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Detecting Plans from Data

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ABSTRACT

Versions of the Missionaries-Cannibals puzzle having a boat capacity of four can be solved via a simple repetitive move sequence. Subjects solving these problems are usually able to infer the strategy, and appear to be planning a solution. Evidence for the existence of a plan was gathered from move patterns latency data, and post-experimental questionnaires.

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Previous work on simple sequential problems in our laboratory (Atwood and Polson, 1976; Jeffries, Polson, Razran, and Atwood, 1977; Polson, Jeffries, Atwood, and Hooke, 1976) has resulted in the construction of a series of models for the solution of these tasks. An important feature of these models has been that we assume that the selection of moves involves no planning; i.e., that the problem solver considers only the immediate successors of his current position in his attempts to select a move. The models do contain quite sophisticated heuristics to constrain the search for a move, but these strategies (i.e., use means-ends analysis, try to select new moves) do not lead to the planning of multi-move sequences.

Other work on problems of this type has concluded that people do plan move sequences (Greeno, 1974; Reed, Ernst, and Banerji, 1974; Thomas, 1974). None of these authors, however, have developed a model of what the move sequences are or how they are discovered by subjects. Our inability to find a satisfactory way for a person to determine a successful plan for solving these problems was the major factor in the assumption that people are not planning. We have since, however, explored some problems in which we think planning does occur. This paper reports on a study that examined behavior on such problems.

Plans can be developed in one of two ways -- top-down or bottom-up. A top-down plan starts from the statement of the problem and expands this into subgoals. The subgoals will either continue to be expanded to the move level, or, if the solver is not able to expand the problem that far, the lowest level subgoals may be solved using general problem solving strategies. Top-down plans are frequently constructed by experienced problem solvers in well structured domains, or by naive solvers in situations where the subgoals are "obvious". For example, Bhaskar and Simon (1977) have shown that a person experienced in thermodynamics develops top-down plans for solving such problems. Polson, Jeffries, Atwood and Hooke (1976) gave naive solvers a problem with two obvious subcomponents. They found that subjects were likely to divide the problem into subgoals if they did not perceive the two components as interacting.

A bottom-up plan, on the other hand, starts at the move level, building up to a structure that will solve the problem. For water jug problems, Atwood and Polson (1976) found that some solvers were able to infer a "dipper" plan for one of their problems. This involved repeatedly using the smallest jug as a dipper to remove water from the largest jug and add it to the middle jug. Their plan presumably did not involve the induction of any subgoals, but rather the discovery of a sequence of moves, which, if repeated would solve the problem. In a sense the discovery of a bottom-up plan is a sequence induction problem which is embedded as an incidental task in the task of solving the problem. Because of the difficulties of such tasks, we should expect bottom-up plans to be discovered only if there exists a solution with some fairly obvious low-level regularities.

The problems we used for examining planning behavior are variants of the Missionaries-Cannibals problem. In all these problems, a group of missionaries and cannibals must cross a river in a boat of limited capacity under the constraint that the cannibals cannot outnumber the missionaries on either bank. The problems used in this experiment involved either 4, 5, or 6 missionaries and a like number of cannibals, and a boat that holds only four passengers. The problem graphs of the legal moves of these problems are shown in Figures 1 to 3. We call these problems the (4,4), (5,4) and (6,4) problems, respectively. Notice how bushy the graphs are; the number of possible moves from each state is large, and from any given state about half of the moves lead closer to the goal. There is a plan that will solve any Missionaries-Cannibals problem with a boat capacity of four: take two missionary-cannibal pairs over, return with one missionary and one cannibal in the boat, repeat until the problem is solved. (We call this the "pairs" solution.) This path through the problem is marked in the figures with a bold line.

The three problems differ in two ways. One is the length of the solution path required to solve the problems. It takes a minimum of 5, 7, and 9 moves to solve the (4,4), (5,4) and (6,4) problem, respectively. The second is the likelihood of stumbling upon the pairs path. It is possible to solve the (4,4) problem without ever reaching a state that is on the pairs path (except the initial and final states, of course). To solve the (5,4) problem, one must eventually intercept the pairs path, but this can occur as late as two steps from the goal. For the (6,4) problem, one must intercept the pairs path no later than the mid-point of the solution. If experience with some segment of the pairs path is a prerequisiste for inferring the pairs plan, then the problems should differ in the ease with which people find such a solution.

There should be three ways we can determine if people are planning in this problem. The first is the actual move choices made. A person who is executing an already formed correct plan should do so without making illegal moves, erasures, or other mistakes. The second form of evidence would be reaction time measures. A plan should be executed comparatively quickly. If the solver discovers the plan after starting the problem, his latencies for moves after he begins executing the plan should be noticeably shorter than they were before the plan. Finally, we can ask a subject whether he had a plan for solving the problem, and what that plan was.

METHOD

The purpose of the experiment was to determine if subjects would solve the four-in-the-boat problems via the pairs solution and if it would be possible to determine whether they were planning. Subjects were given either the Hobbits and Orcs version (HO) of the (4,4), (5,4) or (6,4) Missionaries-Cannibals problem or the Silver and Gold version (SG) of the (6,4) problem. (See Appendix for the text of the HO and SG versions.) Participation in the experiment satisfied part of a course requirement. There were 120 subjects: 21 solved the (4,4) problem, 19 solved the (5,4) problem, and 80 solved the (6,4) problem - 40 attempting each version.

Procedure.

The experiment was run using the Sigma 3/IV:PHASE computing system of the Computer Laboratory for Instruction in Psychological Research. The subject was seated in front of a CRT display terminal upon which was presented the instructions for the problem and the problem itself. Responses were made by pressing one of five buttons on a box in front of the terminal. The computer recorded the subject's move choices and his move latencies, and informed him of illegal moves and other errors. After solving the problem, a subject filled out a questionaire which asked him to describe his strategies for solving the problem.

RESULTS AND DISCUSSION

The number of people who eventually found the pairs strategy varied with the problem. A person was counted as using the pairs strategy if he made a sequence of moves at least three moves long, ending at the goal state, in which he made the correct pairs moves without making any illegal or backwards moves. 52% of those solving the (4,4) problem, 68% of those solving the (5,4) problem, and 90% of those solving the (6,4) problem eventually discovered the pairs strategy. It is likely that two factors contributed to the difference among the groups. First, people were more likely to happen upon the pairs strategy in the problems with longer solution paths. The mean move number at which subjects started making paired moves was 7.5 for the (5,4) problem and 8.3 for the (6,4) problem. The overall number of legal moves to solve the (4,4) problem was only 7.1. Those who found the pairs rule did so in an average of 3.4 moves; those who might have discovered it later apparently solved the problem before they could infer the rule. Second, the structure of the (5,4) and (6,4) problems forces solvers to intercept the pairs solution path. A detailed analysis of the move protocols showed that most subjects began making pair moves on their first entry into a paired state (other than the start state). This most frequently occurred when the move dictated by either a means-ends or choose-new-move strategy resulted in a paired state. If a person made the pair move from this state, he tended to continue with the moves of the pair strategy.

The fact that people make moves that are consistent with a plan is not sufficient evidence that they have inferred t e plan itself. In these problems the moves described by the pairs strategy would be highly desirable even to subjects who had not discovered the plan. We considered a subject to be planning if he executed an errorless, short latency sequence of at least three moves that terminated at the goal state. With this definition we categorized 48% of the subjects solving the (4,4) problem, 68% of those solving the (5,4) problem, and 90% of those solving the (6,4) problem as planners. We also divided the planners into three separate categories: pre-planners, insight planners and gradual planners. Pre-planners are those who moved pairs from the very beginning and also made all moves (except possibly the first move) with very short latencies. We interpreted this pattern as indicating that these subjects discovered the plan either while reading the instructions or before completing the first move, and proceeded to execute a complete plan. A protocol from a pre-planner on the (6,4) problem is shown in Figure 4A. The plan discovered by pre-planners did not have to be the pairs plan; two subjects who solved the (4,4) problem without using a pairs strategy were classified as pre-planners. They both solved in the same way, but their solution does not fit any pattern that we could determine.

Insight planners were those who discovered the pairs strategy in mid-problem, and either at that time or a few moves later exhibited a long latency move, followed by a succession of short latency correct moves. Subjects who made pair moves from the beginning, but whose last long latency move was after the first move of the problem, were also classified as insight planners. Figure 4B shows a prototypical insight planner. Note the 55 sec latency at move 6. We suspect that at this point, the subject is either developing the pairs plan, or checking his newly discovered plan to determine if it has any flaws. Also note that at move 4 this subject intercepts the pairs path at a point where the optimal legal move according to a means-ends strategy is the pair move.

Subjects were classified as gradual planners if they discovered the pairs strategy in mid-problem, but showed no abrupt change in latency; i.e., all moves were of short latency. The protocol of a typical gradual planner is shown in Figure 4C. At move 3 he has intercepted the pairs path. He continues on this path, but no large reaction time differences are evident. We do not know if the people classified as gradual planners are actually a different species of planner, or if they are simply people with a style of making moves rapidly.

In contrast, Figure 4D shows the protocol of a person that was classified as a non-planner. This person was categorized as using a pairs strategy, but was not classified as a planner because of his long latencies up to move 7.

The classification of subjects according to the strategies they used, whether they were planning, and when they began their plan was done independently by two raters. Agreement between the raters was about 85%. Differences in classification were resolved by the two raters together, and, unless the difference was due to an error on the part of one rater, the subject was placed in the more conservative category.

The percentage of subjects solving each problem that were classified as pre-, insight, gradual or non-planners is shown in Table 1 along with the mean number of legal moves to solution in each category. The major difference among groups is in the number of gradual planners. The lack of gradual planners in the (4,4) problem is probably due to that problem's short solution path. Subjects were not classified as planners if the alleged plan was not at least three moves long. If we had included two move plans, there would have several gradual planners in that group. There is also a large difference in the number of gradual planners for the two groups who solved the (6,4) problem (38% for the SG version; 20% for the HO verson). The SG version is known to be more difficult to understand than the HO version, as evidenced by the fact that subjects make twice as many illegal moves on the SG verson of another Missionaries-Cannibals problem (Jeffries et al., 1977). This seems to have resulted in more gradual planners and fewer non-planners, but we can offer no reason for this occurence.

The legal move data show that planners generally solved the problem in fewer moves than non-planners. Pre-planners of course, must solve in the minimum number of moves, as that is a prerequsite for that classification, but no other clear cut differences emerge. In particular, there were no consistent differences between gradual and insight planners.

We also compared our classification of subjects to their responses to the questionaire. We asked subjects two questions. The first requested that they describe their strategy; we divided their responses into those who used a pairs strategy and others. The second question asked if they had discovered this strategy before solving the problem or during the solution. We classified these responses as either pre-planners or non-pre-planners.

We found good agreement between our classification of people as using the pairs strategy and their statements of this as a strategy. 91% of the subjects' avowed strategies agreed with our classification; for 5% we were unable to interpret their strategy description; and 4% claimed to be using trial and error when we classified them as using a pairs strategy. These were all insight planners, and it is possible that they interpreted the question as referring to the strategy t ey started the problem with. For the second question, 95% of our classifications agreed with subjects' own statements. All the errors were people who categorized themselves as pre-planners, but whom we called insight planners.

Intuitively, we feel the data described here are evidence that people do plan in solving four-in-the-boat problems. We realize, however, that we have in no way proved this. Additional evidence is needed to confirm our intuitions. A model similar to our previous models, with no planning assumptions, that cannot account for the moves made by subjects in this experients, or the patterns of latencies, would be strong evidence that some sort of planning must be occurring. This will not be a trivial task. The moves that describe the pairs strategy are highly desirable move choices for persons using other strategies. Where the model is likely to have trouble is in generating a long sequence of such moves; no matter how desirable a given move is, our models will not select it exclusively. Similarly, some simple assumptions to enable the model to make reaction time estimates may be able to replicate the patterns of latencies found in these protocols. A desirable move would be made with a short latency; what will be difficult for the model to predict is the long latency that precedes the short latencies in the case of insight planners.

We are currently in the process of designing a model for these problems that does not plan. As yet we do not know if this endeavor will be successful. In a previous experiment (Atwood and Polson, 1976), our model was unable to fit the protocols of subjects who appeared to be planning (those using the "dipper" solution). We sus ect that a similar result will ensue here.

People are able to discover a plan that will solve this problem, because there is an inherent structure to the problem graph that they can reasonable infer and exploit. In other problems, where we fail to obtain evidence of planning, the structure of the solution graph is not easily discernable from the problem description or from a small amount of experience with the problem. In these problems, we should expect that additional information would be required to enable a person to plan. Extensive experience with similar problems is one source of information that might lead to planning behavior in such problems. In a sense, it is only possible to generate a correct plan for solving any problem if one understands something about the structure of the solution to the problem. In the four-in-the-boat problems, it is necessary to infer that a repetitive sequence of moves will solve the problem. In other problems, the ability to separate the problem into individually solvable components may be what is needed to find a plan.

People develop plans as a way of reducing the amount of search needed to solve a problem. Trial and error search, even guided trial and error search, is a very inefficient way to solve any problem. A plan will greatly reduce the amount of search needed, but only if it does not take an inordinate amount of searching to discover the plan. In a problem that only has a few possible actions at each choice point, it may be more reasonable not to look for a plan, but simply to begin to solve move-by-move. We conjecture that our models for sequential problems represent the fall-back strategies solvers use when they have been unable to uncover a plan for the problem. In problems where is is possible to either generate a plan prior to solving or to discover a plan in mid-problem, we see qualitatively different behavior. The interesting, but unanswered, question is what kinds of information are people able to use to infer plans.

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APPENDIX
Texts of the Problems Given to Subjects

Hobbits and Orcs Version. Once upon a time, in the last days of Middle Earth, three hobbits and three orcs set out on a journey together. They were sent by the great wizard Gandalf to find one of the lost palantiri, or oracle stones.

In the course of their journey, they come to a river. On the bank is a small rowboat. All six travellers need to cross the river, but the boat will only hold two of them at a time.

The orcs are fierce and wicked creatures, who will try to kill the hobbits if they get the opportunity. The hobbits are normally gentle creatures, but are vey good fighters if provoked.

The orcs know this, and will not try to attack the hobbits unless the orcs outnumber the hobbits. That is, the hobbits will be safe as long as there are at least as many hobbits as orcs on either side of the river.

Silver and Gold Version. Once upon a time, in a far away place, there lived a monk who was the guardian of the temple of the three silver and three gold magic talismans. The monk has been ordered to deliver the talismans to the dedication of a new temple. He must cross the enchanted forest of Rangimali to get from one temple to the other.

The talismans must be carried through the forest in a special box. All six of the talismans need to be taken across the forest, but the box will hold only two of them at a time.

The three silver talismans are the sacred symbols of the goddess Silverina. The three gold talismans are those of the god Goldmund.

Silverina will protect any temple which holds one or mor of her talismans. She becomes very upset if, in any temple under her protection, there are more talismans dedicated to Goldmand than to her. If the monk should accidentally allow this to happen, Silverina will strike him dead. Goldmund, on the other hand, is an easygoing god, who doesn´t mind if his talismans are outnumbered.

To overcome the hazards of the forest, the monk must always carry at least one talisman with him whenever he crosses the forest. Either a silver or a gold talisman or both can protect the monk, but he cannot cross the forest alone.

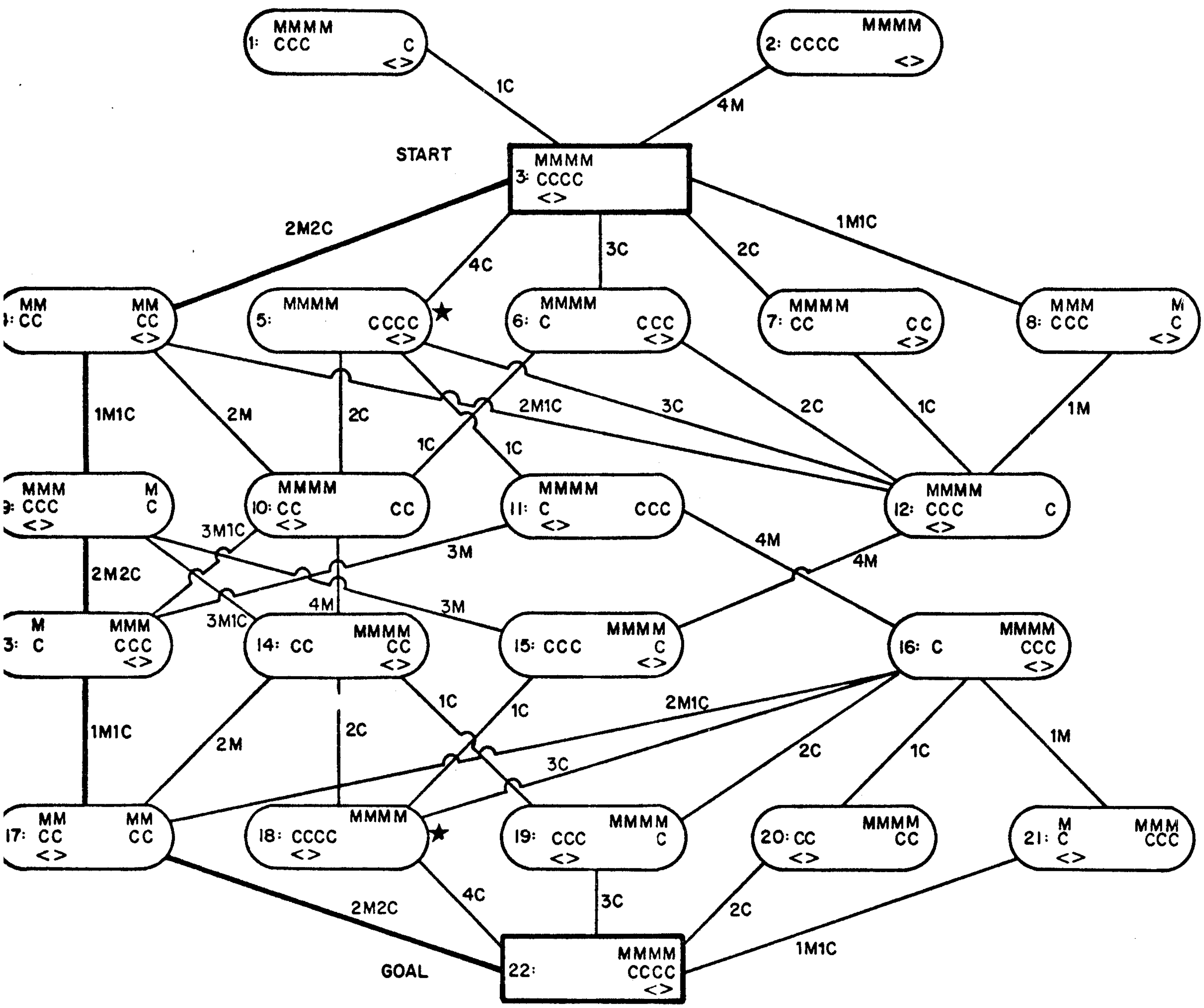

Figure 1. Graph of the legal moves for the (4,4) Missionaries-Cannibals problem.

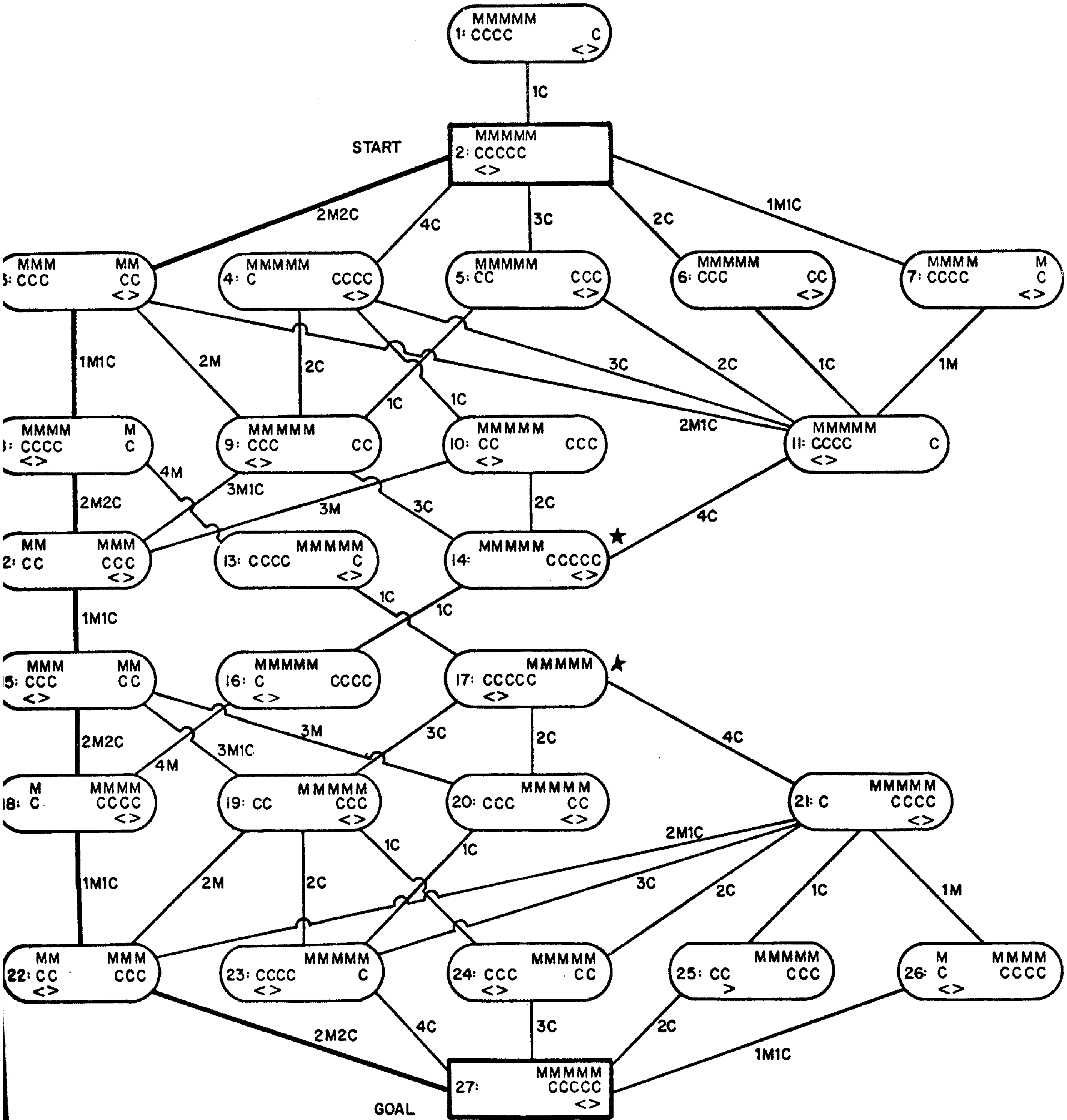

Figure 2. Graph of the legal moves for the (5,4) Missionaries-Cannibals problem.

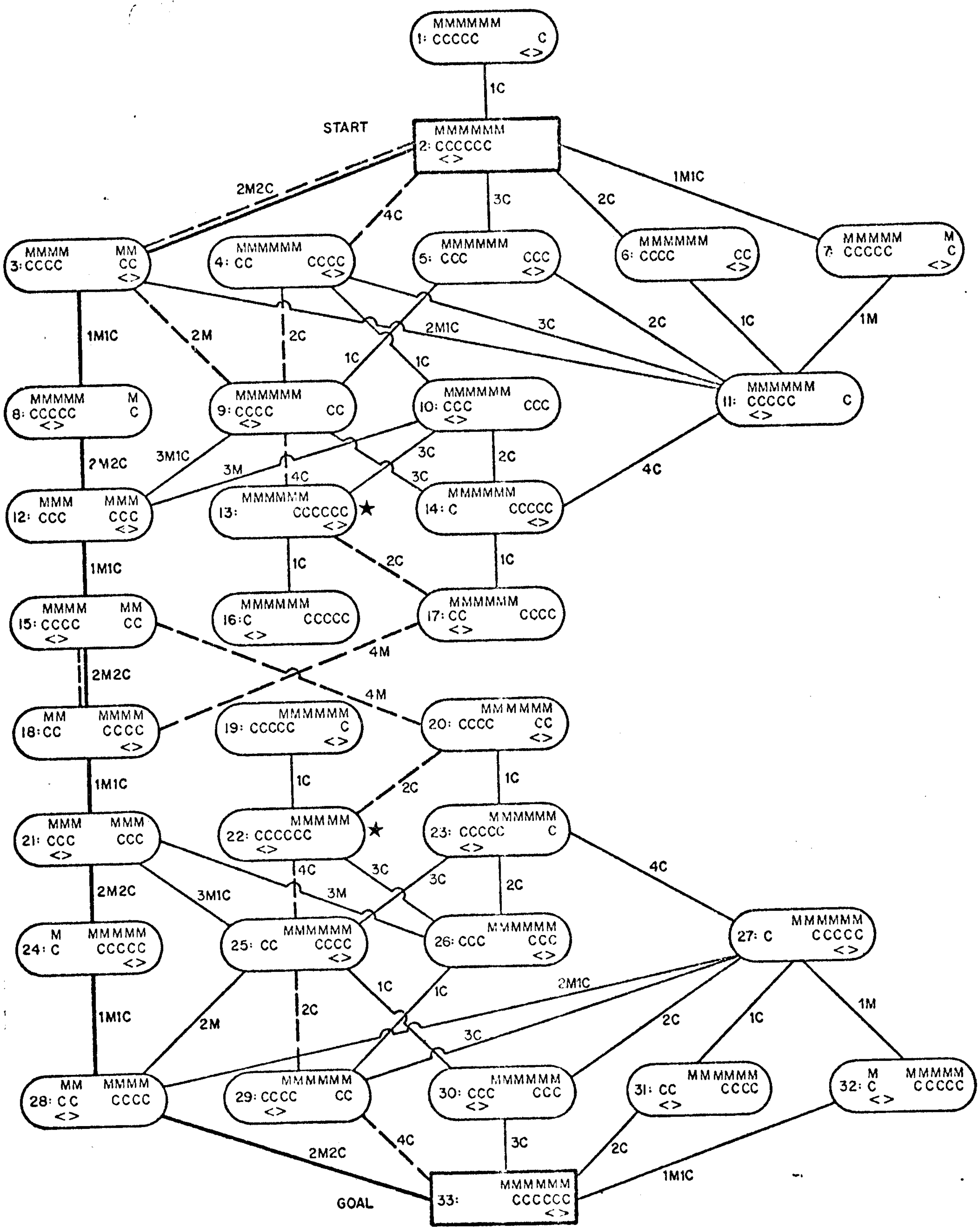

Figure 3. Graph of the legal moves for the (6,4) Missionaries-Cannibals problem.

A. Pre-planner (6,4)

Move No.	Resulting State	Travellers Moved	Latency (SEC)
1	3	M M C C	5
2	8	M C	3
3	12	M M C C	1
4	16	M C	2
5	18	M M C C	2
6	21	M C	1
7	24	M M C C	1
8	28	M C	1
9	33	M M C C	2

B. Insight planner (6,4)

Move No.	Resulting State	Travellers Moved	Latency (SEC)
1	4	C C C C	11
2	10	C	6
3	Erased	C	5
4	12	M M M	1
5	Erased	C	2
6	15	M C	56
7	18	M M C C	2
8	21	M C	4
9	24	M M C C	4
10	28	M C	1
11	33	M M C C	1

C. Gradual planner (6,4)

Move No.	Resulting State	Travellers Moved	Latency (SEC)
1	4	C C C C	20
2	9	C C	10
3	3	M M	10
4	8	M C	8
5	12	C C M M	10
6	15	M C	7
7	18	M M C C	5
8	21	M C	4
9	24	M M C C	5
10	28	M C	3
11	33	M M C C	3

Figure 4. Protocols of typical examples of pre, insight, gradual, and non-planners.

D. Non-Planner (4 ,4)

Move No.	Resulting State	Travellers State					Latency (SEC)
1	Error	M	M	M	C	C	11
2	Illegal	M	M	C			3
3	8	M	C				13
4	12	M					5
5	4	M	M	C			28
6	9	M	C				11
7	13	M	M	C	C		30
8	17	M	C				3
9	22	M	M	C	C		3

Figure 4. Protocols of typical examples of pre, insight, gradual, and non-planners. (cont.)

TABLE 1

Percentages of subjects classified as planners and legal moves for each group.

	PROBLEM			
	(4,4)-HO	(5,4)-HO	(6,4)-HO	(6,4)-SG
Percent classified as:				
Pre-planners	14%	11%	10%	17%
Insight planners:	38%	42%	53%	43%
Gradual planners:	0%	16%	20%	38%
Non-planners:	48%	32%	18%	2%
Legal Moves:				
Pre-planners:	5.0	7.0	9.0	9.0
Insight planners:	6.3	9.0	11.5	11.2
Gradual planners:	---	15.6	12.8	12.3
Non-planners:	8.4	13.3	26.6	15.0