Extended Twin Family Designs: Path Tracing

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The point of this lecture

- Extending Twin Family Designs (ETFDs) can model assortative mating, vertical transmission, and passive G-E covariance, along with other parameters

- This is extremely difficult to do without the magic of path tracing

- This lecture shows how to derive expectations of variances and covariances in ETFDs using path tracing rules

Path Diagrams

- Path diagrams pictorially represent causal models. They aid in deriving the variances and covariances implied by the model.
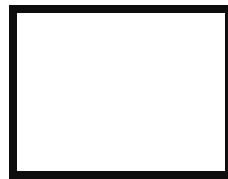
Observed Variables
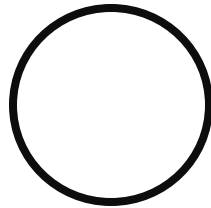
Latent Variables

Causal Paths

(Co)variance Paths

Deriving variances & covariances

1. Identify all legitimate chains (a series of paths) that connect one variable to another (covariances) or connect a variable back to itself (variances)

2. The expected value of a chain is the product of all coefficients associated with each path making up that chain

3. The final expected variance or covariance equals the sum of the values of all legitimate chains

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4. All chains must be counted exactly once and each must be unique. However, order matters: *abc* is a distinct chain from *cba*.
5. Copaths are special. They may only be traversed once per chain, but once crossed, rule 3 resets. A chain therefore must contain a double-headed arrow before traversing a copath and one after traversing a copath.
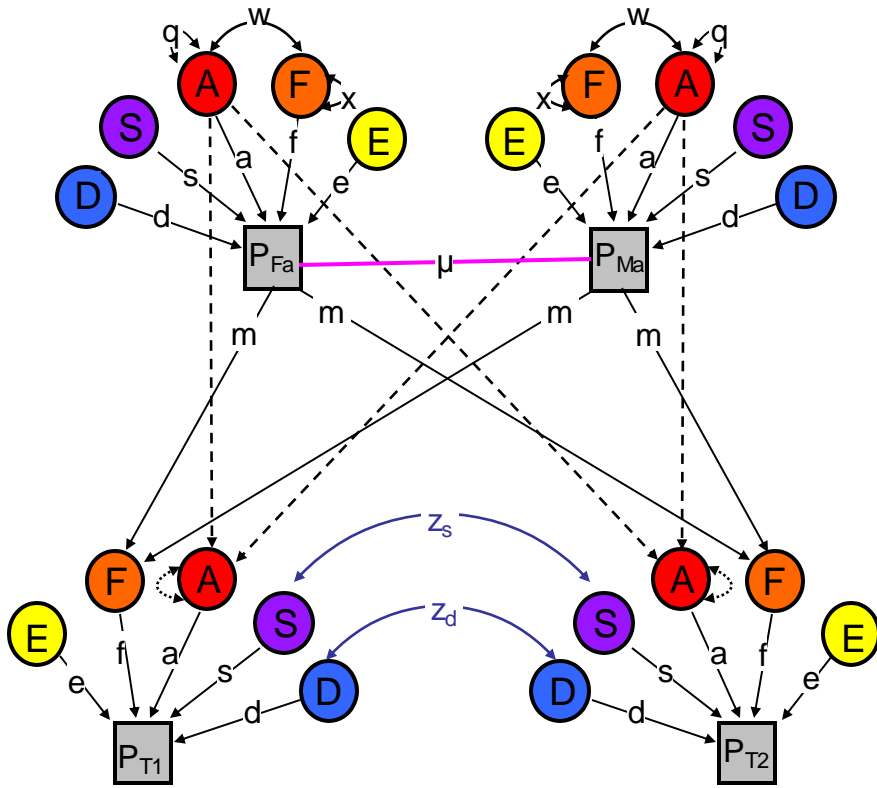
Path Tracing Example

Special considerations in ETFDs

- Typically, we assume that AM and VT have reached equilibrium - i.e., the levels of AM and VT have been stable across multiple generations

- This allows us to set the variances of A & F and the covariances between A & F in the parental generation to be equal to those in the offspring generation

- This results in non-linear constraints: parameters are functions of those same parameters

- Solving these requires iterative procedures, e.g., ML. Closed-form solutions of these estimates are typically not possible.
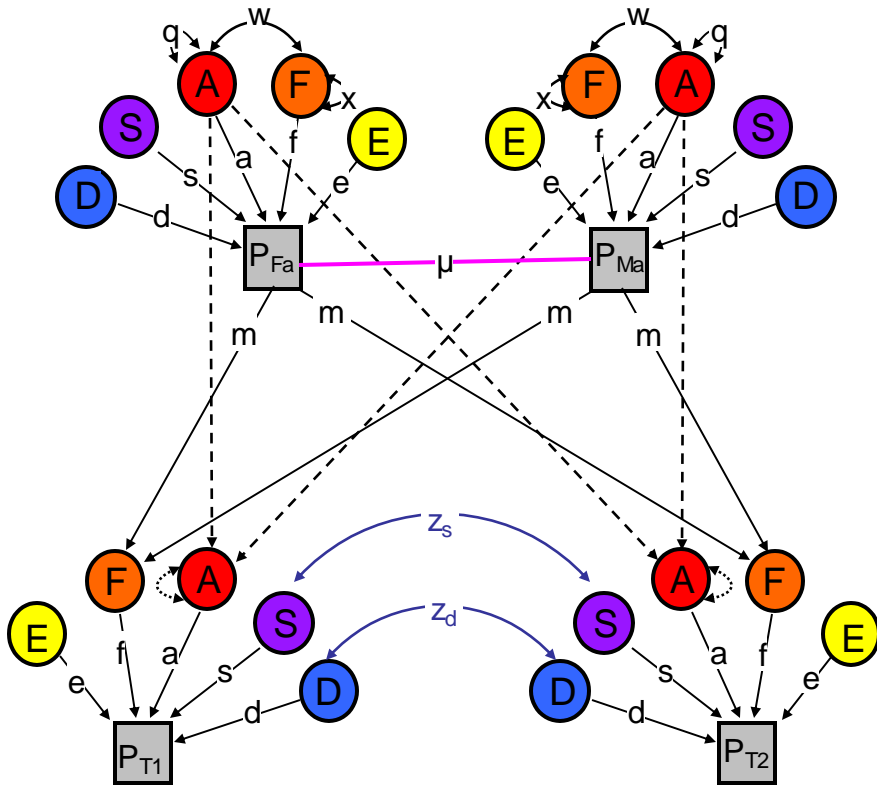
Nuclear Twin Family Design (NTFD)
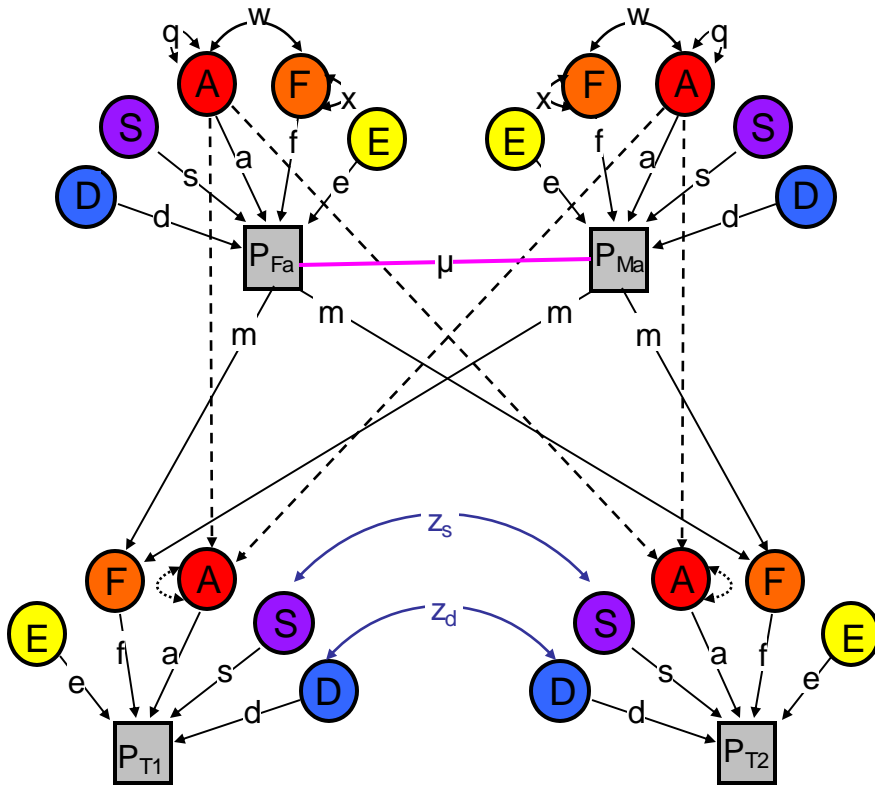
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Conclusions

- A few simple rules from path tracing makes it possible to derive expected variances and covariances when there is VT and AM.

- This would lead to otherwise intractable math arising from recursive relationships caused by AM and VT