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In this paper, we would like to describe our preliminary attempts to understand how individuals set up and manipulate subgoals. First, we want to summarize our previous work which provides a context for this research. We have developed models for a variant of Luchin's (1942) water jug task (Atwood and Polson, 1976) and the Missionaries-Cannibals river crossing problem (Jeffries, Atwood, Polson, and Hooke, 1976). These models are special cases of a General Problem Solver (GPS) like theory of problem solving which assumes that subjects use a means-ends process when evaluating possible moves, that they tend to reject moves that lead to previously occupied problem states, and that subjects do not attempt to plan sequences of moves. The theory asserts that subjects do not set up and manipulate subgoals or attempt to partition a problem into a collection of subproblems. In other words, we assume that subjects use only local information in the process of solving a problem.

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Obviously such a conclusion is not true in general. Studies of complex tasks such as the solution of logic or integration problems have all concluded that subjects reduce such tasks to collections of subproblems (Newell and Simon, 1961). Subjects use global information to define the subproblems in addition to using local information to solve the subproblems.

We conjecture that subjects set up and manipulate subgoals in tasks whose global structure has been learned from repeated experience or in novel problems which are perceived as being composed of recognizable subproblems. Observe that for a naive subject, there are no obvious subgoals in a river crossing problem like Missionaries-Cannibals or in a water jug problem. Recall that GPS has a great deal of difficulty with such problems because it incorrectly assumes that it can set up relevant subgoals.

In light of our previous successful model building efforts and the immediately preceding discussion, we were led to develop a series of problems with the following characteristics. First, the problem must have two obvious subproblems. Second, one or both of the subproblems should already be well understood. Third, the new, combined problem must be simple enough that it can be solved using only local information. We started with Hobbits-Orcs, Greeno's version of Missionaries-Cannibals (Greeno, 1973). The second problem is trivial: subjects have to transfer three bags of gold, one at a time, across the river as well as the three Hobbits and three Orcs. The boat, which can hold two travellers, has a strongbox which can contain one bag of gold. As usual, if Orcs outnumber Hobbits on either side of the river, the Orcs will eat the Hobbits. In addition, if all three Orcs and all three bags of gold are left alone on either side of the river, the Orcs will steal the gold. The graph of Hobbits-Orcs and Gold is shown on the first page of the handout; the graph of Hobbits-Orcs is shown on page two of the handout. The instructions and designs for the two experiments we will discuss are presented on page three of the handout.

Our initial conjecture was that there were three possible global strategies for solving Hobbits-Orcs and Gold: ferry the gold over first (across the top of Figure 1 then down the right hand side), get the travellers across first (down the left side and across the bottom), or solve both subproblems concurrently (move diagonally across Figure 1). The diagonal path is the minimal length solution path for Hobbits-Orcs and Gold, eleven moves. Note that the minimal length path for Hobbits-Orcs is also eleven moves. Either of the two subgoaling strategies, gold first or travellers first, defines a longer solution path; what is more, the subject using these strategies will also be forced to detour around an illegal state, i.e., three bags of gold and three Orcs alone on one side of the river.

In Experiment I, we attempted to manipulate subjects' choices of subgoal or overall strategy by varying the task instructions. Subjects in the No Robbers (Control) group were given the Basic Problem plus Addition 1 shown at the top of page 3. The instructions given to the second group, Robbers on Left, were intended to increase the probability that subjects would initially decide to immediately ferry all the gold across the river. The Robbers on Right group's instructions were designed to lead subjects to ferry all the travellers across the river.

Now let us look at the performance of the 75 subjects in the No Robbers (Control) condition of Experiment I. The mean number of legal moves to solve the basic version of Hobbits-Orcs and Gold is 28.7. The corresponding figure for Hobbits-Orcs is 18.6 moves. Recall that both problems have the same length minimal solution path. Thus, the addition of the trivial gold subproblem leads to a dramatic increase in difficulty.

Next, let us consider the kinds of strategies that subjects use to solve the combined problem. The means of the number of visits to each state are shown on page 4 of the handout. These means are laid out in the same order as the graph of the problem shown on page 1. The pattern of state entries suggests that subjects either move across the top and then down the right side (solve gold problem first), or move along the diagonal path (solve both problems concurrently). A more detailed examination of complete protocols supports this impression. In fact, out of the over 250 subjects that we have run on various versions of Hobbits-Orcs and Gold, only three have solved Hobbits-Orcs before they transported all the gold across the river.

We tentatively concluded that subjects use one of two possible strategies in solving Hobbits-Orcs and Gold: the gold first strategy or the combined strategy. We classified a subject as using the gold first strategy if his first solution to the gold subproblem was on entry into states 13 through 43. A subject was labelled a combined solver if he solved the gold subproblem for the first time by entering states 83 through 153. We then calculated various performance statistics conditional on solution type. The results shown in the upper half of page 5 of the handout were the basis for the conclusions given in our abstract: 1) that there are large differences in performance as a function of overall strategy, 2) that it was difficult to manipulate choice of overall strategy, and 3) that there is a strong tendency to solve the gold subproblem first. The only significant effects in Experiment I were the differences due to overall strategy. The percentages of subjects who are classified as gold first did not differ as a function of instructions. The mean number of legal moves to solve the gold subproblem for the gold first subjects seems to vary in the expected manner as a function of instructions. However, these means are not significantly different. Other measures, total trials, i.e. legal moves plus errors to solve the gold subproblem, do vary significantly as a function of instructions.

The results of Experiment II shown on page 5 of the handout suggest that the Orcs stealing gold may have been a primary determinant of choice of strategy. Experiment II compared performance of subjects who received the standard problem (Interaction, No Robbers) with a version of the problem in which the Orcs don't steal gold (No Interaction). The No Interaction version is solved significantly faster, and there are smaller differences in performance between the two types of subjects for this condition. The No Interaction instructions lead to a significantly higher proportion of the subjects choosing the gold first strategy. These results show that subjects do modify their strategies as a function of the nature of the interactions between the components of the two subproblems, and that when there are no constraints on which subproblem to solve first, they choose the easier problem.

As we began to examine our data in more detail, we became interested in trying to pinpoint how the addition of the gold subtask increased the difficulty of the Hobbits-Orcs problem. This led us to consider how subjects did on the Hobbits-Orcs subproblem of the Hobbits-Orcs and Gold task. Page 6 of the handout shows visits to states collapsed over gold for the 75 subjects in the No Robbers condition of Experiment I, as juxtaposed to visits to states in Hobbits-Orcs. Each number in the right column on page 6 represents the mean number of visits to four states, each of which has the identical configuration of Hobbits and Orcs (i.e. represents the same state on the solution path of Hobbits-Orcs) and which differ only in the number of sacks of gold which have been transported to the right side of the river. Thus, the number .40 at the top of the column on page 6 represents the sum of mean visits to states 10, 11, 12 and 13. It will be noted that with two trivial exceptions the numbers in the right column are higher, and frequently quite a lot higher, than those in the left column. When the relative differences between the two columns are examined closely, it becomes clear that the most pronounced effect of adding the gold subproblem is an increase in the number of visits to early states of the Hobbits-Orcs subproblem, i.e. an increase in the difficulty of the transition between state 5 and state 6. This result is in no way surprising since this particular transition has been found to be the most difficult in virtually every one of the various versions of this problem -- what adding the gold has done is to make the most difficult transition even more difficult relative to the other transitions, as well as in absolute terms.

Unfortunately, while this result was very clear in some respects, it served to call into question our interpretation of subjects' performance as being based on their choice of strategy. It should be remembered that our sole criterion for classifying solvers as gold first or combined solvers was whether they had solved the gold subproblem before or after they had made the transition from state 5 to state 6 in the Hobbits-Orcs subproblem. Now our initial conclusion had been that large differences in performance could be attributed to choice of overall strategy as identified by our criterion.

We began to wonder if our criterion differentiated, not between subpopulations with different overall strategies, but between those who had a greater or lesser degree of difficulty making the transition from state 5 to state 6. Those whom we have been calling "gold first" may have been trying to work on both subproblems concurrently, but, due to their difficulty with state 5, solved the gold subproblem before they found the correct move, i.e., into state 6. If our "gold first" subjects were actually just those who had more difficulty with state 5; then it is not at all surprising that this group turned out to be poorer by other performance indicators as well. It has therefore become of prime importance to us to verify our criterion for attributing differential strategies to subjects in this problem.

Obviously, the soundness of our conclusions depends on the validity of our claim that subjects are using one of two different solution processes or subgoal orderings. We realize that it has become fashionable to partition subjects on the basis of various post hoc criteria and argue that large differences in overall performance reflect differences in underlying strategies. We are not immediately concerned with the possible misclassifications of 10 to 20% of our subjects because of faults in the particular criterion reported here. We want some type of validation of our initial conjecture that the very large differences in performance between subjects we classified as gold first and combined in fact reflect differences in underlying strategies.

What we decided to do was to attempt to evaluate a null like hypothesis. We extended our theory for water jugs and Hobbits-Orcs to the Hobbits-Orcs and Gold problem. The new model was the same version of the theory as we developed for Hobbits-Orcs. The following remarks are a preliminary discussion of our ongoing efforts to simulate this data.

We were stunned to discover that we can easily replicate our results for mean legal moves to solution. We took the output of the simulation program and partitioned the simulated data using the same criterion we used to identify gold first and combined subjects. The simulated means for each group were close to the observed means for the same groups in the No Interaction condition of Experiment II. Recall that our theory uses only local information to solve problems, i.e. it does not set up and manipulate subgoals. There are other aspects of the simulated data that are not consistent with our experimental results. However, a simulation model that assumes no individual differences and has no planning mechanisms can reproduce the basic qualitative finding that initially led us to claim the existence of subgoaling processes.

We are pleased with our general approach: the use of problems like Hobbits-Orcs and Gold, the comparison of performance with a model with no subgoaling processes, and attempting to show that the deviations between simulation's and subjects' performance are consistent with subgoaling mechanisms. We are convinced that all of these steps are necessary. The uncritical evaluation of seeming differences in pattern of moves is clearly not an adequate basis for the inference of differences in solution strategies or subgoaling.

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