We demonstrate that the three-dimensional (3D) binary monoclinic oxides HfO$_2$ and ZrO$_2$ exhibit quasi-2D polaron localization and conductivity, which results from a small difference in the coordination of two oxygen sublattices in these materials. The transition between a 2D large polaron into a zero-dimensional small polaron state requires overcoming a small energetic barrier. These results demonstrate how a small asymmetry in the lattice structure can determine the qualitative character of polaron localization and significantly broaden the realm of quasi-2D polaron systems.

DOI: 10.1103/PhysRevLett.108.116403 PACS numbers: 71.38.Ht, 71.15.−m

Quasi-two-dimensional (2D) systems, including surfaces, heterointerfaces, and layered materials, have become a rich playground for discovering exotic electronic effects such as high-$T_c$ superconductivity and novel magnetic phases [1–5]. The formation of polarons and the modification of their properties by reduced dimensionality have been implicated in many of these phenomena [1,6–8]. Although a number of systems, including layered transition-metal oxides [5], surfaces, and thin insulating films [9,10], are suspected to exhibit 2D polaronic behavior, directly probing their properties remains extremely challenging. Much of what is currently understood about polarons in 2D systems is based upon phenomenological or semiempirical theoretical models [11–15]. For example, the existence of an energetic barrier to the formation of small polarons in systems with more than two dimensions can be shown using simple scaling arguments [12,13,16]. However, the 2D case is a marginal one, making it difficult to make reliable predictions about the properties of materials. Experimental evidence, although fairly limited, suggests that there is a barrier in at least some 2D systems [9]. This uncertainty underlines the fact that the theory of polaron self-trapping in anisotropic polar crystals is far from being complete [17,18] and it remains difficult to predict values of critical parameters causing polaron self-trapping in such systems.

In this Letter, we employ fully ab initio theoretical methods to investigate the properties of hole polarons in the binary monoclinic oxides HfO$_2$ and ZrO$_2$. A number of previous studies provide evidence for polaronic hole trapping in these materials, including measurements of the magnetorefractive effect [19], the oxygen pressure dependence of hole conductivity [20,21], and electron spin resonance spectroscopy [22]. Although HfO$_2$ and ZrO$_2$ are usually considered as 3D materials, our calculations show that they exhibit pronounced 2D polaronic properties. This unexpected effect is due to an asymmetry in the crystal structure of these materials, which induces small polaron formation in only one of its two oxygen sublattices, leading to highly anisotropic polaron conductivity. We further demonstrate the existence of an energetic barrier to hole self-trapping and elucidate the 2D-to-0D localization transition. This discovery broadens the field of quasi-2D systems which may exhibit novel electronic properties. Aside from this fundamental interest, both HfO$_2$ and ZrO$_2$ find important applications in electronics and catalysis (respectively), where the trapping and mobility of holes is very important [23,24].

Accurately predicting the character of localization of electrons or holes from first principles is extremely challenging because widely employed density functional theory (DFT) approaches suffer from self-interaction (SI) error, which tends to delocalize holes [25]. A number of SI corrected electronic structure methods have been applied previously to investigate polaronic localization, both in perfect crystals and near point defects, such as DFT + U [26], semiempirical correction schemes [27], and hybrid DFT [28]. Indeed, the latter was used previously to predict hole polaron localization in HfO$_2$ [29]. Although these approaches restore the qualitative picture of localization, none guarantee exact elimination of SI. To make more confident predictions regarding the character of hole localization and mobility in 2D systems requires a more controlled approach to dealing with SI. To achieve this, we employ the recently developed cancellation of nonlinearity (CON) method within a DFT formalism [30]. The main advantage of this method is that the SI correction that is applied is determined unambiguously by ensuring that the correct linear behavior of the total energy with respect to fractional occupation number is obtained. This method has been successfully applied previously to model holes in wide gap semiconductors, including acceptor defects.
4C oxygen ions are geometrically separated in the lattice coordinated (3C) or four-coordinated (4C). The 3C and 4C oxygen ions are almost exclusively of 3C character. The contribution from 4C O ions appears only deeper in the VB. The origin of the splitting in energy of these electronic states can be traced to differences in the Madelung potential. Consequently, the band dispersion near the top of the VB is very small perpendicular to the 3C layers, [100], but high parallel to them [37]. We will focus our further discussion on m-HfO2; however, we find very similar properties for m-ZrO2, and the key properties of both materials are summarized later in the text.

A direct consequence of the layered oxygen sublattice in m-HfO2 is that a hole introduced at the top of the VB is the anion p-orbital. The reference occupation $n_{\text{host}}$ is uniquely determined by requiring $E_{\text{h}} = e_i$. Calculations are performed using a range of $\lambda_{\text{h}}$ for holes localized on both 3C and 4C oxygen sites. A 324 atom supercell is used to ensure holes are separated from periodic images by over 15 Å. To improve accuracy further, the total energies and eigenvalues are also corrected for image-charge and potential alignment effects, as has been described previously [38]. Following a systematic investigation, we find that the SI of the localized hole is eliminated exactly using $\lambda_{\text{h}} = 3.8$ eV.

Using the self-consistently determined SI correcting potential, we have been able to find local minima in the potential energy surface (PES) corresponding to holes localized on both the 3C and 4C oxygen sites. The 4C site is characterized by $E_i = -0.31$ eV; i.e., hole polarons trapped at 4C sites are metastable. We have confirmed by evaluation of the dynamical matrix by density functional perturbation theory that the 4C hole is metastable rather than unstable, as there are no soft vibrational modes present. On the other hand, 3C sites serve as stable hole traps with $E_i = 0.18$ eV. The corresponding charge transition levels, relative to the top of the bulk HfO2 VB, are 1.13 eV (3C) and 0.78 eV (4C), as illustrated in Fig. 2(a). For comparison, we performed calculations using the screened hybrid functional of Heyd, Scuseria, and Ernzerhof [28] and obtained qualitatively similar results [36].

Given the importance of the prediction that hole polarons are only stable on the 3C oxygen sublattice, it is...
instructive to examine the origin of this different stability. For this purpose, we consider a hypothetical process in which we distort the equilibrium geometry of the neutral system into the geometry of the polaron and subsequently remove an electron vertically. We interpolate linearly between the positions of atoms in the ideal lattice ($R_0$) and the positions corresponding to the (meta)stable polaron state ($R_{3C(4C)}$), in the following way,

$$\mathbf{R}(t) = t \mathbf{R}_{3C(4C)} + (1 - t) \mathbf{R}_0.$$  

(2)

Following distortion of the neutral lattice [i.e., following the lower curve in Fig. 2(b) from $t = 0$ to $t = 1$], there is an increase in energy due to elastic strain ($E_s$), which has an approximately quadratic dependence on the degree of distortion. Accompanying this distortion, the local electrostatic potential at the anion, $\Phi$, is also increased. We find that the increase in local potential is directly proportional to the degree of distortion, which justifies a widely employed approximation in analytical models of polaron trapping [39]. On removing an electron, the hole that is formed is attracted towards the anion due to the potential well formed by the dilation and long-range electronic polarization. However, hole localization also implies an increase in kinetic energy and the balance between these various energetic terms determines the magnitude of the trapping energy.

It is reasonable to assume that the long-range electronic polarization and increase in kinetic energy following localization are similar for both the 3C and 4C hole polarons, as the holes are localized to a similar extent. However, the other two terms—the strain energy and the local electrostatic potential—are, in general, dependent on the trapping site. For $\text{HfO}_2$, we find that $E_s$ is 0.14 eV greater for the 4C polaron than the 3C polaron, which is expected, since an additional cation must be displaced. We also find that the electrostatic potential in the polaron geometry is 0.39 V higher on the 3C site than on the 4C site, which is similar to the difference present in the undistorted lattice (0.36 V). Both of these factors tend to favor trapping at the 3C site. This analysis shows that the main effects responsible for site-selective polaron trapping are differences in the electrostatic potential and elastic strain energy between the two anion sites.

To elucidate the localization transition associated with hole trapping, we again consider the process described by Eq. (2), but this time for the positively charged system. Figure 2(b) shows schematically the form of the PES for hole trapping at the 3C oxygen site. We find that there is a small barrier to trapping from the ideal lattice, ~40 meV. Figure 3 shows the corresponding spin density isosurfaces for the hole at a series of points along the distortion path. For a very small distortion ($t = 0.2$) corresponding to a potential energy fluctuation of only 23 meV, the hole has already undergone a transition from 3D to 2D localization within a single layer composed of 3C oxygen ions. This is an indication that the layered structure of $m$-$\text{HfO}_2$ is intrinsically unstable to 2D hole localization. As the distortion increases further ($t > 0.2$), the spatial extent of the hole begins to decrease within the 2D (100) plane, until ultimately it forms a small polaron ($t = 1$). Once fully localized, 76% of the spin density is localized on a single oxygen ion.

![FIG. 2 (color online). (a) Position of the charge transition level for a hole introduced without lattice relaxation (D) and for small polarons localized at 3C and 4C oxygen sites following lattice distortion. The numbers indicate the position of the electronic level relative to the bulk valence band maximum (in eV). The conduction band (CB) is also shown. (b) Schematic adiabatic PESs before ($q = 0$) and after ($q = 1$) electron ionization.](image)

![FIG. 3 (color online). Spin density isosurfaces showing the change in the character of hole localization associated with distortion of lattice ions. A very small distortion in the lattice ($t = 0.2, \Delta E = 23$ meV) is sufficient to induce a transition from 3D to 2D localization within a layer composed of 3C oxygen ions.](image)
An important consequence of this result is that large polarons in $m$-HfO$_2$ have an essentially 2D character, and the formation of small polarons requires a 2D-to-0D localization transition. To assess the mobility of hole polarons, we calculate activation energies for hole hopping, $E_d$, by linear interpolation. We predict hole polarons can move freely between 3C oxygen sites ($E_d = 0.14$ eV); however, the relatively high energy of the 4C localized hole suggests that even the transient formation of a 4C hole is improbable. Therefore, hole polarons, both large and small, must be confined to exist within 2D planes of 3C oxygen ions.

The predicted activation energy for hole mobility is in good agreement with experimental high-temperature hole conductivity data [20,21]. At lower temperatures, these experiments found a higher activation energy, 0.7 eV, which has been attributed to the thermal liberation of holes from cation vacancies, i.e., $V^+_\text{Hf} \rightarrow V'\text{Hf} + h^-$ (in Kröger-Vink notation). Using the CON approach, we find that the binding energy of a hole to a neutral cation vacancy is 0.64 eV, again in very good agreement with Ref. [21]. A significant advantage of the CON approach over hybrid functionals is that the larger cell sizes needed to describe such defects are computationally feasible. Very similar results are also obtained for ZrO$_2$, and the key quantities are summarized in Table I.

In summary, we have investigated the properties of holes in HfO$_2$ and ZrO$_2$ and their ability to self-trap forming polarons using first-principles methods that are free from the usual SI problems inherent to DFT. We have shown that holes introduced to the VB form small polarons that are distributed across 2D layers comprised of 3C oxygen ions. These polarons are predicted to be highly mobile within the 3C layer but face high barriers to move in the perpendicular direction. We also elucidate the factors which influence the formation of hole polarons in this material and study their interaction with cation vacancy defects. Our calculations predict hole conductivity in these materials should be highly anisotropic. Testing this prediction requires experiments on single crystals or on films with defined texture.

We note that in many ways the 2D electronic properties of $m$-HfO$_2$ and $m$-ZrO$_2$ are similar to those pronounced in numerous layered oxides as well as in 2D electron gases that can be formed at interfaces (e.g., LaAlO$_3$/SrTiO$_3$), in high-$T_c$ superconducting cuprates and also at grain boundaries [40]. Recent theoretical calculations on transition-metal dichalcogenides (e.g., ZrS$_2$ and HfS$_2$) suggest that similar 2D polaronic effects may also be present, owing to their layered structure [41]. One interesting consequence of this layered electronic structure is that polaronic hole trapping is (unusually) a 2D-to-0D localization transition. Similar effects can be introduced by strain in otherwise highly symmetric materials [42]. Investigation of the correlated dynamics and interaction of holes confined in these types of material, using techniques such as THz time domain spectroscopy [43] and low-temperature magnetic and electrical characterization, may reveal interesting effects such as superconductivity, hole crystallization [44], or magnetism which may deepen our understanding of these interesting and important phenomena.

K.M. acknowledges support from JSPS (Grant No. 22740192). S.L. and A.Z. were supported by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences, Energy Frontier Research Centers, under Contract No. DE-AC36-08GO28308 to EFRC of inverse design. This work made use of the facilities of HECToR, the U.K.’s national high-performance computing service, via our membership in the U.K. HPC Materials Chemistry Consortium, which is funded by the EPSRC (EP/F067496).

### Table I. The hole trapping energy ($E_t$), vertical charge transition level ($E_{+/0}$), barrier to trapping ($E_b$), and barrier to diffusion ($E_d$) are summarized below for holes trapped on both 3C and 4C oxygen ions.

<table>
<thead>
<tr>
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<th>HfO$_2$</th>
<th>ZrO$_2$</th>
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<td></td>
<td>3C</td>
<td>4C</td>
</tr>
<tr>
<td>$E_t$ (eV)</td>
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<td>-0.31</td>
</tr>
<tr>
<td>$E_{+/0}$ (eV)</td>
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<tr>
<td>$E_d$ (eV)</td>
<td>0.14</td>
<td>0.13</td>
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</tbody>
</table>

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