Can Fitts' Law Be Improved?: Predicting Movement Time Based on More Than One Dimension

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Abstract

In this paper, we critically review the variables assumed to affect the movement time (MT) of simple aiming responses. After discussing some limitations of Fitts' Law, in which important variables for the prediction of MT are the movement amplitude and actual target width, we propose an alternative equation which takes into account the constant and variable errors of the subjects' endpoints in two dimensions as well as the vertical component of the movement trajectory. Using data of a previous experiment (Wallace, 1983), the elliptical area of the subjects' endpoints was substituted for the actual target width in Fitts' (1954) Index of Difficulty (ID). Using this revision, a correlation between MT and ID of $r = .99$ with a y-intercept of approximately zero (19 msec) was achieved. We also demonstrate in two further experiments that (1) endpoint variability perpendicular to the movement, and (2) the vertical component of the movement trajectory affects MT and should be accounted for. It would appear that Fitts' Law can be improved, particularly in cases where accuracy in more than one dimension is involved.

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Introduction

Recently, there has been increased interest in the speed versus accuracy issue concerned with the control of aimed limb movements (e.g., Carlton, 1981; Howarth & Beggs, 1981; Meyer, Smith & Wright, 1982; Newell, 1980; Schmidt, Zelaznik, Hawkins, Frank & Quinn, 1979; Wallace & Newell, 1983; Wing, 1983; Wright & Meyer, 1983; Zelaznik, Shapiro & McColsky, 1981). That spatial accuracy decreases as movement speed increases is well known and was empirically documented many years ago by Woodworth (1899). However, much of the current work involves the quantification of this relationship and an attempt to theorize as to the underlying perceptual and motor processes responsible for it (Meyer et al., 1982; Schmidt et al., 1979).

The first attempts in this regard were conducted by Fitts in two now classic papers (Fitts, 1954; Fitts & Peterson, 1964). Based on information theory, which was popular at the time, Fitts wished to predict the total movement time (MT) of aiming movements given knowledge about the movement's amplitude (A) and the width of the target involved (W). Fitts showed that the following relationship held for both discrete and reciprocal aiming movements:

$$MT = a + b \log_2 (2A/W), \qquad (1)$$

where $\underline{a}$ (y-intercept) and $\underline{b}$ (slope) are empirically determined constants and $\log_2 (2A/W)$ is referred to as the index of difficulty (ID). In the Fitts' procedure subjects are required to make aimed movements at the target as rapidly as possible but to hit the target on 90-95% of their attempts. Using this procedure, Equation 1 has been shown to hold reasonably well across several movement conditions (see Meyer et al., 1982, for a brief review) and has become known as Fitts' Law. Considering that the correlations between ID and MT in most studies are typically greater than .90, it would appear there is little room for improving this equation in the very simple movements that have been studied to date. However, as we will show in this paper, by considering certain variables it is possible to improve not only the correlation between ID and MT, but also the value of the y-intercept in adaptations of the Fitts' Equation. A case is also made in this paper that the variables we introduce to improve the equation may have more dramatic effects on MT prediction when the movements produced are more complex, for example, when accuracy is considered in two or three dimensions.

In an earlier critical analysis of the Fitts' formulation, Welford (1968) indicated several limitations and offered suggestions for improving the quantification of the relationship between MT and the difficulty of the movement. First, he pointed out that in Fitts' (1954) data, the y-intercept was negative, cutting the zero information line below the origin. Of course, this fact has conceptual problems since a negative MT is impossible. Second, the data points at the lower ID values tend to have a shallower slope than the data points at higher ID values, a finding shown subsequently by others (e.g., Klapp, 1975; Wallace & Newell, 1983). Crossman (1957) had earlier suggested that there may be some limiting factor in the Fitts' procedure setting a minimum time per movement regardless of the condition. In observations of his own research, Welford suggested that this limiting factor affects the amount of the target used. In cases where the movement amplitudes are short and the target widths are large, the subject uses much less than the actual target width. To test this notion, it is necessary to record the actual endpoints of subjects' movements. Until fairly recently, the analysis techniques for recording the actual endpoints have been relatively gross, (e.g., using a ruler, Crossman & Goodeve, 1983; or using a bulls-eye scoring scheme, Connolly, Brown & Bassett, 1968). In a recent study of Fitts' Law, Wallace (1983) digitized subjects' endpoints using an x-y digitizer (resolution = .01 cm) and confirmed that when large targets were used, subjects tended to use a smaller portion of the actual target width than when small targets were used. The implication of these findings from an information theory viewpoint as Welford noted, is that the subject transmitted more information in these target situations than Equation 1 suggests because the effective target width chosen by the subject was narrower than the actual target width. This finding, then, may account for the slight flattening of the MT/ID relationship at the low ID values.

In the next section, we discuss alternative formulations of Fitts' Equation by Welford (1968), Crossman (1957), and the present authors. These formulations are designed to improve the prediction of MT on the basis of knowledge about the movement's amplitude and effective target size.

Alternative Formulations

Most studies of Fitts' Law have used tasks for which it is not possible to record subjects' actual endpoints (e.g., Fitts, 1954; Klapp, 1975; Wallace, Newell & Wade, 1978). In these and other studies, the target and surrounding area are often made of metal making it impossible to record exactly where the subject hit the apparatus surface. Several years ago, however, in an obscure paper, Crossman (1957) developed a method which estimated the effective target width based on knowledge of the subject's error rate for a given amplitude-target width condition. The method starts with the assumption that the subject's (or the group's) dispersion of endpoints around the target are normally distributed with a constant error of zero. If these assumptions are met, then the following argument is justified. The information in a normal distribution is $\log_2 \sigma \sqrt{(2\pi e)}$, where σ is the standard deviation of the distribution. The term $\sqrt{(2\pi e)}$ equals 4.133 Z units ($\pm$ 2.062σ) and includes

96% of a normal distribution when an error rate of 4% is assumed. This being the case, $\log_2 W$, where W is the actual target width, accurately represents the information in the distribution of the subjects' endpoints. Thus, in an experiment where the subjects' error rate is 4%, the effective target width would equal the actual target width. The general equation for calculating the effective target width (We) is as follows:

We = Wa x Za/Ze (2)

where Wa = Actual target width

Za = 4.133 (Z units corresponding to 96% of the distribution or 4% error rate)

Ze = Z units corresponding to percent of the distribution after subtracting the obtained error rate from 100%.

Thus, when Ze is larger than Za, We is smaller than Wa. Using Welford's (1968) example, if Wa = 2 in. and the error rate is 1%, then We equals 2 x 4.133/5.152 or 1.604 in. It is also possible to obtain an estimate of the standard deviation (SD) using the logic of Equation 2. Using Equation 2, an estimate of the standard deviation is as follows:

SD = Wa/Ze (3)

One can easily see that the estimated SD is necessarily correlated with We since Za is a constant (see Wallace & Newell, 1983).

Welford also argued that Fitts' Law could be improved by the following equation:

$MT = K \log_2 (A + Wa/2)/Wa$ (4)

Since A is the distance from the starting point to the center of the actual target, the term Wa/2 is added to account for possible movements to the far edge of the target. Notice also that Equation 4 predicts a proportional relationship between MT and $\log_2 (A + Wa/2)/Wa$ with the y-intercept passing through the origin. Welford (1968) re-analyzed the original Fitts (1954) data which showed a negative y-intercept and a flattening of MT at the low ID values when Equation 1 was used. Using error rate information from the Fitts (1954) experiment, Welford calculated We and substituted it for Wa in Equation 4. A near perfect straight line through the origin was achieved when these data were plotted (Welford, 1968, pg. 148).

Limitations of Welford's Formula and Possible Improvements

In spite of its apparent improvements over the Fitts formulation, we feel Welford's formulation also has some limitations. First, the estimate of We is derived from knowledge of percent miss (Crossman, 1957) and we feel this procedure is not now justified unless it is impossible to record subjects' actual endpoints. If the subjects' actual endpoints can be recorded, it is possible to calculate the constant error and standard deviations of the endpoints in two dimensions; (a) for the x-dimension (parallel to the goal movement, and (b) for the y-dimension (perpendicular to x in the horizontal plane) (see Schmidt, et al., 1979). What we have done to calculate We is to multiply the obtained (rather than the estimated) standard deviation by 4.133 (± 2.06) z-units which includes approximately 96% of the distribution of the endpoints. Thus, it is possible to estimate We based on knowledge of the obtained standard deviation of a particular target-amplitude condition rather than to estimate it from the obtained percent miss data. We feel our estimate of We is more appropriate because it takes into account the magnitude of error of both the hit and miss endpoints. To illustrate, suppose for a particular target-amplitude condition of 40 trials, the miss rate is 10% (or 4 trials). In the Crossman-Welford estimation of We, the magnitude of error associated with those 4 trials is not accounted for. Thus, depending on the magnitude of error of the 4 miss trials, it is likely that We will be either under- or overestimated. On a set of data from Wallace (1983) we found that estimates of the SD using Equation 3 consistently overestimated the obtained SDs in each of nine separate target-amplitude conditions.

The second limitation of the Crossman-Welford estimate of We is that it cannot be used when a noticeable constant error is associated with the endpoint distribution. Recall that an assumption in this model is that constant error is 0. When the distribution is normal and the constant error is 0, Crossman and Welford argued that the percent misses are equal on both ends of the distribution. Under these conditions, it is possible to use the logic of Equation 2 to obtain an estimate of We. But if there is a sizeable CE this would mean a disproportionate number of errors are on one side of the distribution. If this is true, the logic used to estimate We in Equation 2 could no longer be used. A sizeable CE would pose no major problem for our estimate of We since in our formula We is calculated around the mean of the distribution regardless of where the mean falls in relation to the center of the actual target.

A major assumption of the Crossman-Welford formulation is that the endpoint distributions are normally distributed. Of course, this assumption is made in our formulation with the view that ± 1 SD will contain approximately 68% of the endpoint dispersions in a given target-amplitude condition. There are two major threats to normality--skewness and kurtosis (Ferguson, 1981). Skewness refers to the asymmetry of a distribution. Larger frequencies at the low end or high end of the distribution would be evidence for positive and negative skewness, respectively. Even with a CE = 0, skewness negates the relationship of ± 1 SD containing 68% of the distribution. Kurtosis refers to the flatness or peakedness of a distribution in relation to a normal distribution. If a distribution is more peaked than a normal distribution, it is considered to be leptokurtic whereas if it is flatter than a normal distribution it is considered to be platykurtic. As with skewness, large kurtosis would result in ± 1 SD containing more or less than 68% of the distribution. Either one of these departures from normality would prevent an accurate estimation of We using the Crossman-Welford or our formulation. Although there are methods available for adjusting the values for area under the curve when varying degrees of skewness and/or kurtosis are present, as far as we know, there is no general method of calculating some standard measure of variation, comparable to the SD, in non-normal distributions. However, it is possible to determine whether and to what extent skewness or kurtosis exists in a distribution (Ferguson, 1981, pgs. 72-74). Thus, once this is known, one could determine whether the obtained SD is an over- or underestimate of the actual percentage contained in it. Any calculation of We would likewise be affected.

Thus far, we have argued that when it is impossible to record actual endpoints in the Fitts type task, estimates of We can be calculated using Equation 2. But if the underlying endpoint dispersions contain significant CE effects or if the endpoint dispersions are not normally distributed, then Equation 2 will not accurately depict We. The result of this may be a poorer prediction of MT using the inaccurate We value in Equation 4. When it is possible to record actual endpoints, we feel our estimate of We is justified because it takes into account the magnitude of errors of both hit and miss endpoints. Significant CE effects will not negatively influence our estimates of We. Significant deviations from normality of the endpoint dispersions would distort an accurate estimate of We and, while this is a problem, statistical procedures exist to determine if, and to what extent, deviations from normality are present. It would at least be possible to better evaluate any predictions of MT in light of the severity of non-normality.

Another limitation of both Equation 4 and the original Fitts' Equation 1, is that it accounts for variability in only the x- dimension. In Equation 4, both A and Wa are defined in terms of x. Crossman (1957) found some evidence using two subjects on a Fitts' task, that the horizontal x-dimension of the target and the target dimension perpendicular to x on the surface of the apparatus (y) both contributed to the prediction of MT. This led Crossman to suggest that the prediction of MT is dependent on the area of the target as well as A, which is equivalent to saying that the amplitude and directional information of the target are additive. Crossman suggested that MT could be predicted on the basis of these parameters in the following expression:

$$MT = b\log_2 (A/Wa) + b\log_2 (A/Da) \qquad (5)$$

where A = amplitude of movement

Wa = actual target width

Da = actual target depth (perpendicular to x-dimension)

b = slope

When plotted, the data from two subjects fitted reasonably well to Equation 5 even though many of the points deviated considerably from the line of best fit and the y-intercept was well below zero. Despite this drawback, we feel Crossman's ideas were important because he appears to be the only person to have considered two target dimensions in the prediction of MT. We should emphasize that although Crossman was cognizant of the fact that the subject could produce an effective target different from the actual target, for some reason he did not include this fact in Equation 5. Thus, Crossman's equation suggests a trade-off between A and the actual target area (TAa), the geometric shape of the latter being rectangular. We feel that two limitations of Crossman's thinking here are that, (1) an effective target area (TAe) probably better represents subjects' endpoint dispersions than TAa and (2) from data collected by Welford (1968) and others, these dispersions resemble an ellipse rather than a rectangle. We will show shortly that a more accurate prediction of MT might be made when these two points are taken into account.

In a similar vein, we feel that up until now, A has been considered in only one dimension. That is, according to Fitts and the Crossman-Welford formulation, A is defined as the one-dimensional distance from the starting point to the center of the target. This actual amplitude (Aa) does not necessarily describe the effective amplitude (Ae) which, of course, is performed in three dimensions by the subject in the typical Fitts' tapping task. While it is likely that movement in the x-dimension contributes the most to the MT, there is reason to believe that movement in the vertical dimension also contributes to the MT. Connolly, Brown and Bassett (1968) observed that children (ages 6, 8 and 10) who had the slowest MT also ". . . lifted the pencil in a high arc as it passed between the two circles . . ." (p. 310) in a reciprocal tapping task. Evidence for a large vertical component of the movement trajectory comes from other informal observations with mentally-handicapped subjects (Wade, Newell & Wallace, 1978) and pre-school populations (Wallace, Newell & Wade, 1978). Perhaps one of the reasons why the vertical component of the trajectory has not been taken into account in the prediction of MT is that it is difficult to record. However, with high-speed cinemaphotography (Carlton, 1981) and other on-line two dimensional techniques (e.g., Hawkins, 1983) it is possible to record this dimension. With this capability, it should be possible to measure or closely estimate the distance of the curved path of the trajectory which will, of course, be longer than Aa.

An Alternative Formula

In this section we present an alternative formula for the prediction of MT in an attempt to overcome some of the limitations of Equations 1 and 4. The formula for the prediction of MT is as follows:

$$MT = b\log_2\left(\frac{\text{effective amplitude}}{\text{effective target area}}\right) \text{ or } MT = b\log_2\left(\frac{Ae}{\pi \cdot We/2 \cdot De/2}\right) \qquad (6)$$

where Ae = effective amplitude

We = effective target width

De = effective target depth

b = empirically defined constant for the slope

Equation 6 predicts a logarithmic trade-off between the effective amplitude (numerator) chosen by subjects and the effective target area (denominator) of the endpoint dispersions. The MT is directly related to $\log_2$ (Ae) and inversely related to $\log_2(\pi \cdot We/2 \cdot De/2)$.

First, let us discuss the rationale for using Ae instead of A (as in Equations 1 and 4). In Equation 6 we have Ae accounting for the amplitude of movement in two dimensions, horizontally in the x-dimension and vertically in the z-dimension. If it is possible to record the movement trajectory in these two dimensions, then it should be possible to pair each (x, z) point, and sum up all the distances to determine the total Ae. If the sampling rate is reasonably high, then one can assume a straight line between each (x, z) pair (Hay, 1983). Again, the rationale for using Ae is that it is more reflective of the trajectory of the movement and thus, hopefully more highly related to MT than is Aa. In cases where only the endpoints of the movement can be recorded, then Ae will be equal to Aa plus or minus any significant constant error (CE) in the x-dimension, as suggested originally by Welford (1968, pgs. 148-149).

The quantity $(\pi \cdot We/2 \cdot De/2)$ is the effective target area, which, on the bases of previous observations, resembles an ellipse. The area of an ellipse is $(\pi \cdot a \cdot b)$ where a and b are one-half the distances of the major and minor axes of the ellipse, respectively. To estimate a and b we divide We and De each by 2.

What we are suggesting in Equation 6, is that information about the variability of the endpoints in both dimensions is important for the accurate prediction of MT.

Some Expected Outcomes

The prediction of movement time. In this section we discuss some of the expected outcomes of predicting MT using the concepts advocated in Equation 6. The first and most important outcome is that Equation 6 should predict MT better than either Equation 1 (Fitts, 1954), Equation 4 (Welford, 1968), or Equation 5 (Crossman, 1957). Not only should there be a higher correlation between MT and the $\log_2$ (effective amplitude/effective target area) with Equation 6, but we also expect the y-intercept to be closer to zero than with the other two formulae. At this time, we would like to present evidence for these expected outcomes with the understanding that the ultimate support or rejection of Equation 6 should be based on more than one investigation. For this purpose, we have used some data reported by Wallace (1983). In this study, nine subjects were asked to perform a discrete tapping task (e.g., Fitts & Peterson, 1964). The reader may refer to the published study by Wallace and Newell (1983) to obtain a more detailed explanation of the methodology used. Subjects in the Wallace (1983) study performed discrete (uni-directional) movements of various amplitudes to circular targets having different diameters. The amplitudes were 3, 6, and 12 in. (7.62, 15.24 and 30.48 cm) and the target diameters were .25, .50, and 1 in. (.64, 1.27 and 2.54 cm), and these resulted in Fitts ID values ranging from 2.58 to 6.58. Subjects performed under visual and non-visual conditions. On the non-visual trials, the desk light illuminating the apparatus extinguished on stylus lift-off from the start position and remained off until contact was made on the target (or surrounding area). Visual and non-visual trials were randomly dispersed over a block of 20 trials for each target-distance condition (10 visual and 10 non-visual). It was possible to record subjects' endpoints because the tip of the stylus penetrated the paper target sheet, leaving a small hole. Subjects' endpoints were digitized by an x-y digitizer and standard deviations and constant errors in both x- and y-dimensions were calculated using computer programs. For the purposes of this paper, we will discuss the results of only the visual data. Future work will address the issue of visual versus non-visual performance in the Fitts-type task (see Wallace & Newell, 1983 for a recent analysis). Figure 1 displays a graphic representation of the group mean constant errors and standard deviations in both x- and y-dimensions and effective target areas for each of the nine target-amplitude conditions. Two major findings emerged. One, the pattern of dispersions as hypothesized were elliptical in nature in every condition--the standard deviation and effective target widths in the x-dimension were approximately 25% larger than those in the y-dimension. Two, also as predicted, the subjects' effective target areas were smaller than the actual target area. The correlation between the standard deviations of errors in the x- and y-dimensions was .85 (r^2 = .72). This finding indicates considerable shared variance in the x- and y-dimension and suggests perhaps a common process or mechanism which controls variability in the two dimensions. However, we shall see in the next experiment, that variability in one dimension can be controlled somewhat independently of the other dimension.

Insert Figure 1 about here

The next analysis of primary concern was the comparison of correlations between MT and the $\log_2$ (expression) of Equation 1 (Fitts), Equation 4 (Welford), Equation 5 (Crossman), and Equation 6 (ours). In Figure 2, this comparison is shown. In this figure we have plotted the group mean MT against the $\log_2$ (expressions) of the four formulae. Thus, each point represents a total of 90 trials (ten trials/subject). It can be observed that the correlations of the four plots are all over .90. However, our formula shows a near perfect correlation of .99. Welford's formula has a y-intercept closest to zero (a = 3.63 msec), however, his correlation is the lowest (r = .93). Our formula has the next lowest y-intercept (a = 19.55 msec).

Insert Figure 2 about here

We also calculated the presence of skewness and kurtosis in the Wallace (1983) experiment using the formulae presented by Ferguson (1981). Using the values of the first four moments around the arithmetic mean, it is possible to calculate relative values of skewness and kurtosis. For skewness, a value of 0 means normality, but large positive or negative values indicate the presence of positive and negative skewness, respectively. For kurtosis, a value of 0 means normality but large positive or negative values suggest that the data are leptokurtic or platykurtic, respectively. Table 1 presents the results of this analysis. For all nine target-distance combinations, it appeared that the data were not severely skewed. The skewness values were close to 0. However, the data appeared to be somewhat platykurtic, and more so in the y-dimension, for all conditions. This suggests that the endpoint distributions were more evenly distributed over the target area than normally distributed. Thus, ± 1 SD is not likely to encompass 68% of the endpoints and our effective target area probably accounts for less than 96% of the endpoints.

Insert Table 1 about here

However, even with this constraint, our formula still shows a correlation of .99, accounting for over 98% of the variance and shows a near proportional relationship between MT and our $\log_2$ (expression). We feel these results are primarily due to the accounting for the variability of endpoints in both x- and y-dimensions and because our formula recognizes the elliptical pattern of the endpoint distribution around the target area in spite of the presence of some platykurtosis.

The contribution of the y-dimension. One expectation from Equation 6 is that some portion of MT is dependent on the variability of the endpoints in the y-dimension (perpendicular to the direction of the movement in the horizontal plane). Crossman (1957) should be credited, however, for first suggesting the importance of this dimension in predicting MT. The major difference between his formula and ours is that our formula suggests that MT is better related to the endpoint dispersions in the y-dimension than to the length of the actual target in that dimension. This would be the case particularly if subjects' effective target depth in the y-dimension is considerably shorter than the actual target depth in the y-dimension. Using two adult subjects, Crossman (1957) showed that restricting the target depth affected MT in the same way as restricting the target width, albeit to a lesser degree. In the experiment, Crossman manipulated both the width and depth of the target (2, 1, 1/2, 1/4, 1/8 in.) to yield 25 rectangular target sizes with a fixed amplitude of 8 in. from the starting point to the center of the target. Although an actual correlation between MT and his $\log_2$ (expression) was not reported, plots presented by Crossman for both subjects revealed a fair amount of deviation around the line of best fit. Also, the y-intercept was approximately -100 msec for one subject and -200 msec for the other. Because subjects' endpoints were not recorded it is likely that their effective target areas were less than the actual target area, resulting in a less than desirable prediction for both subjects. However, Crossman was able to show that the target dimensions in the y-dimension was responsible for some changes in MT.

We set out to replicate Crossman's findings that the y-dimension is important in contributing to the MT. However, we recorded subjects' endpoints and calculated constant error and standard deviations in both x- and y-dimensions as well as the effective target area for the following conditions. Six subjects performed under three experimental conditions. In all three conditions, A was equal to 12 in. (30.48 cm) and the actual target width in the x-dimension was 1 in. (2.54 cm). In the Circle condition, the actual target depth was also 1 in. In the Large Ellipse condition, the actual target depth was .5 in. (1.27 cm). In the Small Ellipse condition, the actual target depth was .25 in. (.64 cm). As in the Wallace (1983) experiment, subjects were instructed to move as rapidly as possible but to keep misses under 10%. Subjects were told that any endpoint within the target would be considered a hit.

We performed analyses of variance on MT, the standard deviations, and the effective target areas among the three conditions. The means and standard deviations for these dependent variables in the three conditions are shown in Table 2.

Insert Table 2 about here

As we anticipated, MT increased as the y-dimension of the target decreased and this effect was significant, $F(2,10) = 23.26$, $p = .0003$. Using Tukey's post hoc procedures, it was determined that MT for the Small Ellipse condition was significantly slower than MT for the Circle condition. The constant errors in both dimensions were very close to zero for all three conditions and were not significantly different from one another, $F < 1$. While there was a trend for standard deviations of the x-dimension to reduce as a function of the y-dimension of the target, this effect was not significant, $F < 1$. The standard deviations in y-dimension, however, significantly reduced as the y-dimension of target decreased, $F(2,10) = 13.42$, $p = .001$. The effective target area significantly reduced as the y-dimension of the target decreased, $F(2,10) = 13.12$, $p = .001$. Tukey's post hoc tests performed on the above confirmed that the Circle and Small Ellipse conditions were significantly different.

We also performed a correlation between the standard deviations in the x-dimension and those in the y-dimension. The resulting correlation was .533 with an r^2 of .284. This indicates very little shared variance between the processes which control variability in the two dimensions. Of course, it is possible that this correlation may be due to a small number of observations or a limited range of possible endpoints. Clearly, more work is needed in the future to investigate this issue. However, the results suggest that when only the y-dimension of the target is constrained, the subject can somewhat independently control the errors in that dimension.

As in the Wallace (1983) experiment, we calculated the skewness and kurtosis values in this experiment. The most prominent feature of the data in this respect was that endpoint distributions appeared to be rather platykurtic, particularly in the y-dimension. The data did not appear to be severely skewed and these results are consistent with those in the Wallace (1983) experiment.

Insert Table 3 about here

In summary, this experiment provided evidence that the y-dimension of the target contributes to MT. Also, the data indicate that when only the depth of the actual target is manipulated, subjects tend to restrict variability primarily in only that dimension. This finding, plus the fact that there appears to be a low correlation between variability in the two dimensions, suggests that the processes which control variability in those dimensions are, to some degree, independent.

The effective amplitude. At the present time, we do not have the capability to measure the actual trajectory of aimed movements in three dimensions. However, we performed the following experiment to examine the importance of the vertical (z) dimension in affecting MT. We had six subjects perform the Fitts task under two conditions. In both conditions, after several practice trials, subjects performed 40 experimental trials attempting to hit a 1 in (2.54 cm) diameter circular target at a distance of 12 in (30.48 cm). In the No Hurdle condition, subjects were instructed to maintain a low trajectory throughout the movement. In the Hurdle condition we placed a 10 cm-high cardboard barrier halfway between the starting point and the middle of the target. Subjects were told to move the stylus over the barrier without touching it and, as in the No Hurdle condition, to move as rapidly as possible and hit any portion of the target. Subjects were allowed four misses per condition in order to maintain a 90% hit rate (Fitts and Peterson, 1964). The results of this experiment are shown in Table 4. The results indicated that the hurdle manipulation only significantly affected MT, $t(5) = 10.86$ $p < .02$. MT of the Hurdle condition was significantly longer than in the No Hurdle condition.

Constant errors were essentially zero in the x- and y-dimensions for both conditions. As in the previous two experiments, standard deviations in x-dimension were larger than in the y-dimension. Comparisons of the standard deviations and the effective target areas between the Hurdle and No Hurdle conditions produced no significantly different results.

Insert Table 4 about here

As in the first two experiments, we found no evidence for skewness in the data in either dimension with values close to zero. Also in agreement with the earlier experiments, the data from this experiment were platykurtic particularly in the y-dimension.

Insert Table 5 about here

The results of this experiment showed that the distribution of endpoints was equivalent in the Hurdle and No Hurdle conditions. However, in an effort to keep their errors at this level, subjects performing in the Hurdle condition increased their MT because of a larger vertical component of their trajectory. Thus, these results suggest that the vertical component of the movement trajectory may also need to be considered in future attempts at predicting MT.

General Discussion

The general purpose of this paper was to evaluate the original Fitts (1954) equation, focussing primarily on the variables considered to be important in the prediction of MT. In the original Fitts equation, only the diameter of the actual target and the movement amplitude in the x-dimension were considered as potent variables. Extending the logic of Welford (1968) and Crossman (1957), we developed a formula which takes into account more than one dimension of the movement and more than one dimension of movement endpoints. Our formula predicts that the MT is dependent on the $\log_2$ of the ratio between the effective amplitude divided by the effective target area. The effective amplitude is the distance of the curved path of the movement trajectory and takes into account the x (horizontal) and z (vertical) dimensions. The effective target area takes into account both the x- and y-dimensions of the subjects' endpoints and, if these are normally distributed, will reflect 96% of the endpoint variability. However, the data from all three experiments reported here are platykurtic, particularly in the y-dimension. This means that the effective target areas we have calculated probably underestimate 96% of the endpoint variability because the data are more <u>evenly</u> distributed than normally distributed.

The level of platykurtosis could not have been too severe since the prediction of MT with our equation was excellent. Using data from Wallace (1983), we found the correlation to be .99 between MT and our $\log_2$ (expression), and the y-intercept to be close to zero ($\underline{a}$ = 19.55 msec). Our formula predicts a zero y-intercept so the 19.55 value raises some concern. As discussed recently by Wright and Meyer (1983), positive intercepts not predicted by equations make it more difficult to develop a complete understanding of the processes involved in the speed-accuracy tradeoff. Wright and Meyer (1983) speculated that the positive intercept in their data as well as others (Howarth, Beggs, & Bowden, 1971; Schmidt et al., 1979; Zelaznik et al., 1981) might be due to tremor or, a misperception of the target which might randomly vary across trials. In addition, Schmidt et al. (1979) speculated that the positive intercept might be due to measurement error. While these authors were trying to predict accuracy as a function of movement speed, these arguments could also be used to explain the positive intercept obtained in our data where the goal was to predict MT. At the present time, we have no concrete explanation for the presence of the slight positive intercept. Clearly, more work needs to be done to determine whether this is a consistent finding. But the point we wish to make is that extremely large positive y-intercept values such as those found by Hay (1981) or large negative y-intercepts (Crossman, 1957) should not be overlooked when one is trying to account for prediction of MT.

In the second experiment reported, we found some evidence that variability in the y-dimension of the target contributes to the total MT. It was also shown that the correlation between variability in the x- and y-dimension was low. These findings suggest that the control of movement in the two dimensions may be somewhat independent. Although providing no real rationale, Howarth and Beggs (1981) have stated the error measured in the direction of movement "...is related to the accuracy with which a movement can be stopped. Error at right angles to the movement is the error of aiming..." (pg. 91). It is not clear from their description whether the processes which control accuracy and aiming are the same or different. Our data suggest that the processes are somewhat independent and they may be affected differently by different variables. For example, Zelaznik, Hawkins and Kisselburgh (1983) have recently shown that the use of visual feedback in the control of movement is not exactly the same in both x- and y-dimensions. Control of movement in the x-dimension seemed to be more affected by the removal of feedback and more sensitive to changes in the subjects' knowledge of whether visual feedback would be present or not on a given trial. This kind of effect may indicate that the underlying processes which control the two dimensions are somewhat independent.

We provided tentative evidence in the Hurdle experiment that the vertical component of the movement trajectory is important to consider in the prediction of MT. The experiment showed that subjects maintained accuracy at the expense of slowing MT over a higher trajectory. Ideally, it is important to quantify the distance of the curved path of the movement trajectory and we are currently working on this problem. Also, the fact that large vertical components of the trajectory have been noticed previously (Connolly, et al., 1968; Wade, et al., 1978) makes the analysis of this component seem necessary in the study of special populations such as young children and the mentally handicapped.

One possible direction this research could take is the prediction of MT in three-dimensional space. It might be possible that the MT necessary to grasp an object in three-dimensional space is dependent upon its three-dimensional size. This is a difficult problem because other factors such as the number of fingers used, surface area of the fingers, and their orientation in space might need to be taken into account. Related work on this problem has been done by Jeannerod (1981) in grasping three-dimensional objects. Jeannerod found that the size of an object affects the grasping (fingers) component but not the trajectory or transport component of the movement toward the object. However, these experiments were done without explicit accuracy requirements. Perhaps if one defines the accuracy requirements in this type of task (e.g., grasping the object with precise control), it may be possible to predict MT based on knowledge of movement amplitude, three-dimensional target size and perhaps other variables such as object orientation. Extending the ideas in this paper into three dimensions does seem to be a logical step, particularly since there is a keen interest in robotics and in the basic processes involved in grasping (e.g., Wing and Fraser, 1983).

The ideas in this paper impact upon some of the theoretical models which have been used to explain Fitts Law (e.g., Crossman & Goodeve, 1983; Keele, 1968; Meyer et al., 1982). These various models are essentially one-dimensional. For example, in the Keele (1968) discrete correction model, each visual correction which is made during the movement reduces the distance to the target by approximately 93%. Keele's model is one-dimensional and a strict interpretation of it might predict that error in the x-dimension (direction of movement) is the only dimensional error reduced as a result of visual corrections. When a two-dimensional target is considered, however, it would seem necessary to consider the likelihood that error corrections are made in both dimensions. How and when corrections are made in both dimensions would be an important line of future research.

Another issue of theoretical importance is why MT is related to a logarithmic tradeoff rather than to a linear tradeoff. This question has been recently addressed by Meyer et al. (1982) and Wright and Meyer (1983). One possibility is that in tasks such as Fitts' which minimize spatial but not temporal variability, the movement is controlled by a pre-programmed series of overlapping force pulses and should lead to a logarithmic but not a linear tradeoff. Our data (Wallace, 1983) yielded a logarithmic tradeoff between the effective amplitude and the effective target area, and as such would tend to support the Meyer et al. (1982) model. In addition, the Meyer et al. (1982) model does not attribute this logarithmic tradeoff to the processing of visual feedback. Their model predicts a logarithmic tradeoff even under degraded visual conditions. Support for this prediction can be found in a study by Wallace and Newell (1983) which found a logarithmic tradeoff in non-visual discrete movements. The Meyer et al. (1982) model appears to be gaining empirical support and is a clear alternative to the discrete corrections model developed by Keele (1968). Future work is needed to further test this model, perhaps by extending its concepts to more than one dimension.

In summary, we set out to re-examine Fitts Law and the variables assumed to affect the prediction of MT. We have provided some evidence that to accurately predict MT, one must take into account more than one dimension of the target aimed at and the variability of the errors produced as well as more than one dimension of the movement trajectory to the target. While these ideas and findings are preliminary, we feel they serve as a basis for developing a more comprehensive relationship between MT and variables which affect it.

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Author Notes

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Table 1

Skewness and Kurtosis Values* in x- and y-Dimensions

of Wallace (1983) Experiment**

Target-Distance	Skewness		Kurtosis	
Condition	x	y	x	y
1" - 3"	.08	.40	-.80	-1.80
.5" - 3"	.08	-.16	-1.74	-2.90
1" - 6"	.13	.40	-1.80	-2.71
.25" - 3"	.42	.83	-.50	-1.96
.5" - 6"	.00	-.27	-2.07	-2.70
1" - 12"	.36	.76	-1.88	-2.49
.25" - 6"	.08	-.37	-.15	-2.79
.5" - 12"	-.38	-.33	-1.90	-2.72
.25" - 12"	.11	-.25	-1.18	-2.71

*For skewness, 0 = normal distribution, large positive value = positively skewed and large negative = negatively skewed. For kurtosis, 0 = no kurtosis, large positive = leptokurtic and large negative = platykurtic.

**Values collapsed over nine subjects.

Table 2

Means and Standard Deviations (in parentheses) of MT, CE (cm), Standard Deviations in x- and y-Dimensions (cm), and Effective Target Areas for the Three Target Conditions.*

Dependent Measures	Circle	Large Ellipse	Small Ellipse
MT	268	323	334
	(63.5)	(56.2)	(72.2)
CEx	-.010	.070	-.030
	(.19)	(.16)	(.17)
SDx	.476	.445	.418
	(.09)	(.07)	(.10)
CEy	-.100	-.010	-.020
	(.09)	(.08)	(.06)
SDy	.336	.259	.206
	(.09)	(.03)	(.03)
Eff Target Area	2.210	1.540	1.180
(cm^2)	(.96)	(.37)	(.46)

*See text for exact target-dimensions.

Table 3

Skewness and Kurtosis Values* in the x- and y-Dimensions**

	Skewness			Kurtosis		
Dimension	Circle	L. Ellipse	S. Ellipse	Circle	L. Ellipse	S. Ellipse
x	-.87	.33	.11	.33	-.69	-.86
y	.09	.80	.16	-2.88	-2.51	-2.88

*For skewness, 0 = normal distribution, large positive value = positively skewed and large negative = negatively skewed. For kurtosis, 0 = no kurtosis, large positive = leptokurtic, and large negative = platykurtic.

**Values collapsed over six subjects.

Table 4

Means and Standard Deviations (in parentheses) of MT, CE (cm),
Standard Deviations in x- and y-Dimensions (cm), and Effective Target Areas
for the Hurdle and No Hurdle Conditions

Dependent Measures	Hurdle	No Hurdle
MT	264	231
	(20.07)	(31.05)
CEx	-0.50	.070
	(.09)	(.14)
SDx	.625	.639
	(.04)	(.05)
CEy	-.029	-.003
	(.09)	(.10)
SDy	.494	.490
	(.08)	(.09)
Eff Target Area	4.148	4.165
(cm^2)	(.73)	(.53)

Table 5

Skewness and Kurtosis Values* in the

x- and y-Dimensions of Hurdle Experiment**

	Skewness		Kurtosis	
Dimension	Hurdle	No Hurdle	Hurdle	No Hurdle
x	.53	.38	.09	-1.35
y	.02	.05	-2.61	-2.86

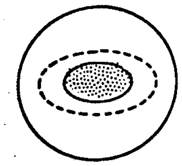
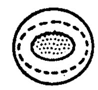
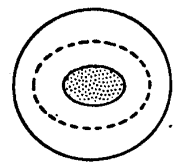
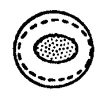
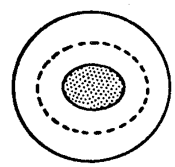
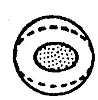
*For skewness, 0 = normal distribution, large positive value = positively skewed and large negative = negatively skewed. For kurtosis, 0 = no kurtosis, large positive = leptokurtosis and large negative = platykurtic.

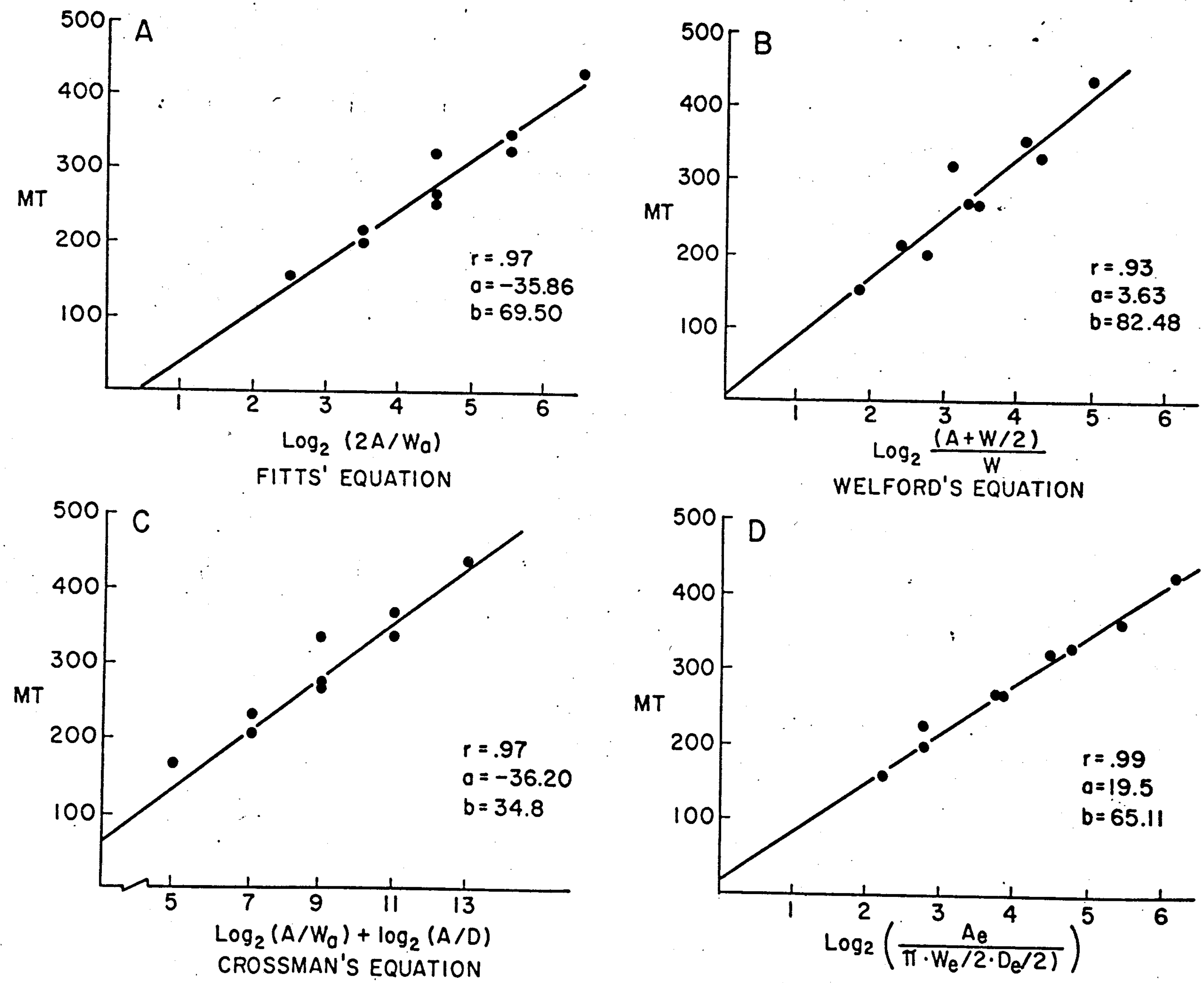
**Values collapsed over six subjects.

Figure Captions

Figure 1. A graphic representation of the effective target areas (dotted ellipses) and standard deviations (shaded ellipses) in the x- and y-dimensions for all nine amplitude-target size conditions. The constant errors in both x- and y-dimensions were essentially zero. Thus, the centers of the ellipses coincide with the centers of the actual circular targets. In every case, standard deviations were greater in the x-dimension. Also, particularly in the larger target conditions, the subjects' effective target areas were smaller than the actual target areas.

Figure 2. A comparison of the four equations used to predict MT. In *A* is Fitts' equation. In *B* is Welford's equation. In *C* is Crossman's equation. In *D* is our equation.

Fitts ID	Target Diameter (cm)	Amplitude (cm)	
2.58	2.54	7.62	
3.58	1.27	7.62	
3.58	2.54	15.24	
4.58	.64	7.62	
4.58	1.27	15.24	
4.58	2.54	30.48	
5.58	.64	15.24	
5.58	1.27	30.48	
6.58	.64	30.48	



A
MT
500
400
300
200
100
1 2 3 4 5 6
r = .97
a = −35.86
b = 69.50
Log2 (2A/Wa)
FITTS' EQUATION
B
MT
r = .93
a = 3.63
b = 82.48
Log2 (A+W/2)/W
WELFORD'S EQUATION
C
MT
5 7 9 11 13
r = .97
a = −36.20
b = 34.8
Log2 (A/Wa) + log2 (A/D)
CROSSMAN'S EQUATION
D
MT
r = .99
a = 19.5
b = 65.11
Log2 (Ae / (π · We/2 · De/2))