Skin Effect in Multiphase Concentric Cylinders

Celestine Ananda and Francisco López Jiménez
Ann and H.J. Smead Department of Aerospace Engineering Sciences
University of Colorado Boulder, CO, USA

***Abstract*—This study addresses the challenges posed by the skin effect in modeling heterogeneous materials. Focusing on radially symmetric circular composites, we employ numerical simulations to explore the relationship between material properties, geometry, and frequency to model electrical current density distribution. We introduce a novel variable, $\hat{\delta}$, to represent the depth that contains 90% of total current density, and present a frequency-independent model for this revised skin depth definition.**

I. Introduction

Heterogenous materials, such as fiber composites, are not only used due to their stiffness-to-weight ratio, but also for the possibility of tailoring their electromagnetic properties, with applications ranging from antennas to shielding [1] - [3]. Several homogenization techniques exist to model them (*e.g.*, Maxwell-Garnett [4]), often based on the assumption that the size of the macroscale is much larger than the characteristic length of the microscale (*e.g.*, the diameter of the fibers in a composite). However, a third length-scale emerges in problems with time-varying fields. The skin effect, an implication of Faraday's law of magnetic induction, implies that a current density gradient occurs in conductors subjected to alternating electric potentials. As frequency increases, the electric current density moves closer to the surface. The skin depth is defined as the distance at which current density falls to $\frac{1}{e}$ (or $\approx 37\%$) of its maximum value on the surface of the conductor, in which e is Euler's number [5]. This third length-scale adds complexity to homogenization models, particularly in cases when the skin depth is of the same order as the typical length-scale at the microscale.

This work aims to numerically explore this phenomena and find scalings explaining the relationship between material properties, geometry, and the distribution of electrical current density within heterogeneous solids. We focus for now on the skin effect in radially symmetric circular composites in which two distinct materials are arranged concentrically.

II. Analytical Models and Numerical Data

Analytical models for current density distribution in conducting mediums exist with varying degrees of accuracy based on material properties and frequency regimes. A common closed-form analytical expression for skin depth is derived in Cartesian coordinates using time-dependent Maxwell's equations; this model solves for the attenuation of an electromagnetic wave impinging on a conductive medium, as follows:

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}}\sqrt{\sqrt{1+(\rho\omega\epsilon)^2}+\rho\omega\epsilon} \tag{1}$$

in which ω is frequency, μ is permeability, ϵ is permittivity, and ρ is resistivity [6].

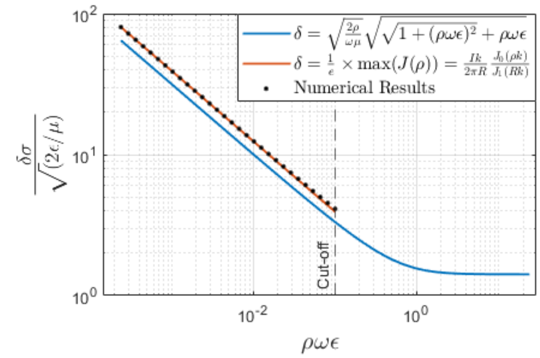

Fig. 1. Normalized skin depth vs. normalized frequency for the three modelling methods discussed. The three methods show good agreement in the range considered in the present study.

The current density distribution in the cross-section of a homogeneous conducting cylinder can be modeled as:

$$\mathbf{J}(\rho) = \frac{Ik}{2\pi R}\frac{J_0(\rho k)}{J_1(Rk)} \tag{2}$$

in which ρ is the radial coordinate, R is the cylinder radius, I is total current in the cross-section, k is the wavenumber, J_0 is the Bessel function of zeroth order and first kind, and J_1 is the Bessel function of first order and first kind [7].

We utilize these models to produce values for the expected skin depth in a circular cross section subjected to an alternating electric potential, and compare against numerical results in Finite Element Analysis (FEA) solver COMSOL Multiphysics. The FEA simulations are configured such that 1A of total electric current distributes across the 2D cross-section and determines the radial current density profile. To achieve generalized results, we normalize the response, as shown in Fig. 1, where we compare results from (1) and (2). To focus on the case in which the skin depth is a function of frequency, we restrict our analysis to parameters in the $\rho\omega\epsilon < 0.1$ range.

III. Skin Effect in Multiphase Materials

In multiphase materials, the maximum current density is not necessarily at the surface of the composite. We take a different approach to the traditional definition of skin depth by introducing a new variable, $\hat{\delta}$, which we define as the depth at which 90% of the total current density is contained. We consider a radially symmetric circular composite in which two distinct materials are arranged concentrically, and excited at a single frequency. The material properties of the inner cylinder (R_i) are considered fixed, and the outer material varies. The outer (total) radius (R_o) is set to a hundred times the skin depth predicted by (1) using the *inner* material conductivity and the frequency selected for the simulations. The outer cylinder's conductivity varies proportionally to the inner cylinder, spanning from 10^{-5} less conductive until it is equivalent to the inner material conductivity, effectively producing a homogeneous material. To produce a variety of current density distributions, we vary the radius of the inner cylinder, spanning from 10% to 99% of the outer radius.

For low values of σ_o/σ_i, the electric current density is mainly concentrated in the internal material, close to the surface. It then transitions to the surface of the outer material as its conductivity increases, as shown in Fig. 2 for a value of $R_i/R_o = 0.5$. In Fig. 3, the normalized values of $\hat{\delta}$ for this scenario are presented for all radii ratios; since we define the radius using the excitation frequency, our normalization for $\hat{\delta}$ shows no dependence on frequency. In all cases, the current density transitions from the inner to the outer material as the ratio of conductivities increases.

We then consider the alternate case, such that the outer material properties remain constant and instead the inner material's electric conductivity varies. In this scenario, the inner cylinder's conductivity spans from 10^{-5} less conductive up to and now *exceeding* the outer material's conductivity, such that it is 10^{25} times larger than the outer material conductivity at maximum. In this case, the outer radius is set to a hundred times the skin depth predicted by (1) using the inner material's *maximum conductivity* and the selected frequency. Again, a sweep is performed to set the inner cylinder radius to vary as a fraction of the outer radius, and it is determined that the same normalization negates frequency-dependence in Fig. 4.

In this case, we find three regimes. First, for $\sigma_i/\sigma_0 < 1$, the profile is equivalent to that described by Figure 3; the current density concentrates in the outer material. Progressing to $\sigma_i/\sigma_0 > 1$, the inner material is more conductive, and the current is distributed uniformly through its radius. As σ_i further increases, the inner material develops its own skin depth, and the current density concentrates to a thin layer underneath the inner radius R_i.

We aim to expand our analysis to complex multi-phase solids with more than one inclusion, to gain insight that can be applied in the modeling of fiber composites.

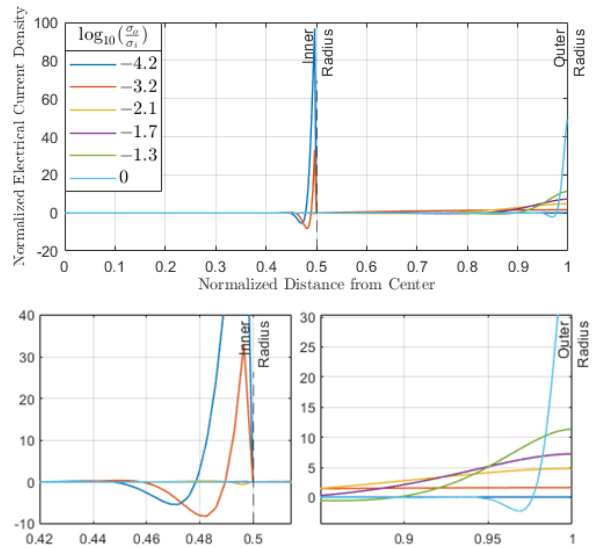

Fig. 2. Electric current density is plotted radially from the center of the concentric cylinders to the outer edge, normalized by the constant distribution corresponding to the 0Hz, DC case. As the outer material's conductivity increases, the current density transitions from entirely within the inner cylinder to majority in the outer, until both materials have the same conductivity and the classical skin effect is observed.

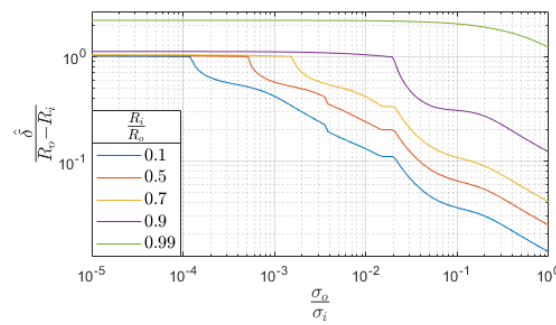

Fig. 3. Normalized $\hat{\delta}$ as a function of the ratio of conductivities, for five different radii ratios. At lower conductivity ratios, the current density is entirely contained within the internal cylinder, and as the ratio increases so does the amount of current density in the outer cylinder until the conductivities are equal and the skin effect for a homogeneous material is observed.

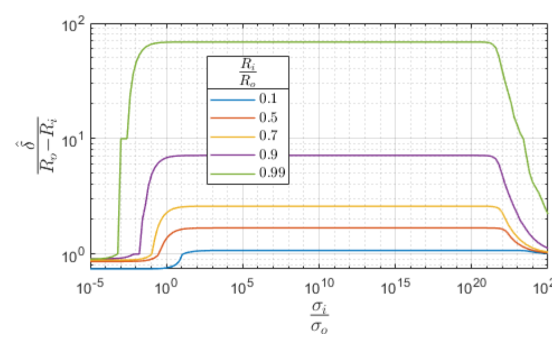

Fig. 4. Normalized $\hat{\delta}$ as a function of the ratio of conductivities, for five different radii ratios. In low conductivity ratios, the majority of current is contained in the outer material, and at higher ratios the current density transitions to the inner material. As this ratio increases, $\hat{\delta}$ decreases due to the electric current density condensing at the internal surface of the inner material.

References

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