# Spectral Performance and Noise Theory of Nonlinear Transmission Line Frequency Multipliers

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***Abstract*— In this paper the theoretical and measured spectral performance of nonlinear transmission lines (NLTLs) and their applicability to frequency multiplication is presented. NLTLs, through nonlinear dispersion in large signal, create a sharp edge from a sinusoid that is rich in harmonics, making a very effective frequency multiplier. While the design of NLTLs has been widely published [1-2] for high harmonic content, prior to this work, the noise performance of NLTLs has only been published in [3]. The results presented in this paper demonstrate the near ideal $20 \cdot \log_{10}N$ phase noise multiplication as referenced to the additive phase noise of the fundamental with N being the multiplicative factor. Theoretical analysis of the noise properties of NLTLs show the effect of DC biasing and the overall change in phase shift of the NLTL in sum with AM source noise has the most dramatic influence in additive phase noise and is confirmed by measurement. An NLTL design with a -187dBc/Hz input referred phase noise is presented.**

***Keywords—phase noise, NLTL, multiplier, additive, residual***

## I. Introduction

Low phase noise RF frequency generation is commonly accomplished through multiplication of a very high quality lower frequency source, typically a crystal oscillator. Frequency doubling, tripling and mixing are all utilized. Comb generation, or integer creation of harmonics, from the fundamental source is advantageous in many systems. Step recovery diodes have historically been used but the high drie levels and noise effects have been problematic at times. NLTLs, due to their passive nature in which they sharpen a wave, create high harmonics with potentially little added noise. The varactor is the key nonlinear component and has been shown to have negligible self noise in reverse bias [4]. This work develops both a theory to the dominant noise source and a measurement technique and results to confirm it. Design of a 100MHz input NLTL with measurements at the $10^{th}$ harmonic is presented with an input referred phase noise of -187dBc/Hz.

Measurements and theory published in [3] were completed at the fundamental and $10^{th}$ harmonic using a cross correlation system. Recent publications suggest cross-correlation measurements may be suspect at or near thermal limits [7]. This work is an extension of [3] with a new comb design and measurements at the $10^{th}$ harmonic using a single channel phase noise system. A lower noise floor was achieved with this new design.

## II. NLTL Design and Operation

NLTLs operate fundamentally different than step recovery diodes [5], frequency doublers or triplers [6]. Step recovery diodes use charge storage in the intrinsic region to generate a snap back effect, observed as a pulse and require high RF drive levels. Diode doublers and triplers operate via zero crossings, essentially a switch but cannot produce all integer harmonics. Nonlinear transmission lines operate on a large signal voltage variable delay. Line phase velocity decreases at higher voltages and increases at lower voltage. As a sinusoidal wave propagates along the line, the higher voltages travel faster relative to the lower voltages. The result is a sharp leading-edge rich in harmonics. The trailing edge has a gentler slope and a saw tooth wave is the best shape approximation. Additional filtering can manipulate the time domain saw tooth into an impulse, forming nearly equal integer harmonics.

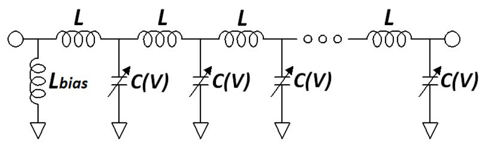


Fig. 1. Simplified NLTL model showing it acts like a variable phase shift delay line. A DC bias condition is added through an inductive bias.

### A. Varctor Noise

An NLTL has only three fundamental components: inductors, varactors and some method to bias it. Air and ceramic core inductors ($u_r$=1) do not exhibit any significant voltage noise [4]. Voltage noise from a varactor in reverse bias was carefully measured by the system shown in Fig. 2 to be negligible in Fig. 3 consistent with [4]. In forward bias, when current is flowing, the noise becomes more significant. In this paper, the NLTL is biased almost completely out of forward conduction.

### B. NLTL Design at 100MHz

The design of a uniform NLTL is relatively simple. The capacitance-voltage characteristics and size of available varactors determine the drive power and available harmonic power by the number of stages for a given input frequency. A capacitance range of approximately 4:1 has been found to be a good balance of compression vs source match and length of line. A 4:1 capacitance change yields a 2:1 input impedance change by Eqn 1. Optimized for 50Ω, the impedance varies from 35Ω to 70Ω or an S11 < -15dB. This design is for a 100MHz input with harmonics to at least to 1GHz. A hyper-abrupt varactor (used as a pair), Toshiba 1SV285, was chosen for a 4:1 capacitance change over approximately 5V. An 18dBm drive power will reach full the full swing of varactor. The C-V characteristics of a varactor will continue down to forward conduction. To maximize the C-V characteristic a midpoint of 2V bias for 50ohms operation was selected.

Table 1 is the voltage vs capacitance relationship of the varactor as a pair of diodes. A midpoint bias of 2V required a 16nH inductor for 50Ω operation. The right-hand column of the table is the voltage vs delay per element by Eqn 2. The difference between the min and max voltages is the compression per stage. A 4:1 capacitance ratio yields a 2:1 compression per unit delay.

$$Z_0 = \sqrt{\frac{L}{C}} \quad (1)$$

$$\tau_{element} = \sqrt{C \cdot L} \quad (2)$$

An NLTL design of a 100MHz sinusoid input requires a certain amount of compression in time. The least amount of pulse compression is optimal, it will reduce the sensitivity to voltage noise on the line. The 20-80% rise time of the sinusoid, where most of the pulse compression will occur, happens over approximately 100 degrees, or about 2.8ns at 100MHz. Based on the table, 12 sections compress approximately 2.7ns over 5V. Fig. 4 is the complete schematic of the design and fabrication.

| Voltage | Capacitance | Unit Delay |
|---|---|---|
| 0 | 13pF | 456ps |
| 1 | 9.4pF | 388ps |
| 2 | 6.8pF | 330ps |
| 3 | 5.0pF | 283ps |
| 4 | 4.0pF | 255ps |
| 5 | 3.4pF | 233ps |

Table 1. Voltage vs Capacitance relationship vs unit delay (per L-D element) based on a 2V midpoint and 16nH inductor.

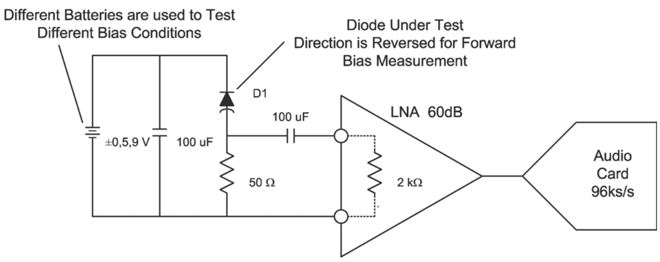


Fig. 2. Test apparatus to measure the noise due to a diode. The varactor diode was measured in both forward and reverse bias conditions. The measurements were all done on batteries

The NLT was modeled in spice, the time domain waveforms shown in Fig. 5 at the input, output and every 4 sections of line, showing the pulse compression along the way. Considerable ripple is always a problem with NLTLs. Inherently, they are unmatched somewhere along the line. Like a mixer, which is either a short circuit or an open circuit and a 'matched' condition is one in which each occurs about 50% of the time. A narrowband drive amplifier is not the best choice for an NLTL, broadband back match recommended.

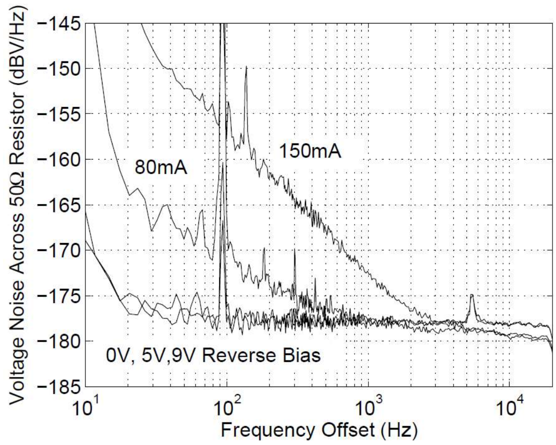


Fig. 3. Noise measurements of a PN junction varactor diode in forward and reverse bias. Current needs to be flowing for noise to be introduced. In reverse bias, the varactors exhibit no noise of their own.

The fabricated NLTL has a bias port and a bias-T with an option for loading a very clean bjt based 1.85V reference (close to the desired 2V) or being driven externally by a function generator or voltage source.

Fig. 6 is the measured harmonic output of the line when biased with 1.85V on-board. Input is 100MHz sinusoid at 18dBm.

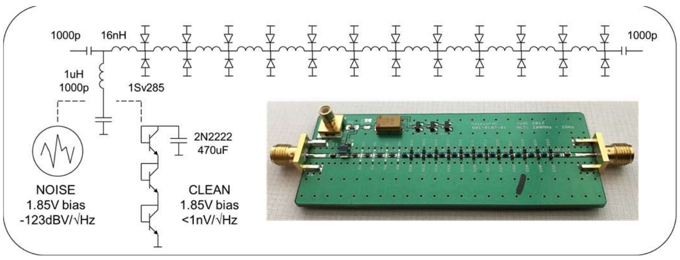


Fig. 4. Schematic and photo of the test NLTL. Two different bias conditions are shown. One uses three bypassed transistors powered by a battery through a current limiting resistor to create an extremely clean low noise 1.85V reference. The second is an external noise source for testing AM-PM conversion.

## III. Theoretical Noise

The NLTL operates as a voltage variable delay line. Provided it is biased out of heavy forward conduction, the dominant noise source is the voltage noise on the line itself. The NLTL operates opposite the function of a phase detector by introducing PM noise based on present AM noise by a rad/V constant in Eqn 3. The 3dB factor is for a single sideband measurement.

$$\mathcal{L}_{fm} = V_{rms\text{-}noise}\,(\text{dBV}/\sqrt{\text{Hz}}) + 20*\log_{10}(\text{rad/V}) - 3\text{dB} \quad (3)$$

The susceptibility to AM noise can be measured by introducing a DC change in the bias condition and measuring the phase output at the harmonic of interest. As a frequency multiplier, the AM-PM conversion of a harmonic may be higher or lower than others. This may be calculated from simulation or measurement.

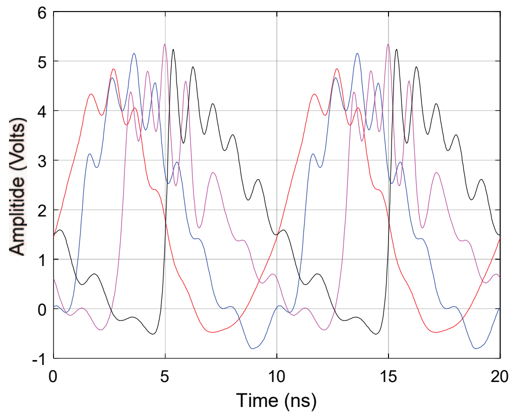


Fig. 5. Time domain analysis along the NLTL in simulation showing points at the beginning, end and every 4 elements. Simulation matches design at a nominal 4ns delay and 3ns of pulse compression.

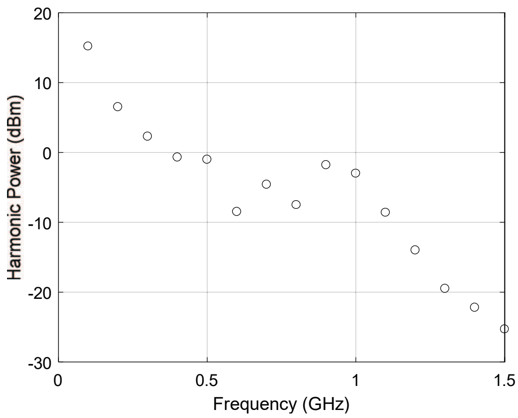


Fig. 6. Measured output harmonics of the fabricated NLTL using the on-board 1.85V bias. Input is a 100MHz sinusoid at 18dBm. The 1GHz comb at -2dBm is the measured harmonic.

## IV. Single Channel Measurement

A basic diagram of the measurement of the NLTL at the $10^{th}$ harmonic is shown in Fig. 7. The 100MHz source is an oven controlled crystal oscillator (OCXO) buffered by a medium power (25dBm) amplifier, both on batteries. The power splitter is broadband, extending well below and above 100MHz. The signal was attenuated to 18dBm after the power splitter, before the NLTL, to improve match and isolation between channels.

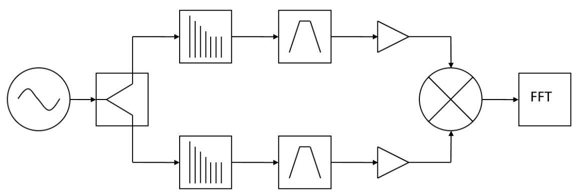


Fig. 7. Single channel measurement system to measure a pair of NLTLs. Diplexing and terminating lower frequencies out of the NLTL is important to maintain pulse compression and desired operation.

Filtering of the harmonic is the most critical part of the measurement system. The NLTL is a transmission line and a narrowband filter is typically reflective. If a filter is put directly behind the NLTL a majority of the propagating wave is reflected back, causing destructive interference. Prior to the 1GHz bandpass filter, a diplexer is used to terminate the first 5 harmonics of the NLTL. The amount of energy reflected from higher harmonics is not enough to disrupt the pulse forming of the NLTL.

The 1GHz filter reduced neighboring harmonics to at least 20dB below that of the desired harmonic. All other harmonics were considerably lower. The output power of the 1GHz harmonic was -3dBm after filtering.

Two HX2400 amplifiers were used to increase the power to the HX3100 phase detector. The amplifiers were attenuated prior to the phase detector to ensure 9dBm to each port, optimal for low phase noise measurements.

Fig. 8 is the results of the phase noise measurement and noise floor characterization of the system. The floor is limited by the power into the HX2400 amplifiers and their inherent 4dB noise figure. 3dB is added when measuring a pair of devices. It is thermally limited by Eqn 4. The calculated -167dBc/Hz is confirmed by measurement in Fig. 8.

$$-167\text{dBc/Hz} = -177\text{dBm}-(-3\text{dBm})+4\text{dB(NF)} +3\text{dB} \quad (4)$$

The NLTLs are measured at -164dBc/Hz, 3dB above the measurement limit. This measurement is consistent based on the AM-PM conversion of the NLTL and will be discussed in the following section.

### *A. AM-PM Conversion*

The NLTL is inherently sensitive to AM-PM conversion. Phase noise is introduced as rad/V based on the compression of the line and frequency. Using the same setup as Fig. 7, the DC bias of the line was varied in one path and the phase shift measured. A 0.1V change resulted in a measured 0.72radian change at the phase detector, a 7.2rad/V AM-PM conversion. This is an extremely high susceptibility to AM-PM conversion. The additive phase noise measurements in Fig. 8 shows when one line is biased with a normal Agilent (Keysight) E3610A power supply and the resultant noise due to it as compared to a clean bias.

Fig. 9 is the result of biasing one of the lines with a noise generator with an offset of 1.85V (on-board 1.85V removed). Based on the theory the line is modulated by the bias voltage on the line at 7.2rad/V, the voltage noise is modulated into phase noise by:

$$\mathcal{L}_{fm} = V_{rms\text{-}noise}\,(\text{dBV}/\sqrt{\text{Hz}}) + 20*\log_{10}(\text{rad/V}) - 3\text{dB}$$

At 7.2rad/V, the measured phase noise will be 14dB higher in dBc/Hz than the voltage noise. The measured result in Fig. 9 confirms this at a measured phase noise of -109dBc/Hz for a voltage noise modulation of -123dBc/√Hz.

With a clean supply (sub 1nV/√Hz) the line still has broadband 50Ω noise around the carrier at -180.9dBV/√Hz. Applying the 14dB conversion would yield a phase noise of -167dBc/Hz each, or -164dBc/Hz, very close to what was measured.

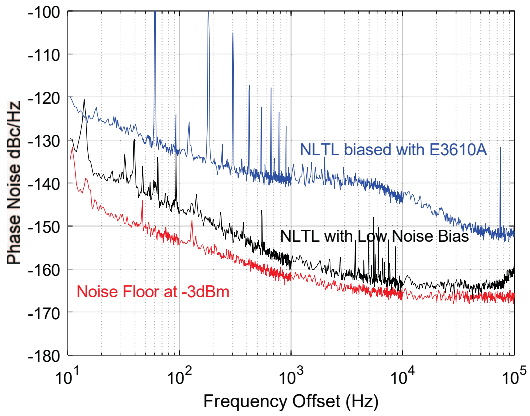


Fig. 8. Measured phase noise of a pair of NLTLs at 1GHz output. The red is the noise floor of the system at the same power. Black is using a very clean bias while the blue is using a bench power supply.

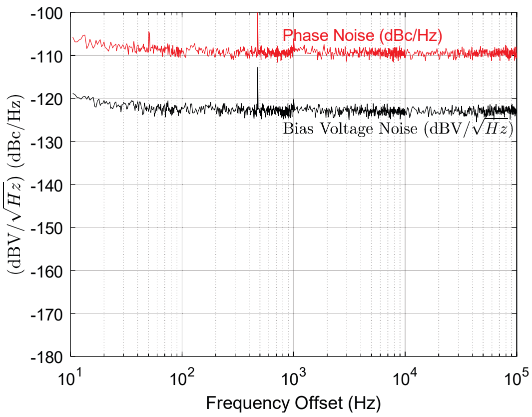


Fig. 9. Measured phase noise (red) when one line is modulated with voltage noise (black). The NLTL shows heavy influence on any voltage noise on the line.

### *B. System Verification of a multiplied crystal*

It has been shown the NLTL is heavily dependent on the voltage noise on the line. It is similarly affected by the AM noise of the source. Fig. 10 is the results of multiplying a 100MHz OCXO with the NLTL and filtering at 1GHz. The drive level was 18dBm and shown in red. The light red plot would be an ideal x10 multiplication of the OCXO. The blue line, actual measurement, shows a deviation from this at offsets of 200Hz out past 1MHz. The additional noise correlates to the AM noise measured of the OCXO and amplifier pair. As the AM noise rolls off or is below the phase noise, the NLTL operates as a near ideal multiplier.

## V. CONCLUSIONS

The NLTL is presented as a low phase noise frequency multiplier, limited by only by the voltage noise on the line. However, the NLTL is extremely susceptible to AM and bias voltage noise contributions and must be implemented carefully. An NLTL with an input referred noise of -187dBc/Hz, provided AM noise is not present, is demonstrated and is at the state of the art for clean frequency multiplication.

The AM-PM conversion does not show up in the additive measurements because it is an electrical property of the line and will be considered common mode in an identical pair of lines. Three identical NLTLs were measured to verify this.

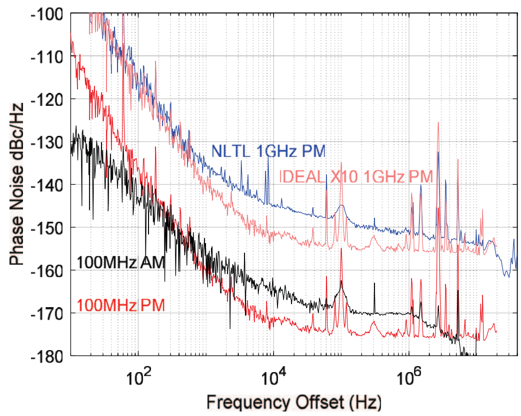


Fig. 10. Measured absolute phase noise of a x10 multiplied crystal oscillator. AM noise is converted to PM noise within the NLTL, degrading the multiplication.


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