Confronting Teachers' Beliefs About Algebra Development: Investigating an Approach for Professional Development

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Abstract

Five high school mathematics teachers from an urban, ethnically and socio-economically diverse community participated in a professional development activity aimed at influencing deep-seated and widespread misconceptions about students' development of algebraic reasoning. As has been found among prior cohorts, participants exhibited a *symbol precedence view* of algebra development, predicting students would solve equations more readily than matched word equations and story problems. Following this belief-elicitation activity, participants engaged in an activity focused on analyses of students' solutions that included common symbolic errors and the contrasting strengths of informal methods that more frequently occur with verbally presented problems. Evidence of pronounced teacher change one month after the professional development suggests that this approach holds promise for future professional development programs.

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Teachers' knowledge and beliefs are powerful mediators of decision-making and action (e.g., Sherin, 2002). For example, teachers generally report that their perceptions of students are the most important factors in instructional planning, and teachers consider student ability to be the characteristic that has greatest influence on their planning decisions (Ball, 1988; Borko & Shavelson, 1990; Borko et al., 1992; Carpenter et al., 1989; Clark & Peterson, 1986; Fennema et al., 1992; Romberg & Carpenter, 1986; Thompson, 1984). Yet teachers' beliefs and expectations of students' behaviors are not always accurate. This paper focuses on a method for influencing the beliefs of five urban high school algebra teachers so that they are more closely aligned with actual student performance data. We report on the initial and changing views exhibited by teachers about the nature of algebraic development and instruction, and discuss why the method has promise for affecting teachers' knowledge of students more generally.

Theoretical Framework: Changing Teachers' Beliefs and Content Knowledge

Teachers possess knowledge of many forms, some explicit, some tacit, some bound to actual practices (e.g., Sherin, 2002). *Pedagogical content knowledge* (PCK) has emerged as a valuable construct for understanding teacher knowledge (Shulman, 1986). PCK is knowledge of content oriented specifically around pedagogical concerns, such as how to illustrate concepts. Some forms of this knowledge cross the blurry line into beliefs and expectations about how students *should* learn. Consequently, some researchers are willing to combine knowledge and beliefs into a common construct (e.g., Pajares, 1992).

Only a few studies have specifically looked at the relation between teachers' beliefs about student reasoning, and students' actual problem-solving performance (e.g., Carpenter et al., 1988; Peterson et al., 1989; Wigfield et al., 1999). For example, members of the Cognitively Guided Instruction (CGI) project found that first-grade teachers' beliefs about student problem solving were usually consistent with the general strategies that students used, but underestimated the frequency with which students used informal counting strategies, while overestimating use of formal methods such as derived facts and direct modeling.

Much of this past research has focused on elementary level mathematics, and has greatly enhanced our understanding of teachers' beliefs. Lacking, however, is a similar emphasis at the secondary level. One exception is the study of the expectations high school mathematics teachers have for algebra students' problem-solving performance. Nathan and Koedinger (2000a) asked teachers to rank order the relative difficulty of mathematics problems that varied along two dimensions (see Table 2): arithmetic-algebra, and presentation as verbal with a context (story problem), verbal with no context (word equation), or symbolic equation. The majority (76%) inaccurate predicted that symbolic equations would be easiest for algebra students; instead they were most difficult, even though they were carefully matched to the story and word equation problems. Teachers justified their rankings by arguing that symbolic reasoning was a necessary precursor to solving story problems, and that symbolic representations were more "familiar," "straightforward," and "pure." This view was termed the *symbol precedence view* (SPV), and its role in algebra teachers' decision making has been independently confirmed with survey instruments (Nathan & Koedinger, 2000b; Nathan & Petrosino, 2002). The replication of both the student performance data (N1=76; N2=171) and teacher expectations (N1=67; N2=105) suggests this is a reliable and widespread view of mathematical development (Koedinger & Nathan, 2003; Nathan & Koedinger, 2000b).

Understanding teachers' views of problem difficulty is important because these beliefs affect instructional planning and the design of assessments. If teachers misperceive the relative difficulties of equations, they may choose to introduce them prematurely, or withhold story problems from a struggling student, because they appear to be out of reach.

Deep-seated beliefs do not easily change. Any attempts to change teachers' views, however, need to explicitly address teachers' beliefs (e.g., Fenstermacher, 1994; Richardson, 1994). This view guided the professional development activity described below.

Method

Participants

Five high school mathematics teachers volunteered to participate in a morning of professional development activities. All participants taught in the same urban school district, which serves a large portion (80%) of minority students, students who are English language learners (20%) and who qualify for free/reduced lunch (75%). They all agreed to stay through the duration of the day's activities, and to respond to a follow up activity that would come in the mail 4 weeks later.

Procedure and materials

(*N. B.* A brief description of the multi-faceted procedure follows to comply with space constraints. A more thorough description will be in the full paper.)

Participants received a professional development packet. The contents is summarized in Table 1. After completing a general cover page, participants responded to a belief elicitation task where they offered predictions of students' relative problem-solving difficulties using a difficulty ranking task (see Table 2). Each participant then presented his or her ranking to the group, and the ranking was recorded on an overhead projection slide for all to see. A brief discussion was moderated.

The professional development team then gave a 30 min presentation that showed the ranking data of other teachers and mathematics educational researchers. As previously observed, the overall pattern of predictions showed the common SPV with similar rationale favoring the development and use of symbolic reasoning before verbal applications. This helped to establish for participants that they had views similar to the mathematics educational community at large; that their views were not anomalous.

Student work was then presented with an overhead projector showing symbol equation use, common conceptual errors in symbolic representation and manipulation (along with frequency data; slip type errors were ignored). Participants also saw student use of alternative solution strategies that led to the verbal advantage, along with frequency, error patterns, and data on likelihood of success when applied to symbolic and verbal algebra problems.

Teachers were then given the following summative account: Despite commonly held beliefs among experienced and knowledgeable educators, algebra students are significantly more likely to correctly solve story problems and word-equations than matched symbolic equations. The differences appear to be due to two factors. First, student performance on symbolic equations is far worse than many educators expect, filled with symbol manipulation and calculation errors. Second, verbally presented problems are far more likely to elicit informal strategies, and these have a substantially higher likelihood of success, due perhaps to certain properties that support meaning making and error correction. Together, these influences lead to demonstrably higher performance on verbal problems than matched problems presented symbolically.

Teachers then participated in a guided activity where they applied a general rubric for evaluating students' written work (Table 3) to four solution examples. The examples were chosen because they captured the major features of the student performance data seen in previous studies (Koedinger & Nathan, 2003; Nathan & Koedinger, 2000a). These were intended to enhance the development of teachers' "algebraic eyes and ears"

(Kaput & Blanton, 1999) by focusing teachers on the problem-solving processes and representations that could be inferred from written work.

Three weeks later, participants were sent, by postal mail, a follow up difficulty ranking task designed to assess the impact of the professional development. The rankings were accompanied by teachers' personal written justifications.

Results and Conclusions

Pre-intervention ranking and justifications

Participants provided problem difficulty rankings individually, using the six items shown in Table 2. The difficulty ranking data were analyzed two different ways. First, rankings from all five teachers were averaged to produce a single group ranking (Table 4). This group ranking was correlated with an idealized SPV ranking that placed symbolic problem solving difficulty lower than that of verbal problem solving—a pattern that repeated for both the arithmetic and algebraic problems given in the ranking task. For the example items in Table 2, the idealized SPV ranking would be `1 2 3 4 5 6`. The correlation between the idealized SPV ranking and the group average ranking was shown to be high, Pearson's r = 0.9.

To corroborate this, the ranking of *each* participant was correlated with the idealized SPV rank, and each Pearson's rank correlation measure was calculated (second row of Table 4). As can be seen in the lower portion of Table 4, the correlations range from 0.49 to 0.9 with a mean of 0.7. This distribution of correlation measures yields a 95% confidence interval that ranges from 0.55 to 0.84 (SD = .17). Like the group level analysis, this analysis shows that participants provide problem difficulty rankings similar to that predicted by the SPV.

The justifications given by participants for their rankings shed further light. The constant comparative method was employed to establish a grounded coding system for teachers' justifications (Glaser and Strauss, 1967). Table 5 shows the resulting categories and reveals that teachers discussed how symbolic representations were favored because they were considered more basic and familiar to students,

The quantitative and qualitative results suggest that these high school teachers appear to draw on a symbol precedence view of algebraic development, predicting that algebra level students will tend to have an easier time solving symbolic equations than solving matched problems presented verbally.

Post-intervention ranking

Teachers participated in a new ranking task one month later. As with the pre-intervention ranking data, two complementary analyses were conducted, one at the group level, and at the individual level. The correlation between the average rankings across all five teachers and an idealized SPV ranking was shown to be low, Pearson's $r = 0.15$. The ranking of each participant was also correlated with the idealized SPV rank, and each Pearson's rank correlation measure was calculated. As can be seen in the lower portion of Table 4, the correlations range from 0.6 to -0.09 with a mean correlation of 0.13 (SD = .27). This distribution of correlation measures yields a 95% confidence interval includes 0. As before, both analyses lead to similar results. However, the analysis of post-intervention rankings now shows little resemblance with SPV.

The justifications (Table 5) reveal how teachers' justifications also changed overtime. There was a greater awareness of the difficulties students have with formal notation and the facilitating effects of verbal representations to provide, for example, context that helps students in their quantitative reasoning.

Importance of the study

Teachers' views of student development must be open to examination. This seems especially important given national (e.g., NAEP) and international data (e.g., TIMSS) showing poor student performance in secondary mathematics topics, and as school districts nationally are exploring how to teach algebra in the primary grades. The ranking task serves as an instrument for assessing one aspect of pedagogical content knowledge for teaching algebra; namely, teachers' expectations about the relative difficulty students actually experience for problems presented in more or less formal representations, while controlling for the underlying quantitative structure. To enhance the validity of this study, teachers were asked to evaluate the relative difficulty of *specific* mathematics problems, rather than making general statements about student reasoning in the abstract.

Our focus is on the effectiveness of a method of professional development as measured by changes in the expectations of a small sample of urban high school teachers. Specifically, we engaged teachers' by eliciting their views about student performance. We showed teachers that their views were held by most educators and that they implied a view of algebraic development, which, while logical and defensible, proved to be empirically inaccurate. We then engaged teachers in a very general, rubric-based activity that allowed teachers to apply their new perspectives on student reasoning. Finally, we observed how teachers' views changed and persisted beyond the confines of the professional development activity. By demonstrating such change, we believe that we have identified a promising approach that can be examined with a greater number of teachers and serve as part of a larger professional development program aimed at enhancing teachers' pedagogical content knowledge.

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Table 1. Summary of the contents of the workshop packet distributed to teachers.

1. Fill out cover sheet with teacher's demographics and contact information.
2. Select among snapshots of classroom climates and curriculum materials.
3. Pre-intervention problem difficulty ranking task (Table 2)
4. Workshop presentation showing ranking data from other teachers and examples of student performance data and student written work.
5. Four-level Rubric for Students' Written Work along with instructions for evaluating student work using the rubric to four examples of student work shown in Figure M-1 (a)-(d).
6. Post-intervention problem difficulty ranking task (*Waiter problem, W x 6 + 66 = 81.90*).

Table 2. Problems in the difficulty ranking task arranged in a 2 x 3 matrix.

Presentation type →	**Verbal**		**Symbolic**
Unknown value ↓	**Story**	**Word**	**Equation**
Result-unknown (Arithmetic)	P3	P2	P1
Start-unknown (Algebra)	P6	P5	P4

Table 3. Rubric given to participants to analyze student written work.

We will use a 4-level rubric to assess students' written work. Please note that there are is no one correct answer for applying this rubric. What is important is that you feel that you can justify your reasons for assigning a particular level, and that you are consistent with your evaluation.

We will share our rubric evaluations. Based on the comments of others, you may elect to change your evaluation. However, do not feel pressured to do so.

Rubric	
Level 4.	Student's written work shows *all* of the characteristics of Level 3, plus at least one of the following: • Student provides a particularly sophisticated solution strategy. • Written work is presented in a very clear and well organized manner. • There are aspects of the solution that indicate a deep understanding of the underlying mathematics.
Level 3. (Standard)	Written work presents a correct answer based on a mathematically sound method.
Level 2.	Written work presents an *in*correct answer that is arrived at by a method that is essentially sound, but with minor error(s) evident. For example, • Computational errors • Copy errors
Level 1.	Written work presents an *in*correct answer that is based on a conceptually flawed

method.

Table 4. Pre- and post-intervention correlation statistics (Pearson's r) with SPV (columns 1 and 2) and VPV (column 3) for the problem difficulty rankings averaged across the group (n = 5), and for each of the individual participants.

	Curricular materials	Pre-intervention r with SPV	Post-intervention r with SPV	Post-intervention r with VPV
Average ranking of group (n = 5)		.9	.15	.88
Mean correlation from individual rankings (n = 5)		.7[a]	.13[b]	.8[c]
Participants				
A	Reform	.9	.6	.94
B	Reform	.49	-0.09	.62
C	Traditional	.71	.03	.83
D	Reform	.56	.03	.83
E	Reform	.81	.09	.77

[a] Average of the individual Pearson's r computed for all 5 participants (SD = .17). The 95% confidence interval extends from .55 to .84.

[b] Average of the individual Pearson's r computed for all 5 participants (SD = .27). The 95% confidence interval includes 0.

[c] Average of the individual Pearson's r computed for all 5 participants (SD = .12). The 95% confidence interval extends from .69 to .90.

Table 5. Participants' coded justifications for the pre- and post-intervention problem difficulty ranking task.

	Students have greater familiarity and skill with symbols than words	Verbal problems are solved using translation to symbolic	Arithmetic word problems tell you exactly what to do	Arithmetic skill strictly precedes algebraic reasoning	Symbol manipulation is difficult (error prone)	Context helps in problem solving
Pre-intervention						
GH	•		•	•		
LM	•	•				
JC			•	•		
SC	•	•				
AH			•			
Post-intervention						
GH			•	•	•	•
LM					•	
JC						
SC				•		•
AH						