight recognition type questions on each block of texts. The second group (Situation Model) received eight verification type questions on each block. The third group (Analogy) received four verification questions, and four analogical questions. The fourth group (Schema) received, four model type questions, four analogical type questions, and one schema type question. The latter two groups received more than one type of question to ensure that the correct relations within texts were grasped prior to comparing texts with each other. In Experiment 2, the verification questions were not presented to the Analogy and Schema groups in order to lessen the time required to complete all the experimental tasks (which were more time consuming in this experiment).

Subjects were run in groups of four, one to a CRT. The experimental tasks immediately followed the orienting task.

Statistical Analyses. In both experiments, significant omnibus F ratios for main effects were followed by Dunn's test for paired comparisons when the set of comparisons were constrained, and Tukey's HSD test for paired comparisons was used when the set was not constrained. Significant omnibus F ratios for interactions were followed by simple effects tests (Keppel, 1973). Significant simple effects involving more than one mean were subjected to the paired comparison tests described above.

EXPERIMENT 1

The experiment described here was designed to test two crucial hypotheses: (1) that schemata can be induced though the processes described in the model, and (2) that schemata induced in this way can facilitate recognition of instances of problem types, just as those possessed by experts are purported to do. The integrity of the hypothesized memory structures was tested by requiring subjects to sort the problems they read according to two criteria. The criteria were sorting the problems according to similarities in topic, and sorting by similarities in problem structure. In addition, all subjects were required to sort new problems--whose structures were the same as the ones previously sorted--by problem structure. The model predicts that all subjects should perform equivalently on the topic sorting task, since all should have produced text-base representations of the problems, which is all that is required to perform this task.

The model also predicts that subjects will vary systematically on the structure sorting tasks based on the type of memory structure internally generated. Schema subjects should produce the best performance since the schematic representation should contain all the information that is necessary to perform these tasks, that is, it should identify the problem structures. The Analogy subjects were expected to produce the next best performance level, since these subjects could use the information in the general models they formed to sort the old problems and to identify analogous structures in the new problems. However, general models can contain much problem-specific information in addition to matches that are irrelevant to the crucial problem structure which will hinder the identification of problems sharing only similarities in structure; therefore, these subjects were expected to produce more errors in the sorting tasks then were the schema subjects. Subjects who verified information in the texts were expected to yield the next level of performance since they will possess rich representations of the situation described in the problem, part of which is the crucial problem structure. However, without the benefit of comparisons across problems, these subject's should find it difficult to identify the important structure within the larger situation. Finally, the text-base group was expected to yield the lowest level of performance, since their understanding of the problems should have been halted at the text-base level.

Method

Subjects. Forty-eight University of Colorado-Boulder undergraduates participated in the experiment in order to fulfill an introductory psychology course requirement. The subjects were randomly divided into four groups as described below. The groups were fairly well matched in mathematics experience. The number of subjects in each group having completed at most high school algebra, college algebra, or college calculus, respectively, is as follows: Recognition--5,2,5; Verification--3,4,5; Analogy--5,3,4; Schema--5,2,5.

Procedure. The first sorting task following the orienting tasks constituted sorting by topic. Subjects were told that the texts they just saw could be categorized into three groups based on similarities in their cover stories or topics. They were shown an example of how to do this using the warm up texts, as follows: The SARAH and BRIAN problems were categorized to gether because they both talked about age; the SMALLTOWN and CAROLINE problems were categorized together because they both talked about elections. These were then removed and three more cards were placed in front of them. On the three cards was typed one of the problems from each of the three topics (Travel,Vat,and Interest). They were told that those three problems represented the three topics by which the remaining problems could be sorted. They were also told that the problems would sort equally, six to a pile, and that they would be given 3 min to sort all the problems. They were given a few minutes to look the problems over and determine what the three topics were. As soon as all subjects indicated their readiness to begin sorting, they were allowed to begin, and were told to raise their hands when they had completed the sorting. The time left was announced following each minute. When time was up, the experimenter recorded each subject's sorting (problem numbers were printed on the backs of the cards and these were recorded).

Following the topic sorting task, subjects were informed that the problems could be sorted another way, according to similarities in their underlying structures, or a principle which describes a fundamental similarity among them. An example based on the four warmup texts was given, as follows: Now, CAROLINE and BRIAN were categorized together because they both described proportions. SARAH and SMALLTOWN were categorized together because they both described two unknowns in two equations. They were carefully instructed in how to look for the lines in the problems that described the problem structures (i.e, 6 out of 8 men, 2 out of 3 voters, Sarah's age is 4 times greater that of her nieces etc.). Once subjects understood what was meant by problem structures and how to find them, the practice problems were removed and the same three experimental problems were then placed before them. These problems represented not only three different topics, but also three different problem structures. The subjects were informed of this, and the fact that, once again, the remaining problems would sort equally into the three piles. The subjects were told that they would be given 5 min to sort the cards, and were allowed a few minutes to look the problems over to develop a strategy. During this time, each subject's pile of cards was shuffled. When all subjects indicated their readiness to begin (usually after 1 min), they were allowed to start. The time remaining was announced every minute after the first 2 min had transpired. When time was up, the subjects' sortings were once again recorded, and the transfer task ensued.

Subjects were given the six transfer problems, and were told that these, too, could be sorted according to the three problem structures represented by the three standard problems. The standard problems were turned face-up, and subjects were given 5 min to read and sort the new problems into the three piles. Sorting times and sortings were once again recorded. Following this, subjects were given a protocol sheet on which they were required to indicate the three topics shared by the problems, and to describe the three problem structures by which they sorted the problems on the structure sorting task.

Results

Orienting-task performance and written protocol information. Subjects performed quite well on the orienting tasks. Average proportion correct responses on each task was as follows: Recognition - 62%, verification - 74%, analogies - 81%, schema questions - 90%.

An important question is whether the different orienting tasks affected subjects' internal representations of the problem types. Evidence that they did comes from subjects' written protocol information. The groups differed systematically when describing the three problem structures. The descriptions they generated could be classified into four categories. The first contains descriptions that refer to the correct problem structures. The second category contains descriptions that reflect a certain degree of analysis of the problems, such as "different start times, same end times". These descriptions indicate an analysis of the problem, but an incorrect interpretation of the structure. The third category includes descriptions that clearly refer to surface aspects of the texts, such as "Speed". The final classification includes those descriptions of subjects who could not remember the structures, did not answer the question, or wrote something unintelligible. The number of descriptions generated by each group in each classification is presented in Table 1.

Insert Table 1 about here

As is apparent in Table 1, Schema and Analogy subjects tended to use structural descriptions more often than the other two groups. Conversely, subjects in the Recognition and Verification Groups tended to use surface descriptions more often than the Schema and Analogy groups. These differences were substantiated statistically, $X(9) = 32.11$, $p < .01$. The groups clearly represented the problem types to themselves differently. The higher level groups defined the types in terms of problem structure; the lower level groups tended to define them in terms of surface features. The descriptions of the Analogy group is particularly interesting since subjects in this group did not have the benefit of the schematic descriptions to help them generate structural descriptions as the Schema group did. They instead generated these descriptions based solely on their experience with mapping relations across problems provided by the orienting task.

In contrast to the structure description task, the groups did not differ in the descriptions they generated of the the three topics, and all descriptions tended to coincide with surface aspects of the texts, e.g., "travel", "liquids", "interest".

Sorting performance. The number of problems correctly sorted on the topic sorting, problem-structure sorting, and transfer sorting tasks were calculated for each subject. Also computed was the number of problems that overlapped between a given subject's topic sortings and structure sortings. For example, if subjects tended to sort the same problems together in both sorting, the overlap between the two sortings would be very high. The greater the overlap, the more misled the subject was by similarities in surface characteristics of the problems. The data for each of these dependent variables on each task was subjected to an analysis of variance involving the following variables: Orienting task (Text-base, Verification, Analogy or Schema). In the topic sorting task, problem topic (Travel, Interest and Vat) was also included as a repeated measures variable. On the structure sorting task, problem structure (Catch-up, Dilution, and Facilitation-Interference) was included as a repeated measures variable. The mean number correct on the sorting tasks and the mean overlap scores were converted to proportions for ease of comparison across measures and are presented in Table 2.

Insert Table 2 about here

Topic Sorting Task. The groups did not differ on the topic sorting task $F < 1$. Moreover, performance was quite good; the overall mean score was 5.78, with a maximum of 6.0 possible. The topics of the problems differed in terms of ease of sorting, however, $F(2,88) = 4.83$, $MSe = .429$, $p < .025$. Tukey's test of pairwise comparisons indicated that the Interest problems were significantly easier to sort than the Travel problems (Interest-Travel=.271), while the Vat problems were not significantly more difficult than either of the other two (Vat - Travel = .062; Interest-Vat= .209). (The required difference for significance at the .05 level rejection probability was .219.) The ordering of difficulty is interesting in that the problems could be viewed as differing to the degree with which they share common entities. For example, the Interest problems all contain an investment account, and they were the easiest to sort. All of the Vat problems describe a liquid container of some kind, although the nature of the container varies (e.g., bathtub, water bed etc.), and they followed the Interest problems in ease of sorting. The Travel problems, however, represent the most heterogeneous set of entities, (e.g., plane, jogger, bicycle) and these were the most difficult problems to sort by topic. This interpretation suggests that subjects notice most readily similarities among entities in texts.

Problem-structure- Sorting Task. Analysis of the number of problems correctly sorted by problem structure returned two significant results: First, the problems differed in regard to the accuracy with which they were sorted $F(2,88) = 4.37$, $MSe = 1.88$, $p < .02$. Using Tukey's HSD test, the Dilution problems were found to be sorted more accurately than both the Catch-Up and the Facilitation-Interest problems, while the latter two did not differ (DL - CU = .312, DL - FI = .333, CU - FI = .021; required difference was .303).

Second, and more importantly, the groups were found to differ in performance $F(3,44) = 19.23$, $MSe = 2.66$, $p < .0001$. Using Dunn's test, the Text-base group was found to correctly sort significantly fewer problems than the Verification group (V - T= .694), the Verification group sorted fewer correctly than the Analogy group (A - V= .945), and the Analogy group sorted fewer correctly than the Schema group (S - A = 1.11; the required difference between the means was .481).

Transfer Sorting Task. The analysis of number of problems correctly sorted here returned a single significant result, that of group $F(3,44) = 5.66$, $MSe = .95$, $p < .01$. Using Dunn's test, the two lower level groups were not found to differ significantly (T - V = .056), the Analogy group was found to differ significantly from the Verification group (A - V= .528), and the Schema group did not differ significantly from the Analogy group, although the obtained difference was marginal (S - A = .278; required difference between the means was .287).

The overall pattern of results for this task indicates that the higher level groups were well capable of using the knowledge representations they acquired during processing of the original problems correctly to sort new problems on the basis of shared problem structures. The lower level groups could not.

Overlap Scores. The trend in the overlap scores was in the expected direction, that is, the lowest level group produced the greatest overlap between topic and structure sortings, and the overlap gradually decreased as the level of processing increased. The analysis of these data produced a significant effect of group $\underline{F}(3,44) = 10.26$, $MS\underline{e} = 3.48$, $\underline{p} = .0001$. Dunn's test indicated that the Text-base group produced significantly greater overlap between it's sortings than did the Verification group (T - V = .778), the Verification group produced more overlap than did the Analogy group (M - A = 1.11), and the Analogy and Schema groups did not differ (A - S = .250; required difference between the means was .549). Thus, the groups were shown to differ in terms of the degree with which they could ignore similarities in surface features when sorting problems on the basis of similar structures.

Correlations between orienting task and sorting performance

Another question pertinent to interpretation of these results is the degree of influence that orienting task performance had on subsequent tasks. Table 3 presents the correlations between proportion correct responses on the orienting tasks and proportion correct sorting performance on both old and new problems. (Only the correlations for analogy questions are reported for the analogy group, and schema questions for the schema group since these are the most questions for these groups).

Insert Table 3 about here

The results are fairly clear: Recognition and verifications questions were not related to proper sorting performance; analogy and schema questions were related in a moderate and positive manner to sorting old problems. Accurate transfer performance was significantly correlated with accurate performance on the schema question task.

Discussion

The combined results of the sorting, overlap, and protocol data provide support for the sufficiency of the processes comprising the proposed model for producing schematic memory structures as well as influencing recognition of shared problem structures. The groups were found to differ systematically in their abilities to sort problems on the basis of problem structure, and their abilities to transfer this skill to new problems. Moreover, they were found to differ systematically in their abilities to ignore surface features when sorting problems on the basis of similar structures. Like experts, the Schema and Analogy groups performed exceptionally well when required to sort problems on the basis of similar structures; in contrast the Text-base and Verification groups were unable to perform adequately when required to use this criterion for sorting. Moreover, they were more likely to misled by surface features when sorting than were the Analogy and Schema groups. Analysis of subjects' written protocols provided further evidence of qualitative differences among the problem-type representations internally generated by subjects in the four groups. Schema and Analogy subjects generated descriptions that reflected an understanding of the problems' underlying structures, in stark contrast to the names used by the other two groups which reflects instead surface characteristics of the problems. This pattern of results is strikingly similar to those obtained in expert versus novice studies employing sorting tasks. In the present experiment, this difference was obtained with novices after exposure to a twenty minute orienting task.

EXPERIMENT 2

Expertise in complex domains involves not only the ability to recognize problem structures, but also the ability to translate those structures into formal representations which can then be manipulated using standard techniques (e.g., calculus, trigonometry). Not surprisingly, a second major difficulty novices encounter when attempting to solve formal problems is translating them from a natural language representation to a mathematical or symbolic one (Clement, Lockhead, & Monk, 1979; Myers, Hansen, Robson, & McCann, 1983; Paige & Simon, 1968). There is reason to believe that experts may rely on problem-type schemata when generating formal representations. Schemata are extremely useful in generating problem representations not only because of the information they do contain about a problem type (i.e., important facts and their relations to one another), but also because of what they do not contain (i.e., irrelevant facts). Since only the problem's crucial structure is represented, the task of generating an adequate representation is substantially simplified; a simple translation or matching of the facts and relations to symbolic notation is all that is required.

The purpose of Experiment 2 was to test the hypothesis that schemata would facilitate recognition of symbolic representations of problems belonging to the class represented by the schemata. This hypothesis was tested by requiring subjects to match problems to sets of equations that could be used to solve problems within the three classes described in Experiment 1 (i.e., Catch-Up, Dilution, and Facilitation-Interference problems), and then to use the equations to solve a subset of the problems. Three orienting tasks were used to manipulate the memory structures constructed by separate groups of subjects while reading the problems (i.e., Verification, Analogy and Schema); in addition, a group of mathematics experts was asked to perform the same task, as was a control group of novices who simply read the problems prior to matching and solving. The experts served as as standard of comparison for ceiling performance on sorting and solving the problems. Since experts were not required to perform an orienting task, but were instead simply allowed to read the problems, a control group of novices who simply read the problems was included as well.

It was predicted that the Schema group would not differ from the experts on the matching task. The Analogy group was expected to perform less well than the Schema group but better than the Verification group in that the analogies they were asked to consider while reading the problems should have been useful in discovering the crucial structures of the problems. Finally, the control group was expected to exhibit performance somewhere between the Verification and Analogy groups; it was assumed that under normal conditions, people look for and notice similarities among stimuli, that without special guidance on stimuli as complex as the problems used in this study, they would have difficulty determining which similarities were important.

With regard to problem solving success, experts were expected to have little difficulty obtaining the correct answers using the equations, since their training provided vast experience with manipulating symbols. No predictions were made concerning novice's abilities to obtain the correct answers, since individual ability for manipulating symbols could not be controlled. It should be noted that if subjects successfully match problems to their symbolic representations (equations), they essentially have the means to solve the problems before them, and believe that this is the case. In other words, if they have matched a problem to the correct equation, they believe that the problem can be solved using that equation. If they fail to derive a correct solution using that equation and the example solution provided them, it is a reflection of an inability to manipulate equations--not an inability to recognize which equations fit the problems.

Method

Subjects. Sixty subjects were recruited from campus newspaper advertisements for participation in the experiment. Experts consisted of eight mathematics graduate students, one physics graduate student, one molecular biology graduate student, and two seniors in computer science. The majors of the novices were many and varied; however, there were no preponderances of engineering, mathematics, or non-science majors in any particular group. All subjects were paid $7.50 for their time (approx. 1 1/2 hours).

Materials and Apparatus. The same problems and apparatus was used in this experiment as in the previous one. In addition, three solution procedures corresponding to the three problem types were constructed, which included symbolic representations of the problem types. They are included in Appendix A. The three solution procedures were typed on separate sheets of paper. Solution procedures for the two sample problems were also constructed.

Procedure. The novices were randomly assigned to four groups, a Verification group, an Analogy group, a Schema group, and a control group. The first three groups went through the orienting task procedure. The control group and the experts simply read the problems on cards prior to sorting them.

All subjects were told prior to reading the problems that they would be required to match some word algebra problems to a set of solution procedures and to solve a subset of the problems using the procedures.

Control subjects and experts were given the stack of cards with the problems typed on them and given as much time as they needed to read them. Most required 15-20 min to read all eighteen problems and four practice problems. When subjects had finished reading the problems (or performing the orienting task), they were shown how to match the sample problems to the appropriate solution procedures.

Following this, the solution procedures for the rest of the problems were distributed. These were explained in detail. When all subjects indicated that they were ready to begin sorting, they were allowed to begin matching the problems to the appropriate solution procedure by laying the cards on the bottom of the solution procedure sheets. They were given 10 min to complete the sorting. When they were finished, their sortings were recorded. The problems were stacked in one pile face down on the table. Then they were given the six transfer problems, and were asked to read and sort these new problems. They were given 5 min to complete the transfer sorting. Their sortings were recorded, and the transfer problems added to the stack. They were then given sheets of paper on each of which was written a problem number and the number of the solution procedure they sorted that problem to. They were required to attempt to solve the problem using the solution procedure they sorted the problem to. They were told that if they changed their minds and wanted to use another procedure to do so, but to record the number of the solution procedure they were now using. It was stressed that they should write down the values they associated with each variable, to form equations as shown, and to manipulate the equations to obtain their answer.

Each subject was required to solve six problems, one old problem and one transfer problem from each problem type. The problems each was to solve was determined in advance. Pains were taken to ensure that all problems were attempted by an equal number of subjects in all groups, and problem order was random for each subject. Subjects were given 30 min to solve the problems, and were notified every 10 min of the time left. Following this, they were asked to write down any rules or strategies they used to perform the sorting tasks. In particular, they were told to write down what sorts of things they looked for in the problems to make their sorting decisions.

Results

Orienting Task and Written Protocol Information. Once again, orienting task performance was quite good. Correct performance averaged 72% on the verification task, 85% on the analogies, and 89% on the schema question task.

Table 4 presents a breakdown of the types of strategies used by subjects while matching the problems to the solution procedures, along with the number of subjects in each group who reported using each strategy on each problem type, along with the proportion of problems correctly matched on the first sorting task. Examples of the protocols included in each strategy are also provided. Four strategies were identified. The first strategy involved descriptions or attempts at descriptions of the problem structures and a matching of these verbal structures to the symbolic ones. The second involved a direct comparison of verbal lines from the problem text to the symbolic structures, essentially matching substructures within the problem text to symbolic substructures. The third classification involved a counting or keyword approach, where certain keywords such as "lower" were noted or the number of rates were counted. The last classification includes classifications based on guessing or "intuition". Subjects tended be consistent in their choice of classification strategies for all problem types, but strategy switches were sometimes observed. For example, some subjects used a structure strategy for two of the three problem types, and used the third type as a default category, categorizing problems that did not have the two known structures into the third category.

Insert Table 4 about here

Group strategies differed reliably, X (12) = 27.43. Ordering of frequencies indicated that experts and schema subjects tended to use structural strategies when classifying the problems, analogy subjects distributed themselves fairly evenly across the strategy types, and the remaining two groups tended to use key-word or guessing strategies. Even more striking is the level of success subjects met with when attempting to use the strategies. Overall, use of a structural strategy produced better matching performance. However, the Schema, Analogy, and Expert groups tended to exhibit better performance than the other two groups in each strategy classification. Thus even when these subjects could not verbalize a strategy, as in the "guess" category, their pre-sorting experience still aided them in their classification endeavors. In contrast, the Verification and Novice-Control groups tended to perform poorly even when attempting a structural strategy. These results strongly suggest marked differences in the way that the groups represented the problem types to themselves. In order to use a structural strategy effectively, the correct structures must be internally available. Failure to use this strategy or to use it effectively suggests that the correct structural representations were not available for use.

The number of problems correctly matched to the solution procedures on the first sorting task and the transfer task was calculated for each subject. An analysis of variance was conducted on each of these dependent variables and on the sorting times for each task. The analysis contained the following variables: group (Verification, Analogy, Schema, Expert, or Novice-Control), and problem type (CU, DL, andFI), with repeated measures on the last variable. The proportions of problems correctly matched on the two tasks are presented in Table 5.

Insert Table 5 about here

Matching Task. Like the results of the problem structure sorting task in Experiment 1, this analysis returned two significant results, the main effect of group $F\ (4,55) = 8.41$, $MSe = 4.006$, $p < .01$, and the main effect of problem type $F\ (2,110) = 3.24$, $MSe = .567$, $p < .05$. Five paired comparisons of performance among the groups were conducted using Dunn's test. The predicted pattern of results was obtained: The Analogy group sorted more problems correctly than the Verification group (A - V = 1.111), and the Schema group sorted more problems correctly than the Analogy group (S - A = .779). More importantly, however, the Schema group's performance did not differ significantly from that of the Experts (E - S = .360; the required difference between pairs of means was .639 at the .05 rejection probability level). The Schema subjects were therefore as adept as Experts in matching problems to their symbolic representations.

The Novice control group's performance was compared to that of two groups: (a) Experts, since, like experts, they were allowed to process the problems freely while reading, and (b) the Verification group, since without guidance, they were expected to perform at most rudimentary comparisons across problems and at least an adequate representation of the situation described in the problem text. These subjects sorted significantly fewer problems correctly than did the Experts (E - NC = 1.916), and performed equivalently to the Verification group (NC - V = .334). It is particularly interesting to note that this group's performance lay mid-way between the performance levels of the Verification and Analogy groups, a result that is consistent with the type of processing they were expected to engage in.

The problem types were not all equally easy to match to symbolic representations, as the significant main effect indicated. Paired comparisons using Tukey's test indicated that these differences were reliable only for the two extreme values, that is, CU problems were more accurately sorted than DL problems.

Transfer Task. The main effects of group and problem type were also significant here F (4,55) = 3.41, MS_e = .861, $p < .05$, F, (2.110) = 15.834, MS_e = .141, $p < .001$, respectively. Once again, the Verification group performed less well than did the Analogy group (A - V = .362), but, unexpectedly, the Analogy and Schema groups yielded similar performance levels (S - A = -.028). Moreover, the Experts matched more new problems correctly than did the Schema group (E - S =.527; the required difference between the means using Dunn's test was .296). Thus, while the Schema group rivalled the Experts when sorting problems they were previously acquainted with, they could not perform as well as experts when given new problems to sort.

Novices in the control group did not perform as well as experts on this task (E - NC = .499), but they did surpass the Verification group (NC - V = .362). Informal comparison of the means showed that the Schema, Analogy, and Novice Control groups all performed equivalently on this task. A plausible interpretation of this effect is that comparisons across problems was an important predictor of performance on this task, since two of these groups explicitly compared problems during reading while the Verification group did not.

The problem types ordered themselves according to increasing difficulty as follows: CU, DL, and FI. Tukey's test indicated that the FI problems were matched correctly significantly fewer times than both the CU and DL problems (CU - FI = .170; DL - FI = .113; the required differences between the mean proportions was .102). The CU problems and the DL problems were matched correctly an equivalent proportion of times (CU - DL = .057).

Solutions. Subjects' solution protocols were scored as follows: For each variable assigned a correct value, one point was given. Since the solution required computing one of the variable's values, a correct solution would also receive one point. The maximum possible score for the CU and FI problems was 4, since there were four variables included in each problem, including one whose value had to be computed. The maximum score for the DL problems was 6, since there were six variables, including one whose value had to be computed. Each subject was required to solve two problems of each type, one old problem and one transfer problem. The maximum scores for each problem type for a given subject was therefore 8, 12, and 8 for the CU, DL, and FI problems, respectively; the maximum total score was 28. Each subject's score for each problem type was computed and converted to a proportion. The mean proportion scores for the five groups are presented in Table 6. An analysis of variance was computed on these data using the same variables as in the previous two analyses, that is, group and problem type.

Insert Table 6 about here

The analysis returned two significant results, the main effects of group and problem type F (4,55) = 4.74, MS_e = .123, $p < .01$, and F (2.110) = 8.42, MS_e = .054, $p < .001$, respectively. Paired comparisons indicated that the novice groups all performed equivalently, while the experts cleanly surpassed them (E - A = .207; A - S = .079; S - V = .019; M - NC = .018; E - NC = .287; the required difference using Dunn's test was .112). It should be noted that the performance levels on this task were quite high. Experts received an average score of .91, while novice performance ranged from .62 to .70.

The problem types were not all equally easy to solve. Significantly lower scores were obtained on the FI problems than on both the CU problems (CU - FI = .170) and the DL problems (DL - FI = .113; the required difference using Tukey's test was .102). The CU and DL problems, however, did not differ (CU - DL = .057). It is of some importance that the interaction of group and problem type was not significant ($\underline{F} < 1$); even the Experts found the FI problem type to be tricky.

Solution Error Analysis. As pointed out earlier, no predictions concerning solution performance could be made since deriving correct answers once the correct solution procedure was chosen depended heavily on subjects' ability to manipulate equations, an ability that could be be controlled. Analysis of the types of errors subjects made, however, strongly suggests important differences in their conceptualizations of the problems they attempted to solve.

Insert Table 7 about here

Table 7 presents the type of errors committed by the groups while attempting solutions, and the number of subjects in each group who committed them. Errors are divided into two major types: Conceptual errors, those that suggest a failure to understand the structure of the problem, and Mechanical errors, those that reflect rusty algebra or arithmetic skills. Under conceptual errors are two subtypes: (1) Associating an incorrect value with a variable in the equation, and choosing the wrong equation in attempting to solve the problem. Errors of these types indicate a misunderstanding of the structure of relations among variables and values in the problem. Under mechanical errors are arithmetic errors, algebra errors (e.g., 100 + r1 = 100r1), and other errors. The category includes running out of time, or using an obscure solution method whose logic or purpose was not readily apparent. (Only two solution attempts involved obscure methods.) As is apparent, while all novices committed a comparable number of mechanical errors, the groups differed substantially in the number of conceptual errors committed. Schema subjects committed the fewest number of these, Verification subjects committed the greatest number, and Analogy and Control subjects fell somewhere between. Thus, it appears that the higher level groups had a better grasp of the problem structure, as well as a better understanding of how to go about solving the problems; they simply did not have adequate mechanical skills to produce the correct answers.

Correlations between orienting-task performance and solution performance. Correlations between orienting task performance and performance on the matching and solving tasks provided stronger evidence that experience on the orienting tasks contributed to accurate performance on matching and solving tasks. The correlations between proportion correct responses on the orienting task and problem matching, transfer, and solution performance are presented in Table 8.

Insert Table 8 about here

The results are straightforward: Proportion correct answers on schema and analogy orienting tasks correlated significantly and positively with matching and solution performance; verification performance was not related to either. Moreover, problem-matching performance correlated strongly and positively with solution performance for all three groups, and with transfer performance for the schema and analogy groups. These results strongly suggest that the information gained from performing well on the analogy and schema orienting tasks contributed to subject's abilities to match and solve problems they so analyzed. The ability to sort new problems on the transfer task was unrelated to orienting task performance, but was strongly correlated to performance on the matching task in two of the groups. The results suggest a that schema and analogy subjects used the same criterion for sorting on both the matching and transfer tasks. Verification subjects, on the other hand, seemed to switch strategies between these two tasks in an unsuccessful attempt to perform better.

GENERAL DISCUSSION

The aim of the present research was to outline and test a model of schema acquisition. It begins with a natural language representation of a problem and demonstrates how memory structures arising from comprehension this type of representation can be compared and processed to yield a schematic representation of a problem type. A primary question concerning the model is whether the processes described by it are sufficient to produce representations of problem types. In order to test the sufficiency of the model, subjects were induced to carry out one or more of the processing stages contained in the model via orienting tasks. The effect of these tasks on subjects' abilities to identify and sort old and new problems according to underlying problem structures and their description of these structures was then observed.

Experiment 1 provided strong support for the sufficiency of the proposed model. Like experts, subjects who answered schema and analogy type questions performed exceptionally well when required to sort problems on the basis of similar structures; in contrast, the subjects who answered recognition or verification type questions were unable to perform adequately when required to use this criterion for sorting. The latter two tended instead to be misled by surface features when making sorting decisions. Moreover, analysis of written protocols revealed systematic differences in the ways subjects represented the problem types to themselves. Schema and Analogy subjects tended to describe the problem types in terms of commonalities in underlying problem structures, Recognition and Verification subjects, on the other hand tended to define problem types in term of commonalities among surface features. This pattern of results is strikingly similar to those obtained in expert vs. novice studies employing sorting tasks. Subjects who focussed their attention _within_ problems (Recognition and Verification subjects) displayed a pattern of performance typically obtained with novices, while subjects who were induced to compare relations and structures _across_ problems (Analogy and Schema subjects) exhibited sorting behavior that was characteristic of experts.

Similar results were obtained when subjects were required to match problems to their symbolic (algebraic) representations (Experiment 2). Schema subjects performed as well as experts and better than Analogy subjects on this task. Verification subjects exhibited a performance level that was inferior to the analogy group, and novices who simply read the problem prior to attempting the matches performed no better than the Verification group. The strategies that subjects used when matching the problems also differed. Experts and Schema subjects tended to attend to problem structures when matching problems; novices and Verification subjects tended to use key word strategies, focussing on surface cues when matching. Like experts, Schema and Analogy subjects also tended to make fewer conceptual errors when attempting to solve problems; the remaining two groups, on the other hand, often made errors that indicated a lack of understanding of the problem structure, such a using the incorrect equation or misapplying values to variables. Finally, a closer look at the correlations between orienting task performance and matching and problem-solving performance indicated that experience with analogy and schema question aided subject in mapping symbolic representations onto natural language representations, while simple rehearsal of intra-problem structures did not.

One disappointing result of Experiment 2 was the failure of schema subjects to perform as well as experts on the transfer tasks, and the low correlations between orienting tasks performance and transfer performance. This result suggests an inability to apply the information and skills obtained from the orienting and matching tasks to new problems. A possible explanation of this failure is the difficulty of the task itself. A major difficulty that novices face in formal domains is understanding what meaning a symbolic representation has, that is, how a symbolic representation maps onto a problem stated in natural language. Novices in the present study undertook this difficult task with only 20 minutes exposure time for problem structure analysis and only 10 minutes to apply this information to new problems. Moreover, the pattern of novice transfer performance was complex. Schema and Analogy subjects performed better than the Verification group, but no better than the control group. The former result suggests a benefit for analogical reasoning. The latter results, however, suggests that it was not so much that Analogy and Schema subjects benefitted from their orienting tasks as that Verification subjects suffered from theirs. Interestingly, this is the pattern of results one would expect if analogical reasoning normally underlies schema abstraction. Inducing subjects to do what they normally do anyway should help their performance at most on the particular instances used during the induction task; preventing them from doing what they normally do should lower their performance on any task that relies on the fruits of that processing.

Perhaps the most intriguing result of these studies was the performance pattern of the Analogy group. While they did not perform as well as the Schema group on the structure sorting and matching tasks, their performance on the transfer tasks and their overlap scores were quite similar to the Schema group. The results suggest that in forming schemata, the importance of comparison across instances cannot be overestimated. While performance seems to benefit to some degree from an analysis of important information within a problem (i.e., the Verification group's performance), the real differences are obtained once comparison across instances is undertaken. This result is particularly relevant to the proposed model in that the model depends on comparisons across instances to weed out irrelevant information and to quantify across particulars (e.g., entities) within problems. Both the Analogy and Schema groups were induced to perform such comparisons, and their performance benefitted accordingly. The Schema questions provided yet another hint as to which matches across structures were important, giving this group yet another boost in performance. Note that the superior performance of this group cannot simply be attributed to a matching of schema descriptions to problems during the orienting task because of their equally superior performance on the transfer problems. Something more than mere association of statement to problem pairs must have occurred.

In addition to accounting for the data in the present experiments, the model makes predictions that are consonant with the results of other studies reported in the literature. For example, the model predicts that surface similarity of instances should affect ease of schema abstraction as well as correctness of the schema ultimately constructed since the more irrelevant matches between situation models, the more baroque and misleading the abstracted schema is likely to be. Ross (1982) noted a similar pattern of behavior. He embedded elementary probability problems of different types in stories, and required subjects to learn to associate formulas with each of the problem types. His subjects found the task far more difficult when the instructional instances for several problem-types shared a story context than when they instances for each problem-type were embedded in stories of differing topics. Gick and Holyoak (1983) also observed this result on an "insight" type problem. Subjects in their study exhibited better transfer performance when they had previously processed two dissimilar analogs to the problem than two similar analogs (Exp. 4 and 6). They concluded that schemata formed from dissimilar instances are more likely to contain only those aspects that are relevant to problem solution, whereas schemata formed from similar analogs contain both relevant and irrelevant aspects. This interpretation is entirely consonant with the mapping process proposed here since only matching elements form the basis of schemata.

The proposed model also readily distinguishes among (1) ease of noticing similarities, (2) degree of perceived similarity, and (3) contribution of similarities to classification selection choices. The comparison rules are ordered according to the ease with which common problem elements are likely to be noticed. Perceived similarity at any point during the acquisition process is a function of the matches that are noticed. However, classification behavior should depend on the integrity of the schema as it exists at the time classifications are made. Thus, as a schema is abstracted, it tends to be used to classify instances and determine similarity more so than comparisons to individual instances. This prediction is also consistent with the pattern of results found by Ross (1982). He noted that subjects were highly likely, in initial stages of skill acquisition, to use such surface cues as similarity of story problem topics in choosing instances on which to base solution attempts. He also noted that these "remindings" occurred frequently and influence significantly the course problem solutions will take early in the learning phase. With practice, however, both the incidence and influence of remindings decreased. Ross suggested that as the number of episodes increases, particular information is lost, and episodes become de-contextualized. The result of these processes is a generalized or abstract representation of problem-types, much as in the proposed model.

Limitations of the model. The model as presented here is at a descriptive stage. It represents more than anything a theoretical framework for exploration of the question of schema abstraction and knowledge acquisition form verbally complex instances. The viability and sufficiency of the processes embodied by the model were tested and received strong support from the results of the studies reported here, an important first step. However, formal modifications of the model are required in order to make more precise predictions concerning schema abstraction. For example, the comparison rules should operate probabilistically. Parameters representing the probabilities of noticing identical and analogous entities, relations, and structures must be included so that predictions concerning the ease of noticing problem features and their contribution to schema building may be tested. Since the abstraction process essentially involves loss of information, parameters representing the probability of retaining matching and non-matching features must be included. With these parameters in place, predictions concerning schema abstraction can be more precisely tested (e.g., memory for matching and non-matching problem features and their relative contribution to classification performance). The present data supported the sufficiency of the processes involved; the next important question is whether subjects in fact employ such processes when left to their own devices. As formalized, the model could be used to make predictions concerning classification learning under more traditional conditions wherein stimulus attributes (i.e., entities, relation, structures) are systematically varied to predict ease of classification performance (instead of systematically varying processing as was done here to test sufficiency).

Comparison to other models. Other models of problem solving and knowledge acquisition that have appeared in the literature vary from descriptive models (like the present one) to fully automated process models. This make direct comparison among models somewhat awkward. Nonetheless, it is useful to compare the theoretical approaches embodied in various other models to the one proposed here, since these various approaches carry with them assumptions about the processes involved in the development of expertise.

As a beginning point, it should be noted that few models of problem solving or knowledge acquisition begin with a natural language representation of a problem, although understanding the meaning of such a representation is often a large part of the problem solving process itself. For example, in physics problems, pulleys etc., may be mentioned and included in the text-base representation of the problem. However, the situation model constructed by the problem-solver may contain entities not mentioned in the text, such as forces, that are vital to achieving a correct solution. The inclusion of such abstract entities in the problem representation depends on their activation in long-term memory by surface cues. Construction of a problem representation depends heavily, therefore, on interactive text comprehension processes, such as those described by van Dijk and Kintsch.

The importance of problem wording is particular relevant in the area of children's problem solving, and attempts to model children's solution behavior reflect this fact (Briars & Larkin, 1984; Kintsch & Greeno, 1984; Riley, Greeno, & Heller, 1983). However, of the models that include verbal processing as an integral part of the problem-solving process, only that of Kintsch and Greeno use something other than a word-by-word translation strategy. The problem text in their model is treated as a structure of verbal information that must be processed in fairly sophisticated fashion in order to produce a problem representation capable of supporting a successful solution attempt. However, like other model of children's problem solving in behavior, their model does not learn from experience. Instead, the schematic structures which allow recognition of problem types are either present of absent, and solution success is predicted on the basis of the number and types of schemata available to the model.

Larkin (1981a;b) has proposed models of physics problem-solving behavior that are schema-utilization models that also learn from experience. Learning is instantiated as a storing of the conditions and results of successful problem-solving episodes which can be used later to match current problem situations. In essence, a problem solver successfully solves a problem in this model by being *reminded* of an earlier, successful attempt. There are three points to be made here. First, given the vast number of problems attempted while developing expertise in a domain, it is unlikely that individual episodes of all problem attempted can be stored or easily accessed. Second, reminding can and do occur often as a function of surface similarities, which will not ensure problem-solving success. Third, Ross's (1982) results strongly suggest that acquisition of problem-solving skill does not involve a simple accumulation of problem-solving episodes with experience, but a construction and refinement of abstract representations of problem types, much as in the proposed model.

While not a comprehensive model of problem-solving, Gentner's (1983) formalism describes how people reason analogically. She calls the process captured by this formalism "structure mapping". The goal of structure mapping is to demonstrate how identical propositional systems apply to two domains with dissimilar objects. Structure-mapping, then, allows knowledge about one domain to be applied to a second domain to further one's understanding of the latter. The mapping process begins by identifying analogous objects across domains. The reasoner then derives analogical predications by applying predicates valid in the base domain to the target domain, using node substitutions dictated by the mapping. Determination of validity depends strongly on the possession of related knowledge by the reasoner. The processes described by Gentner are similar to those used in the present model, but they are applied not to instances for the purpose of knowledge abstractions, but instead to disparate domains --one known the other unknown or new--for the purpose of knowledge transfer. It is not clear whether this intriguing system describes how a reasoner *should* go about transfering information from one domain to another or whether the model represents an hypothesis about how people in fact do it. Nor is it specified how analogous objects in disparate domains can be so identified.

Finally, schema abstraction as outlined in the model bears close resemblance to the processes thought to underlie concept formation and classification learning in that both involve learning to identify category members through exposure to exemplars. Recently, it has been suggested that models of classification learning can be contrasted on two dimensions (Medin & Schaffer, 1978; Medin, Altom, & Murphy, 1984). The first dimension is *independent cue vs. relational coding models*. Independent cue models assume that exemplar attributes are combined additively in determining a classification judgment. Relational coding models assume configural combinations of attributes, often involving a multiplicative rule. The second dimension is *prototype vs. exemplar models*. Prototype models assume that category level information is abstracted and used in classification judgments (Posner & Keele, 1968). Exemplar models assume that, regardless of the category information abstracted, judgments are often made through comparing probes to stored exemplar information (Medin & Smith, 1981). Recent evidence suggests that while subjects tend to use relational encoding as opposed to independent cues, they rely on both category level and exemplar information when making classifications (Elio & Anderson, 1981; Homa, Sterling, & Trepel, 1981).

The proposed model is a relational coding model in that the structure, or configuration, of problem attributes determines class membership. Moreover, it assumes that both exemplar and category level information is used during schema acquisition, but at different times. More particularly, it is assumed that initially exemplar information strongly influences judgments (i.e., matches among situation models), but that with greater exposure to exemplars over trials, the developing schema begins to exert a stronger influence on the classification process. The eventual outcome of this process is expert performance: classification on the basis of similarity to schemata.

REFERENCES

Anderson, J.R. (1983). The architecture of cognition. Cambridge, MA: Harvard University Press.

Anderson, J.R., Farrell, R., & Sauers, R. (1983). Learning to program in LISP. Office of Naval Research Technical Report #ONR 157-465, Carnegie Mellon University.

Anderson, J.R., Greeno, J.G., Kline, R.J., & Neves, D.M. (1981). Acquisition of problem-solving skill. In J.R. Anderson (Ed.), Cognitive skills and their acquisition. Hillsdale, N.J.: Erlbaum.

Anderson, J.R., Kline, R.J., & Beasley, C.M. (1979). A general learning theory and its application to schema abstraction. In G.H. Bower (Ed.), The psychology of learning and motivation, Vol. 13. New York: Academic Press.

Briars, D. J., & Larkin, J.H. (1984) An integrated model of skill in solving elementary word problems. Cognition & Instruction, in press.

Bull, K. (1982). Expert and novice differences in solving algebra word problems. Unpublished doctoral dissertation. University of Colorado, Boulder.

Chi, M.T., Feltovich, P.J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. Cognitive Science, 5, 121-152.

Chi, M.T., Glaser., R., & Rees, E. (1982). Expertise in problem solving. In R. Sternberg (Ed.), Advances in the psychology of human intelligence. Hillsdale, N.J.: Erlbaum.

Clement, J., Lochhead, J., & Monk, G. (1979). Translation difficulties in learning mathematics. Technical Report. University of Massachusetts, Dept. of Physics and Astronomy, September.

Elio, R., & Anderson, J.R. (1981). The effects of category generalizations and instance similarity on schema abstraction. Journal of experimental psychology: Human learning and memory, 7, 397-417.

Gick, M.L., & Holyoak, K.J. (1983). Schema induction and analogical transfer. Cognitive Psychology, 15, 1-38.

Hinsley, D.A., Hayes, J.R., & Simon, H.A. (1977). From words to equations: Meaning and representation in algebra word problems. In M.A. Just & P. A. Carpenter (Eds.), Cognitive processes in comprehension. Hillsdale, N.J.: Erlbaum.

Homa, D., Sterling, S., & Treppel, L. (1981). Limitations of exemplar- based generalization and the abstraction of categorical information. Journal of Experimental Psychology: Human Learning and Memory, 7, 418-439.

Keppel, G. Design and analysis: A researchers handbook. Englewood Cliffs, N.J.: Prentice-Hall.

Kintsch, W., & Greeno, J.G. (1983). Understanding and solving word arithmetic problems. Psychological Review, in press.

Larkin, J. (1981a). The role of problem representation in physics. Technical Report No. CIP-429, Carnegie-Mellon University.

Larkin, J. (1981b). Spatial reasoning in solving physics problems. Technical Report No. CIP-434, Carnegie-Mellon University.

Larkin, J., McDermott, J., Simon, D.P., & Simon, H.A. (1980a). Expert and novice performance in solving physics problems. Science, 208, 1335-1342.

Larkin, J., McDermott, J., Simon, D., & Simon, H. (1980b). Models of competence in solving physics problems. Cognitive Science, 4, 317-345.

Lewis, M.W., & Anderson, J.R. (1983). Discrimination of operator schemata in problem solving: Learning from examples. Department of Psychology Technical Report, Carnegie Mellon University.

Medin, D.L., Altom, M.W., & Murphy, T.D. (1984). Given versus induced category representations: Use of prototype and exemplar information in classification. Journal of Experimental Psychology: Learning, Memory, and Cognition, 10, 333-353.

Medin, D.L., & Schaffer, M.M. (1978). Context theory of classification learning. Psychological Review, 85, 207-238.

Medin, D.L., & Smith, E.E. (1981). Strategies and classification learning. Journal of Experimental Psychology: Human Learning and Memory, 7, 241-253.

Myers, J.L., Hansen, R.S., Robson, R.C., & McCann, J. (1983). The role of explanation in learning elementary probability. The Journal of Educational Psychology,

Paige, J., & Simon, H. (1968). Cognitive processes in solving algebra word problems. In B. Kleinmuntz (Ed.), Problem-solving research, methods, and theory. New York: Wiley.

Posner, M.I., & Keele, S.W. (1968). On the genesis of abstract ideas. Journal of Experimental Psychology, 77, 353-363.

Riley, M.S., Greeno, J.G., & Heller, J.I. (1983) Development of children's problem-solving ability in arithmetic. In H. Ginsberg (Ed.), The development of mathematical thinking. New York: Academic Press.

Ross, B.H. Remindings and their effects in learning a cognitive skill. Technical report No. CIS-19, Xerox-Parc, Palo Alto, CA.

Rumelhart, D.E., & Norman, D.A. (1981). Analogical processes in learning. In J.R. Anderson, (Ed.), Cognitive skills and their acquisition. Hillsdale, N.J.: Lawrence Erlbaum.

Schoenfeld, A.H., & Herrmann, D.J. (1982). Problem perception in expert and novice mathematical problem solvers. Journal of Experimental Psychology: Learning, Memory, and Cognition, 8, 484-494.

Silver, E.A. (1981). Recall of mathematical information: Solving related problem. Journal for research in mathematics education, 12, 54-64.

Simon, D.P., & Simon, H.A. (1978). Individual differences in solving physics problems. In R . Siegler (Ed.) Children's thinking: what develops? Hillsdale, NJ:Erlbaum.

Sweller, J., Mawer, R.F., & Ward, M.R. (1983). Development of expertise in mathematical problem solving. Journal of Experimental Psychology-General, 112, 639-661.

Thorndyke, P.W. (1977). Cognitive structures in comprehension and memory of narrative discourse. Cognitive Psychology, 7, 77-111.

Thorndyke, P.W., & Hayes-Roth, B. (1979). The use of schemata in the acquisition and transfer of knowledge. Cognitive Psychology, 11, 82-106.

van Dijk, T.A. & Kintsch, W. (1983). Strategies of discourse comprehension. New York: Academic Press.

Author Notes

This research was supported by a grant from the Martin Marietta Corporation, Denver, CO. These experiments formed part of a doctoral dissertation submitted to the University of Colorado, Boulder. Thanks are due to Lyle E. Bourne, Jr., Walter Kintsch, Anders Ericsson, Peter Polson, Georges Rey, Robert Cummins, and Stephen Leeds for commenting on portions of this manuscript, and to Kelly Maurice for programming assistance.

Requests for reprints should be sent to Denise Dellarosa, Department of Psychology, University of Colorado, Boulder 80309.

Table 1

Classification of Problem Type Descriptions Produced by Subjects in Experiment 1

	Group			
Description	Text-base	Verification	Analogy	Schema
Structural	4	6	16	21
Analytic	5	3	4	3
Surface	10	11	5	0
Other	17	16	11	12

Note: n = 12 subjects X 3 problem-types = 36

Examples of Descriptions:

Structural - "Catch-up situation", "Went to fast or too much and must slow down", "Both increases and decreases"

Analytic - "Different start times, same end times", "Involves an average or three rates", "Some relation between two rates"

Surface - "Speed or travel", "Filling or liquid", "Interest rates or percents"

Table 2
Mean Proportion of Problems Correctly Sorted on the Basis on
Similarities in Topics and Problem Structures, and
Mean Overlap Scores in Experiment 1

Mean Proportion Correct:

Group	Measure			
	Topic Sorting	Problem Sorting	Transfer	Overlap
Text-base	.976	.398	.527	.773
Verification	.944	.514	.500	.644
Analogy	.976	.671	.763	.458
Schema	.972	.857	.903	.417

Note: n = 12
Proportions based on the following:
Maximum scores - Topic & Problem Sorting - 18
Transfer - 6
Overlap - 18

Table 3

Correlations Between Proportion Correct Responses on the Orienting Tasks and Problem Sorting Performance in Experiment 1

	Group and Question Type			
	Recognition	Verification	Analogy	Schema
OTP-PS	.047	.074	.498	.362
OTP-TR	-.337	-.163	.446	.653*
PS-TR	-.041	.338	.423	.771*

* p < .05
\+ .05 > p < .10

Note: Correlations based on 12 observations.

OTP - Proportion correct on Orienting Task
PM - Problem matching performance
TR - Transfer sorting performance
PS - Problem solving performance

Table 4

Classification of Strategies Used on the Matching Task

and Proportion Correct Problem Matching Performance

in Experiment 2

	Group				
Strategy	Verification	Analogy	Schema	Expert	Novice-Control
Structural	6 (.638)	9 (.870)	12 (.852)	21 (.865)	9 (.667)
Mapping	9 (.528)	9 (.370)	12 (.778)	9 (.815)	9 (.571)
Keyword	9 (.167)	6 (.555)	3 (.764)	3 (.778)	12 (.436)
Guess	12 (.444)	9 (.750)	9 (.759)	3 (.778)	6 (.722)

Note: n = 12 X 3 problem types = 36

Numbers in parentheses are the mean proportion of problems correctly sorted on the first matching task.

Examples:

Structural - "Have too much, need less and reverse; change in standard"
Mapping - "Fit words to equations", "Compared wording to parts of equations"
Keyword - "Counted number of rates"; "Looked for 'higher' or 'lower'"
Guess - "Intuition"; "Punted"

Table 5

Mean Proportion of Problems Correctly Matched to Symbolic Structures in Experiment 2

Mean Proportion Correct:

Group	Measure	
	Old Problems	Transfer Problems
Verification	.472	.472
Analogy	.648	.653
Schema	.778	.639
Expert	.834	.903
Novice-control	.518	.653

Note: n = 16
Proportions based on the following:
Maximum scores - Old problems - 18
Transfer - 6

Table 6

Proportion Correct Answers on the Solution Attempts in Experiment 2

Mean Proportion Correct:

Group	Problem Type		
	Catch-Up	Dilution	Facilitation-Interference
Verification	.771	.604	.438
Analogy	.750	.701	.656
Schema	.718	.652	.500
Experts	.906	.938	.885
Novice-control	.694	.660	.510

Note: n = 12
Proportions based on the following:
Maximum scores - Catch-up Problems - 4
Dilution Problems - 6
Facilitation-Interference - 4
Two observations were obtained for each problem-type for each subject.

Table 7

Classification of Errors Committed During Solution Attempts in Experiment 2

	Conceptual Errors			Mechanical Errors			
Group	Wrong Values	Wrong Equation	Total	Arith Error	Algebra Error	Other	Total
Verification	9	9	18	5	4	22	31
Control	7	6	13	5	6	16	27
Analogy	6	6	12	7	7	14	28
Schema	4	3	7	11	8	17	36
Experts	4	1	5	8	1	3	12

Note: Numbers represent the number of occurrences of each error type within each cell. The maximum number of errors of any given type that could be committed = 6 problems X 12 Subjects = 72.

Table 8

Correlations Between Proportion Correct Responses on the Orienting Tasks and Problem Matching and Solving Performance in Experiment 2

	Group and Question Type		
	Verification	Analogy	Schema
OTP-PM	.262	.726*	.772*
OTP-TR	-.088	.293	.535+
OTP-PS	.095	.572	.579+
PM-TR	.059	.510	.528+
PM-PS	.506	.425	.639*

* $p < .05$
\+ $.05 > p < .10$

Note: Correlations based on 12 observations.

OTP - Proportion correct on Orienting Task
PM - Problem matching performance
TR - Transfer sorting performance
PS - Problem solving performance

Figure Captions

Figure 1. A descriptive model of schema abstraction.

APPENDIX A

Examples of Acquisition Problems:

Practice Problems

Problem Structure: Proportions

Topic--Elections Caroline is running for president of her freshman class. She's sure she should have no trouble winning since her opponent is a real nerd. In fact, Caroline estimates that 2 out of every 3 students will undoubtedly vote for her. When the election votes came in, 1500 votes had been cast. Caroline won by the margin she was expecting to. She also fell in love with the nerd and married him. How many votes did Caroline win?

Topic--Age When Brian was in grade school, his teachers let him skip 4th grade. Brian is now in college, and is a year younger than most of his friends. In fact, 6 out of every 8 men in Brian's fraternity are over 21, while Brian is only 20. There are 240 men in Brian's fraternity, counting Brian. This is are real problem for the younger guys because the drinking age in Brian's state is 21. On Friday nights, the guys over 21 all go out to the bars together. Brian and the rest stay home and do illicit drugs. How many men are over 21?

Problem Structure: Two Unknowns in Two Equations

Topic--Elections The residents of Smalltown voted in a law that required all citizens to wear clean underwear everyday. Since it was difficult to enforce this law, someone proposed that another law be passed which required all citizens to wear their underwear outside their clothes so the police could check whether the first law was being complied with. At first, 5 times as many school districts supported the law than didn't. 6 more districts joined either side in a later poll and then only twice as many districts supported it. The law passed, and Smalltown made the cover of Vogue magazine. How many school districts did not support the law?

Topic--Age Sarah is giving a birthday party for her niece. As she's about to put the candles on the cake, she realizes she can't remember how old the kid is. She calls here niece's mother, Judy, and asks her. Judy can never give anyone a straight answer. She says "You are 4 times older than your niece. In 4 years, you will be 3 times older than she is." Sarah puts the right number of candles on the cake. When Judy and her kid show up, Sarah pushes Judy's face in the punch bowl. How old is Sarah's niece?

Experimental Problems

Problem Structure: Catch-up (CU)

Topic--Vat. Callie Callifornia is expecting guests at 8:00 for a hot tub party. She plans to start filling the tub at 4:00 at the rate of 60 gallons/hour. At first, she has everything under control. Then, she gets caught in traffic while shopping for Perrier. She doesn't get home until 6:45, and starts filling the tub at a faster rate. At 8:00, Callie's guests are

Topic--Travel. Jill, an aviation technician, is testing a model of an experimental jet plane in a wind tunnel. Flying against the air stream, it takes 10 minutes for the plane to travel the length of the tunnel. Flying with the airstream, it can travel the length in 6 minutes. Jill is amazed at how fast the plane can fly, especially since she knows she set the wind speed in the tunnel to 15 mph. At what speed can the plane fly with the wind turned off?

Topic--Interest. Sean and David each invested the same amount of money in long term bonds both of which pay simple interest. The rates of interest on both bonds were the same. However, Sean had to pay 4% of his annual earnings to his ex-wife, while David's mother gave him an extra 4% annually just for being a good boy. It takes Sean 6 years to earn the same amount of money as David earned in 2. Sean goes berserk, quits his job and becomes a bum. David becomes vice-president of his mother's business. What is the interest rate on the bonds?

Examples of Transfer Problems

Topic--Work

Problem Structure: Catch-up. Steve usually has no trouble filling his work quota by producing 30 widgets per hour during the usual 8 hour work day. This morning, Steve was too hung over to make it to work in time, and showed up 3 hours late. The foreman wasn't too happy about this, but Steve promised to fill his quota by working much faster and not taking a lunchbreak. At what rate did Steve work for the rest of the day?

Problem Structure: Dilution. June's work team goes home early every day by making 8 circuit boards per hour for 6 hours instead of the prescribed 6 circuit boards per hour for 8 hours. Management becomes suspicious, however, and decides to place a spy in the team to report on them. June's team discovers the plan on the guy's first day after 2 hours of work. They decide to slow down so that it will take 8 hours to fill their quota. At what rate should June's team work for the rest of the day?

Problem Structure: Facilitation-Interference. Sarah and John work at a bookstore packing crates. Sarah was recently hired, and John, having a crush on her, decides to help her out by secretly putting one of his packed crates in her output stack every hour. The management, seeing a decline in John's work-rate, fires him. Sarah becomes despondent and packs one crate less per hour than she used to. Sarah now takes 8 hours to pack the same number of crates she used to pack in 5 1/2 hours with John's help. What is her normal work rate?

Topic--Travel. Jill, an aviation technician, is testing a model of an experimental jet plane in a wind tunnel. Flying against the air stream, it takes 10 minutes for the plane to travel the length of the tunnel. Flying with the airstream, it can travel the length in 6 minutes. Jill is amazed at how fast the plane can fly, especially since she knows she set the wind speed in the tunnel to 15 mph. At what speed can the plane fly with the wind turned off?

Topic--Interest. Sean and David each invested the same amount of money in long term bonds both of which pay simple interest. The rates of interest on both bonds were the same. However, Sean had to pay 4% of his annual earnings to his ex-wife, while David's mother gave him an extra 4% annually just for being a good boy. It takes Sean 6 years to earn the same amount of money as David earned in 2. Sean goes berserk, quits his job and becomes a bum. David becomes vice-president of his mother's business. What is the interest rate on the bonds?

Examples of Transfer Problems

Topic--Work

Problem Structure: Catch-up. Steve usually has no trouble filling his work quota by producing 30 widgets per hour during the usual 8 hour work day. This morning, Steve was too hung over to make it to work in time, and showed up 3 hours late. The foreman wasn't too happy about this, but Steve promised to fill his quota by working much faster and not taking a lunchbreak. At what rate did Steve work for the rest of the day?

Problem Structure: Dilution. June's work team goes home early every day by making 8 circuit boards per hour for 6 hours instead of the prescribed 6 circuit boards per hour for 8 hours. Management becomes suspicious, however, and decides to place a spy in the team to report on them. June's team discovers the plan on the guy's first day after 2 hours of work. They decide to slow down so that it will take 8 hours to fill their quota. At what rate should June's team work for the rest of the day?

Problem Structure: Facilitation-Interference. Sarah and John work at a bookstore packing crates. Sarah was recently hired, and John, having a crush on her, decides to help her out by secretly putting one of his packed crates in her output stack every hour. The management, seeing a decline in John's work-rate, fires him. Sarah becomes despondent and packs one crate less per hour than she used to. Sarah now takes 8 hours to pack the same number of crates she used to pack in 5 1/2 hours with John's help. What is her normal work rate?

Appendix A (cont.)

Symbolic Representations of the Three Problem Structures

Catch-up Problems:

$$r1(t1)=A1$$
$$r2(t2) = A2$$

If $A1 = A2$, then $r1(t1) = r2(t2)$

		Rate	Time	Total Amount
Goal	¶	$r1$	$t1$	$A1$
Higher Rate	¶	$r2$	$t2$	$A2$
	¶		$t2 < t1$	$A1 = A2$

EXAMPLE: $r1 = 100$
$t1 = 35$
$r2 = ?$
$A1 = ?$ BUT $A1 = A2$
$A2 = ?$

$100(35) = r2(35-10)$ CHECK: $100(35) = 140(35-10)$
$3500 = r2(25)$ $3500 = 140(25)$
$140 = r2$ $3500 = 3500$

Dilution Problems:

$$r1(t1) = A1$$
$$r2(t2) = A2$$
$$r3(t1 - t2) = A3$$
$$\text{If } A1 = A2 + A3\text{, then } r1(t1) = r2(t2) + [r3(t1 - t2)]$$

	¶	Rate	Time	Total Amount
Goal	¶	r1	t1	A1
Higher Rate	¶	r2	t2	A2
Lower Rate	¶	r3	t1 - t2	A3
	¶			A1 = A2 + A3

EXAMPLE: $r1 = 12$
$r2 = 15$
$r3 = ?$
$t1 = 20$
$t2 = 10$
$A1 = ?$
$A2 = ?$ BUT $A1 = A2 + A3$
$A3 = ?$

$$12(20) = 15(10) + r3(20-10)$$
$$240 = 150 + r3(10)$$
$$90 = r3(10)$$
$$9 = r3$$

CHECK:

$$12(20) = 15(10) + 9(20-10)$$
$$240 = 150 + 9(10)$$
$$240 = 150 + 90$$
$$240 = 240$$

Facilitation-Interference Problems:

$$t1(r1 + r2) = A1$$
$$t2(r1 - r2) = A2$$
$$\text{If } A1 = A2\text{, then } t1(r1 + r2) = t2(r1 - r2)$$

	Rate	Time	Total Amount
Standard Rate	$r1$		
Higher Rate	$r1 + r2$	$t1$	$A1$
Lower Rate	$r1 - r2$	$t2$	$A2$
		$t1 < t2$	$A1 = A2$

EXAMPLE:

$r1 = ?$
$r2 = 20$
$t1 = 3$
$t2 = 6$
$A1 = ?$
$A2 = ?$ BUT $A1 = A2$

$$\begin{aligned} 3(r1+20) &= 6(r1 - 20) \\ 3r1 + 60 &= 6r1 - 120 \\ 3r1 + 180 &= 6r1 \\ 180 &= 3r1 \\ 60 &= r1 \end{aligned}$$

CHECK:

$$\begin{aligned} 3(60 + 20) &= 6(60 - 20) \\ 180 + 60 &= 360 - 120 \\ 240 &= 240 \end{aligned}$$

Appendix A (cont)

Examples of Orienting Task Questions

Recognition

Old: Callie California is expecting guests for a hot tub party

New: Callie Callifornia has invited guests for a hot tub party

Verification

True: She plans to start filling the tub at 4:00

False: She plans to start filling the tub at 6:45

Analogy (Note: * denotes correct answer)

Flying at 180 mph : leaving at 5:00 :: filling the tub at 60 gal/hr : ?

1. starting at 6:45 pm
*2. starting at 4:00 pm

Landing at 10:00 (problem 1) :: ? (problem 2)

*1. filling the tub by 8:00
2. filling the tub by 6:45

Schema (Note: Only two choices were shown following each problem pair)

1. Both problems involve a "catch-up" situation where there is less time than was planned on to achieve some goal.
2. Both problems involve a situation where something has to be lessened or decreased.
3. Both problems involve increases and decreases in a normally equal rate of performance.