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Incorporating Theory-Consistent Endogenous Markups into Applied General-Equilibrium Models

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The incorporation of increasing returns and imperfect competition into applied general-equilibrium (AGE) models, beginning with Harris (1984), led to much larger welfare effects from changes such as trade liberalization. But the imperfect competition side of these industrial organization (IO) developments has often failed to incorporate meaningful strategic behavior, largely ruling out firm-level productivity and scale effects. I show here that the incorporation of theory-based endogenous markups into AGE models is not difficult by employing the template of non-linear complementarity. I first derive the optimal markup equations for Nash Cournot and Nash Bertrand competition in a constant elasticity of substitution (CES) environment with free entry and exit. The first model is a simple closed-economy model where three alternatives are considered: large-group monopolistic competition (LGMC), small-group Cournot (SGC) and small-group Bertrand (SGB). Growth in the economy, a parable for trade liberalization among similar economies, is the experiment used to compare these specifications. The gains to initially small economies are much larger under either small-group assumption relative to LGMC, but diminish relative to LGMC as economies grow large. I also show how the contributions of variety (entry), firm scale (productivity), and markups (distortions) to welfare changes differ substantially among the three alternatives. The second model is a two-country trade model where the experiment is a reduction in trade costs using the SGB case and compares it to LGMC. A fall in trade costs increases firm scale, lowers markups and boosts welfare in spite of a fall in variety. Under LGMC, firm scale and markups are constant, but no fall in variety.

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1. Introduction

In its first several decades, applied general-equilibrium (AGE) modeling relied on the assumptions of perfect competition, constant returns to scale, and Armington product differentiation in trade models, the latter ensuring interior solutions to systems of equations. Many important trade distortions such as quantitative restrictions and real trade costs were converted into ad valorem tax equivalents which, in my opinion, biased the results of counterfactual experiments. I addressed some of these limitations in an earlier paper Markusen (2021), showing how modern tools and software, particularly non-linear complementarity, allow us to build more complex models which are entirely tractable for computing.

This is, in a sense, a companion paper that heads further down that road. It is also a pedagogic work that takes existing theory seriously and shows how AGE models can be made more realistic. The specific target of this paper is the incorporation of theory-consistent imperfect competition into an environment of increasing returns to scale. The incorporation of increasing returns and imperfect competition into AGE models, beginning with Harris (1984) and Cox and Harris (1985) led to much larger welfare effects from simulated changes such as trade liberalization. But the imperfect competition side of these industrial organization (IO) developments has generally failed to incorporate meaningful strategic behavior, largely ruling out firm-level scale and pro-competitive effects.

Specifically, a large share of papers involving imperfect competition use the "largegroup" monopolistic competition specification (LGMC), which assumes that firms are so small that they cannot affect their industry's price index, and hence have constant and exogenous markups. It is bafflingly inconsistent to assume that firms produce with increasing returns to scale, yet have no mass. This has remained true in almost all papers modeling heterogeneous firms, where the most productive firms are very large relative to their industry average. A newly published paper by Balistreri and Tarr (2022) is a major step forward in incorporating heterogeneous firms into AGE models, but it continues to use LGMC. Perhaps my paper can also be seen as a companion paper to Balistreri and Tarr, with the two together pointing the way to modeling heterogeneous firms with non-zero market shares and variable markups added.

There are a number of problem with LGMC that leave our AGE models detached from important realities. (1) trade liberalization (or increased protection) creates no firm-scale effects, no increase (decrease) in productivity. There are no pro-competitive effects, no fall in markups, no strategic behavior. (2) In heterogeneous firm models, all firms charge the same markups. Price ratios are proportional to marginal cost ratios across firms. (3) In models with endogenous multinationals where firms chose between exporting and foreign affiliate production, their choices are not affect by the size of the foreign market. By making firm size constant, growth only adds more varieties at constant scale. No firm will bear the fixed costs of switching to a foreign plant as the foreign market size grows. (4)

To emphasize the point that LGMC is devoid of any of the key features of industrial organization economics, large group monopolistic competition with constant markups is equivalent to a simple formulation of production with industry-level external economies of scale. A literature review is provided at the end of this section.

Why then are endogenous markups and firm scale effects avoided? Endogenous markups create an added complexity for analytical (algebraic) solutions to models. Under typical formulations such as Bertrand and Cournot, the markup depends on a firm's market share, which is of course an endogenous variable. Even with identical firms as in typical monopolistic competition models, the market share depends on the number of firms in equilibrium, and this in turn depends on total income. These dependencies means that the markup must be solved for simultaneously with all other endogenous variables. Yet many authors persist in using LGMC in numerical simulations and counter-factuals. But if we forsake strictly analytical methods for numerical ones, these endogeneity complications are lifted: a non-linear complementarity formulation and solver handles large number of equations and variables, weak inequalities, corner solutions, large parameter changes, and the simultaneity among firm scale, markups and total income. Since AGE modeling focuses on numerical simulation, the incorporating of variable markups should or could be a standard feature of modeling.

As noted above, the purpose of the paper is to demonstrate how our basic AGE models can be extended to incorporate Nash competition in increasing-returns sectors. The first objective is to derive markup formulae under small-group Cournot (SGC) and Bertrand (SGB) conjectures and identify clearly the limiting assumption that leads to constant markups, the LGMC case. Second, I will present a simple general-equilibrium model with endogenous markups and code it into GAMS. Third, I will discuss and identify some awkward calibration and interpretation issues when comparing counter-factual results under Cournot, Bertrand and LGMC alternatives. Specifically, I will present two alternative ways to calibrate the same benchmark data to the three different behavioral assumptions.

Fourth, I will compare simulations under the three alternatives SGC, SGB, and LGMC for each of the two alternative calibration strategies. The experiment is growth in the economy, a parable for trade among similar economies first exploited by Krugman (1979). While the overall effects of growth on welfare are qualitatively similar, the gains to initially small economies are much larger under either small-group assumption relative to LGMC, but diminish relative to LGMC as economies grow large in one calibration but not in the second one.

The drivers of the welfare gains under the three cases are quite different. There is a tension between added varieties and increased firm scale, and therefore productivity. The LGMC model has the largest expansion of firm (variety) numbers from growth, but no change in firm scale, markups or efficiency. SGC under the limiting assumption that varieties are perfect substitutes derives its welfare gains from growth entirely the other way around. There is no variety effect, but firm scale increases, moving firms down their average cost curve to higher productivity. SGB with differentiated products lies in between.

I then develop a two-country trade model where each identical country is the same as the close-economy case just discussed. The experiment is reduction in bilateral trade costs using the SGB case in comparison with LGMC. A fall in trade costs increases firm scale, lowers markups, and boosts welfare, but has a fall in product variety. LGMC has no changes in firm scale, markups or product variety.

2. Related literature

The purpose of the paper is pedagogic, proving what I believe to be an efficient and straightforward way of incorporating Nash Cournot and Nash Bertrand into AGE models using the robust non-linear complementarity framework of Mathiesen (1985) and Rutherford (1985, 1995). As such, it is not directly building on or revising earlier theoretical or AGE work, but nevertheless a (non-comprehensive) literature review adds context. First, there was a substantial literature involving the US-Canada free trade agreement and/or NAFTA in the late 1980s and early 1990s. While it is beyond the scope of my paper to review a very large number of contributions, I recommend two collected volumes that reprinted some of the best known journal articles of that period. These are (editors) Francois and Shields (1994), which focuses on AGE models addressing NAFTA, and Whalley (1985) which focuses on US-Canada and is broader than just AGE models. Second, there were quite a few theoretical papers in the 1980s and 1990s that addressed various aspects of increasing returns to scale and imperfect competition. Many of these concerned strategic trade policy or other normative/policy issues and are not models that slot easily into AGE modeling frameworks. But again, many relevant journal articles are reprinted in two collected volumes, (editors) Grossman (1992) and Neary (1995), the latter a much larger and broader collection than just industrial organization models of trade.

I will mention a few much more recent approaches, with a particular focus on those that address endogenous markups empirically. There are two papers that are closely related to mine. Both are more ambitious than the present paper, but I take things in some different directions. One is Francois, Manchin, and Martin (2013). They have the similar overall goal of considering alternative market structures in AGE models. They derive a markup formula in constant elasticity of substitution (CES) environment that is equivalent to my Bertrand markup. They do not appear to derive the perceived demand elasticity for SGC except for the case of perfect substitutes, a special case of my more general formula. Francois et. al. also don't provide a bridge as to how to incorporate this into a computational model, a major goal here, although Francois in particular has certainly done so in earlier work. Finally, they use a iterative (solve and update) procedure that is not necessary with the non-linear complementarity tools I present here. This paper is also valuable in citing much earlier work that I am skipping.

The second is Heid and Stähler (2023). They have endogenous markups and compare Cournot and Bertrand competition with (non-strategic) monopolistic competition. Reassuringly, their formulas for Cournot and Bertrand are the same as mine. Their experiment is reduced trade costs in a multi-country world, where each country has a "national champion" firm. The reallocation and general fall in firms' market shares, important variables in the markup equations, leads to significantly greater welfare effects under Cournot and Bertrand. There is no entry or exit of firms, while this is a crucial feature of my approach.

There are a number of other related papers, though none of these are AGE models in the usual sense. An important early paper is Levinsohn (1993). He hypothesizes that trade liberalization causes firms to behave more competitively and finds that this is supported in the data. Levinsohn's markup formula is the same as my SGC equation if firm's products are perfect substitutes, but also adds in a multiplicative conjectural variable parameter. Bernard et al. (2003) features variable markups, which they describe as having a Bertrand foundation. This is however a different concept than the one I have here, and I comment more on this a little further down. These two papers are important for documenting that constant markups and firm scale are not consistent with empirical evidence, and that the latter in turn require variable markups.

Atkeson and Burstein (2008) use a Cournot-type assumption about firm behavior in a CES environment. Markup equations quite similar to my Cournot formula and the same as mine if preferences are Cobb-Douglas across sectors. This paper also relies on an iterative procedure which is not needed in non-linear complementarity. Melitz and Ottaviano (2008) introduce variable markups into their heterogeneous firm model. But a drawback, in my view, is that they use quasi-linear preferences which remove any income effects from demand for the sector's output. This seems counter-empirical, but it does remove the simultaneity among firm scale, markups, and income.

Feenstra (2010) identifies an important role of reduced markups in the overall gains from trade. But in arguing for his alternative approach, he seems to suggest that CES functions imply constant markups. My formulation here makes it clear that constant markups and fixed firm scale are not a characteristic of CES preferences, but are due to the *added assumption* that firms have no mass, or alternatively that a firm's market share is zero, even for the largest firms. Behrens and Urata (2012) use a variable elasticity of substitution formulation to address similar issues in a model that includes variable markups and pro-competitive effects. However, the paper also makes the incorrect assertion that a CES formulation has no firm-scale and procompetitive effects. Again, this is due to the zero market-share assumption not to CES.

Amiti, Itskhoki, and Konings (2014) have variable markups in a CES framework. I had some difficulty in understanding how the markup rule is derived from underlying imperfectly competitive behavior, but the role of market shares is quite similar to what I derive. Atkin et al. (2015) show that larger firms charge higher markups, and that the elasticity of markups with respect to firm size is significantly greater than the elasticity of costs. De Loecker et al. (2016) examine how prices, markups, and marginal costs respond to trade liberalization. Markups are estimated empirically, with no theoretical concept imposed on the data. They also show that constant markups are not consistent with the data.

Hsu, Lu, and Wu (2020) have a variable markup mechanism in an environment of heterogeneous firms that seems closely related to the mechanism in Bernard et al. (2003). As noted above, they describe behavior as Bertrand, but this concept of Bertrand is not consistent with the what I am deriving as the classic Nash Cournot and Bertrand mechanisms. In both their papers, the most productive firm in a sector prices at the minimum of either its (Bertrand) monopoly price, or the marginal cost of the second most productive firm. I would characterize this as more a limit pricing or preemption strategy. Hsu et. al. do find that pro-competitive gains do account for a sizeable proportion of the gains from trade-cost reduction which is the important motivation for my paper.[1]

The present paper also overlaps with Impullitti, Licandro, and Rendahl (2022) who model variable markups with free entry in a trade model, but focus on quite different issues from myself. Arkolakis et al. (2019) also overlaps with my basic approach, but use a demand system that makes comparisons difficult. One of their results is that models with variable markups have welfare results not very different from those with constant markups. Here I show that the difference depends very much on the initial size of the economies (again using the growth of one economy as a simple substitute for combining identical economies). Initially small economies, get a much bigger boost under endogenous markups than initially large ones relative to LGMC. Further, other important aspects of general equilibrium (other than welfare) such as firm scale and productivity differ significantly among the three cases.

Finally, while I will stay away from heterogeneous firms in this paper, articles by Balistreri, Hillberry, and Rutherford (2011), Balistreri and Rutherford (2013), and Bekkers and Francois (2013) provide a good measure of what has been accomplished.

[1] I define a Bertrand equilibrium as the solution to Nash best-response behavior where firms view the other firms' prices as fixed. Especially if the goods are poor substitutes, it will generally not be optimal for the most productive firm to lower its price to the marginal cost of the next most productive firm to block entry, and that is not a Nash equilibrium in any case.

3. The CES Marshallian demand function

An appendix to the paper derives the Marshallian (uncompensated) CES demand function, which will be a review for most readers, or an asset for your students to exploit. I do not derive a general case, but stick with a special case which is popular in the extensive theoretical and empirical literatures. This special case involves a two-sector economy, with Cobb-Douglas preferences between the two sectors. One sector, Y, is a homogeneous good produced with constant returns to scale by a competitive industry. The other sector is composed of an endogenous number of symmetric but imperfectly substitutable products, X, with an elasticity of substitution $\sigma > 1$ among the varieties.

Utility or welfare (W) of the representative consumer between sectors, and the symmetry of varieties within a group of goods allows us to write utility as follows ($0 < \alpha, \beta < 1$).

$$W = X_c^{\beta} Y^{1-\beta} \qquad X_c \equiv \left[\sum_i^N X_i^{\alpha} \right]^{1/\alpha} \qquad \sigma = \frac{1}{1-\alpha} \tag{1}$$

where the number of varieties N is endogenous and X_c is often referred to as a composite commodity. X_c is a utility value, not the sum over varieties of the total units produced.

$$Y = (1-\beta)I \quad X_c = \frac{\beta I}{e} \qquad e(p) = \min(X_i) \sum_i p_i X_i \quad st \quad X_c = 1 \tag{2}$$

A convenient feature of the Cobb-Douglas upper nest is that the share of expenditure on each sector is a constant. Let $I_x = \beta I$ be the expenditure on X in aggregate. The appendix solves for the demand for a given X variety and for the price index e. These are given by

$$X_i = p_i^{-\sigma} \left[\sum_j p_j^{1-\sigma} \right]^{-1} I_x \quad e = \left[\sum_j p_j^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \qquad X_i = p_i^{-\sigma} e^{\sigma-1} I_x \tag{3}$$

$$e = N^{\frac{1}{1-\sigma}} p \qquad \text{if all prices equal} \tag{4}$$

An increase in the *range* of goods lowers the cost e of buy a unit of (sub)utility X_c. As one quick check, note that (3) is homogeneous of degree zero in prices and income as it should be.

4. Bertrand price elasticity of demand for an individual good (holds *prices* of other goods constant)

We can now derive an individual firm's perceived price elasticity of demand under the Bertrand assumption that the firm views the prices of other firms (varieties) as constant, and also views total income as constant. The latter also implies the firm views income spent on X goods, I_x, as constant under the assumption that preferences are Cobb-Douglas between X_c and Y. Using (3), we can derive the share of X sector expenditure on variety X_i, denoted s_i.

$$X_i = p_i^{-\sigma} \left(\sum p_j^{1-\sigma}\right)^{-1} I_x, \quad s_i \equiv \frac{p_i X_i}{I_x} = p_i^{1-\sigma} \left(\sum p_j^{1-\sigma}\right)^{-1} \tag{5}$$

To visually simplify the algebra a bit, we will us the following shorthand

$$(\ldots) = \left(\sum p_j^{1-\sigma}\right) \tag{6}$$

The response of demand to an increase in the firm's own price, holding other prices constant and expenditure on the sector constant, is given by

$$\begin{aligned} \frac{\partial X_i}{\partial p_i} &= -\sigma p_i^{-\sigma-1}(\ldots)^{-1} I_x - (1-\sigma) p_i^{-\sigma}(\ldots)^{-2} p_i^{-\sigma} I_x \\ &= -\sigma p_i^{-\sigma-1}(\ldots)^{-1} I_x + (\sigma-1) p_i^{-2\sigma}(\ldots)^{-2} I_x \end{aligned} \tag{7}$$

We can then derive the perceived Bertrand elasticity.

$$\begin{aligned} p_i \frac{\partial X_i}{\partial p_i} &= -\sigma p_i^{-\sigma}(\ldots)^{-1} I_x + (\sigma-1) p_i^{-2\sigma+1}(\ldots)^{-2} I_x \\ \frac{p_i}{X_i}\frac{\partial X_i}{\partial p_i} &= -\sigma + (\sigma-1) p_i^{-\sigma+1}(\ldots)^{-1} = -\sigma + s_i(\sigma-1) \end{aligned} \tag{8}$$

A convention is to define the Marshallian price elasticity as positive in order to aid memory: all variables and parameters in the model are positive (nonnegative for variables).

$$\eta_b = -\frac{p_i}{X_i}\frac{\partial X_i}{\partial p_i} = \sigma - (\sigma-1) p_i^{-\sigma+1}(\ldots)^{-1} = \sigma - s_i(\sigma-1) \quad \text{Bertrand} \tag{9}$$

There are several things to note about this elasticity. (a) as the firm's market share goes to zero, the demand elasticity converges to σ, the elasticity of substitution among the X goods. This is precisely the case of *large group monopolistic* *competition* so widely used in the literature: bizarrely, even though firms have increasing returns to scale, they all have zero market share. But this assumption, though it defies logic, is immensely useful in that it reduces the simultaneity of a model by making the firm's markup exogenous to all other variables in the model. (b) the firm's Bertrand perceived elasticity is decreasing in the firm's market share, and becomes equal to one when the firm is a monopolist in the X sector: $s_i = 1$.

5. Cournot price elasticity of demand for an individual good (holds *quantities* of other goods constant)

The appendix shows that we can solve for the inverse demand functions using the same procedure. The inverse demand function is

$$p_i = X_i^{-\delta} \left[\sum_j X_j^{1-\delta} \right]^{-1} I_x \qquad \delta \equiv \frac{1}{\sigma} \tag{10}$$

which is homogeneous of degree zero in all quantities and income. The expenditure share on good i given by

$$s_i = \frac{p_i X_i}{I_x} = X_i^{1-\delta} \left(\sum X_j^{1-\delta} \right)^{-1} \qquad (\ldots) \equiv \left(\sum_j X_j^{1-\delta} \right) \tag{11}$$

We can now use the same procedure as in the previous section to get the inverse Cournot (holds *quantities* of other goods constant) perceived elasticity of demand

$$\begin{aligned} \frac{\partial p_i}{\partial X_i} &= -\delta X_i^{-\delta-1}(\ldots)^{-1} I_x - (1-\delta) X_i^{-\delta}(\ldots)^{-2} X_i^{-\delta} I_x \\ &= -\delta X_i^{-\delta-1}(\ldots)^{-1} I_x - (1-\delta) X_i^{-2\delta}(\ldots)^{-2} I_x \end{aligned} \tag{12}$$

$$X_i \frac{\partial p_i}{\partial X_i} = -\delta X_i^{-\delta}(\ldots)^{-1} I_x - (1-\delta) X_i^{-2\delta+1}(\ldots)^{-2} I_x$$

$$-\frac{X_i}{p_i} \frac{\partial p_i}{\partial X_i} = \delta - (\delta - 1) X_i^{-\delta+1}(\ldots)^{-1} = \delta - (\delta - 1) s_1 \tag{13}$$

$$\frac{1}{\eta_c} = \frac{1}{\sigma} - \left(\frac{1}{\sigma} - 1 \right) s_i = s_i + (1 - s_i) \frac{1}{\sigma} \tag{14}$$

Invert this to get the Cournot perceived elasticity

$$\eta_c = \frac{\sigma}{\sigma s_i + (1 - s_i)} = \frac{1}{s_i + (1 - s_i)\frac{1}{\sigma}} \qquad \text{Cournot} \tag{15}$$

Although the Cournot elasticity seems quite different from the Bertrand formula in (9), they have the same values at the extremes $s_i = 0$ and $s_i = 1$. Here is the comparison of (9) and (15):

At	$s = 0,$	$\eta_c = \eta_b = \sigma$	(LGMC)
At	$s = 1,$	$\eta_c = \eta_b = 1$	(monopoly)
For	$0 < s < 1,$	$\eta_c < \eta_b < \sigma$	(Cournot is less elastic)
For	$0 < s < 1$ and $\sigma = \infty,$	$\eta_c = \frac{1}{s}, \eta_b = \infty$	(perfect substitutes)

For this last case of perfect substitutes, the Bertrand elasticity is infinite, which implies perfect competition. However, this cannot be supported in equilibrium with increasing returns to scale. The Cournot perceived elasticity becomes simply the firm's inverse market share.

The relationship between the firm's perceived elasticity of demand and its market share are graphed for Bertrand and Cournot in Figures 1a (perceived elasticity) and 1b (inverse - will become the markup) using the common elasticity of substitution $\sigma = 5$. As just noted, they have the same values at the extremes of market share, but the perceived elasticity is much less under Cournot for intermediate values. It is probably well understood that the firm's markup is related to the inverse of the demand elasticity, so this in turn tell us that Cournot will be a less competitive form of behavior, *ceteris paribus*.

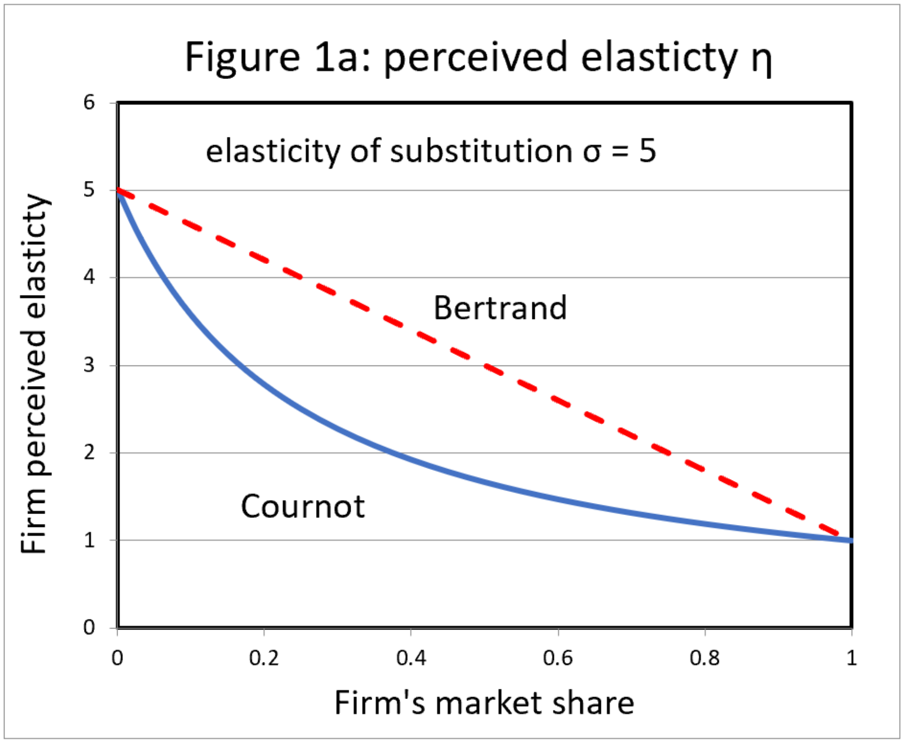
6. Markup formulae

Consider first profits for a Cournot competitor using (10) above for the inverse demand function. Π denotes profit, C the firm's total cost, and mc denotes marginal cost. Firm i maximizes profits with respect to output holding the outputs of other firms constant.

$$\Pi_{ic} = p_i(X_i, \overline{X}_j)X_i - C(X_i) \qquad \text{for } j \neq i \tag{16}$$

Dropping the i subscript for clarity, the first-order condition is given by

$$\frac{\partial \Pi}{\partial X} = p + X\frac{\partial p}{\partial X} - \frac{dC}{dX} = p + p\left[\frac{X}{p}\frac{\partial p}{\partial X}\right] - \frac{dC}{dX} = p\left[1 - \frac{1}{\eta_c}\right] - mc = 0 \tag{17}$$



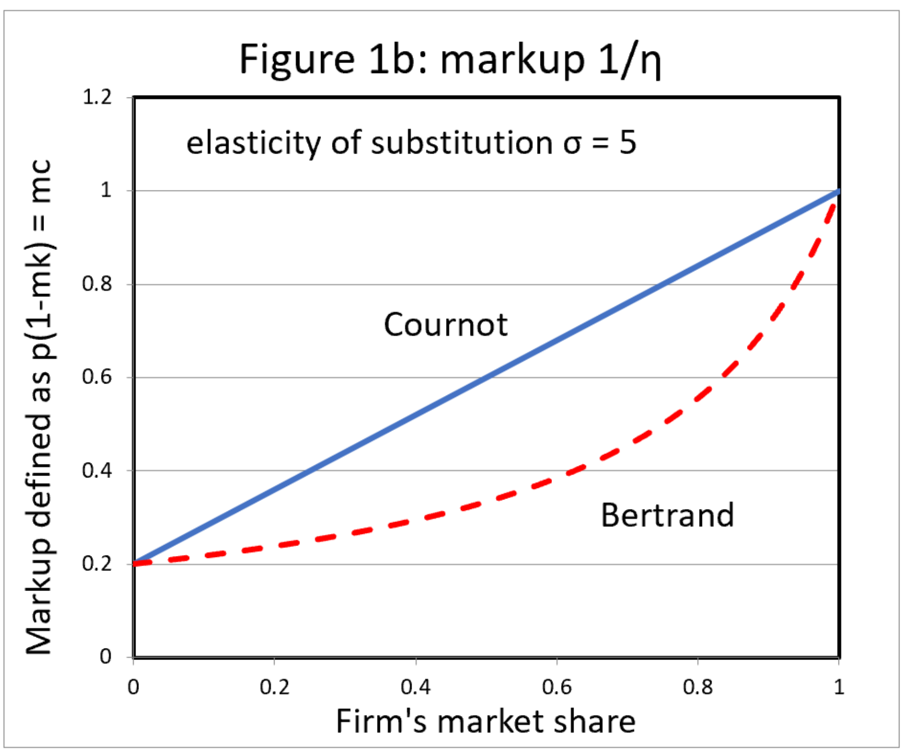


Figure 1: Perceived elasticity and markup.

The markup (m_k), is then simply $mk = \frac{1}{\eta_c}$. Profits and the first-order condition in the Bertrand case are given by

$$\Pi_{ib} = X_i(p_i, \overline{p}_j)p_i - C(X_i(p_i, \overline{p}_j)) \qquad j \neq i \tag{18}$$

$$\frac{\partial \Pi}{\partial p} = X + p\frac{\partial X}{\partial p} - \frac{dC}{dX}\frac{\partial X}{\partial p} = 0 \tag{19}$$

Multiply through by p/X (again defining η as positive)

$$\begin{aligned} &\frac{\partial \Pi}{\partial p}\frac{p}{X} = p + p\left[\frac{p}{X}\frac{\partial X}{\partial p}\right] - \frac{dC}{dX}\left[\frac{p}{X}\frac{\partial X}{\partial p}\right] = 0 \\ &\frac{\partial \Pi}{\partial p}\frac{p}{X} = p - p\eta_b + mc\eta_b = p - \left[\frac{\eta_b}{\eta_b - 1}\right]mc \quad p\left[1 - \frac{1}{\eta_b}\right] - mc = 0 \end{aligned} \tag{20}$$

Summarizing, the markups in the Bertrand and Cournot cases are the same in terms of the firm's (inverse) perceived demand elasticity, but the formula for this elasticity differs in the two cases.

$$p\left[1 - \frac{1}{\eta_b}\right] = mc \quad \text{Bertrand} \qquad\qquad p\left[1 - \frac{1}{\eta_c}\right] = mc \quad \text{Cournot} \tag{21}$$

$$p = \left[\frac{\eta}{\eta - 1}\right]mc \tag{22}$$

7. Calibration subtleties

There are four parameter/variables in the markup formulae, some of which but usually not all of which may be in your data set: p, mc, σ, and s. With identical firms, s is just the inverse of the number of active firms. In a general-equilibrium model, three of these things are variables while σ is always treated as a parameter. The problem is that, if you have data or estimates of all four, then it is almost inconceivable that they will satisfy either the Cournot or Bertrand equations in (21). If you have three of the four, it may still well be the case that the fourth cannot be set at a permissible value that satisfied either markup rule. For example, if you have p, mc and σ, there may not be any value of s between 0 and 1, or at least a realistic one, that satisfies one or both markup rules.

Calibration 1: Suppose that you have p and mc, and therefore the markup, or the markup is calculated independently by usual methods giving the wedge between price and marginal cost. Then for either Cournot or Bertrand, the elasticity of substitution, σ, and market concentration s cannot be chosen independently. You can pick an s as an average firm market share across a distribution of firms (thereby implying the calibrated number of firms = $1/s$), but there is no freedom to choose σ. If instead you have a value for σ, then s is determinant. Does it matter how you divide a calibration between s and σ in this example? Yes it does, and I will try to show this below. But to preview the point, if data is calibrated to a small s (large number of firms), then especially under Cournot competition an increase in market size (as in trade integration) will have weak pro-competitive effects relative to calibrating to a large s (small number of firms).

Now suppose that we want to run scenarios, such as trade liberalization or protection, and compare results under alternative assumptions such as Cournot, Bertrand, and large-group monopolistic competition. Further, imagine that we wish to run these three options by calibrating each case to the initial benchmark data. Finally, suppose that we have estimates of an industry's markup and therefore the wedge between price and marginal cost and we wish to hold this the same in the three calibrations. The problem is that this will require either changing the benchmark s in the three cases ($s = 0$ in large-group mc by definition) and/or changing σ, the elasticity of substitution in consumption. Refer back to the Bertrand formula in (9), and note that for an s of 0.25 (four firms), then we have to use a different σ in the Bertrand and LG cases to make $n_b = n_{lg}$ where the subscript lg refers to LG monopolistic competition.

In our free-entry equilibria, there are two equations which characterized equilibria. One is the optimization condition marginal revenue equals marginal cost, and the second is the free-entry condition that price equals average cost. Noting that the markup is each of our three cases is a (different) function of σ and market share s only, these are given by

$$p(1 - mk) = mc, \quad p = mc + \frac{fc}{X}, \quad \text{together implying} \quad pX = \frac{fc}{mk}, \quad \text{or} \quad mk = \frac{fc}{pX}$$

where $\boldsymbol{mk} = \boldsymbol{mk}(\boldsymbol{\sigma}, \boldsymbol{s})$. Revenue pX is the same value given by the same data in each case. If the three cases have the same markup, then either σ and/or the market share in each case must differ with LGMC having $s = 0$ by definition. Note also from the last equation, that if the markups are the same in the three cases, then the technologies must also be identical in the sense that the markup equals the share of fixed costs in total costs: $mk = fc/pX$ at the calibration point.

Let the observed markup be 0.2 on an output-price basis in (21) (1.25 on a (gross) marginal cost basis (22)). Then the LG elasticity of substitution calibrates to $\eta_{lg} = \sigma = 5$. The Bertrand σ (equation (14)) would have to be $\sigma = 6.333$ if it is calibrated to four firms ($s = 0.25$) to get $\eta_b = 5$. But the dilemma is now that, in comparing counter-factual experiments under LG versus Bertrand, we are using two different consumer preference specifications. So the differences between the counter-factual results are going to be partly due to the pro- (or anti-) competitive effects in Bertrand and partly due to the different preference elasticity.

This tension, and the caution needed to interpret counter-factual comparisons, also occurs with Cournot versus Bertrand. In our example of the (output price) markup 0.20 in (21), if we wish to calibrate Cournot to four firms to match

Bertrand ($s = 0.25$), then there is no σ large enough to produce a perceived elasticity of $\eta_c = 5$. If we set $\sigma = 6.333$ to match Bertrand, then the Cournot market share calibrates to $s = 0.05$ or $N = 20$. But with this many firms under Cournot, there will be virtually no pro-competitive and productivity enhancing effects of growth. If instead we set σ equal to its limiting value of $\sigma = \infty$, then calibrating to five firms ($s = 0.20$) does produce a perceived elasticity $\eta_c = 5$ and a markup of 0.20. The Bertrand-Cournot comparison is then $\sigma = 6.333$, $s = 0.25$ (Bertrand) versus $\sigma = \infty$, $s = 0.20$ (Cournot). There is some interpretative advantage to this. Since the Cournot case now has pure productivity and pro-competitive effects with no variety effect, while LGMC is the other way around, the interpretation of the comparison very clear. Let fcs denote the share of fixed costs in total costs:

$$mk = fcs = fc/pX.$$

Calibration 1 - different preferences, calibrated to identical markups and technologies

LGMC:	$\sigma = 5$	$s = 0$	$N = 4$	$mk = fcs = 0.20$
SGB:	$\sigma = 6.333$	$s = 0.25$	$N = 4$	$mk = fcs = 0.20$
SGC:	$\sigma = \infty$	$s = 0.20$	$N = 5$	$mk = fcs = 0.20$

While this makes for an interesting comparison of SGC and LGMC as just noted, there is a weakness in that a welfare comparison of comparative statics (e.g., over country size) is muddled by comparing consumers with different preferences. A second and more practical problem is that the data may not give any handle on the relative size of fixed versus variable costs (scale economies), markups, or the number of firms (market shares) in an industry.

Calibration 2: Consider an alternative in which we assume that preferences (specifically σ) and the number of firms / market shares in the benchmark are the same for SGC and SGB. Calibration of SGC can replicate the data by assuming a higher degree of scale economies (ratio of fixed to total or variable costs). That is, there is a technology difference rather than a preferences difference between SGC and SGB. Keep SGB is same as above, using its σ for all three conjectures. The value of fixed costs for SGC that replicates the benchmark is 18.4211 (up from SGB = 10.0). This does imply a significantly higher markup of $mk = 0.3684$, but it is likely that this cannot be observed in the data in the first place. Conversely, calibrating LGMC to the same substitution elasticity implies a lower markup and lower fixed-cost share.

Calibration 2 - different technologies, calibrated to identical preferences and number of firms

$$\begin{aligned}
\text{LGMC:} \quad & \sigma = 6.333, & s = 0, \quad & N = 4, & mk = fcs = 0.1579 \\
\text{SGB:} \quad & \sigma = 6.333, & s = 0.25, \quad & N = 4, & mk = fcs = 0.20 \\
\text{SGC:} \quad & \sigma = 6.333, & s = 0.25, \quad & N = 4, & mk = fcs = 0.3684
\end{aligned}$$

We can now specify the model, and consider these alternative calibrations for the same model[2]

8. Translating theory into GAMS using non-linear complementarity

In this section, I will show how to translate theory plus an initial set of micro-consistent data into GAMS. I will keep it brief, because I go through much of the process in my earlier JGEA paper, Markusen (2021). Please refer to that if the exposition here leaves too many gaps. I will go through the equations for the SGB case for the closed economy, the other two differ only in the markup equation (and parameter values for σ in the first calibration and fixed costs in the second). The other programs can be obtained from me directly. After developing the SGB case, we will run and compare this with LGMC and SGC. The trade model is developed in the next section.

The closed economy model has four activities. There are two final goods, X and Y, with Y being the traditional constant-returns, perfect competition sector, and X being the increasing-returns, imperfect competition sector. The third activity produces fixed costs of X-sector firms, with the total output of fixed costs denoted by N, which is also the number of firms (varieties) in equilibrium. There is free entry and exit, so N is an endogenous variable. There is one factor of production L, and units for these three activities are chosen such that one unit of labor produces one unit of X, Y or of fixed costs.

Utility or welfare W is the fourth activity and is treated as a produced good, with inputs X and Y. Utility is specified using the widely-used two-level CES: the top level between X and Y is Cobb-Douglas, with the X sector consisting of symmetric varieties with an elasticity of substitution $\sigma > 1$ among the individual goods. The (representative) consumer's income is denoted I.

[2] There is at least one other way of calibrating Cournot and Bertrand to the same σ and same s (or number of firms $N = 1/s$). This is to use a kluge or fudge factor, which is an exogenous parameter multiplied on the Cournot markup so as to give it the same initial value as Bertrand. As noted above at the end of section 4, $\boldsymbol{\eta_c} < \boldsymbol{\eta_b}$ for any value $0 < s < 1$, so the Cournot markup will be greater than the Bertrand markup for the same values of σ and s. The kluge parameter multiplied on the Cournot markup will thus be less than one to give it the same value as the Bertrand markup. Essentially the same technique under the term "conjectural variation" is to introduce a multiplicative parameter in the markup equation so as to make the theoretical markup such as the ones derived above be consistent with the empirical measure of markup characterizing the initial data (e.g., Levinsohn (1993).

Firm "owners" are treated like consumers, and denoted *ENTRE*. Entrepreneurs receive markup revenues and demand fixed costs. This is how to model free entry and exit: the level of this agent's income and expenditure is the number of firms N in equilibrium, and profits are zero. Markup revenue equals fixed costs, and thus consumer income is just the value of the labor endowment (no profit income).

Finally, the markup equation is the Bertrand formula given by the inverse of (9) above. This involves the substitution parameter σ and the variable s, which is an individual firm's market share (all firms are symmetric and identical, but their number N is variable). Since all (active) firms produce the same output and sell at the same price, the market share s reduces to $s = \frac{1}{N}$, so s is not specified as an additional variable.

Table 1. Variables and parameters of the simulation model

Variables:		
	X	quantity of one X good (variety)
	N	number of X sector firms/goods
	Y	total output of Y
	W	welfare
	p_x	price of one X variety
	p_n	price of one unit of fixed costs f_c
	p_y	price of one unit of Y
	p_w	price of one unit of welfare W (consumer price index)
	p_l	price of one unit of labor L
	I	income
	$ENTRE$	entrepreneur income
	mk	markup on an X good
	p_e	price index for composite X goods (sub-utility)
Parameters:		
	L	labor endowment
	σ	elasticity of substitution among X goods
	fc	fixed cost of X in units of labor L

Table 2. Simulation model formulated as a non-linear complementarity problem

$p_l \geq p_x (1 - mk)$	$\perp$	X		(23)
$p_l \geq p_n$	$\perp$	N	pricing inequalities	(24)
$p_l \geq p_y$	$\perp$	Y		(25)
$p_e^{0.5} p_y^{0.5} \geq p_w$	$\perp$	W		(26)
$X \geq p_x^{-\sigma} \left(p_e^{(\sigma-1)} \right) I/2$	$\perp$	p_x		(27)
$fc\ N \geq ENTRE/p_n$	$\perp$	p_n	market-clearing inequalities	(28)
$Y \geq I/\left(2p_y\right)$	$\perp$	p_y		(29)
$W \geq I/p_w$	$\perp$	p_w		(30)
$L \geq Y + NX + fc\ N$	$\perp$	p_l		(31)
$I \geq p_l L$	$\perp$	I	income balance inequalities	(32)
$ENTRE \geq p_x\ mk\ XN$	$\perp$	$ENTRE$		(33)
$mk \geq [\sigma - (1/N)(\sigma - 1)]^{-1}$	$\perp$	mk	definitional inequalities (Bertrand markups shown here)	(34)
$\left[N p_x^{1-\sigma}\right]^{(1/(1-\sigma))} \geq p_e$	$\perp$	p_e		(35)

Table 1 gives the variables and parameters of the model. Table 2 gives the non-linear complementarity problem matching inequalities and variables. The simulation model then consists of 13 non-linear weak inequalities in 13 non-negative complementary variables. Weak inequalities are orthogonal to their complementary variable. Labor L is chosen as numeraire, so that with the choice of units mentioned above, the marginal costs of X, Y, and N are all equal to one. Budget share of X and Y are chosen equal to 0.5 each.

Pricing inequalities are complementary with quantities: if marginal cost exceeds price in equilibrium, then the quantity is zero. Market clearing inequalities are complementary to prices: if supply exceeds demand in equilibrium, then the price is zero (free good). Please refer to the GAMS file at the end of the paper.

The top of the file gives a small data matrix, with production activities and income levels the columns, and markets as the rows. Markets are labeled with their prices, which are complementary to market clearing equations. A column is a list of output (positive) and inputs (negative) in value terms. A row lists supply (positive) and demand (negative) in that market. Thus micro-consistency requires that both row and column sums are zero.

There is a single factor of production, labor. The first column is the technology for the total units of X produced, with inputs (costs or expenditures) being 160 units of labor and 40 units markup, also units of labor. We will choose the price of labor, $PL = 1$ as numeraire ($PL.FX = 1$ in GAMS), with the 200 units of X interpreted as 160 units at a price $PX = 1.25$. Fixed costs are treated as a production activity with the (scaled) output being N, the number of firms/varieties. 40 units of labor produce 40 units of fixed costs, and this will be interpreted as 4 firms with a fixed costs of 10 (parameter FC) per firm. The benchmark markup MK will then be 0.20 (40/200), using the output-basis formula in (21) to define our markup equation.

The Y sector is perfectly competitive, using 200 units of labor to produce 200 units of Y, so PY =1 with L being the numeraire. Welfare (W) is treated as a produced good: inputs of 200 each of X and Y in value produce 400 units of welfare (utility).

The final two columns are consumer (CONS) income and entre (ENTRE) income. The consumer is endowed with 400 units of labor and spends it all on "buying" utility W. The entrepreneur receives markup revenues as income and spends all of that income buying fixed costs (starting firms). If markup revenues go up, there is an increased demand for fixed costs, which translates into an increased number of firms. The number of firms then translates into a change in the markup, which feedback to the other variables including firm scale in general equilibrium.

There are three parameters in the model, with SI being the elasticity of substitution in consumption, set at SI = 6.333 for Bertrand (in both calibrations) as discussed above. Fixed costs per firm are FC = 10, and the labor endowment is ENDOWL = 400. Then there are the lists of equations and variables, and I have listed them in the same order according to their complementarity relationship. Please refer to my earlier paper, Markusen (2021) which explains some of the other GAMS code notation.

As noted above, I have also prepared two other versions of the model calibrated to the same data, but having different values of σ and N (calibration 1). These are SGC, LGMC. In all cases, I run the same experiment to see how the results differ from one another. The experiment I report here is to repeatedly increase the size of the economy and re-solve, specifically 25 values of the labor endowment, considering values both smaller and larger than the benchmark value.

Results for the three alternatives are shown in Figures 2-5 for calibration 1. The common calibration point is normalized to size = 1, with size running from 0.2 to 2.0 on the horizontal axis. I will first describe all four set of results and then offer some interpretations about their similarities and differences.

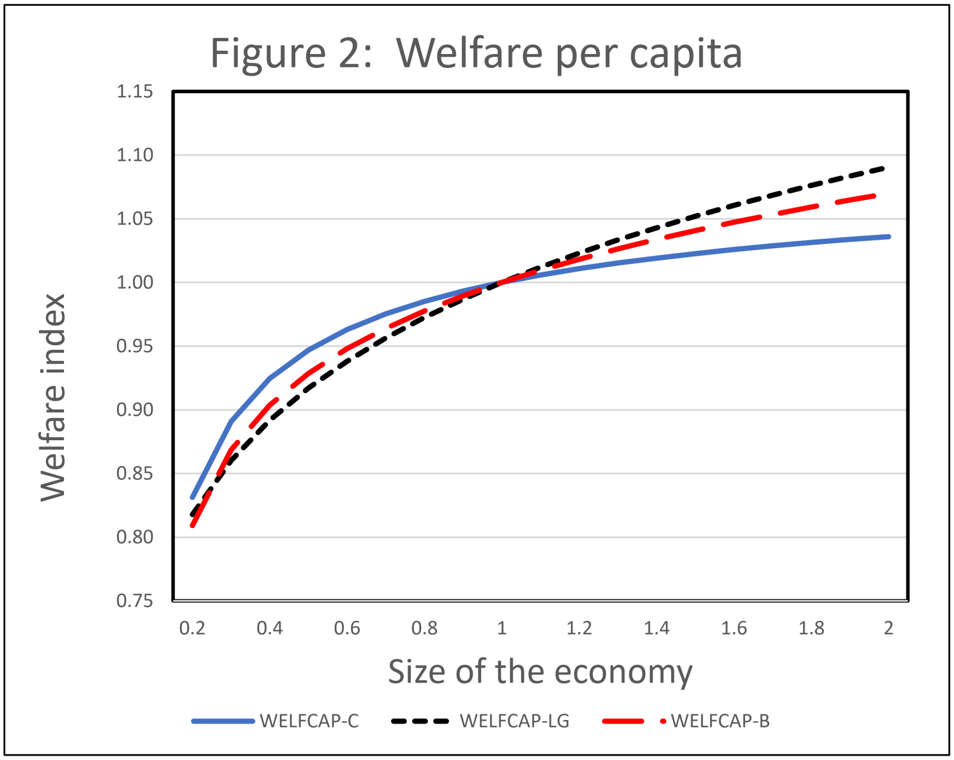
Figure 2 gives the result for welfare per capita. All curves are particularly steep for small economies, but especially for the two small-group cases. At at the upper end, welfare for the Cournot case flattens out, while LGMC is concave but steeper.[3]

Figures 3-5 are closely intertwined and simultaneously determined. Figure 3 gives an index of firm numbers. For LGMC, the number of firms is just linear in the size of the economy: double the size, double the number of firms (varieties). SGB is a slightly flatter curve, but it is also linear. SGC is concave: for small economies entry respond significantly to an increase in size, but this effect runs out of steam for large economies. A major difference among the three cases is firm size/scale (which is also productivity) as shown in Figure 4. As I'm sure is well known (and which I have complained about), LGMC produces a constant output per firm so that there is no firm-scale effect and no firm-level productivity increase from a larger economy. SGC produces a large firm-scale effect. Figure 5 compares the firm markups in the three cases. The markup is constant for LGMC as is well know. As noted before, this is the major attraction of LGMC since it removes an important endogeneity in the general-equilibrium model. As in Figures 2-4, SGB lies between LGMC and SGC in Figure 5, the latter having the biggest effect.

Although the models capture the general-equilibrium simultaneity among firm numbers, firm scale, and markups, it is still a stylized model. The important properties in Figures 2-5 should be viewed as qualitative, and the quantitative differences shown should not be given too much emphasis. We now turn to interpreting these differences in Figure 2-5. I will stick to comparing the LGMC and SGC cases, since SGB generally lies between the two.

The three per-capita welfare curves in Figure 2 are qualitatively similar, but note that there is a substantial difference for a very small economy (e.g., size 0.2-0.6), a point I will return to shortly. The basic tension between LGMC and SGC is the number of varieties (firms) versus firm scale, the latter corresponding to productivity. A larger number of varieties raises consumer welfare as is very well understood: half as much each of twice as many varieties raises utility. But there are no production efficiency effects with LGMC. SGC, with the calibrated value of $\sigma = \infty$ (perfect substitutes), has no variety effect, but has a strong productivity effect in the growing economy. Firms move down their average cost curve. One approach generates direct utility gains, the other direct production efficiency

[3] This result that the elasticity of *per-capita* welfare with respect to economic size is a constant under LGMC is a general result for the functional form of welfare used here, but the specific *value*. shown here is due to the assumptions of Cobb-Douglas shares on Y and X of 0.5 and on the assumption of an elasticity of substitution of 5. This elasticity is also constant under external economies of scale.



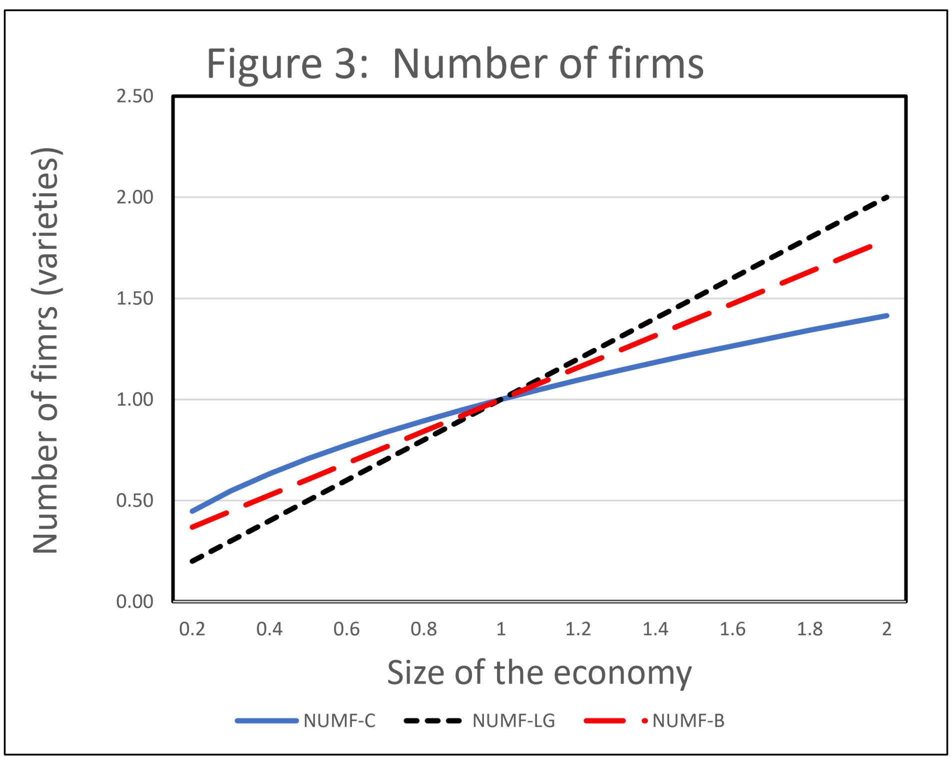
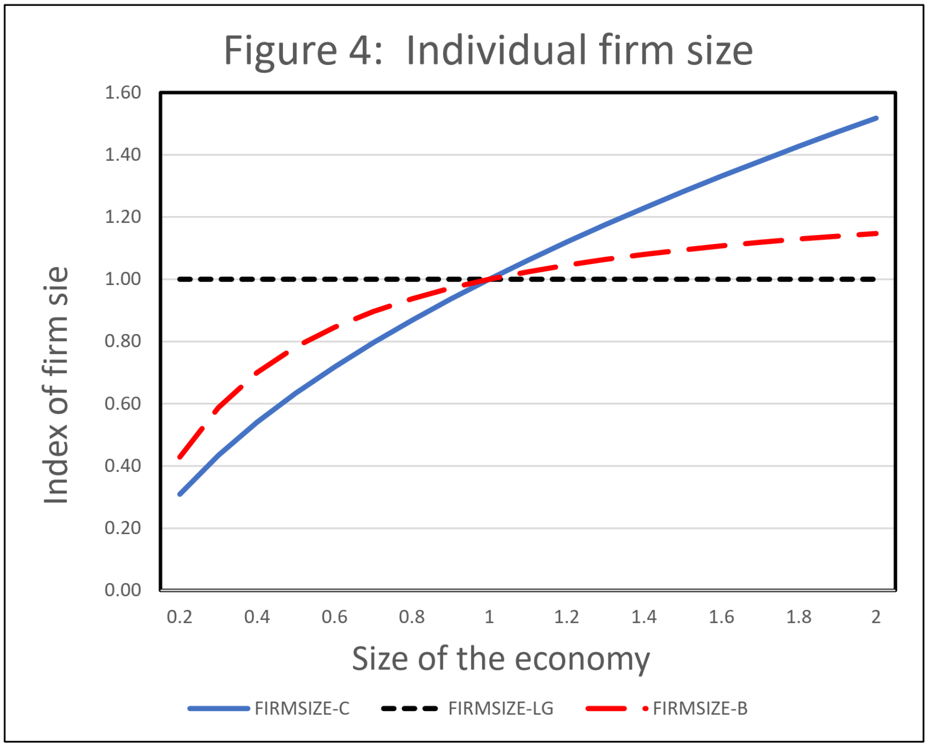


Figure 2 and **Figure 3**: Welfare and firm numbers, calibration 1.



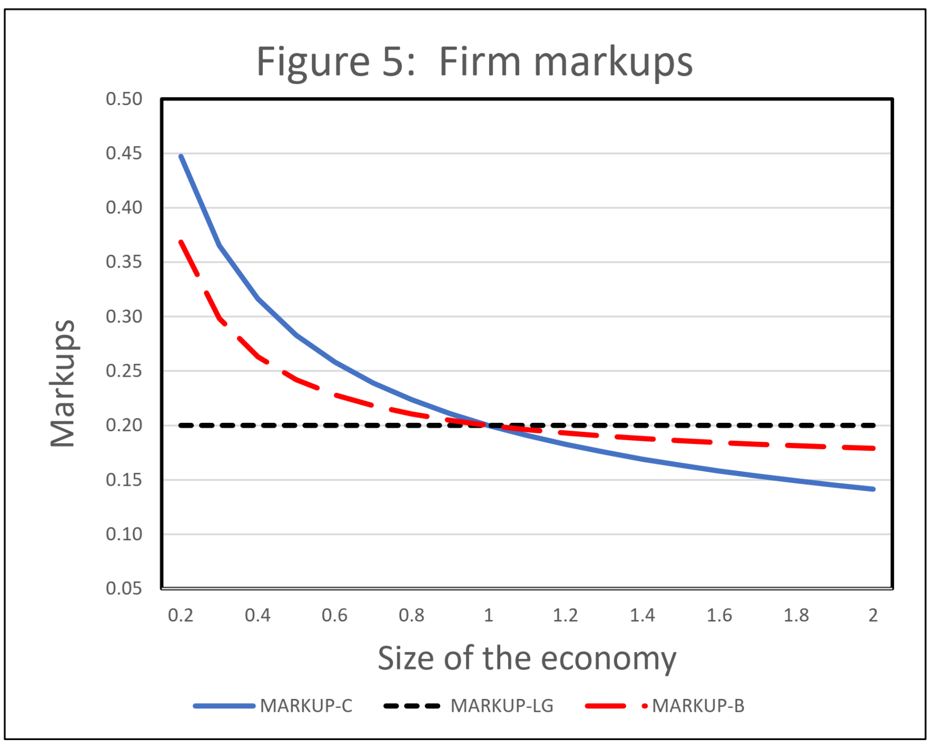


Figure 4 and **Figure 5**: Firm size, firm markups, calibration 1.

gains. But although these very different effects result in similar per-capita welfare effects in Figure 2, I do not make any claim that this would hold in more complex models.

In our free-entry zero-profit models, firm scale and firm markup are inextricably linked. But conceptually, there are two separate welfare effects which could show up separately in a model without free entry. First, there is a pure production efficiency effect: when a firm increases output, it moves down its average cost curve, increasing productivity. Second, there is a distortion-reduction gain as the wedge between price and marginal cost is reduced. This can be termed a pro-competitive effect. In this paper with zero profits in equilibrium, these cannot be separated: the lower (gross) markup defined as in (22) is just equal to *p/mc* = *ac/mc* where mc is constant. But consider, for example, a model with a monopolist in sector X (the elasticity of substitution between X and Y will now need to be greater than one, Cobb-Douglas won't work). As the economy expands, X output will expand lowering average cost, leading to a per-capita welfare gain. But there will be no change in the markup.

I do think that there is some intuition from theory as to why LGMC generates a larger welfare gain relative to SGC starting at the calibration point size = 1, and a smaller gain starting at the left of Figure 2 with a very small economy. Figures 2-5 indicate that for small-group cases, particularly SGC, the effect of growth on per-capita welfare diminishes as the number of firms gets much beyond 5. If a model is calibrated with 10 firms, for example, then increases in size will show very little welfare effects in the SGC while effects in the LGMC case will continue to be strong. For SGC, the effect of growth on the crucial variable output per firm (falling average cost) diminishes as output grows. The average cost curve is a rectangular hyperbole. The per-capita effect of LGMC due to added varieties is more sustained as the economy grows.

The corresponding results for calibration 2 are shown in Figure 6a-d. Many of the underlying explanations and intuition are very similar, so I'll try to avoid repetition. Referring back, all three cases in calibration 2 have the same σ, so any variety gains from more firms are now the same in all three cases. Figure 6a shows that we now get an even bigger boost to growth in a small economy under Cournot. I think that this is due to the fact that with much higher fixed costs (bigger scale economies), firms in a small economy move down a steep average cost curve (move quickly to greater productivity) than in the other two cases.

Panel 6b shows that the variety gains under LGMC are the largest with growth, SGC has the least with SGB in between. Panel 6c is intuitively linked to these other effects. Similar to Figure 3, Cournot is delivering its welfare gains though firm efficient effects (though now there are variety welfare gains as well), whereas LGMC is producing gains solely through added varieties. The biggest difference between the two calibrations is seen in comparing Figure 5 with Figure 6d. Because we are now calibrating to different technologies in calibration 2 rather than different preferences in calibration 1, markups are different at the benchmark point. The SGC markup is always much larger than the other two. The sharp fall in the markup and the sharp rise in output per firm in panel 6b are the intuition behind the strong rise in welfare for a small economy as growth proceeds.

To illustrate the importance of falling average costs and pro-competitive gains for a small economy, we can renormalize welfare in Figures 2 and 6a, giving each case a welfare value of one at size 0.2. Calibration 1 is shown in Figure 7a, and calibration 2 in Figure 7b. Let's think of these as three different economies, have three different initial welfare levels. What we are doing is looking at the *proportional* gain in each of those welfare levels starting at size = 0.2. The gains from growth are larger for SGC and SGB starting at a small size for calibration 1 and far larger in calibration 2. This is, I believe, due to the fact that a given expansion in firm scale as shown in Figures 4 and 6c generates a large decline in average cost (increase in productivity). Or we can put it another way, which is that the expansion in firm scale causes a sharp decline in the wedge between price, equal to average cost in a free-entry model, and (constant) marginal cost, which is the steep fall in the markup as shown in Figure 5 and 6d. On the right-hand section of Figure 7a, the Cournot economy runs out of steam because the average cost curve is now very flat and there is no variety effect. But in Figure 7b for calibration 2, the fact that SGC continues to deliver strong welfare gains for a much larger economy is at least partly due to the fact that it is now delivering the same variety gain from an added firms as the other two behaviors while it does not under calibration 1.

9. Two-country Trade Model

Now turn to a two-country trade model, where the experiment will be reducing trade costs. To keep things as straightforward as possible, each country individually will be identical to the single-economy version of the previous sections (and therefore identical to one another). I will analyze the Bertrand case and compare it to LGMC. The Cournot case has very similar qualitative properties to Bertrand. Once again, an important objective here is to argue that incorporating endogenous markups into general-equilibrium models is not difficult, rather than trying to highlight specific formulations and results. Here is a list of the components and assumptions of the model:

- Two identical countries h and f, each the same as in the previous sections. Iceberg trade costs on X, no trade costs on Y, no fixed cost to exporting.
- Firms set prices independently in domestic and export markets (segmented markets).
- Small-group Bertrand competition contrasted to integrated-markets LGMC

The code for this model is given in an appendix to the paper. While I don't believe it is valuable to run through the entire specification of the model as in

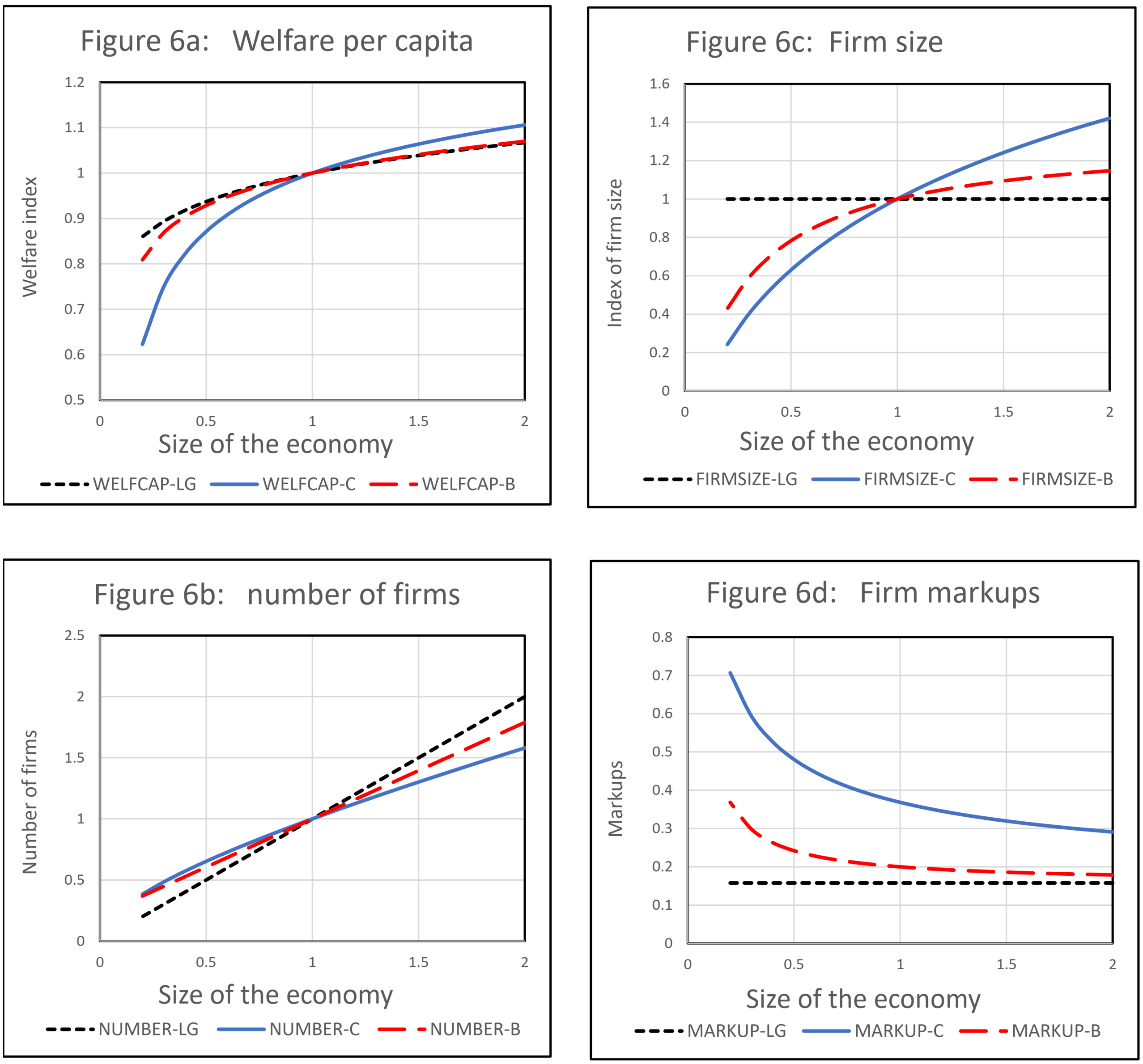
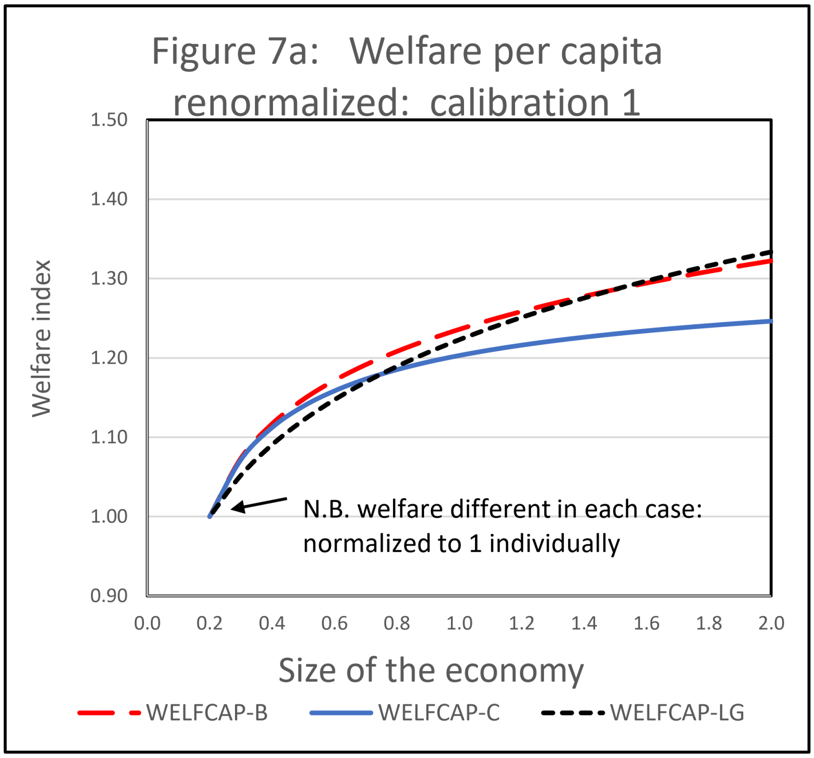


Figure 6: Calibration 2 - technology rather than preferences.

Notes: Bertrand same as in Figures 2-5.



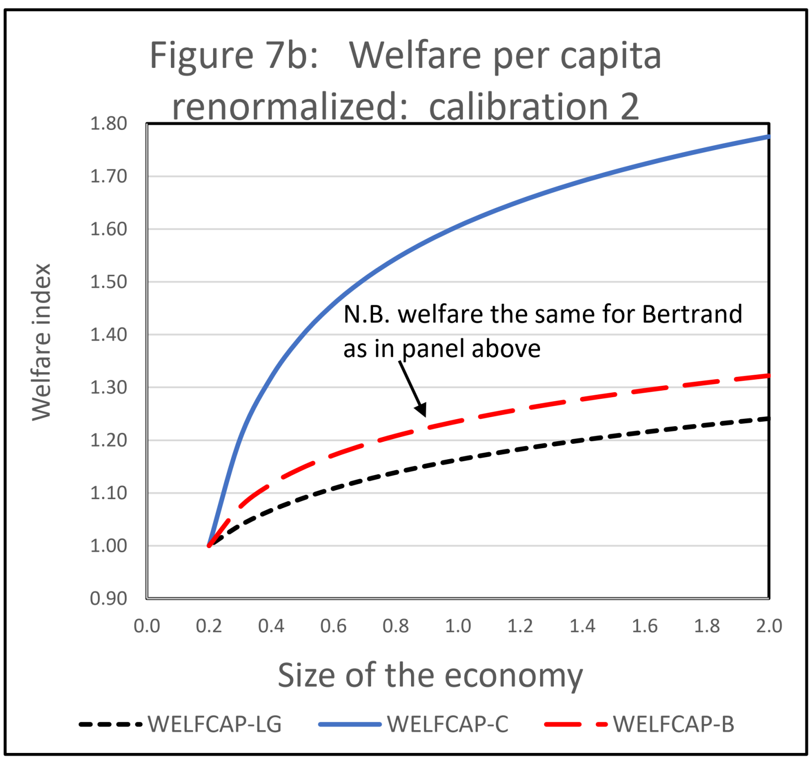


Figure 7: Comparison of calibrations 1 and 2.

Table 2 above, I will just note that the important modifications are in the market share terms in the markup equations and entrepreneur's income and entry conditions. Drop the X subscript on p_x (avoiding triple subscripts) and let the double subscript (ij) denote a variable such as output or markup shipped from country i to j. Gross trade cost (one plus the trade cost rate) tc increases the marginal cost of producing in i and exporting to j to $p_i tc$, and reduces the amount produced, X_i, to the quantity received and consumed in the other country to X_i/tc. For the X firms in country h, the pricing equations for domestic and export production are as follows, symmetrically for country f (for modeling trade or transactions cost versus a tax, see Markusen, 2021).

$$p_l \geq p_{hh}(1 - mk_{hh}) \qquad p_l tc \geq p_{hf}(1 - mk_{hf}) \qquad \perp \quad X_{hh}, X_{hf} \tag{36}$$

$$X_{hh} \geq p_{hh}^{-\sigma}(p_{eh}^{(\sigma-1)})I/2 \qquad X_{hf}/tc \geq p_{hf}^{-\sigma}(p_{ef}^{(\sigma-1)})I/2 \qquad \perp \quad p_{hh}, p_{hf} \tag{37}$$

The markup inequalities for domestic and export sales for the country h firms have the same Bertrand formula, but more complex market share equations.

$$mk_{hh} \geq [\sigma - s_{hh}(\sigma - 1)]^{-1} \qquad s_{hh} = \frac{p_{hh}X_{hh}}{p_{hh}N_h X_{hh} + p_{fh}N_f X_{fh}/tc} \tag{38}$$

$$mk_{hf} \geq \left[\sigma - s_{hf}(\sigma - 1)\right]^{-1} \qquad s_{hf} = \frac{p_{hf}X_{hf}/tc}{p_{hf}N_h X_{hf}/tc + p_{ff}N_f X_{ff}} \tag{39}$$

The equations for entrepreneur income (spent buying fixed costs) now includes markup revenues from home and foreign.

$$ENTRE_h \geq mk_{hh}p_{hh}X_{hh} + mk_{hf}p_{hf}X_{hf}/tc \tag{40}$$

Figure 8 presents results in four panels (similar to Figure 6) for a simulation which loops over (gross) trade costs, running from a high of 2.0 on the left to 1.0 (costless trade) on the right. The size of each country is held constant and, with the countries identical, results need only be presented for one country. The model uses calibration 1 above, and each country's (fixed) size is one-half that in the previous section's calibration. Thus in free trade on the right-hand side of Figure 8 panels, the model is effectively identical to the closed economy version with size = 1.

Welfare per capita is convex in decreases in the trade cost (panel 8a), and very similar for the SGB and LGMC cases. Panel 8b shows that the number of firms in each country actually falls with liberalization while the output per firm rises as shown in Figure 8c under SGB, while both firm numbers and firm scale don't change under LGMC. The overall increase in firm scale is composed of falling domestic sales in competition with foreign imports outmatched by increasing export sales under both SGB and LGMC. (I haven't shown the breakdown for LGMC to avoid too much clutter.) It is worth noting that with CES product differentiation and no fixed costs of entering exporting, any firm that produces domestically will always export something no matter how high the trade costs. This is because the demand price with CES goes off to infinity as the quantity (exported) goes to zero. This effect is also present with the Armington assumption, with both leading to the counter-empirical prediction that every country exports something in every industry: there are no zeros in the trade matrix.

Panel 8d shows the effect of the segmented markets assumption, which is that trade costs are partly absorbed by the exporting firm though a lower export markup under SGB. At trade costs tc = 2 at the left edge, the domestic sales markup is 50 percent higher than the export sales markup. The convergence of markups is working through the changes in market shares implied by panel 8c: the falling domestic market share reduces the domestic markup and the increasing share in the foreign market raises the markup. For this particular example and paramterization, the "pass through" of the trade cost at the left-hand value of tc = 2 is 81 percent; that is, phf = 1.81*phh whereas full passthrough would give phf = 2*phh. I follow the usual assumption that there is a single "factory gate" price under LGMC: markups are the same on domestic and export sales as shown in panel 8d, so there is full pass through.

As noted above, the welfare graphs for SGB and LGMC are quite similar in Figure 8a. But the source of these gains from falling costs are quite different as we also discussed in the earlier single-economy section. SGB features increased firm scale and decreased markups as trade costs fall, with a small decrease in firm numbers (variety). LGMC has no increase in firm scale or fall in markups, but maintains the original number of varieties. Why then does LGMC show an increase in welfare? With goods being imperfect substitutes, more even quantities in consumption of home and imported goods increases utility even if the total quantity of goods is constant. For example, consuming 9 units of a representative home good and 1 unit of an imported good yields significantly lower utility than consuming 5 units of each. More balanced consumption as shown in panel 8c offsets a lower variety (SGB) or constant variety (LGMC) in panel 8b, perhaps entirely, as trade cost fall.

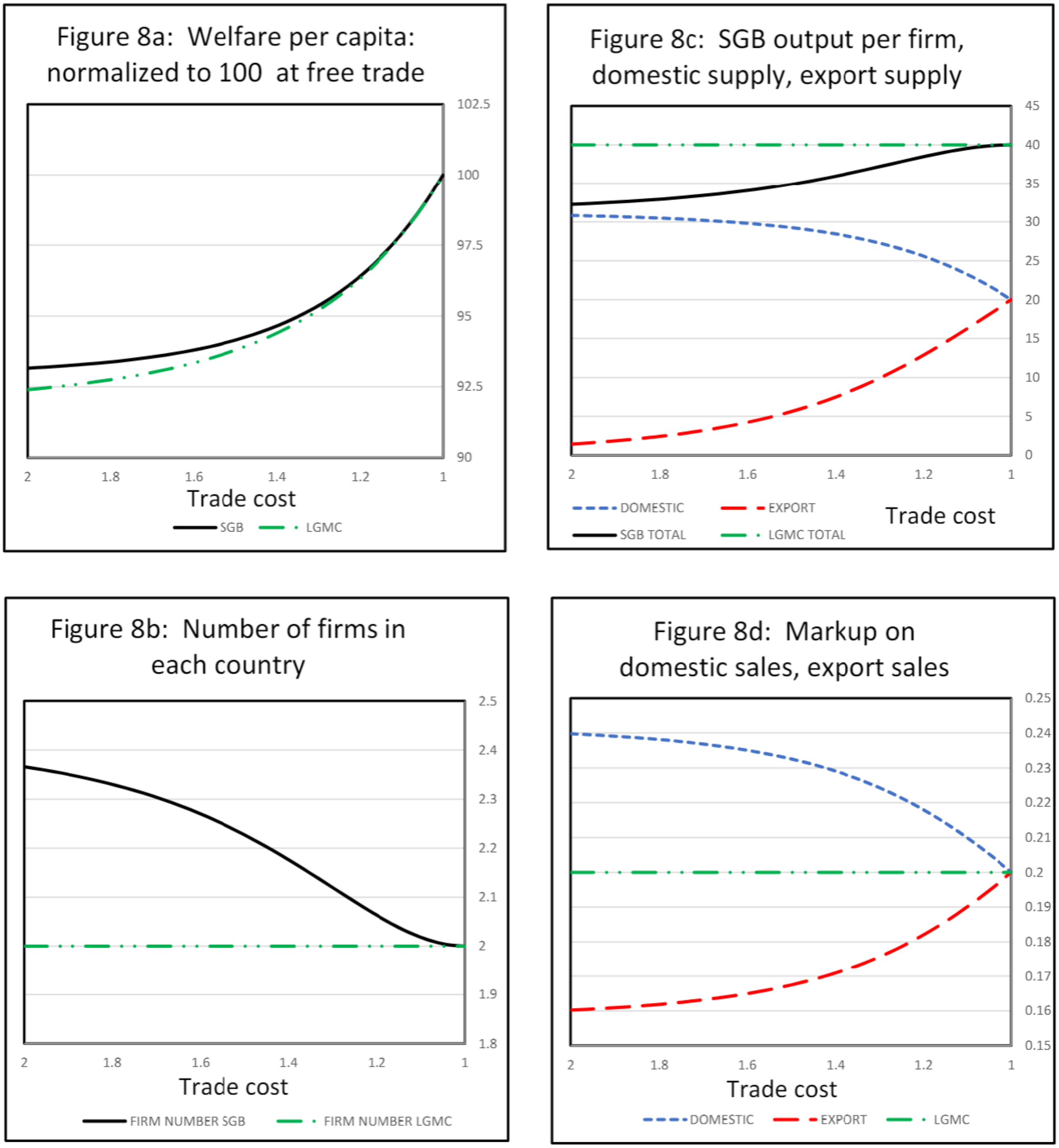


Figure 8: Two-country trade: Bertrand competition, identical countries.

Notes: Green dot-dash lines: large group monopolistic competition: Calibration 1

10. Summary

My motivation for writing this paper has both a "carrot" and a "stick" aspect. The stick motive is a very long dissatisfaction with the use of the large-group monopolistic-competition idea in both analytical and applied (numerical) general-equilibrium models. This is used in an environment of increasing returns to scale and imperfect competition, but it implies that firms have zero market shares under Nash Cournot or Bertrand conjectures: increasing returns to scale but no scale effects. Yet the industries used to motivate the models and even more so in heterogeneous firm models are typically dominated by a small number of large firms. My assertion is that the LGMC assumption is a kluge to avoid the added simultaneity that a proper theoretical approach would entail.

The carrot is to show that introducing positive firm size and market share, along with Cournot and/or Bertrand behavior by these firms is not a big challenge in numerical modeling. And it can be done without resorting to alternative kluges such as quasi-linear preferences in which the relative industries are borderline inferior goods. I show that the markup rules for Bertrand and Cournot involve only the elasticity of substitution (parameter) among the industries varieties and the equilibrium market shares of the active firms (endogenous variable). Although I treat a special case of symmetric firms, it is clear that the markup rules apply perfectly in a world of heterogeneous firms with differing market shares.

The next section of the paper shows how to code such a generalization into a numerical model using GAMS and its non-linear complemetarity solver PATH. The three forms of behavior, LGMC, SGB, and SGC are then compared via a simulation in which the key parameter is change in market size, a parable for trade liberalization among similar economies first exploited by Krugman (1979) . I discuss the subtleties of calibrating the same data to different competitive assumptions and simulate two different alternatives. The difficulty is that changing the competitive assumption requires changing something else, preferences or technology, in order to reproduce the same benchmark data.

While we cannot be sure how the results generalize, the simulation show that, for initially quite small economies, the positive effects of growth on welfare are much stronger under the small-group assumptions than under LGMC. But conversely, for initially quite large economies a corresponding proportion growth (e.g., doubling in size), the effect of growth on welfare is smaller for Cournot under calibration 1, but remains larger under calibration 2. I suppose that this indicates caution in choosing how to balanced a change in the behavioral assumption. These results suggest a policy implication for modelers, which is that the use of LGMC may significantly underestimate the gains from liberalization for small countries. And these are likely precisely the economies that often have highly concentrated sectors under protective policies.

The final section of the paper presents a two-country trade model, where each countries is identical to the economy of the previous section. The experiment is reduction in trade costs, contrasting the SGB and LGMC cases. Employing a segmented-markets assumption in the SGB case falling trade costs imply increasing firm scale, a fall in total varieties (at least in this parameterization) but more even consumption of domestic and imported goods, partial pass through of trade costs via different domestic and export markups, and a significant increase in welfare for the identical economies (no comparative advantage gains). The LGMC has no firm scale effects, no fall in markups, and a constant number of firms (varieties). Welfare gains from falling trade costs in the LGMC case are entirely from a more balance consumption of domestic and foreign goods, a result completely analogous to the traditional Armington assumption.

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Appendix A. Derivation of the Marshallian CES demand functions

Solve for the demand for a given X variety, and for the price index e. The consumer's sub-problem maximizing the utility from X goods subject to an expenditure constraint (using λ as a Lagrangean multiplier) and first-order conditions are:

$$\max X_c = \left[\sum_i X_i^{\alpha}\right]^{\frac{1}{\alpha}} + \lambda\left(I_x - \sum p_i X_i\right) \Rightarrow \frac{1}{\alpha}\left[\sum_i X_i^{\alpha}\right]^{\frac{1}{\alpha}-1} \alpha X_i^{\alpha-1} - \lambda p_i = 0 \quad \text{(A.1)}$$

Let σ denote the elasticity of substitution among varieties. Dividing the first-order condition for variety i by the one for variety j,

$$\left[\frac{X_i}{X_j}\right]^{\alpha-1} = \frac{p_i}{p_j} \qquad \frac{X_i}{X_j} = \left[\frac{p_i}{p_j}\right]^{\frac{1}{\alpha-1}} = \left[\frac{p_i}{p_j}\right]^{-\sigma} \qquad \textit{since} \quad \sigma = \frac{1}{1-\alpha} \qquad \text{(A.2)}$$

$$X_j = \left[\frac{p_i}{p_j}\right]^{\sigma} X_i \qquad p_j X_j = p_j p_j^{-\sigma} p_i^{\sigma} X_i \quad \sum p_j X_j = I_x = \left[\sum p_j\right]^{1-\sigma} p_i^{\sigma} X_i \quad \text{(A.3)}$$

Inverting this last equation, the demand for an individual variety i:

$$X_i = p_i^{-\sigma}\left[\sum p_j^{1-\sigma}\right]^{-1} I_x \qquad \sigma = \frac{1}{1-\alpha}, \qquad \alpha = \frac{\sigma-1}{\sigma} \qquad \text{(A.4)}$$

Use X_i to construct X_c and then solve for e, noting the relationship between α and σ.

$$X_i^{\alpha} = X_i^{\frac{\sigma-1}{\sigma}} = p_i^{1-\sigma}\left[\sum p_j\right]^{\frac{1-\sigma}{\sigma}} I_x^{\alpha}$$

$$\sum X_i^{\alpha} = \left[\sum p_i^{1-\sigma}\right]\left[\sum p_j^{1-\sigma}\right]^{\frac{1-\sigma}{\sigma}} I_x^{\alpha} = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{\sigma}} I_x^{\alpha} \qquad \text{(A.5)}$$

$$X_c = \left[\sum X_i^{\alpha}\right]^{\frac{1}{\sigma}} = \left[\sum X_i^{\alpha}\right]^{\frac{\sigma}{\sigma-1}} = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{\sigma-1}} I_x$$

$$e = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \qquad \text{if all prices equal:} \quad e = N^{\frac{1}{1-\sigma}} p \qquad \text{(A.6)}$$

An increase in the *range* of goods lowers the cost of buy a unit of (sub)utility. Having derived e, we can then use equation (A6) in (A4) to get the demand for an individual variety using e.

$$X_i = p_i^{-\sigma} e^{\sigma-1} I_x \qquad \text{since} \qquad e^{\sigma-1} = \left[\sum p_j^{1-\sigma}\right]^{-1} \tag{A.7}$$

A similar procedure allows us to derive the inverse demand function which is used in the Cournot markup. Using the same first-order condition (A2), this is as follow

$$\begin{gathered} X_j = \left[\frac{p_i}{p_j}\right]^{\sigma} X_i \quad p_j^{\sigma} = p_i^{\sigma} X_i X_j^{-1} \quad p_j = p_i X_i^{\frac{1}{\sigma}} X_j^{\frac{-1}{\sigma}} = p_i X_i^{\delta} X_j^{-\delta} \quad \delta \equiv \frac{1}{\sigma} \\ \sum p_j X_j = I_x = p_i X_i^{\delta} [\sum X_j^{1-\delta}] \end{gathered} \tag{A.8}$$

This gives the inverse demand function as

$$p_i = X_i^{-\delta} [\sum X_j^{1-\delta}]^{-1} I_x \tag{A.9}$$

with the expenditure share on good i given by

$$s_i \equiv \frac{p_i X_i}{I_x} = X_i^{1-\delta} \left(\sum X_j^{1-\delta}\right)^{-1} \qquad (\ldots) \equiv (\sum X_j^{1-\delta}) \tag{A.10}$$

Appendix B. GAMS Code

```
$TITLE: SGB.GMS James R. Markusen, University of Colorado, Boulder
* Bertrand Small-Group Monopolistic Competition

$ONTEXT
  P(1 - MK) = MC, markup MK =  1/(SI - 1/N*(SI - 1))
  N  = endogenous number of firms,  1/N = firm market share
  SI = elasticity of substitution (exogenous)

  calibrate to the same data below as LGMC SI=5, MK=0.2;
  calibration for SGB is then  SI = 6.3333, N = 4,
  MK = 1/(sigma - (1/N)(sigma - 1) = 0.20

                        Production Sectors          Consumers
  Markets   |   X       N        Y        W    |  CONS   ENTR
---------------------------------------------------------------
       PX   |  200                      -200    |
       PY   |                   200     -200    |
       PN   |          40                       |          -40
       PW   |                            400    |  -400
       PL   | -160    -40     -200              |   400
       MK   |  -40                              |           40
$OFFTEXT

PARAMETERS
  SI          SIGMA: elasticity of substitution
  FC          parameter setting the level of fixed costs
  ENDOWL      endowment of labor;

SI = 6 + 1/3;
FC = 10;
ENDOWL = 400;

NONNEGATIVE VARIABLES

  X       Activity level for X (output per firm)
  N       Number of X sector firms (variety measure)
  Y       Activity level of Y output
  W       Activity level for welfare

  PE      Price index for X goods (unit expenditure function)

  PX      Price of an individual X variety
  PN      Price of fixed costs (price of entering)
  PY      Price of Y
  PW      Price index for utility (consumer price index)
  PL      Price of labor

  CONS    Income of the representative consumer
  ENTRE   Income of firm owners spent on fixed costs
  MK      Markup;

EQUATIONS

  PRICEX    MR = MC in X (associated with X output per firm)
  PRICEN    Zero profits - free entry in X (associated with N)
  PRICEY    Zero profit condition for Y (PY = MC)
  PRICEW    Zero profit condition for W (PW = MC of utility)
```

```
  INDEX     Definitional equation for the price index PE

  DX        Supply-demand balance for X (individual variety)
  DN        Supply-demand for firms N: markup rev = fexed cost
  DY        Supply-demand balance for Y
  DW        Supply-demand balance for utility W (welfare)
  DL        Supply-demand balance for labor

  ICONS     Consumer income
  IENTRE    Firm owner markup income
  MARKUPB   Bertrand markup equation;


PRICEX..   PL =G= PX*(1 - MK);

PRICEN..   PL =G= PN;

PRICEY..   PL =G= PY;

PRICEW..   (PE**0.5)*(PY**0.5) =G= PW;

INDEX..    ((N/4)*PX**(1-SI))**(1/(1-SI)) =G= PE;

DX..       X*80  =G=  PX**(-SI)*(PE**(SI-1))*CONS/2;

DN..       N*FC  =G= ENTRE/PN;

DY..       Y*100 =G= CONS/(2*PY);

DW..       200*W =G= (1.25**0.5)*CONS/PW;

DL..       ENDOWL =E= Y*100 + N*X*20 + N*FC;

ICONS..    CONS =E= PL*ENDOWL;

IENTRE..   ENTRE =E= MK*PX*X*20*N;

MARKUPB..  MK =E= 1/(SI - 1/N*(SI - 1));


MODEL SGB /PRICEX.X, PRICEY.Y, PRICEW.W, PRICEN.N, INDEX.PE,
           DX.PX, DN.PN, DY.PY, DW.PW,
           DL.PL, ICONS.CONS, IENTRE.ENTRE, MARKUPB.MK/;

*    set initial values at benchmark for replication check

PE.L = 1.25;  CONS.L = 400;  ENTRE.L = 40;
X.L = 2;  Y.L = 2;  N.L = 4;  W.L = 2;
PX.L = 1.25; PN.L = 1; PY.L = 1; PW.L = 1.25**0.5; PL.L = 1;
MK.L = 0.20;

*    choose the price of good Y as numeraire
PY.FX = 1;

*    check for calibration, starting-value errors with ITERLIM=0
SGB.ITERLIM = 0;
SOLVE SGB USING MCP;
*    free up iteration limit, set to 1000
SGB.ITERLIM = 1000;
SOLVE SGB USING MCP;
```

```
* show welfare as a function of the economy's size

SETS I indexes 25 different size levels  /I1*I25/;

PARAMETERS
 SIZE(I)         benchmark I16 SIZE=2 ENDOWL=400)
 RESULTS(I, *)  summarizes results;

MK.L = 0.2;

LOOP(I,

SIZE(I) =  5.2 - 0.2*ORD(I);
ENDOWL = 200*SIZE(I);

SOLVE SGB USING MCP;

RESULTS(I, "SIZE") = SIZE(I);
RESULTS(I, "WELFARE-B")  = W.L;
RESULTS(I, "WELFCAP-B")  = W.L/SIZE(I);
RESULTS(I, "FIRMSIZE-B") = X.L;
RESULTS(I, "NUMBERF-B")  = N.L;
RESULTS(I, "MARKUP-B")   = MK.L;

);

DISPLAY RESULTS;

* Write parameter RESULTS to an Excel file SGB.XLS,
* starting in Sheet3, cell A3

Execute_Unload 'SGB.gdx' RESULTS
execute 'gdxxrw.exe SGB.gdx par=RESULTS rng=SHEET3!A3:J29'
```

```
$TITLE SGB-trade6  James R. Markusen, University of Colorado, Boulder
* two country (h and f) trade model, small group Bertrand competition
* no comparative advanatage, one factor labor
* data calibrated to sigma = 6.333, ENDOW = 200 for each country, costless trade
* which calibrates to four type 1 firms in costless trade
* 2 firm each in h and f,  markup 0.20 at TC = 1
* this version - integrated (no trade cost) world market for Y simplifies model

$ONTEXT
                  Production Sectors                Consumers
  Markets   |  XHH   XHF     N        Y        W    |  CONS   ENTR
--------------------------------------------------------------
       PX   |  50    50                      -100  |
       PY   |                       100      -100  |
       PN   |              20                      |          -20
       PW   |                                 200  |  -200
       PL   | -40   -40    -20      -100              200
       MK   | -10   -10                          |             20
$OFFTEXT

SETS I firm types differ by marginal costs /I1*I1/;
ALIAS (I,II);

PARAMETERS
 ENDOWH, ENDOWF    Endowment scale multiplier
 MC                marginal cost for firm types same across countries
 TC                trade cost gross basis (1 + trade cost rate)
 SIG               elasticity of substitution among X goods
 FC                fixed cost per firm;

ENDOWH = 200; ENDOWF = 200;
TC = 1.00001;
SIG = 6 + 1/3;
FC = 10;
MC = 1;

NONNEGATIVE VARIABLES

  XHH      Production by an h firm of type i for sale in h
  XHF      Production by an h firm of type i for export to f
  XFF      Production by an f firm of type i for sale in f
  XFH      Production by an f firm of type i for export to h
  NH       Number of X sector firms of type i in h
  NF       Number of X sector firms of type i in f
  YH       Level of Y output in country h
  YF       Level of Y output in country f
  WH       Welfare of h
  WF       Welfare of f

  PXHH     Price of good Xh sold in h
  PXHF     Price of good Xh sold in f
  PXFF     Price of good Xf sold in f
  PXFH     Price of good Xf sold in f
  PY       World price of Y
  PWH      Price index of utility in country h
  PWF      Price index of utility in country f
  PLH      Price of labor in country h
  PLF      Price of labor in country f

  CONSH    Income of the representative consumer in country h
```

```
  CONSF    Income of the representative consumer in country f
  ENTRH    Income of the agent ENTRE for firm type i in h
  ENTRF    Income of the agent ENTRE for firm type i in f

  PEH      Price index for X composite in h
  PEF      Price index for X composite in f
  MARKHH   Markup of a type i h firm for sale in h
  MARKHF   Markup of a type i h firm for sale in f
  MARKFF   Markup of a type i f firm for sale in f
  MARKFH   Markup of a type i f firm for sale in h;

EQUATIONS

  PRXHH    Pricing inequality for XHH
  PRXHF    Pricing inequality for XHF
  PRXFF    Pricing inequality for XFF
  PRXFH    Pricing inequality for XFH
  PRNH     Pricing inequality for NH
  PRNF     Pricing inequality for NF
  PRICEYH  Pricing inequality for YH (PY = MC)
  PRICEYF  Pricing inequality for YF
  PRICEWH  Consumer price index for country h
  PRICEWF  Consumer price index for country f

  MKTXHH   Supply >= demand for XHH
  MKTXHF   Supply >= demand for XHF
  MKTXFF   Supply >= demand for XFF
  MKTXFH   Supply >= demand for XFH
  MKTFY    Export supply = import demand for Y
  MKTWH    Supply-demand for WH
  MKTWF    Supply-demand for WF
  MKTLH    Supply-demand balance for labor LH
  MKTLF    Supply-demand balance for labor LF

  ICONSH   Consumer income in h including profits of Xh firms
  ICONSF   Consumer income in f including profits of Xf firms
  IENTRH   Entrepreneur's profits (markup revenues) in h
  IENTRF   Entrepreneur's profits (markup revenues) in f

  PINDEXH  Price index for X goods in h
  PINDEXF  Price index for X goods in f
  MKHH     Markup inequality for XHH
  MKHF     Markup inequality for XHF
  MKFF     Markup inequality for XFF
  MKFH     Markup inequality for XFH;


PRXHH..      PLH*MC =G= PXHH*(1 - MARKHH);
PRXHF..      PLH*MC*TC =G= PXHF*(1 - MARKHF);
PRXFF..      PLF*MC =G= PXFF*(1 - MARKFF);
PRXFH..      PLF*MC*TC =G= PXFH*(1 - MARKFH);
PRNH..       FC*PLH =G= ENTRH;
PRNF..       FC*PLF =G= ENTRF;
PRICEYH..    PLH =G= PY;
PRICEYF..    PLF =G= PY;
PRICEWH..    ((PEH)**0.5)*(PY**0.5) =G= PWH;
PRICEWF..    ((PEF)**0.5)*(PY**0.5) =G= PWF;

MKTXHH..     XHH =G=  PXHH**(-SIG)*(PEH**(SIG-1))*0.5*CONSH;
```

```
MKTXHF..     XHF/TC =G=  PXHF**(-SIG)*(PEF**(SIG-1))*0.5*CONSF;
MKTXFF..     XFF =G= PXFF**(-SIG)*(PEF**(SIG-1))*0.5*CONSF;
MKTXFH..     XFH/TC =G=  PXFH**(-SIG)*(PEH**(SIG-1))*0.5*CONSH;
MKTFY..      YH + YF =G= 0.5*CONSH/PY + 0.5*CONSF/PY;
MKTWH..      PWH*WH =G= CONSH;
MKTWF..      PWF*WF =G= CONSF;
MKTLH..      ENDOWH =G= YH + NH*(XHH+XHF)*MC + NH*FC;
MKTLF..      ENDOWF =G= YF + NF*(XFF+XFH)*MC + NF*FC;

ICONSH..     CONSH  =G= PLH*ENDOWH + NH*ENTRH - NH*PLH*FC;
ICONSF..     CONSF  =G= PLF*ENDOWF + NF*ENTRF - NF*PLF*FC;
IENTRH..     ENTRH =G= MARKHH*PXHH*XHH + MARKHF*PXHF*XHF/TC;
IENTRF..     ENTRF =G= MARKFF*PXFF*XFF + MARKFH*PXFH*XFH/TC;

PINDEXH..    PEH =E= (NH*PXHH**(1-SIG) + NF*PXFH**(1-SIG))**(1/(1-SIG));
PINDEXF..    PEF =E= (NF*PXFF**(1-SIG) + NH*PXHF**(1-SIG))**(1/(1-SIG));

MKHH..       MARKHH =G= 1 / (SIG - (SIG-1)*PXHH*XHH/
                    (NH*PXHH*XHH + NF*PXFH*XFH/TC));

MKHF..       MARKHF =G= 1 / (SIG - (SIG-1)*PXHF*XHF/TC/
                    (NF*PXFF*XFF + NH*PXHF*XHF/TC));

MKFF..       MARKFF =G= 1 / (SIG - (SIG-1)*PXFF*XFF/
                    (NF*PXFF*XFF + NH*PXHF*XHF/TC));

MKFH..       MARKFH =G= 1 / (SIG - (SIG-1)*PXFH*XFH/TC/
                    (NH*PXHH*XHH + NF*PXFH*XFH/TC));


MODEL M52 /PRXHH.XHH, PRXHF.XHF, PRXFF.XFF, PRXFH.XFH, PRICEYH.YH, PRICEYF.YF,
        PRICEWH.WH, PRICEWF.WF,
        MKTXHH.PXHH, MKTXHF.PXHF, MKTXFF.PXFF, MKTXFH.PXFH, PINDEXH.PEH, PINDEXF.PEF,
        MKTFY.PY, MKTWH.PWH, MKTWF.PWF,
        MKTLH.PLH, MKTLF.PLF,
        ICONSH.CONSH, ICONSF.CONSF, PRNH.NH, PRNF.NF,
        IENTRH.ENTRH, IENTRF.ENTRF,
        MKHH.MARKHH, MKHF.MARKHF, MKFF.MARKFF, MKFH.MARKFH/;


* set initial values of variables for solver

CONSH.L = 200; CONSF.L = 200;
XHH.L = 20; XFF.L = 20; XHF.L = 20; XFH.L = 20;
YH.L = 100; YF.L = 100; WH.L = 200; WF.L = 200;
NH.L = 2; NF.L = 2;

PXHH.L = 1.25; PXHF.L = 1.25; PXFF.L = 1.25; PXFH.L = 1.25;
PY.L = 1; PY.L = 1; PLH.L = 1; PLF.L = 1; PWH.L = 0.9818; PWF.L = 0.9818;
PY.L = 1; PEH.L = 0.9639; PEF.L = 0.9639;
ENTRH.L = 10; ENTRF.L = 10;
MARKHH.L = 0.20; MARKHF.L = 0.20; MARKFF.L = 0.20; MARKFH.L = 0.20;

* choose PYH as numeraire, and check calibration
PY.FX = 1;
TC = 1.00001;

M52.ITERLIM = 0;
SOLVE M52 USING MCP;
```

```
M52.ITERLIM = 1000;
SOLVE M52 USING MCP;


SETS j indexes 25 different trade cost levels  /J1*J25/;

PARAMETERS
 TCOST(J)
 FIRMNUMB(J,I)
 MARKUPO(J,I)
 RESULTS1(J,*), RESULTS1a(J,*), RESULTS1b(J,*), RESULTS1c(J,*), RESULTS1d(J,*),
 RESULTS1e(J,*), RESULTS1f(J,*), RESULTS2(J, *), RESULTS3(J, *);

LOOP(J,

TCOST(J) =  2.0 - 0.041667*ORD(J) + 0.041667;
TCOST("J25") = 1.0001;
TC = TCOST(J);
ENDOWH = 200; ENDOWF = 200;

SOLVE M52 USING MCP;

RESULTS1(J, "TCOST") = TCOST(J);
RESULTS1(J, "WELFCAP") = WH.L;
RESULTS1a(J, "FIRM NUMBER") = NH.L;
RESULTS1b(J, "DOMESTIC SALES") = XHH.L$(NH.L GT 0);
RESULTS1c(J,"EXPORT SALES") = XHF.L$(NH.L GT 0);
RESULTS1d(J, "DOMETIC MARKUP") = MARKHH.L$(NH.L GT 0);
RESULTS1e(J, "EXPORT MARKUP") = MARKHF.L$(NH.L GT 0);
RESULTS2(J, "PROFSRH") = (SUM(I, ENTRH.L*NH.L - NH.L*PLH.L*FC))/CONSH.L;
RESULTS3(J, "X SUBWELFARE") =0.5*CONSH.L/PEH.L;
);

DISPLAY RESULTS1, RESULTS1a, RESULTS1b, RESULTS1c, RESULTS1d, RESULTS1e,
        RESULTS2, RESULTS3;

Execute_Unload 'RESULTS6.gdx' RESULTS1
execute 'gdxxrw.exe RESULTS6.gdx par=RESULTS1 rng=SHEET1!A3:C36'

Execute_Unload 'RESULTS6.gdx' RESULTS1a
execute 'gdxxrw.exe RESULTS6.gdx par=RESULTS1a rng=SHEET1!E3:F36'

Execute_Unload 'RESULTS6.gdx' RESULTS1b
execute 'gdxxrw.exe RESULTS6.gdx par=RESULTS1b rng=SHEET2!B3:C36'

Execute_Unload 'RESULTS6.gdx' RESULTS1c
execute 'gdxxrw.exe RESULTS6.gdx par=RESULTS1c rng=SHEET2!E3:F36'

Execute_Unload 'RESULTS6.gdx' RESULTS1d
execute 'gdxxrw.exe RESULTS6.gdx par=RESULTS1d rng=SHEET2!H3:I36'

Execute_Unload 'RESULTS6.gdx' RESULTS1e
execute 'gdxxrw.exe RESULTS6.gdx par=RESULTS1e rng=SHEET2!K3:L36'

Execute_Unload 'RESULTS6.gdx' RESULTS2
execute 'gdxxrw.exe RESULTS6.gdx par=RESULTS2 rng=SHEET2!N3:O36'
```