'Fractonicity' from Elasticity

Boulder

Center for Theory of Quantum Matter

CTQM

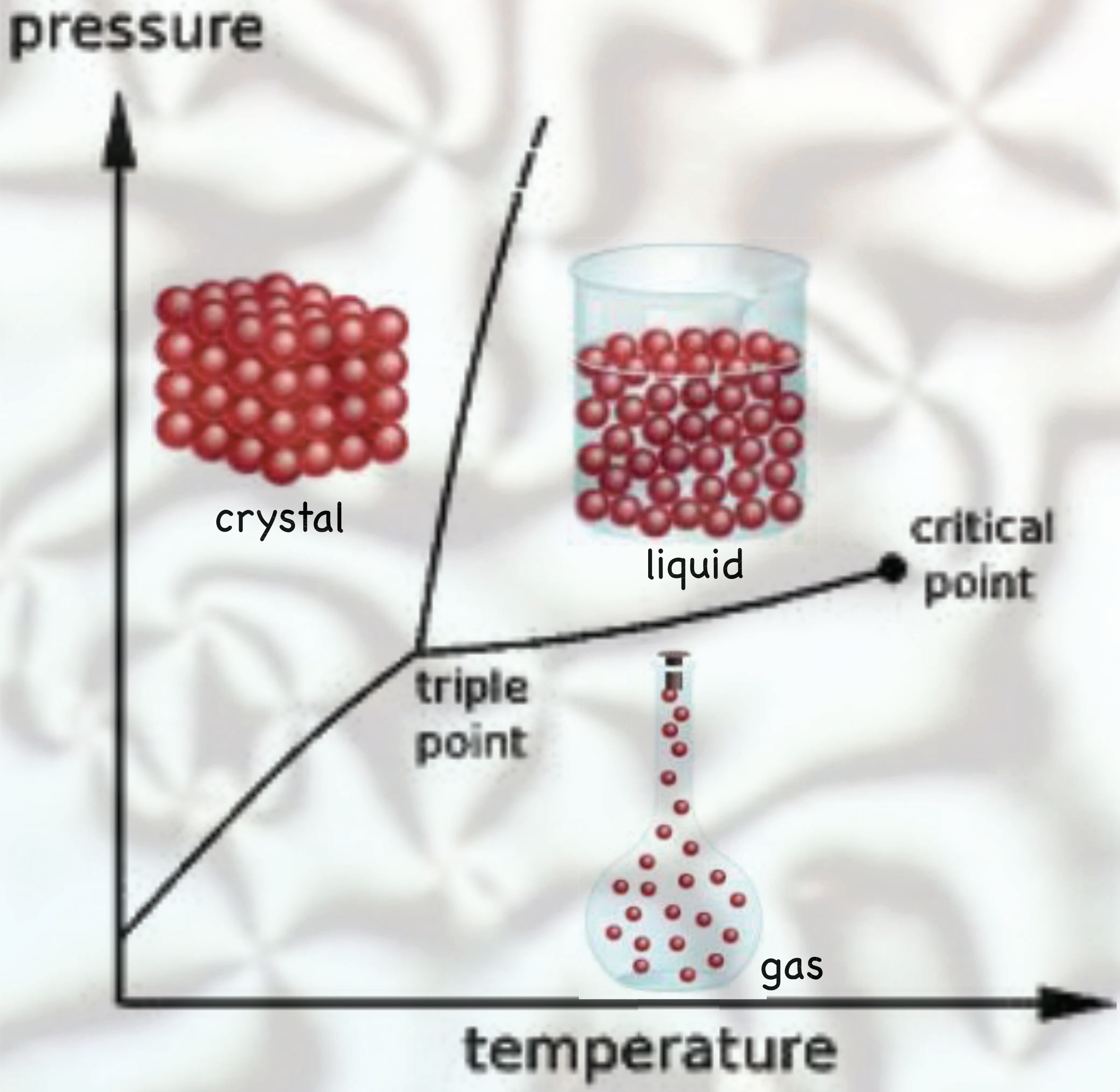
M. Pretko and L.R., PRLs 2018
Z. Zhai and L.R., PRB 2019
L.R. and M. Hermele, PRL 2020
L.R., PRL 2020
Z. Zhai and L.R., AOP 2021

$: Simons Investigator, NSF-MRSEC

SMRC

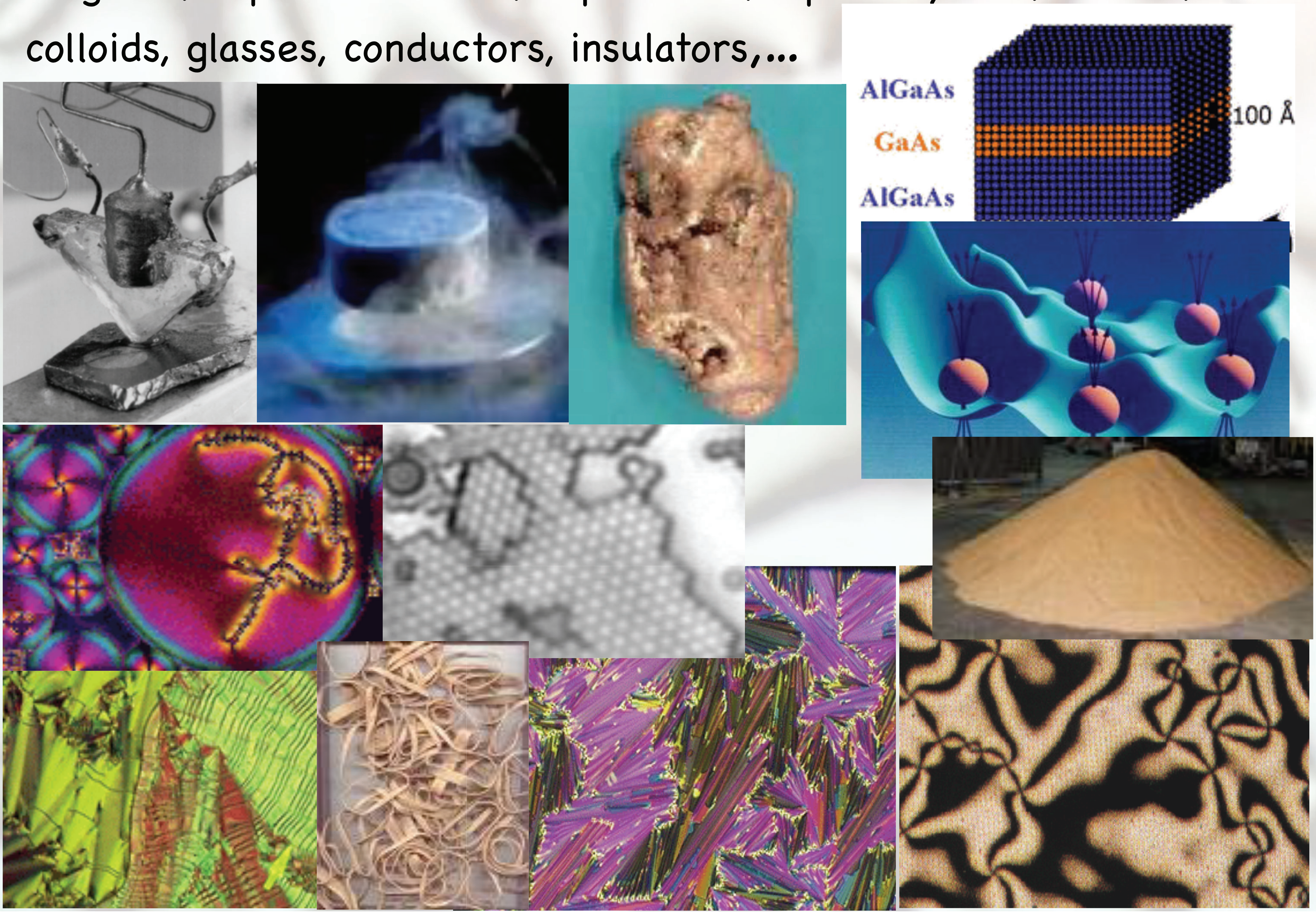
Physics Colloquium, Harvard, 1 February, 2021

"White lies" about phases of matter



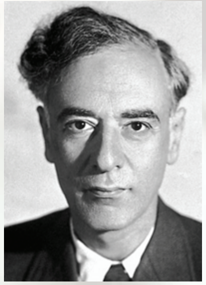
States of condensed matter in nature

- magnets, superconductors, superfluids, liquid crystals, rubber, colloids, glasses, conductors, insulators,...



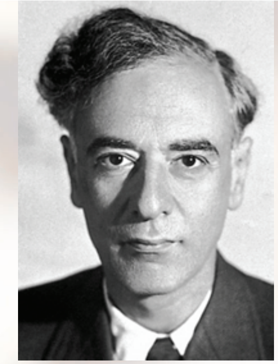
Emergence of richness, simplicity & universality

- Richness in nature -> "More is different" (P. W. Anderson)
- Emergent laws of nature & universality (L. Landau, E. Noether, K. G. Wilson) *(effective field theories)*

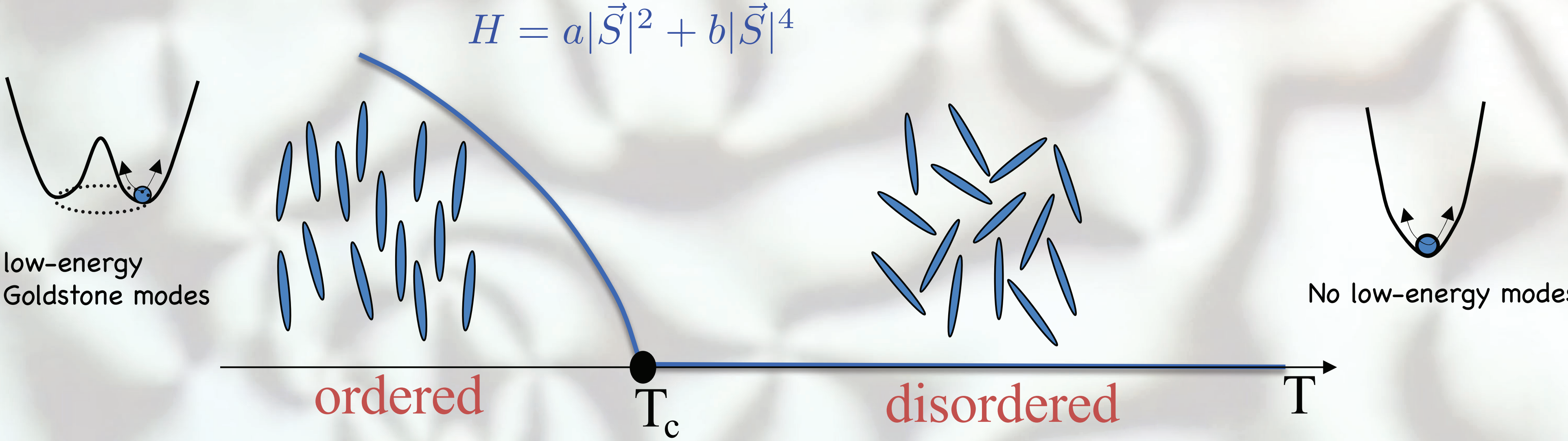
  - *Navier-Stokes hydrodynamics for fluids -> waves, shear flow, viscosity,...*
  - *Elasticity for crystals -> phonons, transverse sound,...*
  - *Landau-Lifshitz for ferromagnets -> spin-waves,...*
  - *Generalized elasticity -> superfluids, liquid crystals, rubber,...*
  - *Ginzburg-Landau for superconductors -> vortices, Meissner (Anderson-Higgs)...*
  - *Thermodynamics, critical phenomena,...*
  - *Einstein's gravity...*

States of (bosonic) matter: Landau paradigm

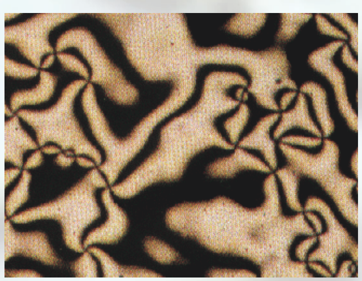


Lev Landau

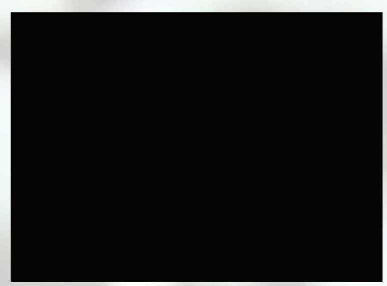
- "conventional" *ordered* states, *e.g., AFM, SF, liquid crystals...*
  - *local* order parameter, *S(r)*
  - classified by patterns of *spontaneously broken symmetry*
  - *short*-range entangled



(ferromagnet, nematic)



(paramagnet, isotropic)

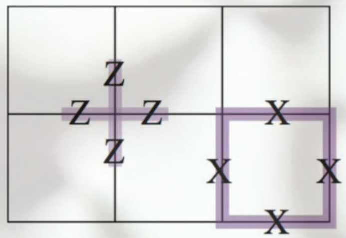


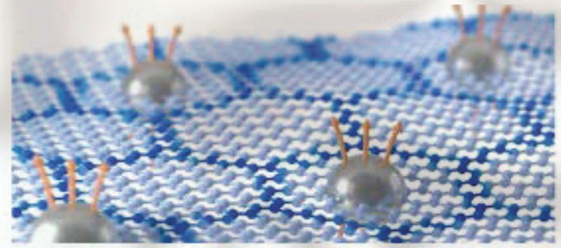
Anderson
Laughlin
Wen
Kitaev
Sachdev
Fisher
…

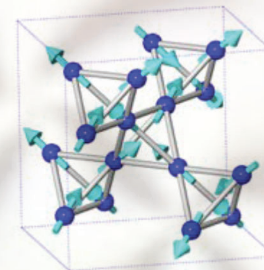
States of quantum matter

(beyond symmetry breaking)

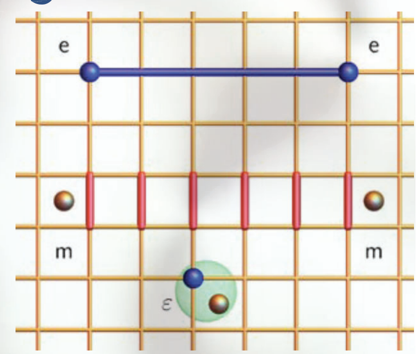
"conventional" quantum *'liquid'* states, e.g., FQHE, spin ice, toric code, …







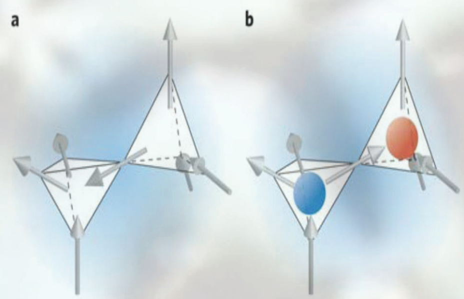
- Non-local, fractionalized bulk excitations as ends of strings: anyons – *free to move but with statistical "interaction"*



- Topological order with O(1) gs degeneracy

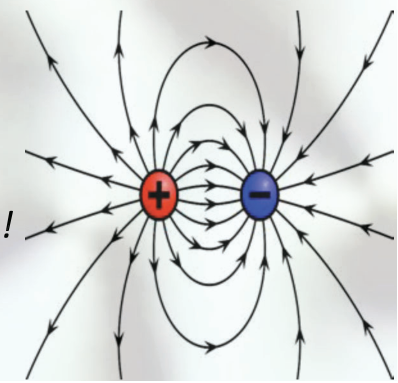
- Long-range entangled

Geometric frustration (e.g. "spin-ice rules")



- Gauge theory (Z_2, U(1), …)

Emergent electromagnetism!



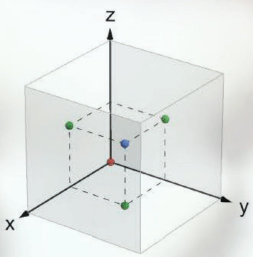
Outline

- New type of quantum ‘liquids’: “***fractons***”
- Symmetric *tensor* gauge theories
- Elasticity
- Duality
- Fractons from *vector* gauge theories
- Symmetry-enriched fractons from a *supersolid*
- Melting into *super-hexatic, super-smectic*

C. Chamon, 2005
A. Rasmussen, et al., 2016
J. Haah, 2011, '13
S. Bravyi, et al., 2011
B. Yoshida, 2013
S. Vijay, L. Fu, 2015, '16
...

Fracton quantum matter

- *new class of* quantum *'liquids': Z_2 fractons*, e.g., Haah's code, X-cube, lattice rotors,...

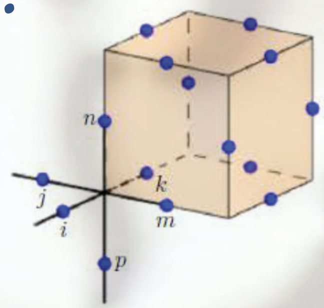


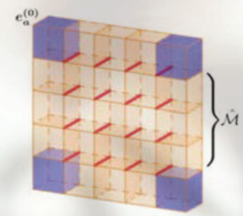
$$A_c = \prod_{n\in\partial c} \sigma_n^x$$

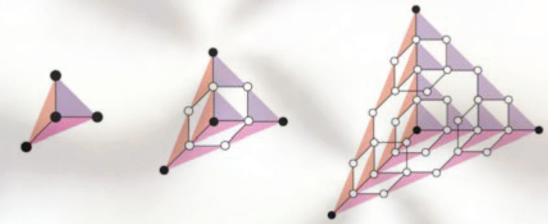
$$B_s^{(xz)} = \sigma_i^z\sigma_p^z\sigma_n^z\sigma_k^z$$

$$B_s^{(xy)} = \sigma_i^z\sigma_j^z\sigma_k^z\sigma_m^z$$

$$B_s^{(yz)} = \sigma_j^z\sigma_n^z\sigma_m^z\sigma_p^z$$



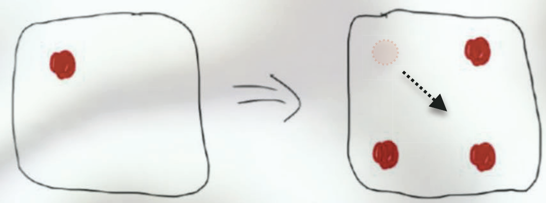




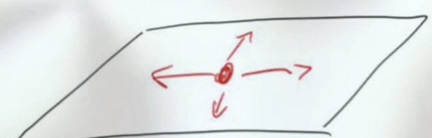
Fractal operator

  - Non-local, fractionalized excitations with restricted mobility and exponential topological degeneracy, beyond TQFT description,...
    - -> at corners of extended objects: fractons – *immobile* in isolation

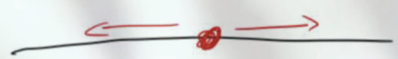
fracton



    - -> at ends of undeformable string: dipoles – *subdimensional*



planons

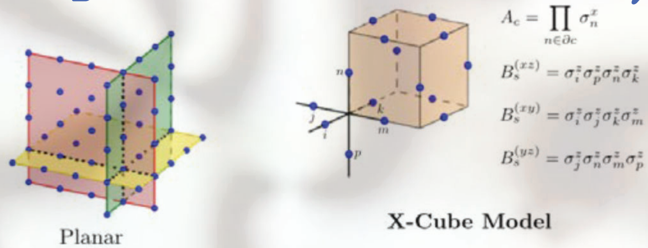


lineons

S. Vijay, et al., 2015, '16
M. Pretko, 2016, '17
T. Hsieh, et al., 2017
K. Slagle, Y. B. Kim, 2017
H. Ma, et al., 2017
X. Chen, et al., 2017, '18

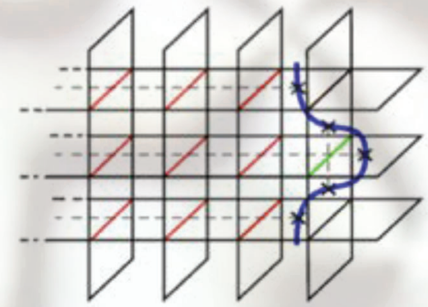
Fracton developments

- *"gauging" global Z_2 subdimensional symmetry spin model*



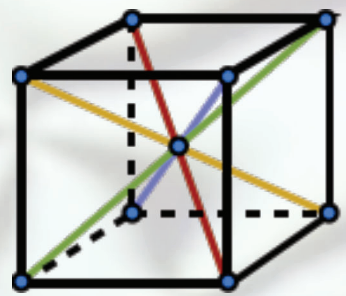
S. Vijay, J. Haah, L. Fu, 2016

- *Coupled-layers construction*



Ma, Lake, Chen, Hermele, 2017

- *Coupled-chains construction*



Halasz, Hsieh, et al., 2017

- *Parton construction*

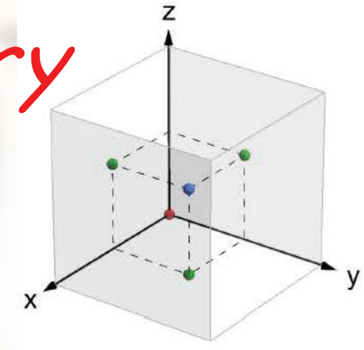
- *Higher rank tensor gauge theory* $\partial_i \partial_j E_{ij} = \rho_f$

M. Pretko, 2016

Rasmussen, You, C. Xu, 2016

Pretko, 2016

Fractons via **tensor** gauge theory



- U(1) symmetric tensor gauge theory (2+1D):

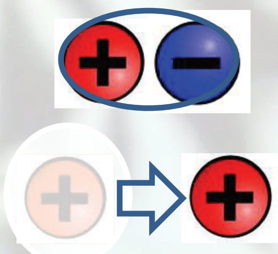
$$\mathcal{H} = \frac{1}{2}E_{ij}E_{ij} + \frac{1}{2}B_iB_i \qquad [E^{ij}, A_{ij}] = i\delta^{(2)}(\mathbf{x}) \qquad B^i = \epsilon_{jk}\partial^j A^{ki}$$

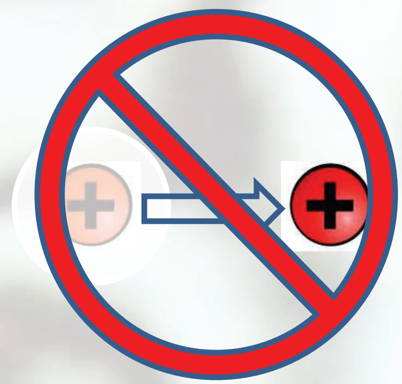
- Gauss' law: $\partial_i\partial_j E^{ij} = \rho$

- Conservation of charges and of *dipoles* ---> *fracton* phenomenology!

-> moving charge changes dipole moment -> forbidden by dipole conservation

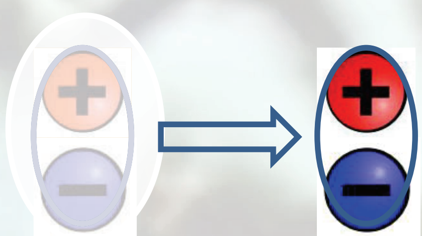
- immobile





-> dipole motion constrained

- subdimensional



Fractons

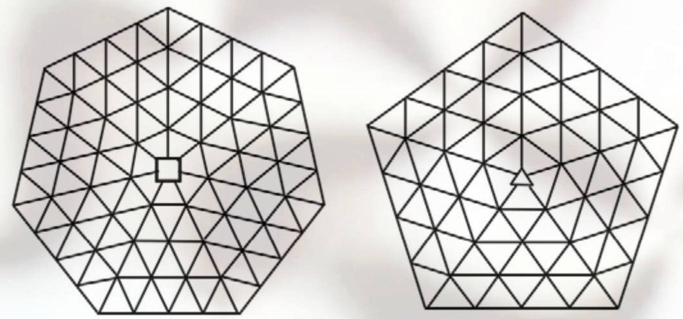
¿ any physical realizations ?

***YES*: 2D quantum crystal!**

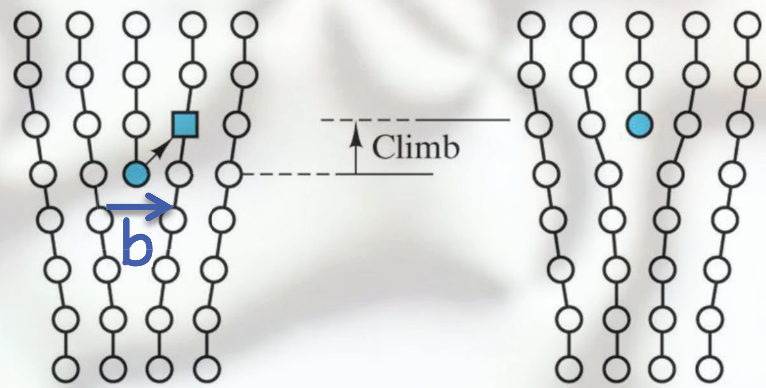
L. R., 2016
classical duality

Topological defects in a crystal

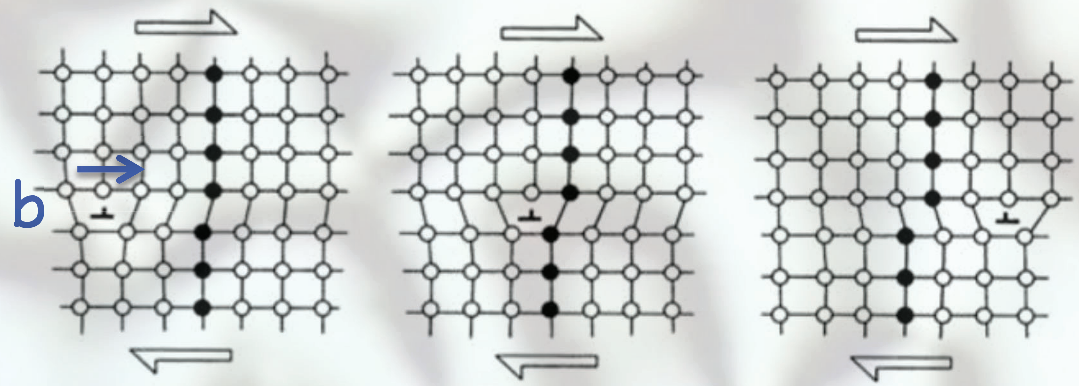
- *Disclination:*
  immobile



- *Dislocation climb:*
  constrained by
  v/i diffusion



- *Dislocation glide:*
  subdimension (d-1)
  motion



L. R., 2016
classical duality

Topological defects in a crystal

- *Disclination:*
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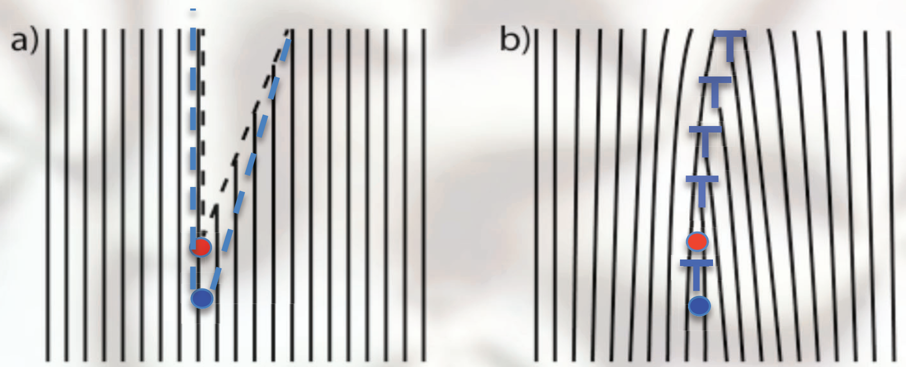

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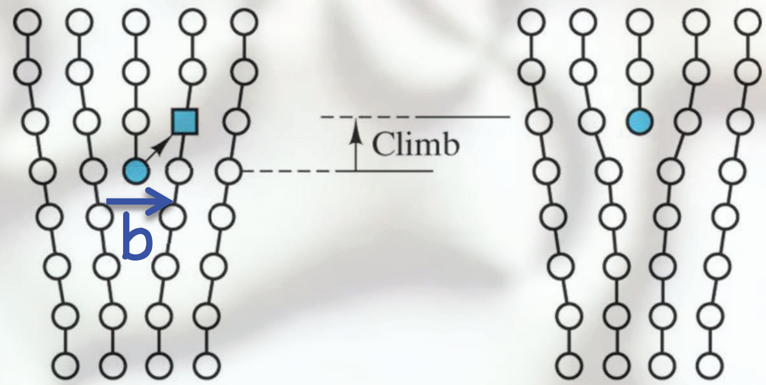
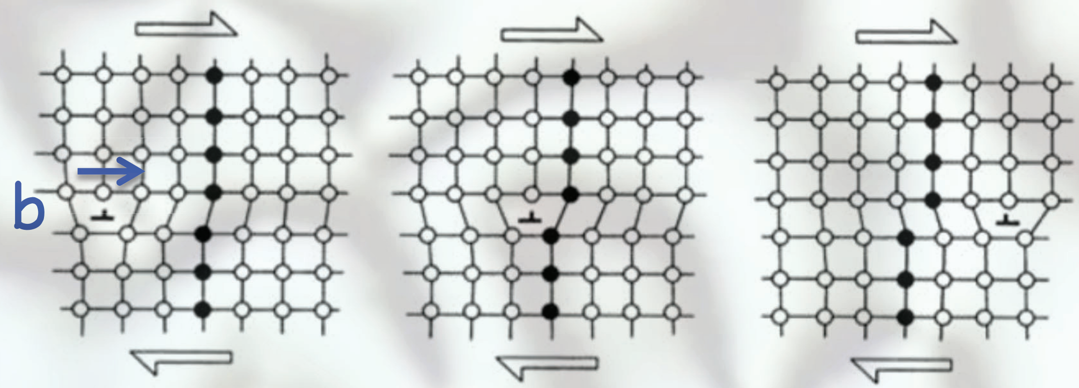
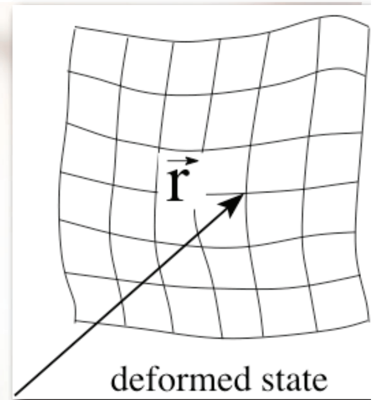


- *Dislocation glide:*
  subdimension (d-1)
  motion

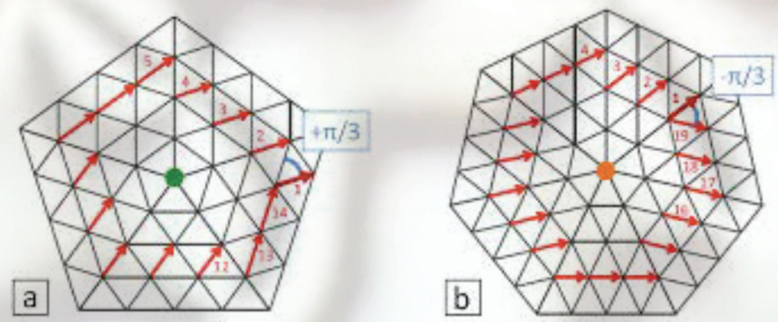


Elasticity theory and defects

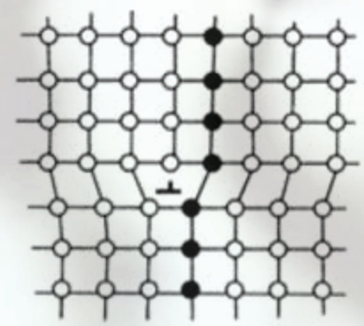


- **Eulerian phonons:** $\vec{r} = \vec{R} + \vec{u}(\vec{r})$
- **Strain:** $u_{ij} = \frac{1}{2}(\partial_i \vec{R} \cdot \partial_j \vec{R} - \delta_{ij}) \approx \frac{1}{2}(\partial_i u_j + \partial_j u_i)$
- **Hamiltonian:** $\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2} C_{ij,kl} u_{ij} u_{kl}$
- **Topological defects**
  - Disclinations: $\nabla \times \nabla \theta = s\,\delta^2(\vec{r}) \equiv s(\vec{r})$

    (bond angle: $\theta = ½\, \varepsilon_{ij} \partial_i u_j$)

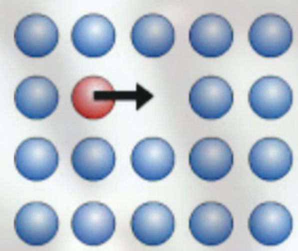


  - Dislocations: $\nabla \times \nabla u_i = b_i \delta^2(\vec{r}) \equiv b_i(\vec{r}).$



  - Vacancies/interstitials:

$$n_d = n_v - n_i$$

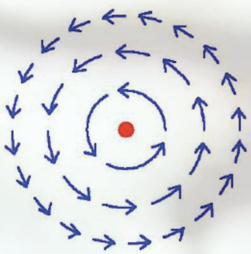


Dasgupta, Halperin
Fisher, Lee

Boson-vortex duality

Superfluid

vortices



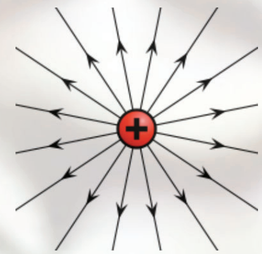
topological
winding

$$\nabla \times \mathbf{j} = \rho$$

$$\mathbf{j} = \nabla\phi$$

Maxwell Gauge Theory (with matter)

particles



Gauss's law:

$$\nabla \cdot \mathbf{E} = \rho$$

Goldstone mode

$$H = \frac{1}{2}\int d^2x \left[|\nabla\phi|^2 + n^2\right]$$

$$[n, \phi] = i$$

photon

$$H = \frac{1}{2}\int d^2x \left[|\mathbf{E}|^2 + (\nabla \times \mathbf{A})^2\right]$$

$$[A_i, E_j] = i\delta_{ij}$$

M. Pretko, L. R.,2017
PRLs 2018

Fracton-elasticity duality

see also
Zaanen, et al
other contexts

- Elastic Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial_t u_i)^2 - \frac{1}{2}u_{ij}^2$
- Disclinicity: $\partial_i^* \partial_j^* u_{ij} = s(\mathbf{x}) + \hat{\mathbf{z}} \cdot \nabla \times \mathbf{b}(\mathbf{x})$
- Momentum conservation (Newton) constraint: $\partial_t \pi^i - \partial_j \sigma^{ij} = 0$
- "Electric", "magnetic" fields: $B^i = \epsilon^{ij}\pi_j \quad E_\sigma^{ij} = \epsilon^{ik}\epsilon^{j\ell}\sigma_{k\ell}$

-> Faraday law: $\partial_t B^i + \epsilon_{jk}\partial^j E_\sigma^{ki} = 0$

-> Gauge fields: $B^i = \epsilon_{jk}\partial^j A^{ki} \qquad E_\sigma^{ij} = -\partial_t A^{ij} - \partial_i \partial_j \phi$

-> Gauge freedom: $A_{ij} \rightarrow A_{ij} + \partial_i \partial_j \alpha \qquad \phi \rightarrow \phi + \partial_t \alpha$

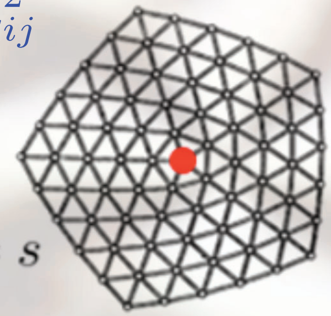
-> Peach-Koehler force: $F_i = E_{ij}\, p_j$

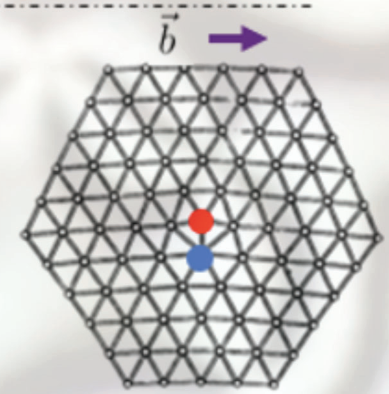
M. Pretko, L. R., 2017
PRLs 2018

Fracton-elasticity duality

$$\mathcal{H} = \frac{1}{2}\pi_i^2 + \frac{1}{2}u_{ij}^2$$

Fracton $\partial_i \partial_j E^{ij} = \rho$	Disclination $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s$
Dipole	Dislocation
Gauge Modes	Phonons
Electric Field E_{ij}	Strain Tensor u_{ij}
Magnetic Field B_i	Lattice Momentum π_i





$$\partial_t B^i + \epsilon_{jk}\partial^j E_\sigma^{ki} = 0. \Longleftrightarrow \partial_t \pi^i - \partial_j \sigma^{ij} = 0$$

Faraday <-> Newton

L. R.,
Hermele
PRL 2020

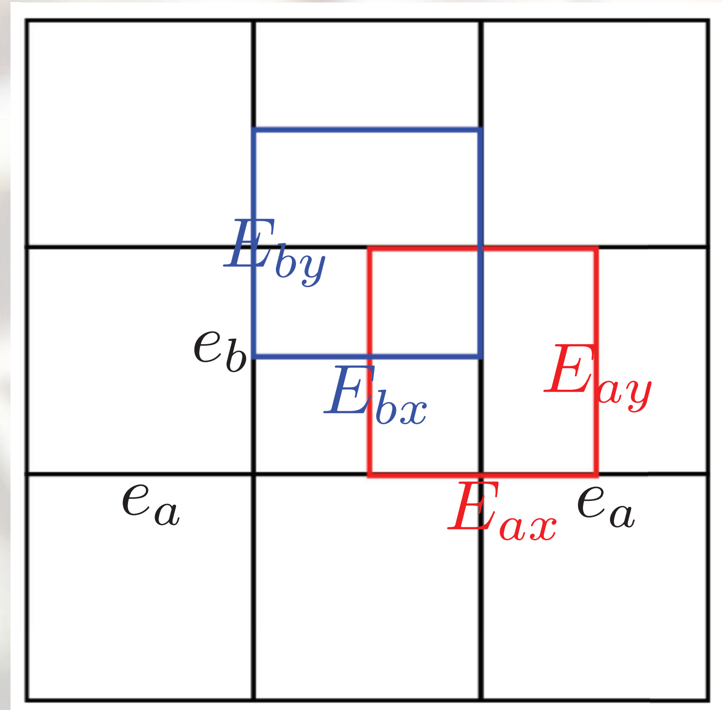
Fractons via **vector** gauge theory

- Lattice fractonic *vector* gauge theory:

$\mathbf{E}_a$ $\mathbf{E}_b$ $\mathbf{e}$

$$[\hat{A}_{ik}, \hat{E}_{jk'}] = -i\delta_{ij}\delta_{kk'}\delta^2(\mathbf{x}-\mathbf{x}'),$$

$$[\hat{a}_i, \hat{e}_j] = -i\delta_{ij}\delta^2(\mathbf{x}-\mathbf{x}')$$



- Gauss' law:

$$\nabla\cdot\mathbf{e} = s \qquad \nabla\cdot\mathbf{E}_k = e_k$$

(k = a, b)

$$\tilde{\mathcal{H}} = \frac{1}{2}C|\mathbf{E}_k|^2 + \frac{1}{2}(\nabla\times\mathbf{A}_k)^2 + \frac{1}{2}K|\mathbf{e}|^2 + \frac{1}{2}(\nabla\times\mathbf{a} - A_{[ij]})^2 - \mathbf{A}_k\cdot\mathbf{J}^b_k - \mathbf{a}\cdot\mathbf{j}^s$$

gauge invariance demands $\partial_t p_k + \nabla\cdot\mathbf{J}_k = j_k \longrightarrow \mathbf{j} = 0$

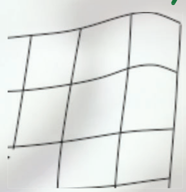
"Fractons via higher-form symmetry breaking"
Qi, L. R., Hermele, AOP 2020

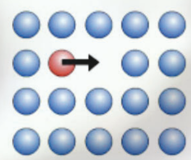
M. Pretko, L. R., PRL2018

2D *Bosonic crystal*

- Coupled elasticity and bosonic vacancies/interstitials:

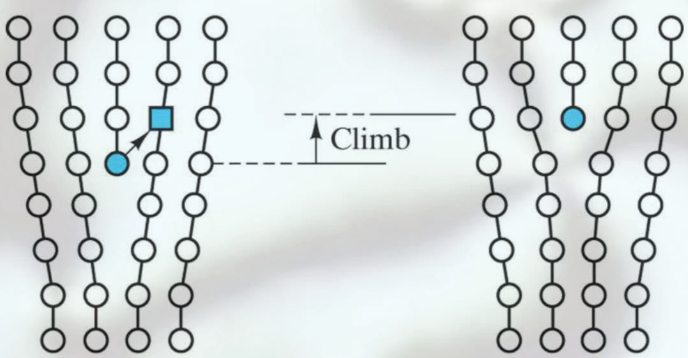
$$\hat{\mathcal{H}} = \underbrace{\frac{1}{2}\hat{\pi}^2 + \frac{1}{2}\hat{u}_{ij}^2}_{\text{elasticity}} + \underbrace{\frac{1}{2}(\nabla\hat{\phi})^2 + \frac{1}{2}\hat{n}^2}_{\text{vacancies/interstitials}} + \underbrace{\nabla\hat{\phi}\cdot\hat{\pi} + \hat{n}\hat{u}_{ii}}_{\text{coupling}}$$





-> *Commensurate* (Mott-insulating) crystal

-> *Incommensurate* (supersolid) crystal



$$\partial_t n_d + \partial_i J_d^i = -J_i^i$$

(→ Ampere's law)

Marchetti, L.R. 1998

M. Pretko, L. R., PRL2018

2D **Symmetry-enriched fractons**

Kumar Potter, 2018

- Hybrid U(1) vector-tensor gauge duality

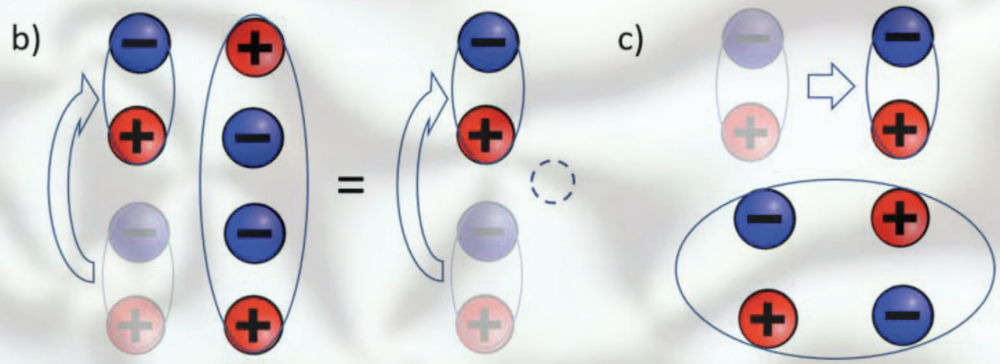
$$\mathcal{H} = \underbrace{\frac{1}{2}(E_{ij}^2 + B_i^2)}_{\text{elasticity}} + \underbrace{\frac{1}{2}(\mathbf{e}^2 + b^2)}_{\text{bosons}} + \underbrace{g(\mathbf{B}\cdot\mathbf{e} + E_{ii}b)}_{\text{"axion" coupling}} + \underbrace{J^{\mu\nu}A_{\mu\nu} + j^\mu a_\mu}_{\text{charges}}$$

-> supersolid -> "fracton superfluid" (mobile dipoles) F

-> normal crystal -> "fracton Mott insulator" (confined dipoles) $F_{U(1)}$

$\hat{a}_{\mathbf{r}}\hat{b}^\dagger_{x,\mathbf{r}+\hat{\mathbf{y}}}\hat{b}_{x,\mathbf{r}}e^{iA_{yy}}$

$E_{yy} \neq 0$



$\hat{b}^\dagger_{x,\mathbf{r}+\hat{\mathbf{x}}}\hat{b}_{x,\mathbf{r}}e^{iA_{xy}}$

$E_{xy} \neq 0$

- Vortex condensation: F --> $F_{U(1)}$

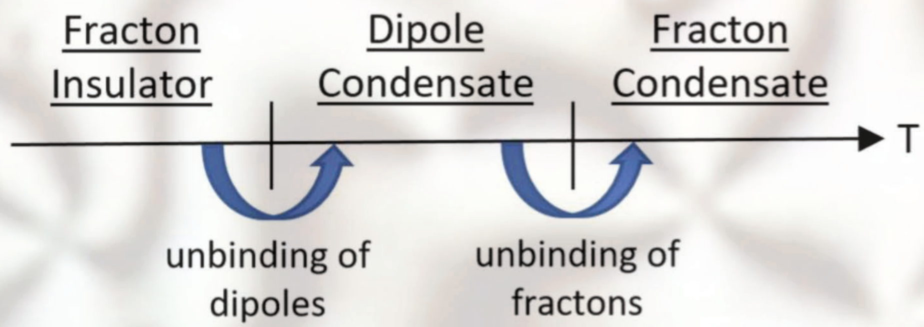
-> fracton dipole dimensional confinement

-> superfluid to Mott-insulating fracton transition

Zhai, L. R., 2018

Fracton condensation transition

2D

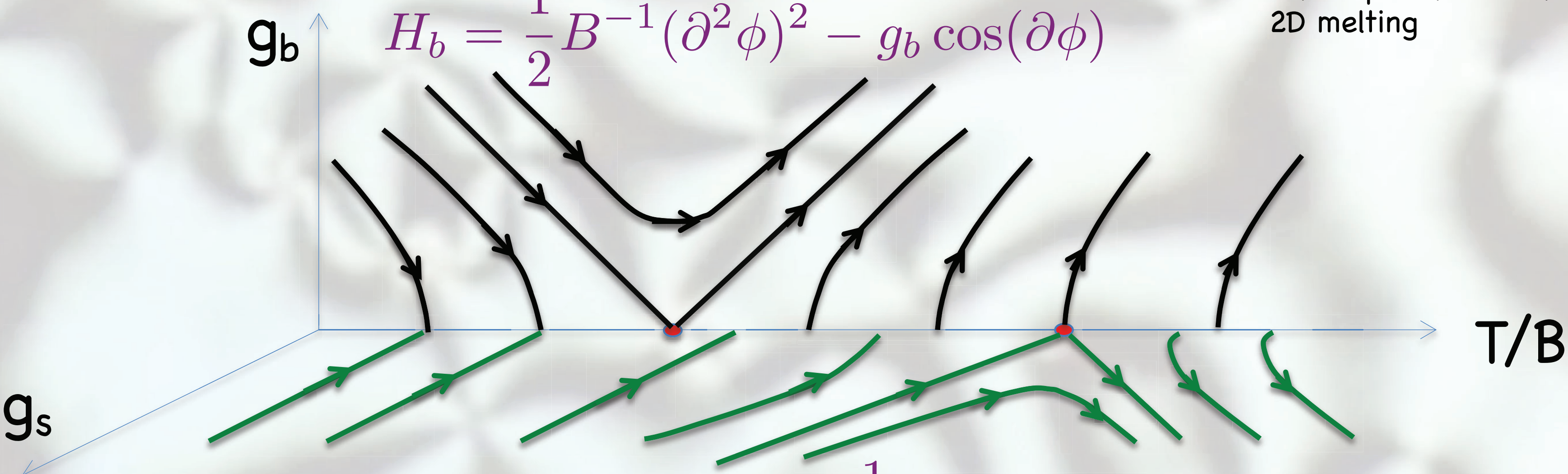


2D scalar fracton model:

$$\tilde{\mathcal{H}} = \frac{1}{2}B^{-1}(\nabla^2\phi)^2 - \underbrace{g_s \cos\left(\frac{2\pi}{6}\phi\right)}_{\text{charges}} - g_b \sum_{n=1,2,3} \underbrace{\cos\left(\mathbf{b}_n \cdot \hat{z} \times \nabla\phi\right)}_{\text{dipoles}}$$

KT, Halperin, Nelson, Young '79
2D melting

$$H_b = \frac{1}{2}B^{-1}(\partial^2\phi)^2 - g_b \cos(\partial\phi)$$



$$H_s = \frac{1}{2}g_b(\partial\phi)^2 - g_s \cos(\phi)$$

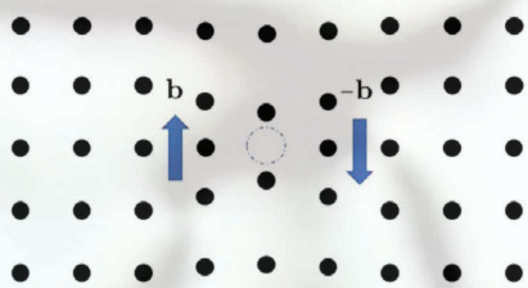
M. Pretko, L. R., 2018

2D *Quantum melting fractons*

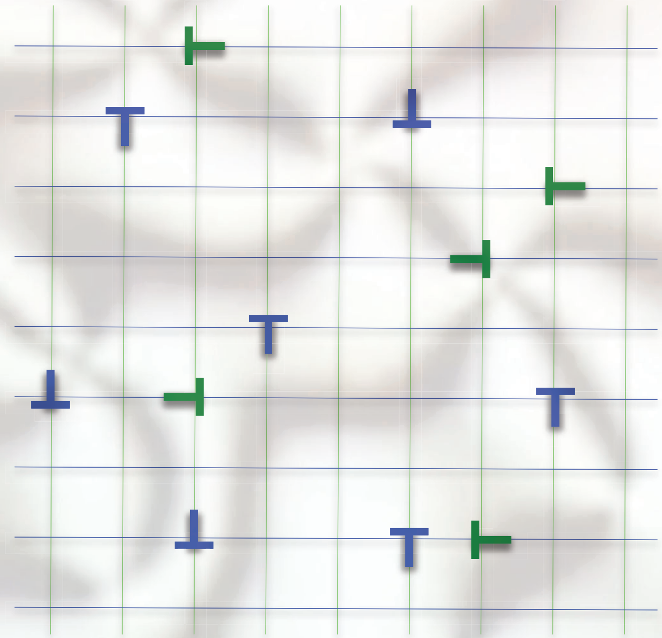
Kumar Potter, 2018

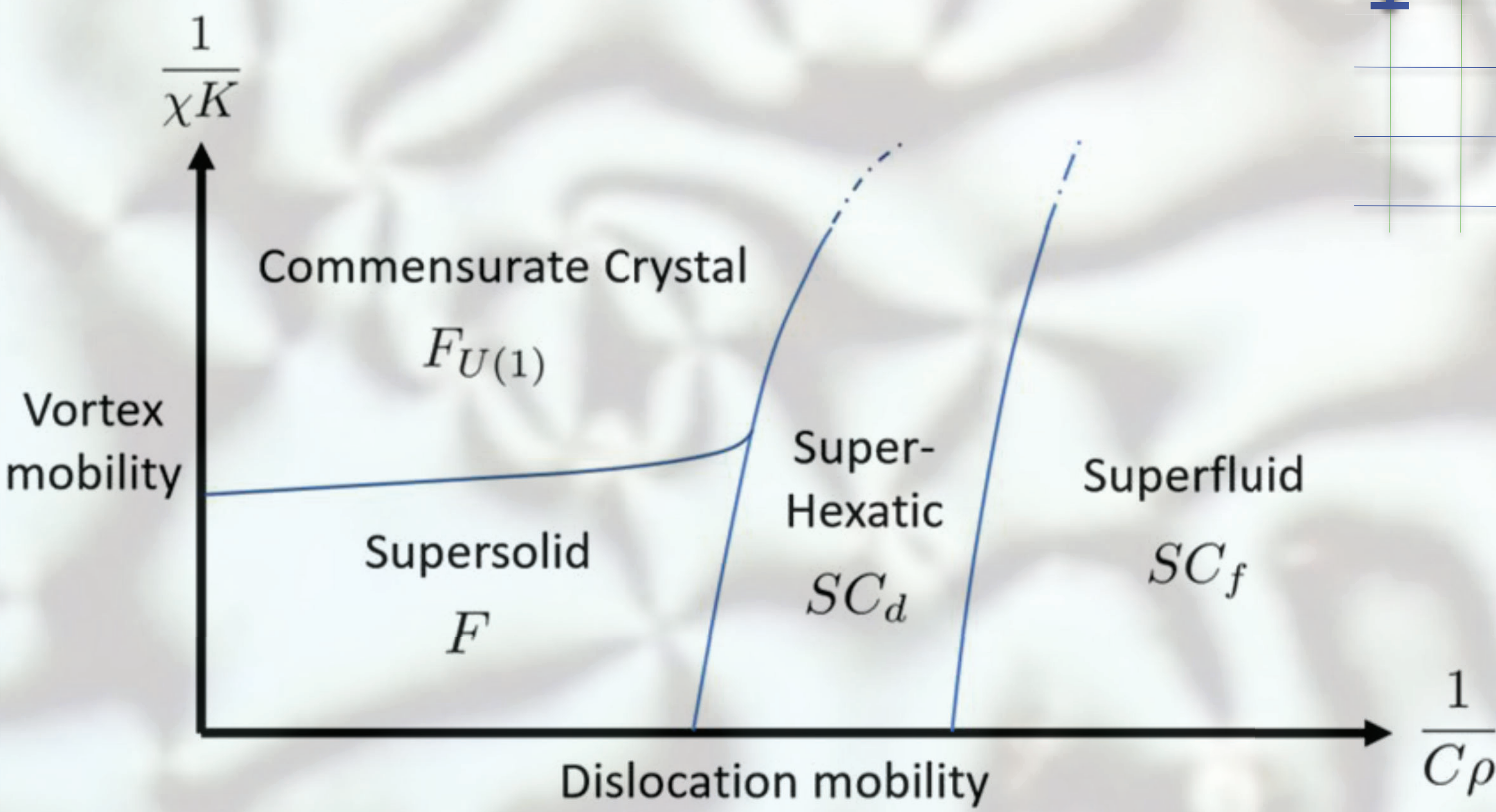
- Fracton dipoles $b_n = \sqrt{n_d} e^{i\theta_n}$ condense: -> ***super**-hexatic*

$$\mathcal{L} = \frac{1}{2}E_{ij}^2 - \frac{1}{2}B_i^2 - \cos(\partial_t\theta - A_0) + g\cos(\partial_i\partial_j\theta - A_{ij})$$



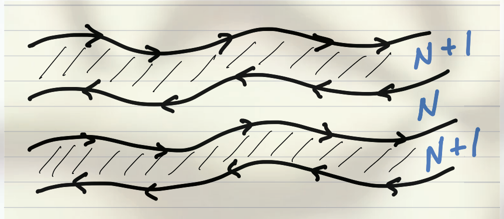
$\hat{b}^\dagger_{\mathbf{b}}\hat{b}^\dagger_{-\mathbf{b}}\hat{a}$

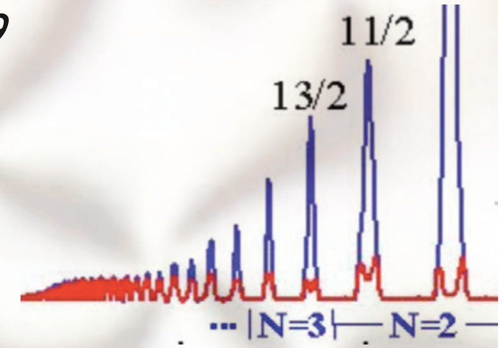




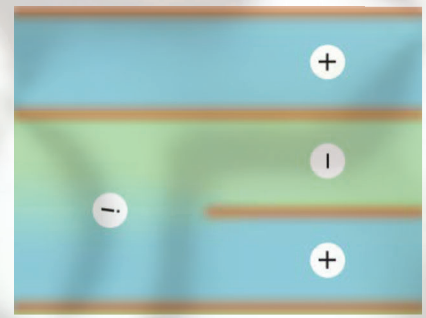
Quantum liquid crystals

- Quantum Hall *Fogler, et al. '96, Moessner, Chalker '96, Fradkin, Kivelson '99, MacDonald, Fisher '99, L.R., Dorsey '02,…Eisenstein, et al. '99*

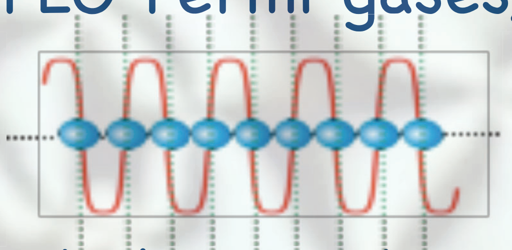




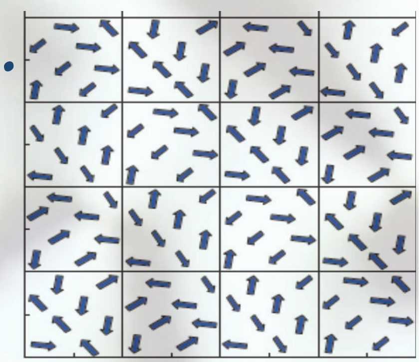
- SDW, CDW, PDW in doped Mott insulators
  Tranquada, et al., '97, Kivelson, Fradkin, Emery '98, Sachdev,…



- Imbalanced FFLO Fermi gases, SOC Bose gases, *L.R. et al. '09,'11, Zhai '15*



- Helical, frustrated magnets, e.g., MnSi, FeGe, AB_2X_4,…
  Pfleiderer, et al.' 09, Bergman, et al. '07

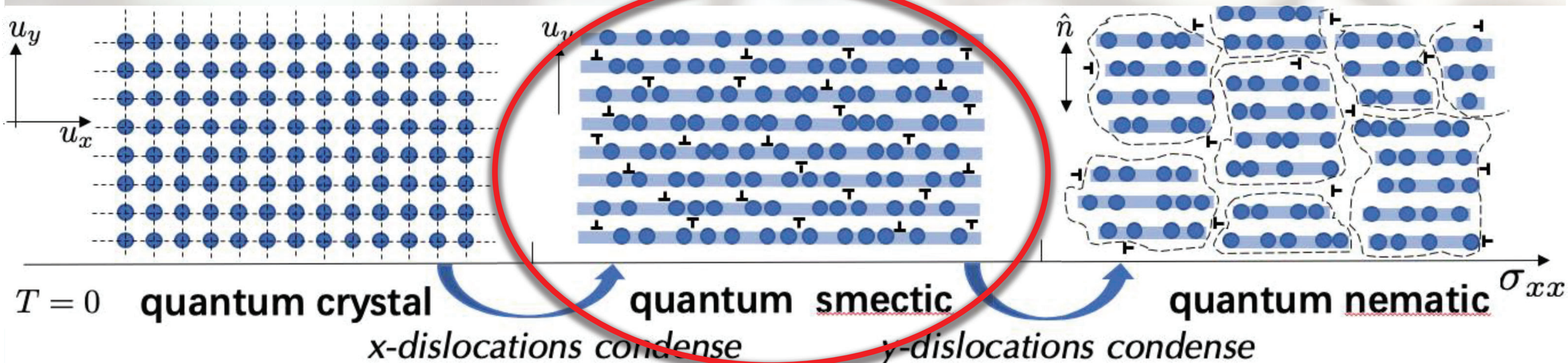


L.R PRL 2020
L.R., Z. Zhai
AOP 2021

Anisotropic quantum melting

Ostlund, Halperin '81

- Crystal: $\mathcal{H}_{cr} = \frac{C}{2} u_{ij}^2 + \frac{1}{2}\pi^2$
- Condense x-dislocations: $b_x = \sqrt{n_d} e^{i\theta_x}$ -> ***super**-smectic*

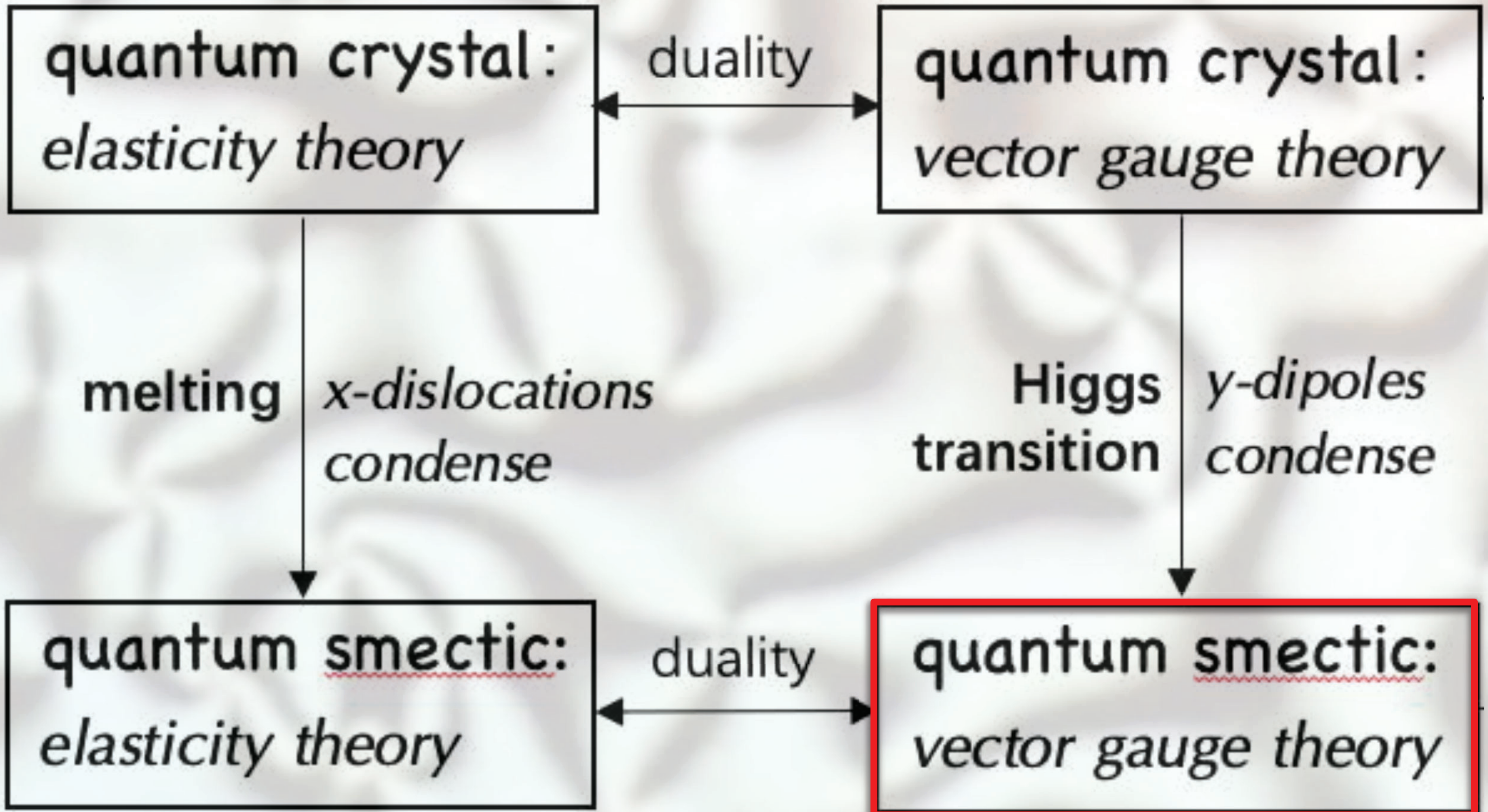


- Super-smectic:

$$\mathcal{H}_{sm} = \underbrace{\frac{C}{2}(\nabla u_y - \theta \hat{\mathbf{x}})^2 + \frac{K}{2}(\nabla\theta)^2 + \frac{1}{2}\pi^2 + \frac{1}{2}L^2}_{\text{quantum smectic elasticity}} + \underbrace{\frac{1}{2}(\nabla\phi_s)^2 + \frac{U}{2}n^2}_{\text{bosonic atoms}}$$

L.R PRL 2020
L.R., Z. Zhai
AOP 2021

Crystal – smectic – gauge duality



L.R PRL 2020
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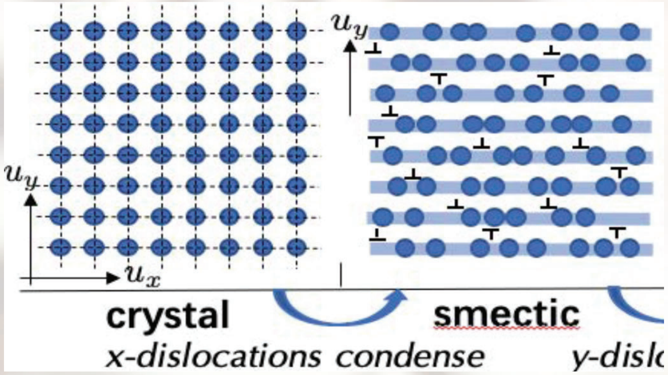
Higgs'ing crystal gauge dual -> smectic gauge dual



- Crystal gauge dual:

$$\tilde{\mathcal{H}} = \frac{1}{2}C|\mathbf{E}_k|^2 + \frac{1}{2}(\nabla \times \mathbf{A}_k)^2 + \frac{1}{2}K|\mathbf{e}|^2 + \frac{1}{2}(\nabla \times \mathbf{a} - A_{[ij]})^2 - \mathbf{A}_k \cdot \mathbf{J}_k^b - \mathbf{a} \cdot \mathbf{j}^s$$

$$\tilde{\mathcal{H}}_{cr} = |(i\nabla - p_k \mathbf{A}_k)\psi_k|^2 + V(\psi_k) + \tilde{\mathcal{H}}_{Max}[\mathbf{A}_k, \mathbf{E}_k]$$

- ➢ Higgs transition - condensation of y-dipoles (x-dislocations)

$$\psi_x = 0, \quad \psi_y \neq 0 \quad \rightarrow \quad \mathbf{A}_y \approx 0 \ \text{ gapped}$$

- Smectic gauge dual: $\tilde{\mathcal{H}}_{sm}[\mathbf{A}^x, \mathbf{a}] \approx \tilde{\mathcal{H}}_{cr}[\mathbf{A}^x, \mathbf{A}^y = 0, \mathbf{a}]$

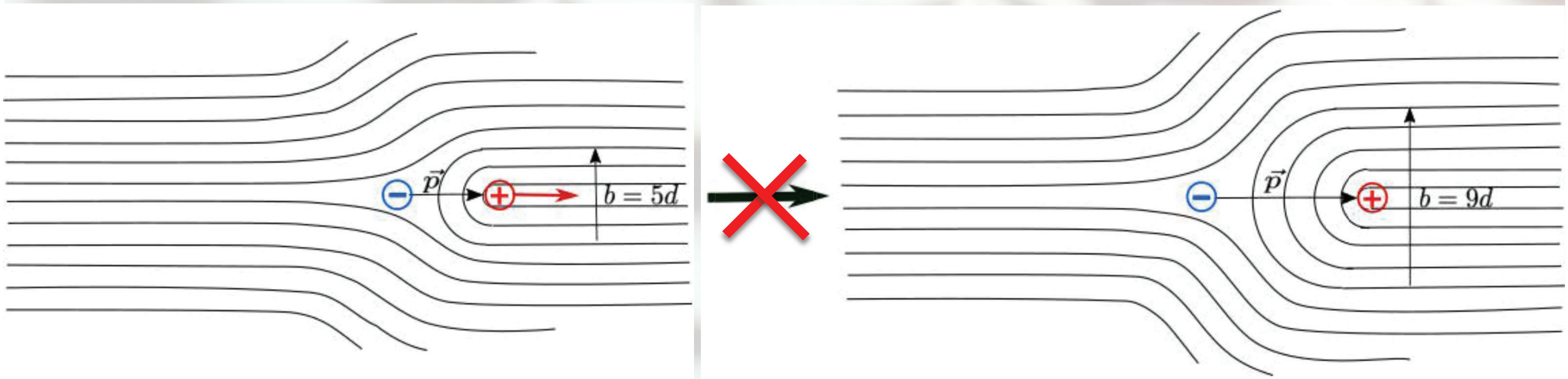
$$\tilde{\mathcal{H}}_{sm} = \frac{1}{2}C\mathbf{E}^2 + \frac{1}{2}(\nabla \times \mathbf{A})^2 + \frac{1}{2}K\mathbf{e}^2 + \frac{1}{2}(\nabla \times \mathbf{a} - \hat{\mathbf{y}} \cdot \mathbf{A})^2 - \mathbf{A} \cdot \mathbf{J} - \mathbf{a} \cdot \mathbf{j}$$

L.R PRL 2020
L.R., Z. Zhai
AOP 2021

Restricted disclination mobility

gauge invariance demands $\partial_t p + \nabla \cdot \mathbf{J} = \hat{\mathbf{x}} \cdot \mathbf{j} \longrightarrow j_x = 0$

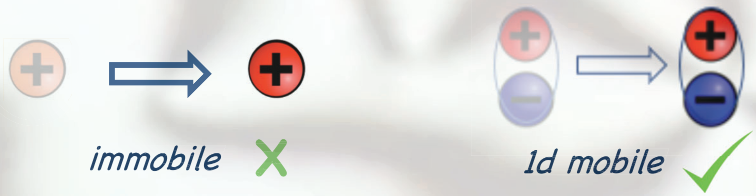
- Fractonic restricted dynamics *via disclination microscopics:*



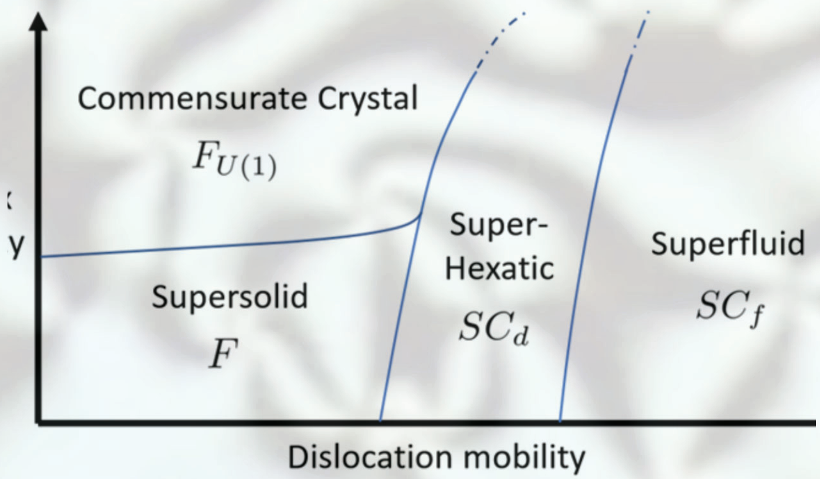
requires a nonlocal process of adding a pair smectic half-layer per lattice constant of disclination separation

Summary and conclusions

- *New class of fractonic quantum liquids*
  excitations w/ restricted/fractionalized mobility

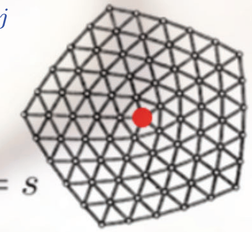


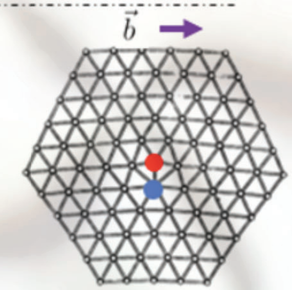
- *Fractons – elasticity duality*
  realized as defects in quantum crystal
- *Fractonic phases and transitions:*

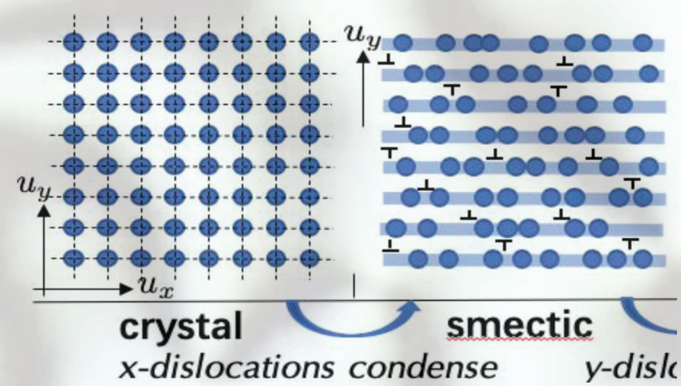


$$\mathcal{H} = \frac{1}{2}\pi_i^2 + \frac{1}{2}u_{ij}^2$$

Fracton $\partial_i\partial_j E^{ij} = \rho$	Disclination $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s$
Dipole	Dislocation
Gauge Modes	Phonons
Electric Field E_{ij}	Strain Tensor u_{ij}
Magnetic Field B_i	Lattice Momentum π_i







- *Quantum melting criticality?*
- *QH smectic ?* (anisotropic melting, via Higgs transitions, LR 'PRL20)
- *Elastic nonlinearities ?*
- *Classification, relation to Z_2 models, higher form symmetries, …?*
- *Animation dynamics and editing of tessellated surfaces ?*

Thank you

L. R., 2018

2D *T-R breaking fractons*

Kumar
Potter, 2018

- Wigner crystal in B-field elasticity (also vortex lattice)

$$\hat{\mathcal{H}} = \frac{1}{2} C^{ijk\ell} \hat{u}_{ij} \hat{u}_{k\ell} \qquad [u_x(\mathbf{r}), u_y(\mathbf{r}')] = i\ell^2 \delta^2(\mathbf{r} - \mathbf{r}').$$

- T-R breaking fracton phase

$$\hat{\mathcal{L}} = \frac{1}{2} \mathbf{B} \times \partial_t \mathbf{B} - \frac{1}{2} C^{ijk\ell} E_{ij} E_{k\ell}$$

$$\rightarrow \quad \omega \sim q^2$$

M. Pretko, L. R., 2017

Fracton-elasticity duality

- Elastic Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial_t u^i)^2 - \frac{1}{2}C^{ijk\ell}u_{ij}u_{k\ell}$
- Disclinations: $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s(\mathbf{x})$
- Electric, magnetic fields: $B^i = \epsilon^{ij}\pi_j \qquad E^{ij}_\sigma = \epsilon^{ik}\epsilon^{j\ell}\sigma_{k\ell}$

$B^i = \epsilon_{jk}\partial^j A^{ki} \qquad E^{ij}_\sigma = -\partial_t A^{ij} - \partial_i\partial_j\phi$

-> Fracton Hamiltonian: $[E^{ij}, A_{ij}] = i\delta^{(2)}(\mathbf{x})$

$$\mathcal{H} = \frac{1}{2}\tilde{C}^{ijk\ell}E_{ij}E_{k\ell} + \frac{1}{2}B^i B_i + \rho\phi + J^{ij}A_{ij}$$

-> Fracton charges, dipole currents:

$$\rho = s \qquad J^{ij} = \epsilon^{ik}\epsilon^{j\ell}(\partial_t\partial_k - \partial_k\partial_t)u_\ell = \epsilon^{(i\ell}v^{j)}b_\ell$$

-> Gauss' law, continuity: $\partial_i\partial_j E^{ij} = \rho \qquad \partial_t\rho + \partial_i\partial_j J^{ij} = 0$

-> Ampere's law: $\partial_t E^{ij} + \frac{1}{2}(\epsilon^{ik}\partial_k B^j + \epsilon^{jk}\partial_k B^i) = -J^{ij} \qquad \partial_t n_d + \partial_i J^i_d = -J^i_i$

Marchetti, L.R. 1998

M. Pretko,
L. R., 2017
PRLs 2018

Fracton dual "superconductor"

- trace over fractons and dipoles:

$$\mathcal{L} = \frac{1}{2}E_{ij}^2 - \frac{1}{2}B_i^2 - \cos(\partial_t\theta - A_0) + g\cos(\partial_i\partial_j\theta - A_{ij})$$

- Fractons in "normal" Coulomb phase (*crystal*)

- Higgs transition out of fracton phase (*liquid*)

L. R. 2016

Fractons via **vector** gauge theory ?

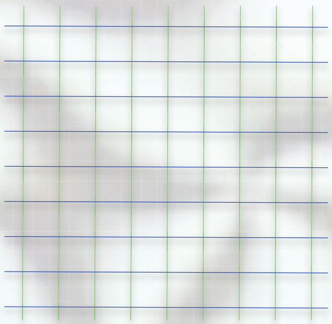
- *Flavored* xy model -> vector gauge duality *(no fractons)*

$$\mathcal{H} = \frac{1}{2}n_k^2 + \frac{1}{2}|\nabla\phi_k|^2 \longrightarrow \tilde{\mathcal{H}} = \frac{1}{2}(\nabla \times \mathbf{A}_k)^2 + \frac{1}{2}|\mathbf{E}_k|^2$$

- Reformulate elasticity into coupled xy models: $u_{ik} \longrightarrow \partial_i u_k$

$$\mathcal{H} = \frac{1}{2}\pi_k^2 + \frac{1}{2}Cu_{ik}^2 \longrightarrow \mathcal{H} = \frac{1}{2}\pi_k^2 + \frac{1}{2}C(\partial_i u_k - g\theta\epsilon_{ik})^2 + \frac{1}{2}L^2 + \frac{1}{2}K(\nabla\theta)^2$$

- Target space rotational symmetry:



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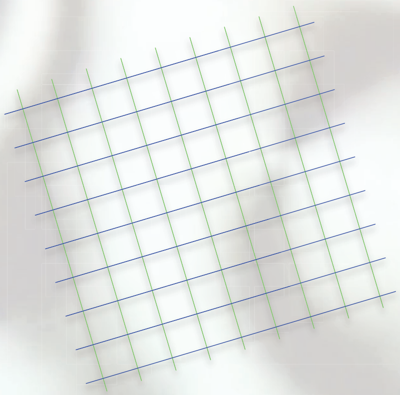

-> $u_x = x(\cos\theta - 1) + y\sin\theta$

-> $u_y = -x\sin\theta + y(\cos\theta - 1)$

L. R., 2018

Fracton "sliding phase"

Incompressible crystal -> "fracton Mott insulator"

$F_{U(1)}$

$$\hat{H} = \sum_{\mathbf{r}} \left[-t_x \hat{b}^{\dagger}_{x,\mathbf{r}+\hat{\mathbf{x}}} \hat{b}_{x,\mathbf{r}} e^{iA_{xy}} - t_y \hat{b}^{\dagger}_{y,\mathbf{r}+\hat{\mathbf{y}}} \hat{b}_{y,\mathbf{r}} e^{iA_{xy}} + \frac{1}{2} B_i^2 + \frac{1}{2} C_{ij} E_{ij}^2 \right]$$

- dispersionless along lines
- stability to interactions?
- $D_{climb} \sim e^{-\Delta/T} << D_{glide}$