Lecture 11

Wave nature of matter

Announcements:

- lecture 10 is posted
- homework 6 (*due Feb 25, in class)* solutions are posted on CULearn
- homework 7 (*due March 4, in class*) is posted on CULearn
- reading for this week is:
  - Ch 6 in TZD

Last Time

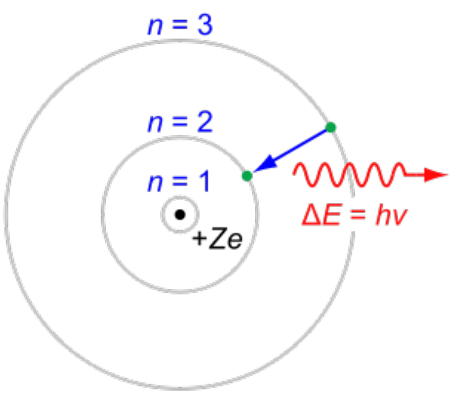
recall lecture 10:

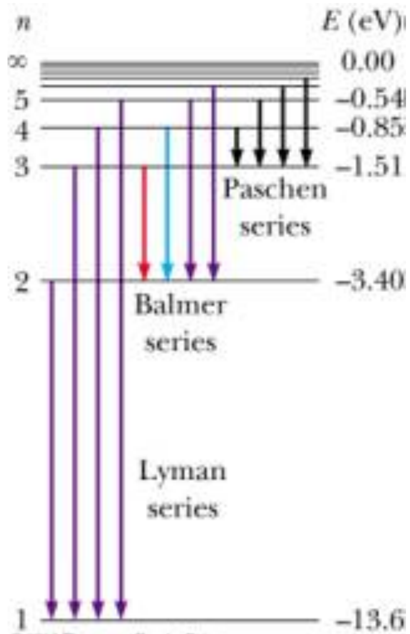
Problems with classical physics: *atomic spectra*

- Atomic spectra:

Balmer series of Hydrogen (n->2 transitions):

- Atomic instability in classical theory
- Bohr's theory of atomic spectra:





$$E_n = -\frac{Z^2 e^4 m}{2\hbar^2 n^2} = -\frac{13.6eV}{n^2} Z^2$$

Today

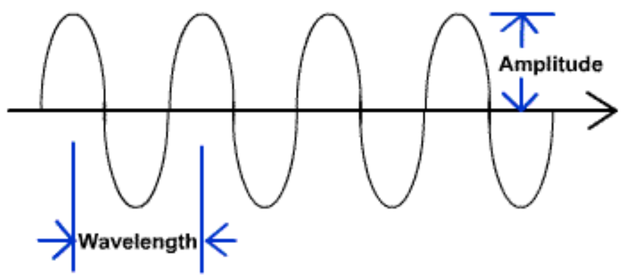
Wave nature of matter

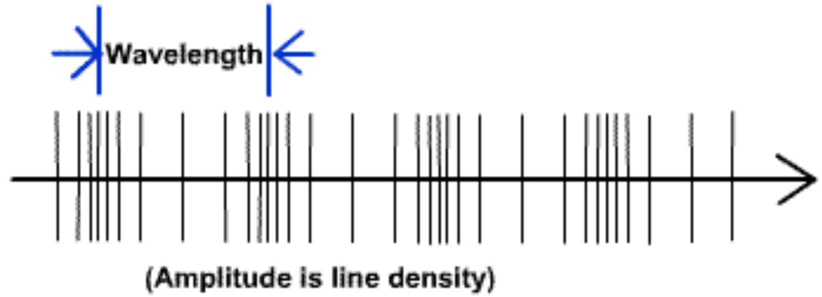
- waves primer
- Young's double-slit experiment
- electron diffraction Davisson-Germer experiment
- deBroglie matter waves
- wavefunction and its interpretation
- Heisenberg uncertainty principle

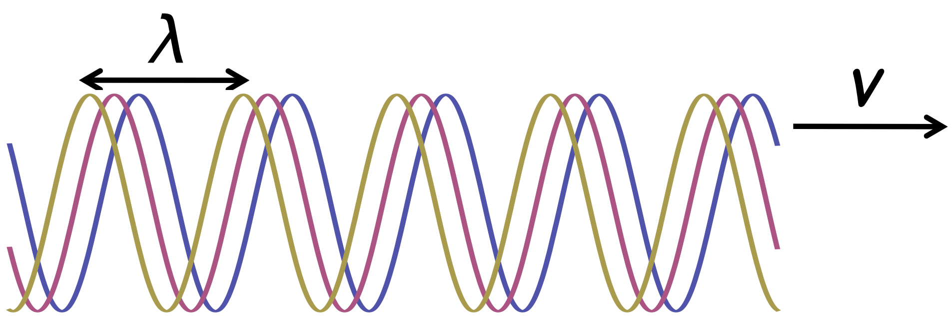
Waves primer: basics

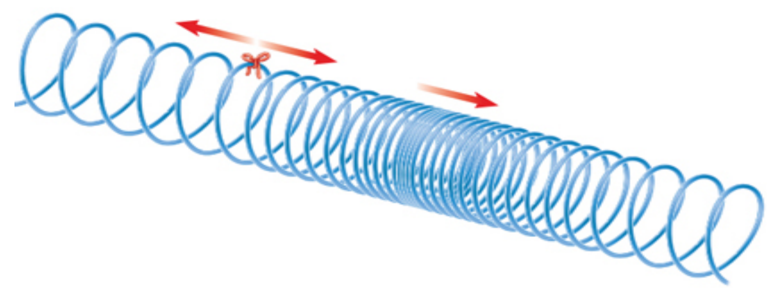
- periodic *(spatially-temporally extended)* disturbance

  e.g., sound, water, EM waves (in gas, liquid, solid, vacuum)









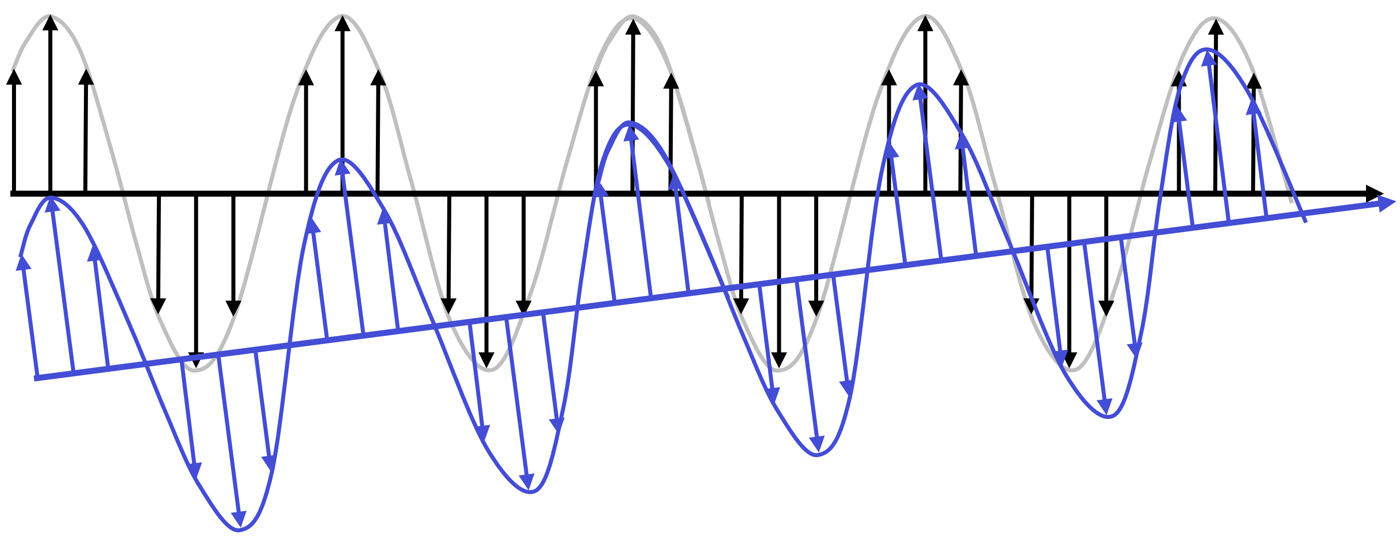
$$\mathcal{E} = \mathcal{E}_0 \cos(kx - \omega t) = \mathcal{E}_0 \cos[k(x - vt)]$$

- *frequency:* $\omega = 2\pi\nu$
- *wavevector:* $k = 2\pi/\lambda$
- *phase velocity:* $\omega = v_p\, k$

Interference

- key wave property: *interference*

- constructive
  destructive



Mathematics of interference (I)

- wave interference:
$$\begin{aligned} I_{12} &= \mathcal{E}_{12}^2 = (\mathcal{E}_1 + \mathcal{E}_2)^2 \\ &= \mathcal{E}_1^2 + \mathcal{E}_2^2 + 2\mathcal{E}_1\mathcal{E}_2 \\ &= I_1 + I_2 + 2\mathcal{E}_1\mathcal{E}_2 \neq I_1 + I_2 \end{aligned}$$

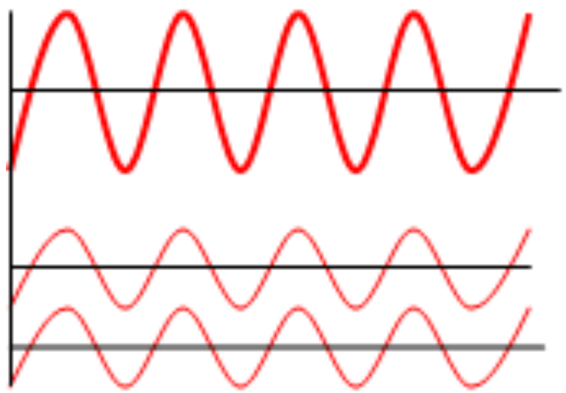
  - adding two phase-shifted waves:

in-phase

$\mathcal{E}_{12} = 2\cos(kx)$

$\mathcal{E}_1 = \cos(kx)$

$\mathcal{E}_2 = \cos(kx)$



$$\begin{aligned} I_{12} &= 4\cos^2 kx \\ &= \cos^2 kx + \cos^2 kx + 2\cos^2 kx \end{aligned}$$

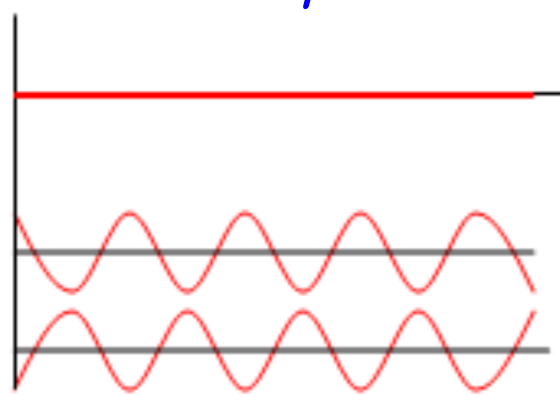
constructive interference

out-of-phase

$\mathcal{E}_{12} = 0$

$\mathcal{E}_1 = \cos(kx + \pi)$

$\mathcal{E}_2 = \cos(kx)$

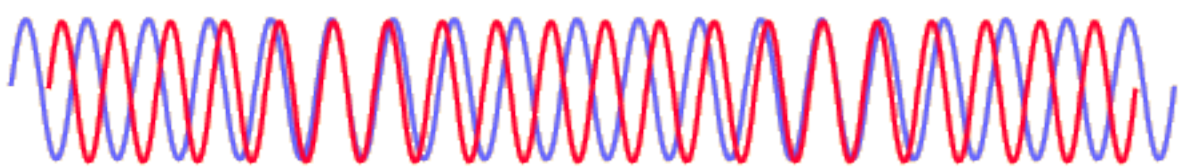


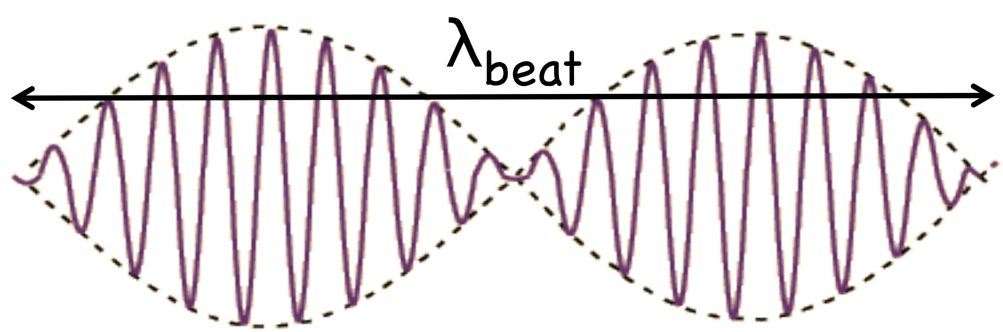
$$\begin{aligned} I_{12} &= 0 \\ &= \cos^2 kx + \cos^2 kx - 2\cos^2 kx \end{aligned}$$

destructive interference

Mathematics of interference (II)

- wave interference: $$\begin{aligned} I_{12} &= \mathcal{E}_{12}^2 = (\mathcal{E}_1 + \mathcal{E}_2)^2 \\ &= \mathcal{E}_1^2 + \mathcal{E}_2^2 + 2\mathcal{E}_1\mathcal{E}_2 \\ &= I_1 + I_2 + 2\mathcal{E}_1\mathcal{E}_2 \neq I_1 + I_2 \end{aligned}$$
    - adding two different wavelengths, k_1, k_2 waves:



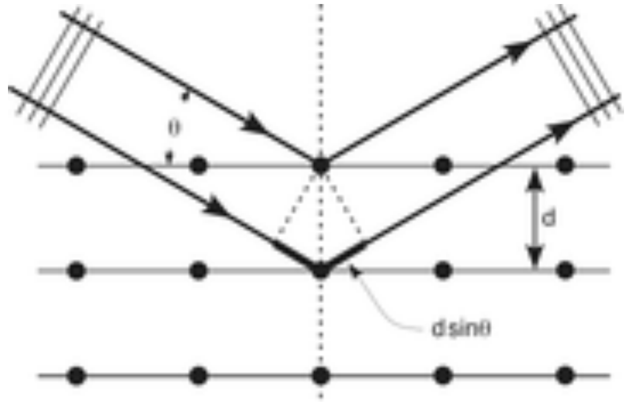


$$\begin{aligned} \mathcal{E}_{12} &= \cos(k_1 x) + \cos(k_2 x) \\ &= 2\cos\left[\frac{1}{2}(k_1 + k_2)x\right]\cos\left[\overbrace{\frac{1}{2}(k_1 - k_2)}^{k_{beat}}x\right] \end{aligned}$$

beating phenomena (tuning piano, FM modulation,…)

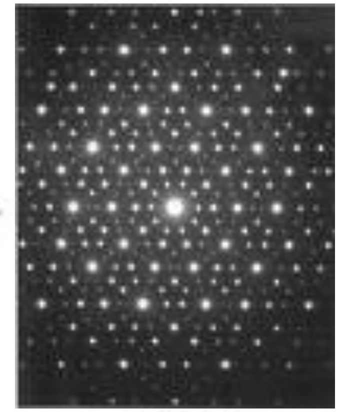
Diffraction and refraction

- Bragg diffraction:
  - *spectroscopy*

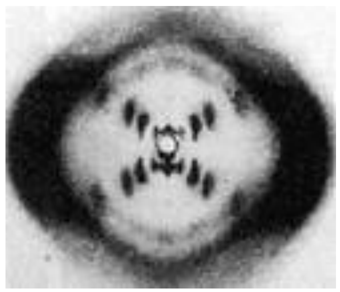


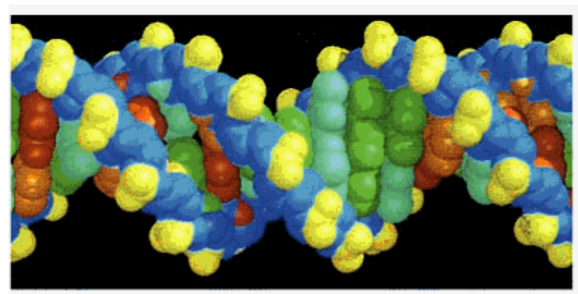
Bragg condition:

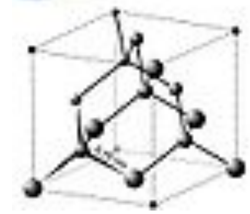
$2d\sin\vartheta = n\lambda$

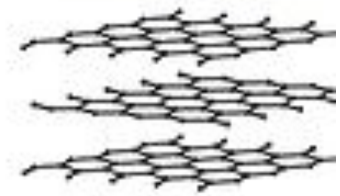


  - *crystallography:*
    - *crystals*
    - *DNA*
    - *Proteins*

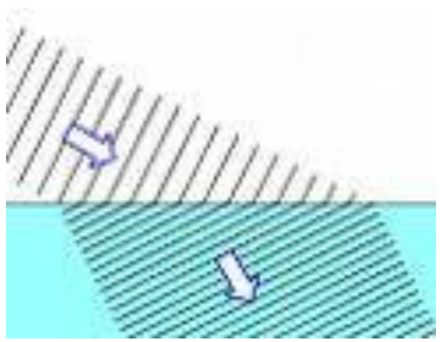


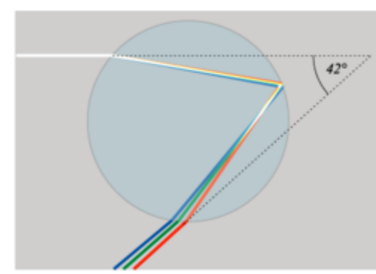


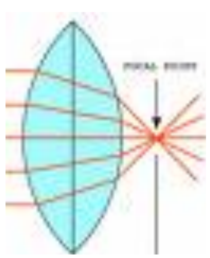


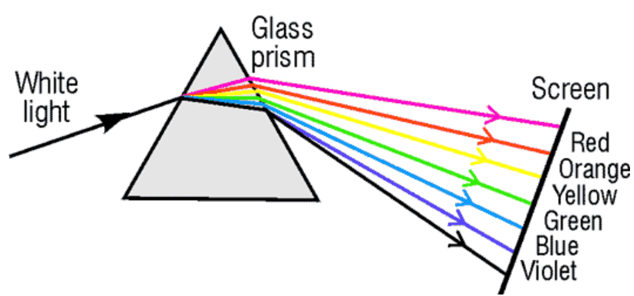


- refraction:
  - *rainbow*
  - *prism*
  - *lens*



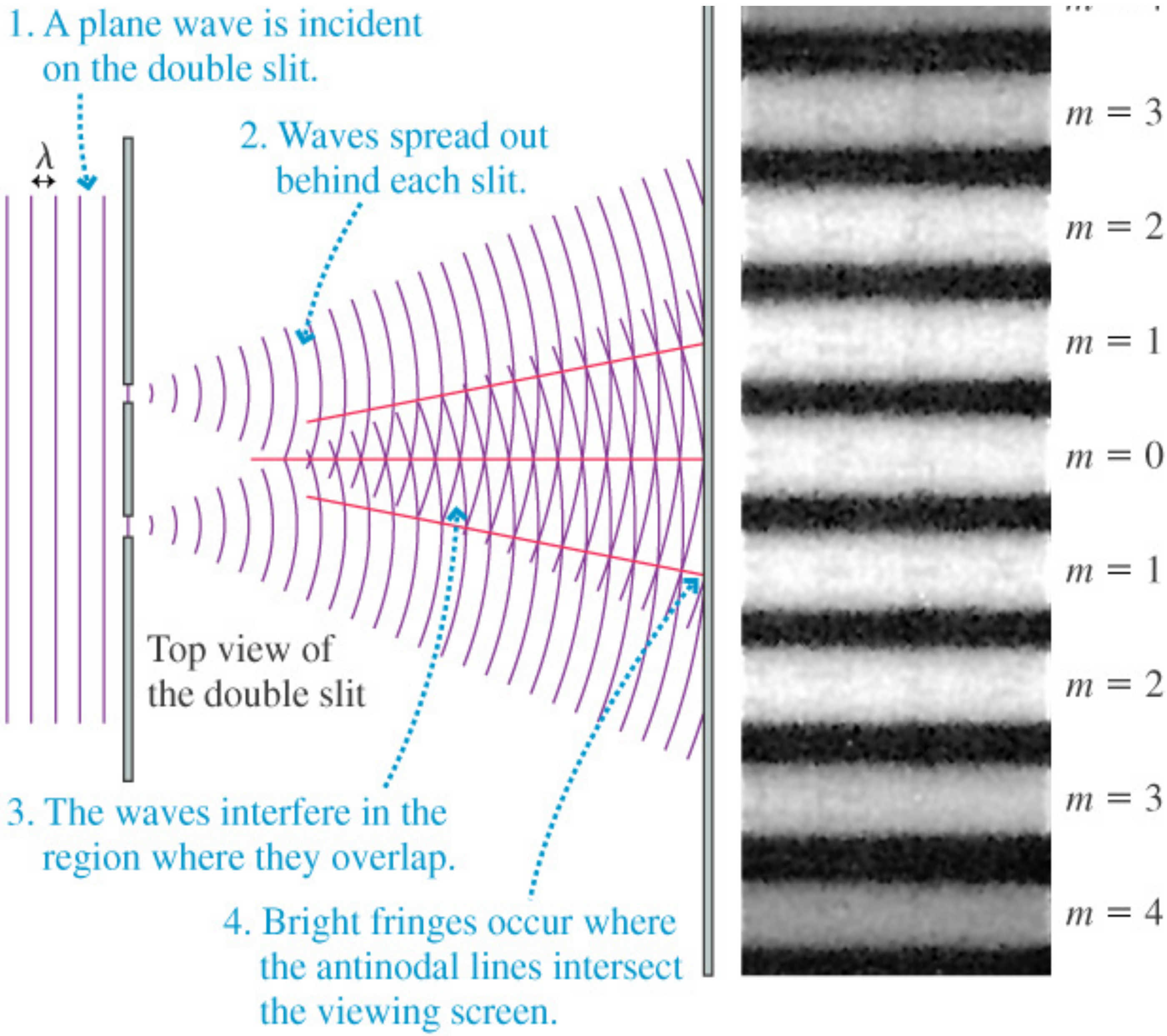




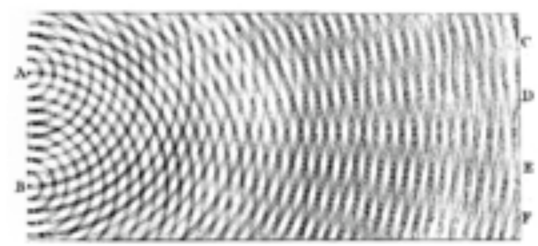


Young's double-slit experiment

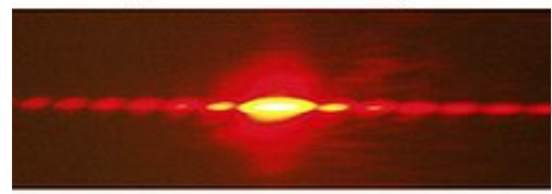
wave character of light:



T. Young
1773-1829



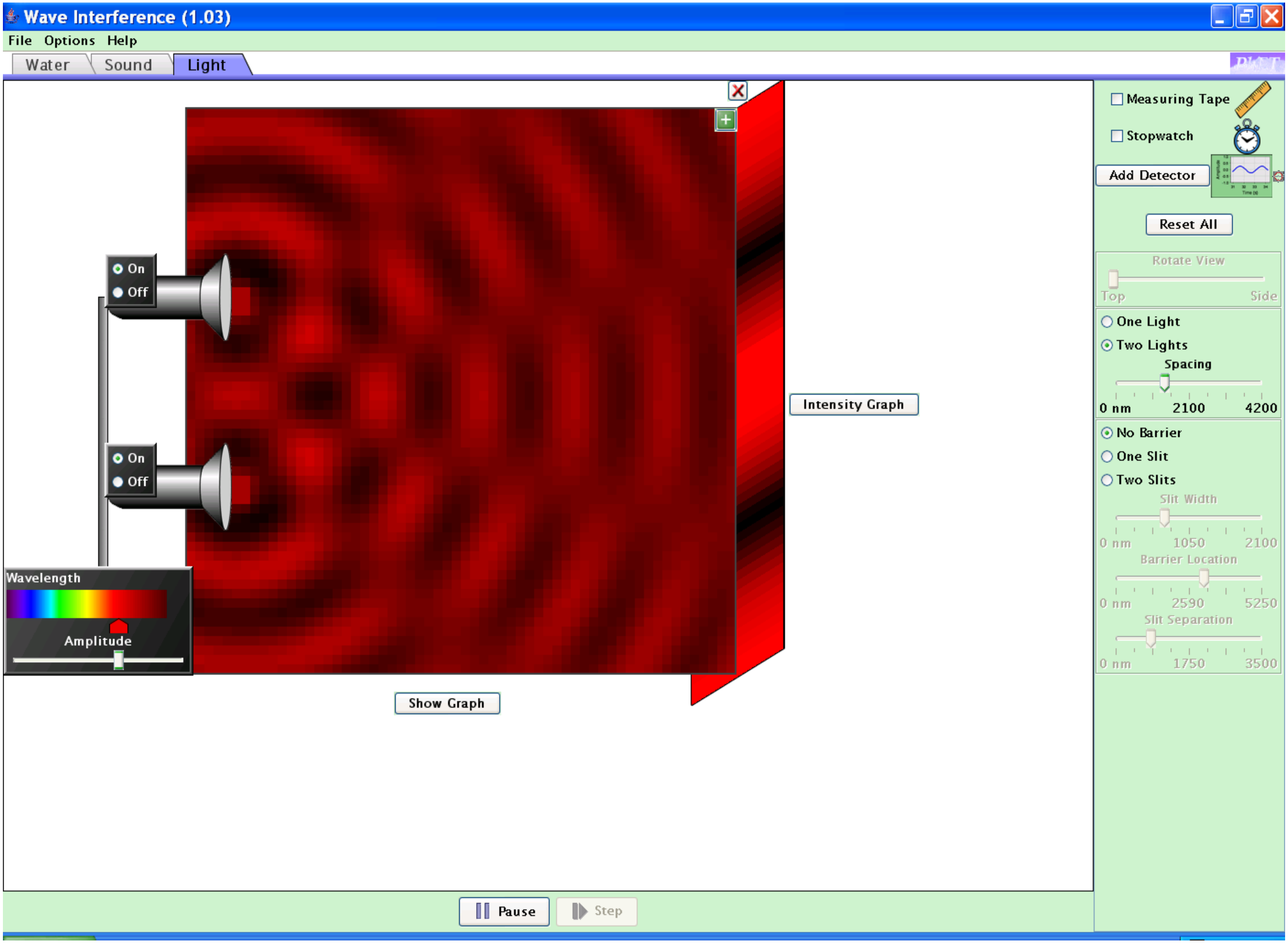
Pattern produced from a single slit.



Pattern produced from a double slit.

Interference applets

http://phet.colorado.edu/new/index.php

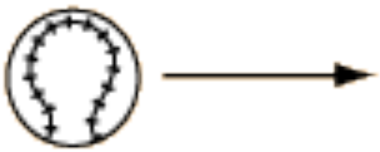


deBroglie waves

- particle-wave duality:
  - photon: $\underbrace{E = pc}_{\text{relativity}} = \underbrace{h\nu}_{\text{Planck}} = \underbrace{hc/\lambda}_{\nu = c\lambda} \rightarrow \boxed{p = h/\lambda}$
  - particle: true as well (deBroglie, 1924)

L. deBroglie
1893-1987

Does this relationship apply to all particles? Consider a pitched baseball:



$$v = 40m/s \approx 90mi/hr$$

$$m = 0.15kg \qquad \lambda = \frac{h}{mv} = \frac{6.626x10^{-34}\,J\cdot s}{(0.15kg)(40m/s)} = 1.1x10^{-34}m$$

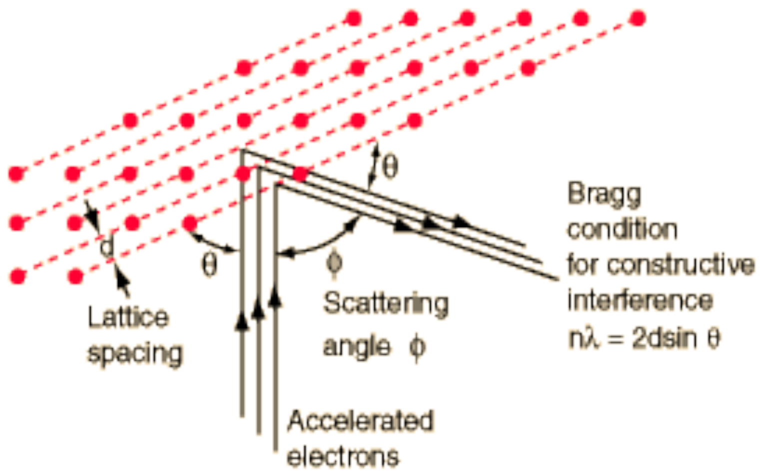
For an electron accelerated through 100 volts: v= $5.9x10^{6}m/s$

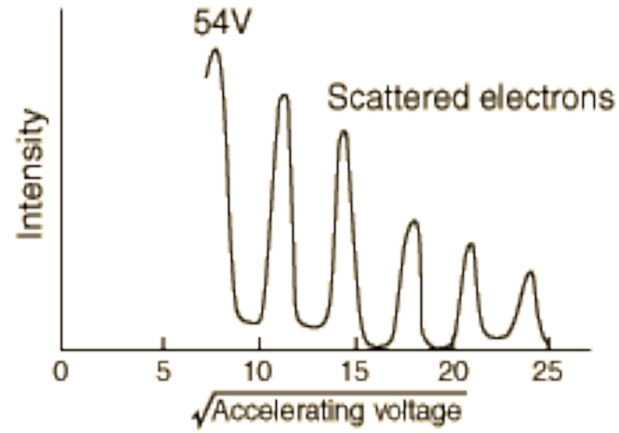
$$\lambda = \frac{6.626x10^{-34}\,J\cdot s}{(9.11x10^{-31}\,kg)(5.9x10^{6}\,m/s)} = 1.2x10^{-10}\,m = 0.12nm$$

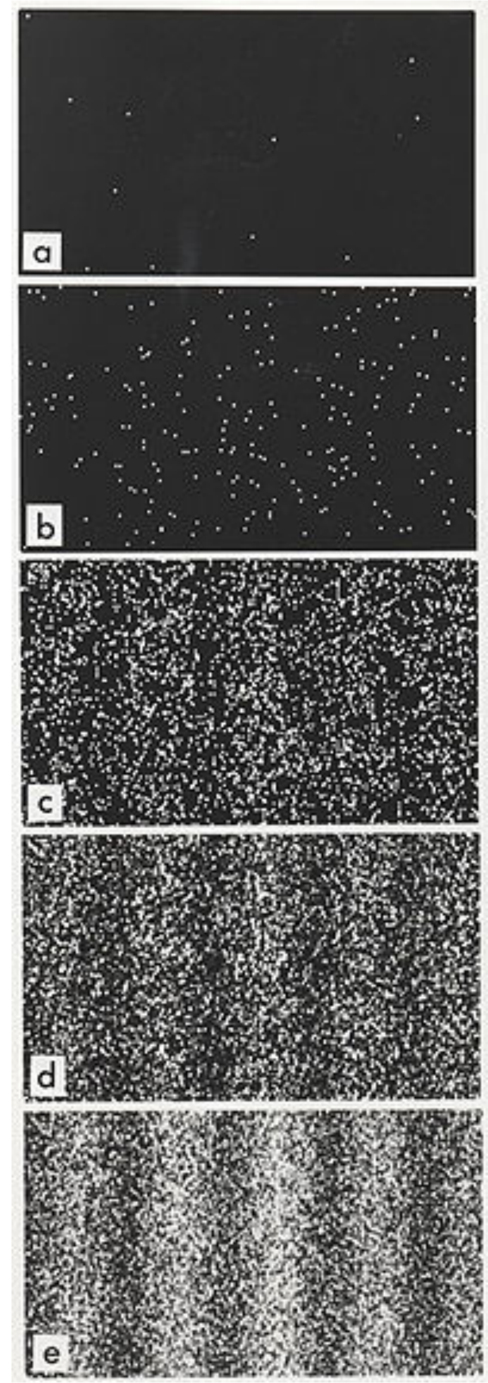
This is on the order of atomic dimensions and is much shorter than the shortest visible light wavelength of about 390 nm.

Electron diffraction: Davisson-Germer experiment

- wave character of electrons (and all matter) (1927)
  - *diffraction of e's off nickel crystal:*





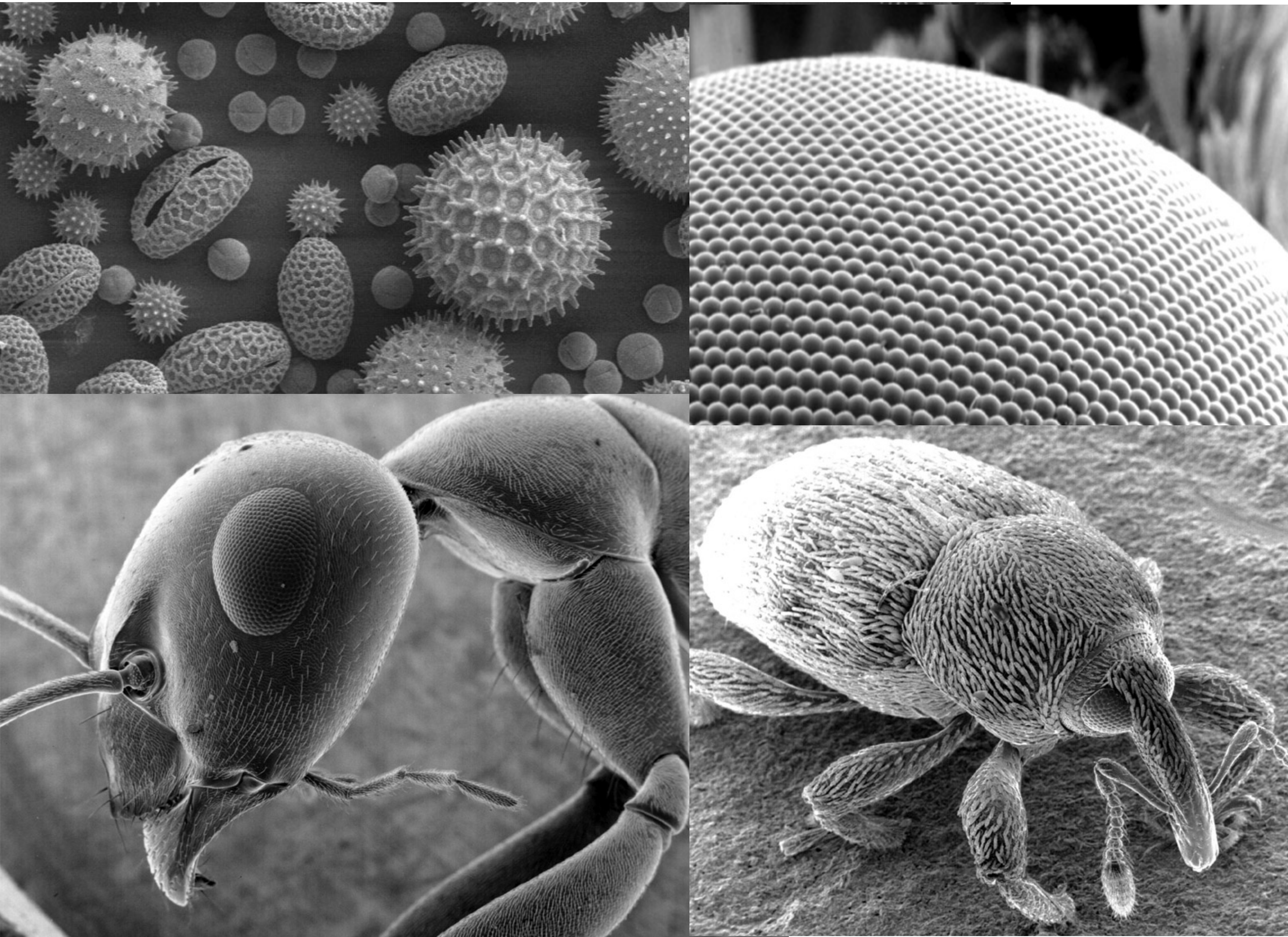


$$\frac{1}{\lambda} = \frac{n}{2d\sin\theta} = \frac{p}{h} = \frac{\sqrt{2mE}}{h} = \frac{\sqrt{2meV}}{h}$$

Electron wavelength — *Bragg law* — *deBroglie relationship* — *Acceleration through voltage V*

  - *estimate of e wavelength: 1 Angstrom for 100 eV*
  - *if detect which slit, diffraction pattern disappears*
  - *double-slit experiment with: e, p, n, molecules,...*
- electron microscope:

Electron microscope



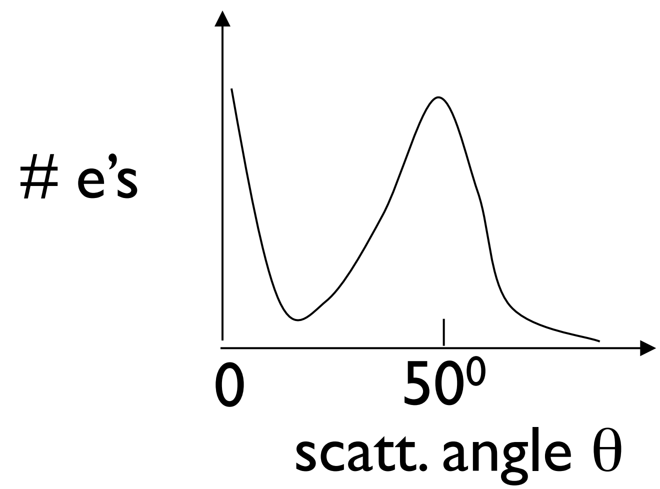
clicker question

Electron waves

Q: *To further prove the de Broglie wave hypothesis, D-G increased the Electron energy. If de Broglie's theory is correct, what will happen?*

Bragg condition:

$2d\sin\theta = n\lambda$



a) The peak will get larger
b) The peak will get smaller
c) $\boxed{\text{The peak will shift to smaller angle}}$
d) The peak will shift to larger angle
e) No change will take place

A: *(c) Increasing energy increases momentum, which decreases the angle: $\lambda = h/p$ and $\theta \approx \lambda/d = h/(pd)$*

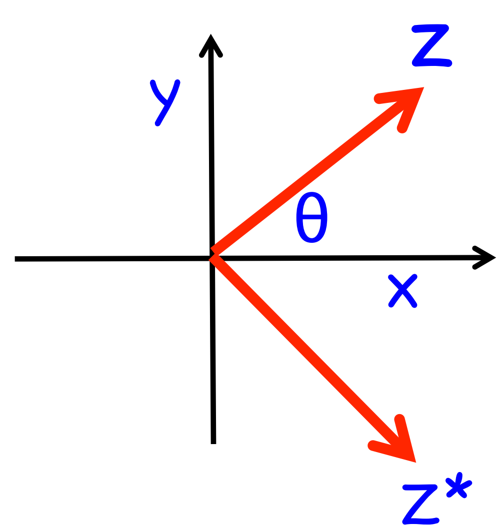
Richard Feynman QM lecture

Richard P. Feynman
1918–1988

"What do you care what other people think?"

Complex numbers

- complex number: $z = x + i\,y$
  - $i = \sqrt{-1} \longleftrightarrow i^2 = -1$
  - complex conjugate of z: $z^* = x - i\,y$ (change $i \rightarrow -i$; $i^* = -i$)
  - magnitude of z: $|z|^2 = z^*z = (x + iy)(x - iy) = x^2 + y^2$ (no cross term)
  - $z = x + i\,y = |z|(\cos\theta + i\,\sin\theta) = |z|e^{i\theta}$
  - analogous to a 2D vector: $z = (x, y)$



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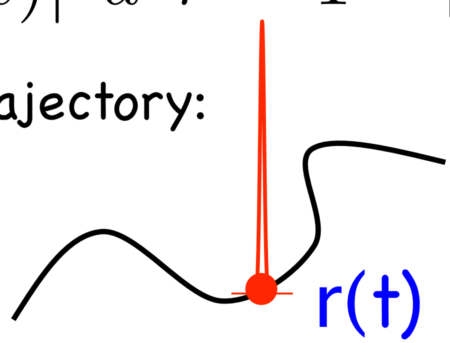
deBroglie matter waves and wavefunction interpretation

- familiar waves:
  - *sound (pressure wave in a gas):* *$P(r,t)$* – pressure/molecular displacement
  - *water (transverse wave in liquid):* *$h(r,t)$* – up/down displacement
  - *string (transverse wave in a string):* *$y(s,t)$* – up/down displacement
  - *EM wave (E, B oscillating in vacuum):* *$E(r,t)$, $B(r,t)$* – fields

    $I = |E|^2$ – intensity = number of photons per second landing on a unit area

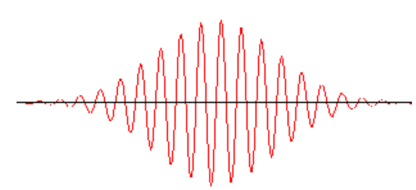
    probability of photon arrival
- deBroglie matter waves:
  - $\psi(r,t)$ – not physical, complex wavefunction *(psi)*
  - $P(r,t) = |\psi(r,t)|^2$ – probability density of finding a particle at r, at time t

    Copenhagen interpretation, Max Born (1926)
  - $$\int |\psi(r,t)|^2 d^3r = 1$$ – probability normalization = particle is somewhere
  - classical trajectory:



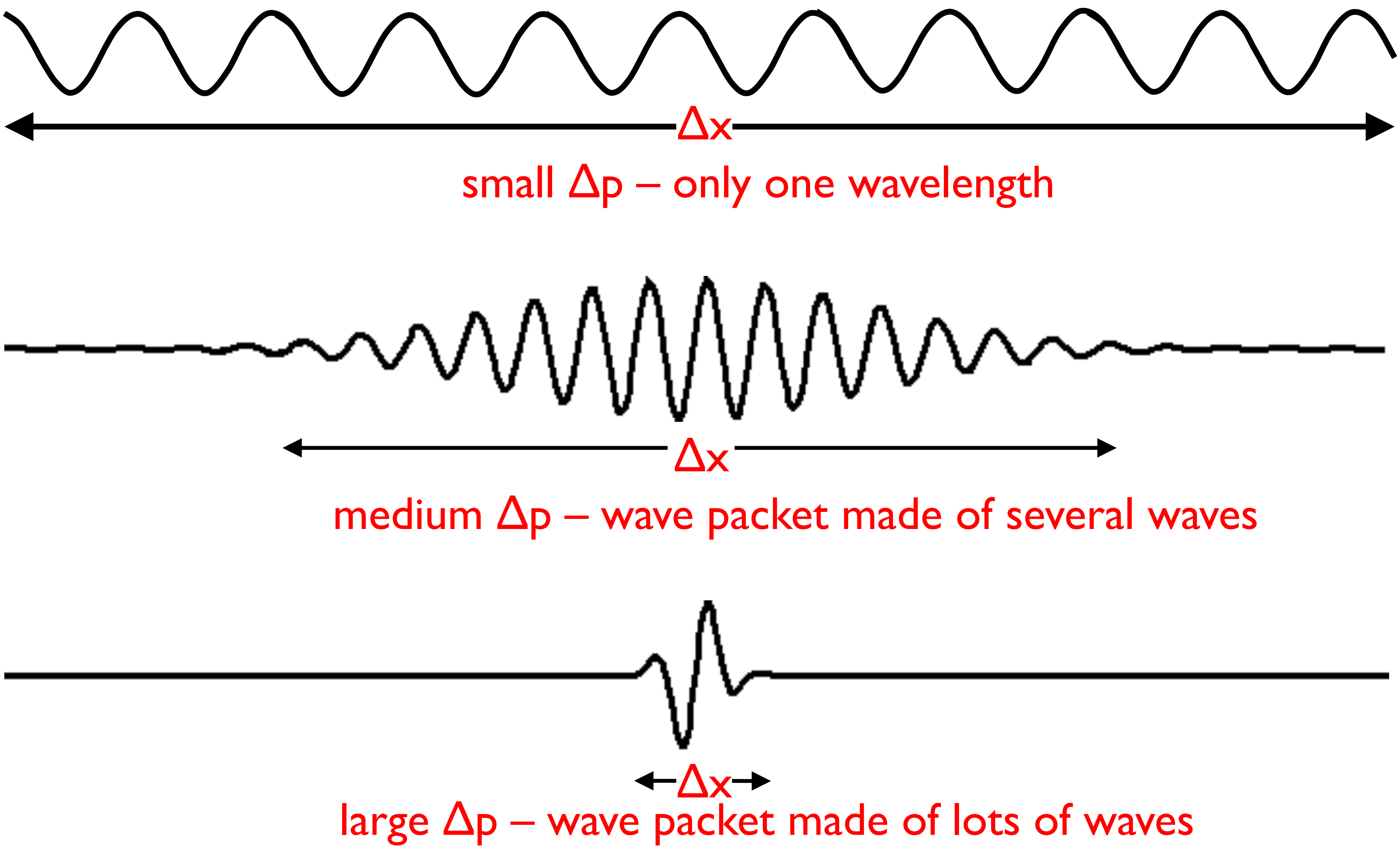
r(t)

quantum wavepacket:

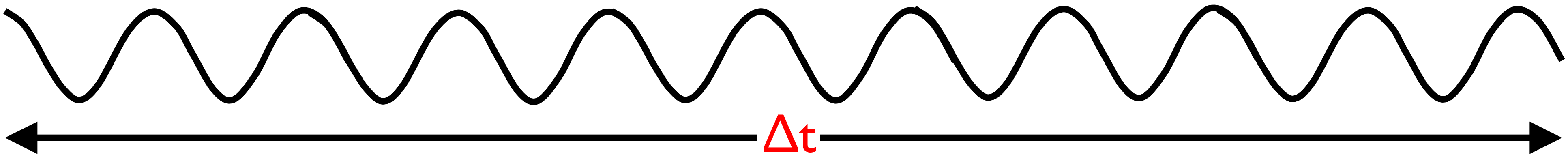


$\psi(r,t)$

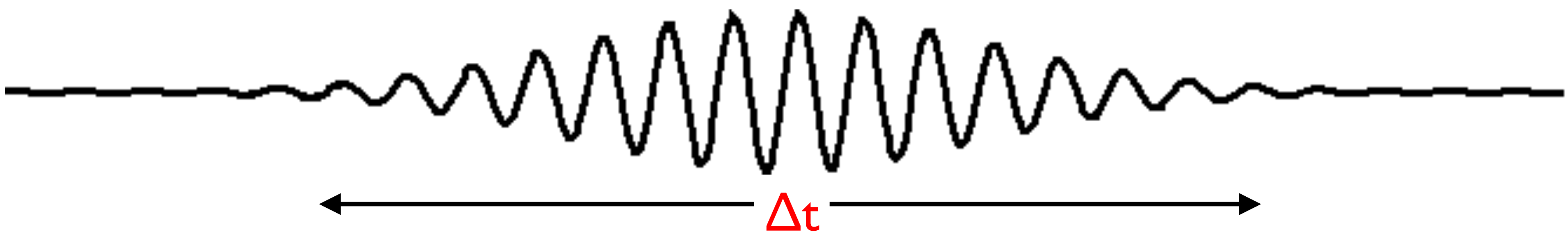
Localization of waves in space



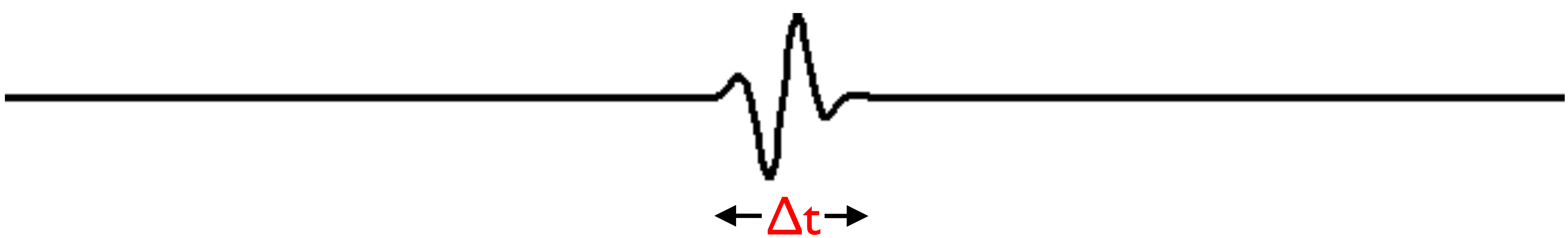
Localization of waves in time



small ΔE – plane-wave in time, only one frequency (energy, E)



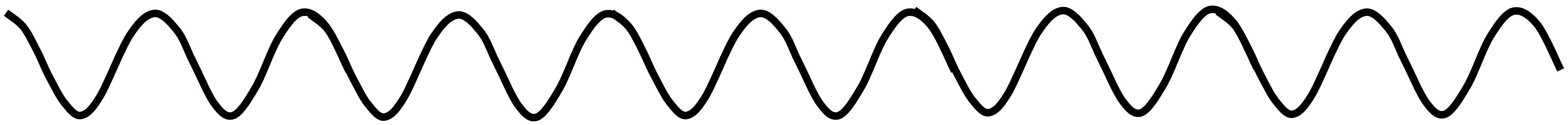
medium ΔE – wave packet in time made of several frequency (E) waves



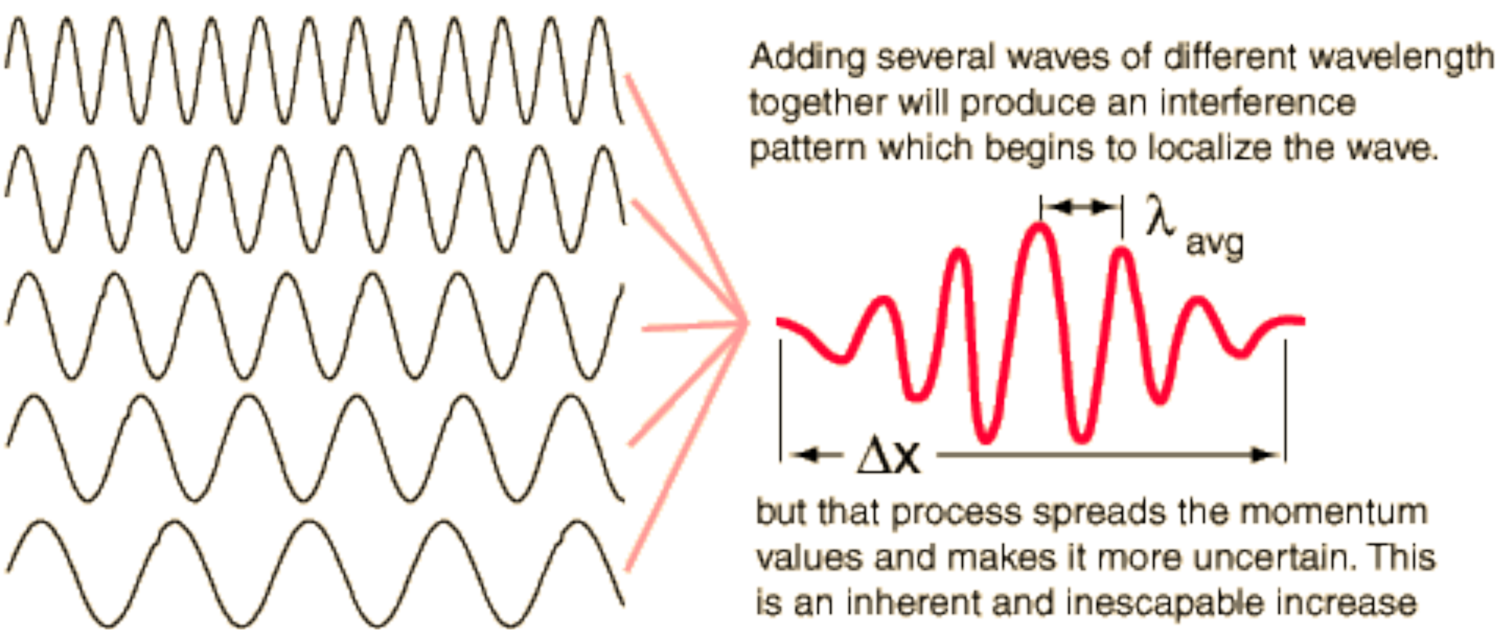
large ΔE – wave packet in time made of lots of frequency (E) waves

Plane-waves vs wave packets

- plane wave: $\Psi(x,t) = Ae^{i(kx-\omega t)}$



- wave packet: $\Psi(x,t) = \sum_{n=0}^{\infty} A_n e^{i(k_n x - \omega_n t)}$



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LA Program

You are cordially invited to attend an Informational Session to learn more about becoming a Learning Assistant.

***When:* Wednesday, March 9, 2011, at 6 p.m.**

Where: UMC 235 (hall right of Reception Desk)

RSVP: By March 4 to olivia.holzman@colorado.edu

Refreshments will be served, while they last.

Applications for Fall 2011 available March 9 - 23

Goto: http://laprogram.colorado.edu/applications

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MCDBiology   Chemistry   Geological Sciences
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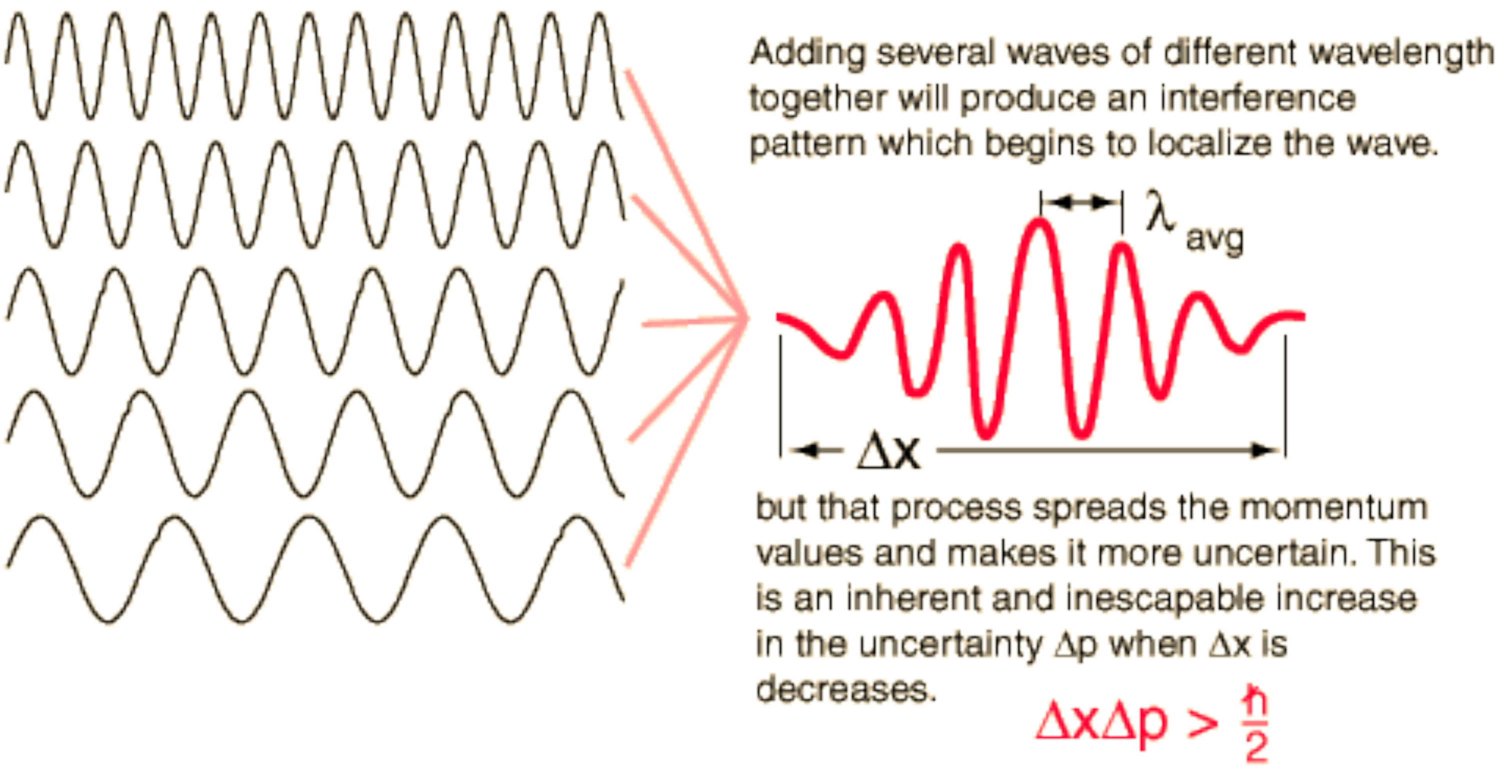
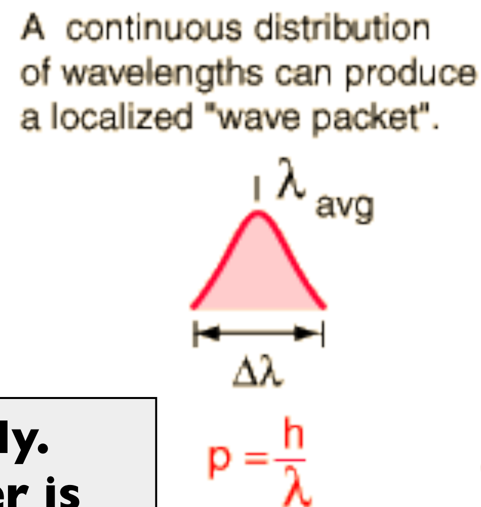
Heisenberg uncertainty principle

- position-momentum and time-energy:



W. Heisenberg
1901-1976



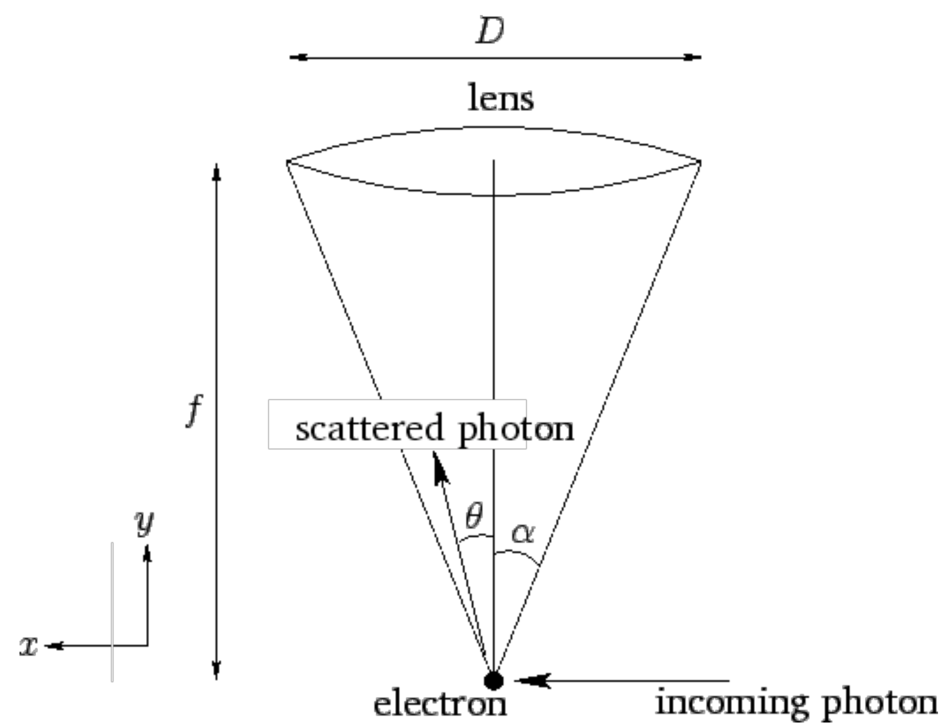
$$\Delta x \Delta p > \frac{\hbar}{2} \qquad \Delta E \Delta t > \frac{\hbar}{2}$$

The position and momentum cannot both be determined precisely. The more precisely one is determined, the less precisely the other is determined.

- true for all waves, consequence of Fourier transformation

Heisenberg microscope

- thought-experiment: *observation of electron's position by a photon gives it a kick and thereby perturbs its momentum*
  - optical resolution: $\Delta x = \lambda f/D = \lambda/\text{N.A.} \approx \lambda/\alpha$
  - momentum kick from photon: $\Delta p \approx \hbar k \alpha = h \alpha/\lambda$
  - $\Longrightarrow$ $\boxed{\Delta x \Delta p > \frac{\hbar}{2}}$
  -



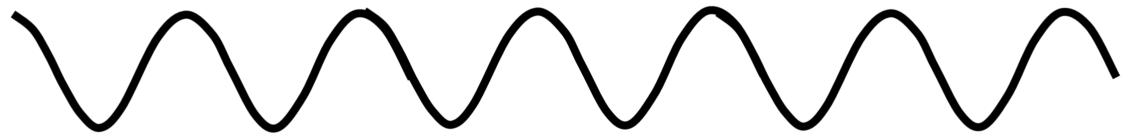
Smaller wavelengths allow a better measurement of *x* but the photons have larger momentum giving larger kicks to the particle, making the momentum more uncertain.

clicker question

Position-momentum uncertainty

Q: *For which type of a wave is the momentum and position most well defined?*

Plane wave: $\Psi(x,t) = Ae^{i(kx-\omega t)}$



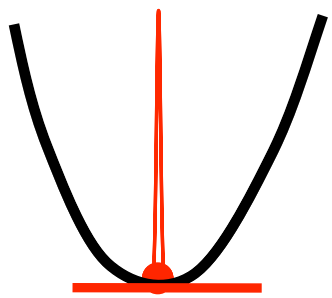
Wave packet: $\Psi(x,t) = \sum_{n=0}^{\infty} A_n e^{i(k_n x-\omega_n t)}$

a) $\boxed{\text{p is well defined for plane wave, x is well defined for wave packet}}$
b) x is well defined for plane wave, p is well defined for wave packet
c) p is well defined for one, but x is well defined for both
d) p is well defined for both, but x is well defined for one
e) both p and x are well defined for both

A: *(a) plane wave has a well-defined wavelength and therefore describes particle with well-defined momentum. However, its position is not well defined as the particle is infinitely delocalized.*

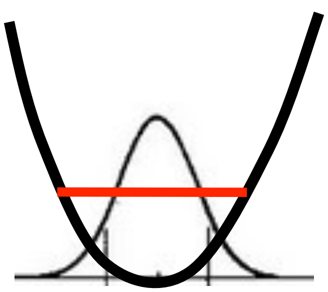
Zero-point energy

- E = $p^2/2m + V(x)$



- *uncertainty:* $\Delta x \Delta p > \hbar \Longrightarrow \Delta p \approx n\hbar/\Delta x$

- *cannot settle down to classical energy minimum:*



- *zero-point energy:*

$$E \approx \frac{\hbar^2}{2mx^2} + V(x)$$