Leo Radzihovsky, Spring 2016

PHYS 5260: Quantum Mechanics - II

Homework Set 1

Issued January 11, 2016
Due January 25, 2016

Reading Assignment: Shankar, Ch.16

1. Harmonic oscillator

   Use a variational theory with a Gaussian trial wavefunction $\psi(x) = Ne^{-\alpha x^2}$ (with N as proper normalization) to determine the ground state wavefunction, i.e., the optimum α_0, and the corresponding ground state energy E_0 by minimizing $E(\alpha) = \langle H \rangle$ (where H is the harmonic oscillator Hamiltonian) with respect to the variational parameter α. Compare your answers here with the exact results for α_0 and E_0.

2. (Shankar 16.1.3) For the attractive delta function potential $V = -aV_0\delta(x)$, use the variational principle with a Gaussian trial wavefunction to calculate the upper bound on E_0 and compare it to the exact answer $-ma^2V_0^2/2\hbar^2$ (from last semester, a problem that you should review).

3. Use the variational method with a variational function $\psi(x) = Nxe^{-ax}$ (with N as proper normalization) for a 1D particle of mass m in a potential $V(x)$ to determine the optimum value a_0 of parameter a and the corresponding ground-state energy $E_0 = E_{gs}(a_0)$.

   Take $V(x) = \infty$, for $x < 0$ and $V(x) = \epsilon x$, for $0 < x$.

4. Helium atom within Hartree-variational approximation

   Fill in all the steps for the variational estimate (with Z as the variational parameter) of the ground state energy of a Helium atom sketched out in Shankar, Ch.16, that leads to E_0 in Eq.16.1.16.

5. Estimate the tunneling-out probability (decay rate) at energy E for the potential $V(x) = \epsilon x$, for $0 < x < d$, $V(x) = 0$, for $d < x$, and $V(x < 0) = \infty$, using WKB approximation.

6. Use WKB approximation to compute a decay rate out of a 3D attractive short-ranged potential well for a particle in a state of orbital angular momentum ℓ and energy E. How does the result depend on E and ℓ in the universal low E limit?

   Hint: Recall from last semester that the finite orbital angular momentum creates an additional effective centrifugal potential that qualitatively modifies the attractive short-ranged potential by introducing a repulsive power-law tail at large r. At low E you should be able to considerably simplify your computation.