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When Content Knowledge & Pedagogical Content Knowledge Collide

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Abstract

Expertise in a content area is immensely valuable for effective teaching. However, domain expertise can also lead educators to organize their pedagogy in accordance with the structure of the domain rather than the learning needs of novices. We term this phenomenon *expert blind spot* and use historical, psychological, and textual evidence in the area of mathematics education to explore its effect on curriculum and pedagogy. The existence of expert blind spot in education poses a serious dilemma for pre-service and in-service teacher education programs, and for educational researchers involved in curriculum development and teaching reform.

Expert Blind Spot: When Content Knowledge and Pedagogical Content Knowledge Collide

The importance of content knowledge for proficiency in teaching practices is well documented. Research suggests that more content knowledge is always better (Borko et al., 1992; Grossman, Wilson, & Shulman, 1989; Shulman, 1986-a, 1986-b). But is this statement completely unimpeachable? Are there drawbacks for teaching that are due to expertise in content areas? In this paper we draw on historical events, analyses of a corpus of textbooks, and empirical studies of teacher cognition to show evidence for *expert blind spot.* We define expert blind spot as the inability to perceive the difficulties that novices will experience as they approach a new domain of knowledge. In education it is manifest as the tendency for content area experts to perceive the organization of the domain of study as the central structure for organizing students' learning experiences, rather than basing instruction on students' actual developmental processes. The existence of expert blind spot contributes to the distinction made between content knowledge (e.g., knowing how to do mathematics) and pedagogical content knowledge (e.g., knowing how to present mathematics to support novice learning; Shulman, 1986-a). In this article, we do not contend that content knowledge is bad for teaching – it is clearly crucial. Rather, we raise the issue that advanced content knowledge without concomitant advancements in the knowledge for how novices actually learn within a content area can lead toward views of instruction that align more closely with the organization favored by the domain experts than the learning needs of students.

In the following sections, we first briefly discuss the nature of content area expertise and its desirable and undesirable traits. We next review prior research on the roles of content knowledge and pedagogical content knowledge in expert and novice teaching practices. Then, we present evidence for the existence of an expert blind spot in mathematics education, and discuss its impact on the structuring of students' learning experiences. The existence of expert blind spot poses a serious dilemma for pre-service and in-service teacher education programs, and we offer several recommendations to address it. Finally, we raise questions of the views about learning and teaching held by members of the educational research community, and acknowledge that these views, shaped by our own expert blind spots, may hinder progress in research and educational reform if they are allowed to go uninvestigated.

The Nature of Expertise

Before the launching of the cognitive science research program in the 1950's, experts were considered to be a different breed from others. They were regarded as more intelligent than average people, and as having greater memory capacity and superior intellectual resources (Bruer, 1993). The inferences seemed plausible: Experts in a wide range of fields such as musical and athletic performance, strategic games, and medical practice, reasoned more accurately and more quickly, multi-tasked better, and assimilated far more information than non-experts (Ericsson & Smith, 1991).

However, careful research into the reasoning processes of experts has shown that they function with the same internal constraints as non-experts (Ericsson & Smith, 1991; Frensch & Buchner, 1999). Expert performance has been shown to be due to vast amounts of well-organized, domain-specific knowledge; intense, long-term practice within a narrow field; psychological and physiological adaptations; and the exploitation of regularities of familiar tasks (Bereiter, 1993; Ericsson & Lehmann 1996; Glaser, 1990). For example, in a year-long study, one subject, SF, was able to perform expert memory feats and reliably recall strings of numbers presented one every 2 seconds at ten times the span of most people. Yet his capacity returned to the typical 7 items when letters were used in place of digits (Ericsson, Chase, & Faloon, 1980). Even so, the demystification of expertise does not undermine its significance or its allure for education, and many prominent researchers argue that our educational efforts should be guided by expert achievements (Brown & Campione, 1994; Glaser, 1976; Hatano & Inagaki, 1986; Soloman, 1993; Sternberg, 1996).

Expertise is not without its shortcomings, however. It has been shown that subjects with a large amount of domain knowledge may actually be at a disadvantage when compared to novices on certain tasks such as forming remote associations among disparate concepts. Wiley (1998) argued that this is because experts' knowledge and experiences tend to confine their efforts to highly probable events, and things like disparately related concepts may elude them. In other words, expert domain knowledge can act as a mental set, fixating experts on unproductive solution paths during creative problem-solving tasks while novices may behave more flexibly.

Verbal "think aloud" reports also show that experts are less likely than novices to have access to memory traces of their cognitive processes when engaged in tasks within their domain of expertise, because these highly practiced cognitive and perceptual processes have become automatized (Ericsson & Simon, 1984; Schneider, Dumais, & Shiffrin, 1984). This means there is nothing in memory for experts to "replay," verbalize, and reflect upon. Among novices, these processes tend to be deliberate and stepwise, and so they leave a memory trace that is inspectable and verbalizable.

One particularly striking example of this phenomenon comes from a case study of an expert in literature training to be a secondary school reading teacher (Holt-Reynolds, 1999). The subject matter expertise of this teacher was well established in the study: She was a committed, life-long reader with straight A's in her college literature classes, and was valedictorian of a large high school. In interviews, she demonstrated sophisticated literary analyses for a range of texts (using such techniques as inter-textual references, and parallel analyses of the writings and the life and times of the author), and had a strong preference for poetry. Yet this woman's expertise did not translate into an understanding of how to model or instruct others in the reading process. Her own reading and analytic processes were so well developed and automated that they left no memory trace to reflect upon. She had no awareness of her own reading process – she did not even see reading as something that she once had learned – and she was unable to transform her own disciplinary knowledge into a form that novice learners could use and apply. As Holt-Reynolds (1999) described, this pre-service teacher apparently imagined all students to be "replications of herself" (p. 41); she simply could not imagine some one not knowing how to read and needing to be taught. This expert blind spot interfered greatly with the teacher's professional development, and contributed to a rather lackluster style of teaching, as evident from follow-up observations of her own classroom.

In summary, research on expertise shows that domain experts develop limited but powerful cognitive structures that allow them to approach familiar tasks deeply and efficiently. This is an admirable achievement, and we do not want to dismiss the importance of such an accomplishment. However, the development of domain expertise leaves people largely unaware of the workings of their own expert behavior and the processes and learning experiences that led to its development.

Content Knowledge and Pedagogical Content Knowledge in Teaching

Expertise in teaching is a complex phenomenon that appears to substantiate many of the general claims about experts described above. Expert teachers differ from novices along several dimensions: They notice different things about the classroom environment, do more planning and plan differently than novices, and organize their knowledge of content, students, and pedagogy in ways that readily facilitate lesson planning and teaching (Borko & Livingston, 1989). Expert teaching practices also seem to be more schema based, tending toward "routinization and consistency" (Leinhardt, 1988, p. 147) to provide high quality instruction in an efficient manner. Even so, characteristics associated with expert teaching behaviors, as with expertise in general, have been shown to be quite fragile, and generally limited to familiar and well-practiced teaching situations (Borko & Livingston, 1989; Rich, 1993).

Content knowledge

Expert teaching behavior is highly dependent upon efficient access to vast, well-managed knowledge structures concerning, among other things, pedagogy and subject matter. For example, among Shulman's (1987-b) case studies from the "Knowledge growth in teaching project," Grossman described the practices of a beginning English teacher, Colleen, whose knowledge of literature was far better developed than her knowledge of grammar.

> In teaching literature, she conducted open-ended discussions, welcoming student questions and alternative interpretations of the text. When teaching a grammar lesson, Colleen looked like a very different teacher. She raced through a homework check at the speed of light, avoiding eye contact, and later admitted that she didn't want to give students the chance to ask questions she couldn't answer. She later explained that in grammar, unlike literature, she wasn't interested in student opinion because the students were usually wrong. (Shulman, 1987-b, p. 15).

Others have also shown the strong tie between content knowledge and teaching practices. In their study of a veteran fifth grade teacher, Stein, Baxter, and Leinhardt (1990) documented how gaps in the teacher's knowledge about graphing and mathematical functions translated into lessons that missed valuable opportunities to form conceptual connections, tended toward mathematical rules that had limited generalizability, and provided a weak foundation for future conceptual development. Lee (1995) found that areas of science instruction that were relatively unfamiliar resulted in one elementary school teacher's excessive use of individual activities (e.g., seatwork) as opposed to group activities that might have led to discussions that strayed outside the teacher's comfort zone. Lee also observed that the teacher relied more heavily on the textbook to guide her curricular choices in areas outside her expertise. This pattern was less evident in lessons on social studies, in which the teacher held a bachelors degree.

Pedagogical content knowledge

While the importance of content knowledge for teaching has long been known, only recently has the educational community become concerned with the knowledge teachers possess for how to teach that content to novices. Shulman introduced the term "pedagogical content knowledge" (PCK) to describe the "blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented and adapted to the diverse interests and abilities of learners for instruction" (Shulman, 1987-a, p. 8). As the name implies, PCK is highly related to content knowledge. For example, physical education teachers with greater content knowledge tend to be more comfortable in their teaching roles, and plan and execute lessons that are more accommodating to students with a wide range of abilities (Schempp, Manross, Tan, and Fincher, 1998). Science teachers with higher levels of chemistry knowledge also make better use of representations that are helpful for concept learning, and are more sensitive to the learning obstacles that may arise for students (Clermont, Borko, and Krajcik, 1994). Consequently, the expert chemistry teachers studied showed greater flexibility than did non-experts in simplifying their science demonstrations to facilitate conceptual development in their students.

However, it is not the case that educators who have an advanced and well-developed body of content knowledge *necessarily* have well-developed PCK. As suggested in one study of science teaching, content knowledge seems to be a prerequisite for well-developed PCK, but it appears that PCK develops out of classroom teaching experiences as well as subject area knowledge (van Driel, Verloop, and de Voss, 1998; see also Rich, 1993). This notion is underscored by a study in which teachers of varying levels of experience openly analyzed a video of classroom events (Copeland, Birmingham, DeMeulle, DeMidiocaston, & Natal, 1994). Regardless of their knowledge of the content, non-educators analyzed the instruction in relatively simplistic ways, describing mostly surface-level characteristics of teaching behaviors. In contrast, educators with varying degrees of actual experience tended to focus on the central purposes of the instruction and the connections between the teacher's actions and goals and student responses. The most highly experienced and most decorated teachers gave rich accounts and also offered suggestions for the videotaped teacher for how to improve student involvement and learning.

Expert Blind Spot in Mathematics Education

Pedagogical content knowledge has indeed been established as a principal component for effective teaching. But if pedagogical content knowledge is the confluence of knowledge of content and pedagogy, then expert blind spot is where these bodies of knowledge collide. In this collision, domain-centered expert knowledge dictates pedagogical decisions even though it may conflict with the needs of learners.

We consider three arenas in which people with advanced content knowledge in mathematics tend to make assumptions about student learning that are in conflict with students' actual performance and developmental trajectories. In this way, we highlight particular instances of "Expert Blind Spot" in mathematics instruction. Later, we examine its potential impact on teaching practices, and on research on mathematics and teacher education.

The first example takes an historical perspective and looks at the so-called "New Math" movement of the 1950's in the USA. The second example looks at teachers' and researchers' intuitions regarding students' mathematical development in the domain of early algebra. The third example examines the organizational structure of algebra textbooks. Each example shows how advanced knowledge of mathematical content leads to an expert blind spot for mathematical instruction, whereby knowledgeable mathematicians and educators believe that, like themselves, mathematics learners will find symbolic formalisms of quantitative relations and mathematical concepts most accessible because of their relative parsimony – regardless of the empirical evidence. This view of mathematics learning and development leads these experts to make non-optimal decisions about curriculum and instruction.

New Math

The first example of expert blind spot we consider occurred half a century ago. In the 1950's, the state of mathematics achievement, interest, and instruction in the United States was scrutinized by the National Council of Teachers of Mathematics (NCTM) in its Second Report of the Commission on Post War Plans (NCTM, 1945/1970-b), as well as by the University of Illinois Committee on School Mathematics (UICSM), and the College Entrance Examination Board (CEEB) Commission on Mathematics. The declining enrollment and interest toward mathematics education that began prior to WWII was still continuing, despite the growing importance and marketability of a technical education. By the mid-1950's, the popular press of the time, along with the university mathematicians, declared that the content of K-14 mathematics education had been determined by professional educators for too long, with insufficient progress. To turn this tide, academicians turned their attention to school curricula (NCTM, 1970-a). The fix, they reasoned, was to base mathematics education on the same foundational concepts that were being used to organize the domain of mathematics for university study – set theory and number theory. In 1958 the NCTM, MAA, and AMS empowered the School Mathematics Study Group (SMSG) to produce curricula based on this new conceptual structure. Led by mathematicians, the SMSG is often regarded today as the face of the "New Math" movement. Thr group was very productive in generating curricular outlines and guidelines, and in producing surveys, evaluations, sample textbooks, and enrichment materials that served as a guide for commercial textbooks for many years to follow.

Critics of the New Math curriculum, such as Morris Kline of NYU, and others (e.g., NCTM, 1970-a), argued that New Math pedagogy was poor and often absent; that the curriculum did not motivate students; that it neglected areas of application; that the curricula did not promote active participation by students; and that it failed to develop students' intuitive notions of mathematics necessary to form the mathematical generalizations of concern. Kline criticized what he saw as an over-emphasis on the formal structure and notation of set theory. He also criticized the lack of staff development for teachers, noting that teachers needed to be better informed about the curricular structure and goals.

The New Math program failed, not simply because it offered a poor curriculum, but, as Roberta Flexer, a professor of mathematics education states, because "the mathematicians [who organized SMSG] didn't know a lot about kids or teachers" (personal communication, Oct. 24, 1998). The mathematical content that formed the basis of New Math had been designed by mathematicians to highlight the organization of the domain of mathematics, with little regard to how that domain was to be learned by children, understood by teachers, or taught in classrooms (Loveless, 1997). The mathematical experts who led the New Math movement believed that, by revealing the logical foundations of mathematical structure to the learner, children's understanding would naturally follow Unfortunately, their expertise in mathematics made them blind to the struggles experienced by non-expert teachers and students.

Views of algebra development among teachers and researchers

As a second example of expert blind spot, we consider contemporary educators' views of algebra development. In a recent study (Removed-For-Review-1), investigators compared algebra students' problem-solving performance to teachers' expectations about problem difficulty. Elementary, middle, and high school teachers (n = 105) ranked a set of problems from easiest for their students to solve, to most difficult. The high school teachers in the sample all had college level mathematics degrees or the equivalent. Table 1 shows how the problems given in the ranking task can be organized into six categories. The rows show problems that are either arithmetic (with the result as the unknown) or algebraic (with a starting quantity as the unknown). The columns show the same underlying mathematical relations in one of three forms: a contextualized verbal story problem, a non-contextualized verbal word equation, and a symbolic equation.

Recent research on the problem-solving performance of ninth grade students in two samples ($n_1 = 76$, $n_2 = 171$; Removed-For-Review-2) who had completed a year of formal algebra instruction showed that students generally found symbolically presented arithmetic and algebra problems to be *harder* than verbally presented problems. These students correctly solved fewer than 30% of the symbolic equations, compared to 50% of the verbal problems, leading to a significant advantage for verbal problems in both samples. Students who could solve verbal problems could not necessarily solve matched symbolic problems, but students who accurately solved symbolic problems were very likely to solve the matched verbal problems. This led the investigators to suggest that algebra students may follow a *verbal precedence* model of mathematical development, whereby verbally based reasoning about quantitative relations precedes symbolic reasoning (Removed-For-Review-3; Removed-For-Review-1). The results are also consistent with findings by Case and his colleagues in early number development (Case, 1991; Case & Okamoto, 1996), rational number processing (Moss & Case, 1999), and reasoning about functions (Kalchman, 1998; Kalchman & Case, 1998).

Analyses of students' problem-solving strategies and errors (Removed-For-Review-4) revealed that students generally tried to solve symbolic problems using symbolic strategies (e.g., symbol manipulations), which were fraught with errors and misconceptions and were often abandoned. Verbal problems both with and without a supportive context (i.e., both word equations and story problems in Table 1) tended to elicit more reliable informal strategies, such as guess-and-test and working-backwards. (For a detailed account of students' problem-solving strategies, see Removed-For-Review-4).

When teachers were asked to judge the relative difficulty of the problems, their pattern of responses was clear. First, teachers generally considered arithmetic (result-unknown) problems to be easier for students to solve than algebra (start-unknown) problems, and this pattern was borne out in the student data. Second, among high school teachers ($n = 39$), verbal problems (word equations and story problems like P1, P2, P4, and P5 of Table 1) were considered to be more difficult for students than symbol problems (P3 and P6). In fact, high school teachers considered algebra word and story problems to be most difficult for students. This directly contradicts the student data described above. In their written comments and later interviews, high school teachers reasoned that symbolic problems should be easiest for students because they were written in "pure math," while verbal problems needed to be translated to equations before being solved, and this required understanding the language on top of the mathematics. A representative comment from one high school teacher defending her ranking order captures this well:

Students are used to expressions written algebraically [such as P3 in Table 1] and have typically had the most practice with these. Translating straight from mathematical words [as in the word-equation problems, P2 and P5] to mathematical expressions usually isn't too difficult. Translating "English" or "non-mathematical" words is a difficult task for many students.

The difficulty ranking data were obtained concurrently with data on teachers' beliefs. These data showed that high school teachers tended toward a *symbol precedence* account of mathematical development, held symbolic forms of reasoning in the highest regard, and held the efficacy of students' informal solution methods in relatively low regard. This seemingly traditional view of mathematics instruction was accompanied by a relatively reform-minded view of student learning. Teachers across the grade levels tended to agree with statements like "Mathematical understanding is more clearly shown in a student's reasoning than in the final answer a student produces." This suggests that expert blind spot is highly robust, and can persist even in the face of conflicting views such as those advanced in current educational reforms (e. g., NCTM, 2000).

In contrast with the views of high school teachers, middle school teachers (n = 30) with less post-secondary mathematics education predicted that students would find story and word problems to be easiest. The belief instruments showed that these teachers held students' intuitions in higher regard, and they believed that students were more likely to invent effective problem-solving methods that were not symbol based (Removed-For-Review-1). In addition, middle school teachers were far more accurate than high school teachers at predicting the relative order of students' problem-solving performance on the six problem types (Kendall's rank statistic $\tau(6) = .733, p = .034$). Elementary teachers' predictions were marginally predictive ($\tau(6) = .67, p = .06$). Surprisingly, the ranking provided by high school teachers was not significantly related to student performance at all, despite their more extensive mathematics education, providing further support for expert blind spot hypothesis that greater mathematics knowledge may move these teachers further away from the learning experiences of novices.

To test the scope of this bias toward symbolic precedence, researchers also studied the decision making of a small group of mathematics education researchers (n = 35) who focused on algebra reasoning and instruction (Removed-For-Review-4). Like the teachers, the majority of the respondents (about 66%) accurately predicted that algebra (start-unknown) problems would be consistently harder for students to solve than arithmetic (result-unknown) problems. As a group, researchers were more likely than teachers to judge equations as difficult. But, like the teachers, the researchers still expected equations to be easier for beginning algebra students to solve than verbal problems. Only 23% of the researchers ranked equations as harder than word and story problems.

The findings on teachers indicated that they generally held contemporary views of pedagogy, student learning, and mathematics activities. Yet, mathematically knowledgeable teachers at the high school level did not seem to be guided by these views when they judged how students would perform on a specific set of arithmetic and algebra problems. Their views of student development, along with those of educational researchers, seemed to be guided instead by notions of the centrality of symbolic formalisms and procedures to the field of mathematics, despite student performance data to the contrary.

Sequencing of topics in algebra textbooks

As a third piece of evidence for the expert blind spot hypothesis, we consider the sequencing of topics in algebra textbooks. Researchers in one study (Removed-For-Review-5) analyzed the organization of ten common algebra textbooks, and found that textbooks tended to place symbolic problems and activities (e.g., solving algebra equations) before verbally based tasks. The sample of books included one pre-algebra textbook and one algebra textbook from each of five major publishers (Glencoe/MacMillan/McGraw-Hill; Harcourt Brace Jovanovich; Houghton/Mifflin; McDougal, Littell; and The University of Chicago School Mathematics Project), which comprised al large portion of the US algebra textbook market.

Textbook sections were chosen as the unit of analysis because new material is introduced at this level. Sections devoted exclusively to the review of prior content were excluded from the analyses. In all, 1,083 textbook sections were analyzed. Each section was coded for certain patterns of presentation by examining the "written exercises" portions of the sections.

Topic sequencing within the textbook sections was examined to determine if new topics tended to be presented first symbolically and then verbally, as would be expected by a symbol-precedence view. The first written exercise in each section was first coded as either symbolic (e.g., an equation) or verbal in presentation format (e.g., a story problem). If a written exercise was presented only in Arabic numbers or algebraic notation it was coded as symbolic. If a written exercise contained words and phrases, it was coded as verbal. After the first item was coded, the remaining section items were compared to determine the code for the entire section. If the first written exercise for a section was coded as symbolic, and all of the later exercises in that section were also symbolic, then the section was coded as Symbol-to-Symbol (SS). If a verbal problem followed the initial symbolic exercise, the section was coded as Symbol-to-Verbal (SV). The same method was used for sections that began with verbal problems: Verbal-to-Verbal (VV) sections were all verbal, and Verbal-to-Symbol (VS) sections introduced topics verbally, and then moved on to symbolic problems. From this coding, one of four mutually exclusive pattern designations was possible for each textbook section. Textbooks would be consistent with the symbol precedence hypothesis evident among educators if SV patterns occurred more frequently than VS patterns.

Among the 1,083 chapter sections analyzed, 45% of them (487 sections) initially presented activities symbolically, followed by verbal problems as "applications" or "challenge problems." Fewer than 18% of the sections (194) presented verbal problems before symbolic problems. This pattern differed significantly from what one would expect given no symbol-precedence bias, as indicated by a statistical analysis on the 681 sections that followed both the SV and VS patterns, *Chi-square* $(1, N = 681) = 134.81$, $p < .001$). Symbol-to-symbol (11%) and verbal-to-verbal (26%) patterns made up the remaining 402 sections of the textbooks. These results indicate that a symbol-precedence view was the governing model of students' algebraic development used by the publishers in that sample.

The investigators (Removed-For-Review-5) argued that the dominant SV curricular organization portrays symbolic problems as easier for students to solve, and verbal problems as more demanding. However, when compared to student performance, the symbol precedence view describes the performance of far fewer students than the verbal precedence model. A quantitative comparison of developmental models of mathematics showed that 55% of all the students in one sample ($n = 171$) could be described by either a verbal precedence or a symbol precedence trajectory, because of overlapping states in the hypothetical trajectories. However, only 7% uniquely fit the symbol precedence model, whereas 36% uniquely fit the verbal precedence model (Removed-For-Review-

4). Thus, according to the original student data, the verbal precedence model provides a better quantitative fit than the symbol precedence model.

The curricular approach taken by textbook authors and publishers is yet another example of expert blind spot, in which the symbolic formalisms that help mathematicians to structure their own mathematical thinking are presumed to provide the best model for mathematics pedagogy and the development of mathematical reasoning. This stands in contrast to empirical data on student performance showing that verbal reasoning tends to precede symbolic reasoning.

Discussion

These examples of expert blind spot highlight the distinction between content knowledge and pedagogical content knowledge that have been previously made by Shulman (1987) and others. This distinction has proven to be a powerful one, and has helped to differentiate between knowledge needed to perform well within a domain and the knowledge that is specific to teaching a domain to new learners. The educational community must also be aware that highly developed content knowledge can subtly influence pedagogical content knowledge and teacher decision making. We have presented evidence from historical, psychological, and textual sources indicating that domain expertise in mathematics can lead educators to adopt a symbol precedence view of mathematical development because of the primacy and enormous utility of symbolic formalisms within the field of mathematics. We share the goal to advance learners' understanding of and facility with symbolic representations within mathematics. However, we recognize that students do not tend to develop these formal representations first, and that symbolic reasoning may trail or even depend upon the prior development of verbally based representations and procedures (Kalchman, Moss, & Case, 1999; Removed-For-Review-3; Removed-For-Review-4).

In the current *zeitgeist* of educational reform, many see a need for teachers to have greater understanding of their subject areas in order to be more effective instructors. Some see subject matter preparation as paramount, and put pedagogy in a distant second place (e.g., the Holmes Group, 1986). This view is echoed in the current "Math Wars" between calculation-centered and activity-centered curricula. It has led at least one educationally oriented foundation president to push for higher standards for teachers' content knowledge, and to advocate that the American mathematics research community ("Number one in the world") lead the reform of our mathematics curricula (Goldman, 1997).

There is no doubt, in our view, about the essential role of content knowledge for effective teaching, and we certainly do not argue that more highly developed content knowledge is bad. Yet we raise the issue that advancements in content knowledge without collateral advances in pedagogical content knowledge can lead educators to adopt expert-based views of curricula that shape their pedagogical practices, even though they may be at odds with students' actual learning processes. This tension between content knowledge and pedagogical content knowledge creates a dilemma for teacher educators who must conceptualize and design programs that address both content and pedagogy. Furthermore, this dilemma may not be unique to mathematics instruction, and seems to be evident in other areas of education, such as history (Quinlan, 1999), physical education (Schempp, 1998), language arts (Holt-Reynolds, 1999), and medical school (Krisman-Scott et al., 1998).

We recommend several ways to address this dilemma. First, teacher education programs should provide prospective teachers with the experiences they need to develop a strong base of pedagogical knowledge. Content knowledge is necessary, but not sufficient for this purpose. Extensive practicum experiences observing students' classroom-based learning and analyzing student work must be a part of every teacher education program. In fact, contrary to the directives of the Holmes Group (1986), we believe these experiences should be in place as early as possible in the teacher education program, so that pedagogical content knowledge develops concurrently with subject-matter knowledge. This way, the two bodies of knowledge do not form in isolation from one another and thus have a greater likelihood of becoming convergent.

Second, teacher education programs and the associated Arts and Sciences programs that provide content-based courses should be designed toward what Hatano and Inagaki (1986, 2000) term "adaptive expertise." We have seen that expertise can operate outside of the practitioner's awareness, and can instill the rigidity of behavior that accompanies all automated processes. In most cases, expertise develops through the deliberate practice of routinized problems that can eventually be solved using schema-based approaches (Ericsson & Smith, 1991; Hatano & Inagaki, 1986). In contrast to routinized expertise, *adaptive* expertise is aimed at how people deal effectively with *unfamiliar* situations in dynamic and unpredictable environments, such as becoming a life-long learner or teaching in authentic settings. Whether one becomes a routine expert or an adaptive expert depends on the nature of the extended practice. If practice is oriented exclusively toward skillfully but repeatedly solving a fixed class of problems, people tend to become experts characterized by fast, accurate performance and automatic procedures. They become efficient problem solvers, but also become rigid and relatively non-reflective. When flexibility and adaptiveness are the hallmarks of success, experts tend to focus on understanding the *meaning* of problem representations and solution procedures, and are more able to adapt their knowledge to ever-changing conditions (Hatano & Inagaki, 2000).

This distinction between routine expertise and adaptive expertise is related to Berliner's (1986) distinction between "knowing that" and "knowing how." Well-developed content knowledge (knowing *that*) is certainly essential for the development of either routine or adaptive expertise. But in isolation, content knowledge education tends to lead to routinized expertise. If teachers are to develop a flexible and responsive base of pedagogical content knowledge, content knowledge training must be coupled with educational experiences that focus on teaching for student understanding, rather than exclusively on rapidly generating correct answers or covering the curriculum. Teachers in these settings become more reflective about their own teaching, more sensitive to their own intellectual processes and the obstacles to learning, and more aware of how they know what they know and do what they do (i.e., Berliner's knowing *how*). These teachers also become more attuned to what students actually understand about new material and how students learn (Zech & Davies, 2000).

In addition to developing adaptive expertise, both new and practicing teachers need to be aware that they may hold untested and inaccurate assumptions about student learning, no matter how self-evident these notions may seem. As a third recommendation, we believe that teacher education programs should draw upon empirical support for instructional prescriptions, and should instill in pre-service teachers a critical stance toward prescribed teaching methods. Practicing teachers should consider formative assessment to be an integral and frequent part of their instructional practices (e.g., Shepard, 2000). One approach to this is the use of "embedded assessments," in which curricula and assessments are derived from the same tasks and activities (Snow & Mandinach, 1991). Despite their infrequent use, Black and Wiliam (1998) showed strong positive effects of embedded assessment on student learning. We believe that embedded assessments can improve the accuracy of teachers' conceptions of students' learning and development. Teacher educators should strive to develop embedded assessment tools that can be used by teachers in their classrooms to challenge and advance their own knowledge of student thinking and development.

One example of using classroom-based formative assessment techniques is the use of "Problem Design" materials (Removed-For-Review-6). In this approach, teachers give students comparison tasks (such as pattern generalization tasks with either tables of values or with manipulatives) to see which yields the most student participation, the richest mathematical discourse, and the greatest conceptual learning. Problem design has its roots in curricular design and experimental assessment design aimed at diagnosing what factors make tasks difficult for students. The problem design approach allows teachers to systematically compare the impact of instructional tasks on learning during classroom lessons. Glaser (1976) notes that periods of reform invite teachers to experiment with their instructional practices as they reflect upon and revise their teaching. We see embedded assessment practices as necessary to counter-balance expert blind spot and identify inaccurate assumptions educators may hold about student reasoning.

Lastly, we acknowledge the limitations of the knowledge and intuitions held by members of the educational research community. There is data showing that we, too, are susceptible to expert blind spots and that we sometimes hold inaccurate assumptions about student behavior that can negatively influence our research activities and curricular contributions. For example, as reported above, researchers in mathematics education also tend to hold to a symbol precedence view of algebra development that guides their decision making. This may also be true for educational researchers in other content areas. Because of the active role that the research community plays in classroom instruction and teacher education, unchecked assumptions at this level invite misconceptions that may influence curriculum design and the development of new teachers' pedagogical content knowledge. Members of the research community must exercise a strong critical stance on taken-for-granted views of learning and approaches to instruction. Assessment and experimentation are natural mechanisms for challenging false assumptions. In addition, researchers and practitioners must actively exchange research findings, classroom observations, and theoretical advances to further refine the knowledge base of the entire educational community.

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Tables

Table 1: The design structure of the six problem types given to students and teachers, with example entries.

Table 1: The design structure of the six problem types given to students and teachers, with example entries.

Presentation →	Verbal problems		Symbolic
Unknown value ↓	Story	Word Equation	Equation
Result-unknown (Arithmetic)	P4. When Ted got home from his waiter job, he took the \$81.90 he earned that day and subtracted the \$66 he received in tips. Then he divided the remaining money by the 6 hours he worked and found his hourly wage. How much per hour does Ted make?	P5. Starting with 81.9, if I subtract 66 and then divide by 6, I get a number. What is it?	P6. Solve for X: $(81.90 - 66) / 6 = X$
Start-unknown (Algebra)	P1. When Ted got home from his waiter job, he multiplied his hourly wage by the 6 hours he worked that day. Then he added the \$66 he made in tips and found he earned \$81.90. How much per hour does Ted make?	P2. Starting with some number, if I multiply it by 6 and then add 66, I get 81.9. What number did I start with?	P3. Solve for X: $X * 6 + 66 = 81.90$