Lecture 10

Atomic spectra

Announcements:

- lecture 9 is posted
- homework 5 (*due Feb 18, in class*) solutions are posted
- homework 6 is *due Feb 25, in class*
- reading for this week is:
  - Ch 5 in TZD

Last Time

recall lecture 9:

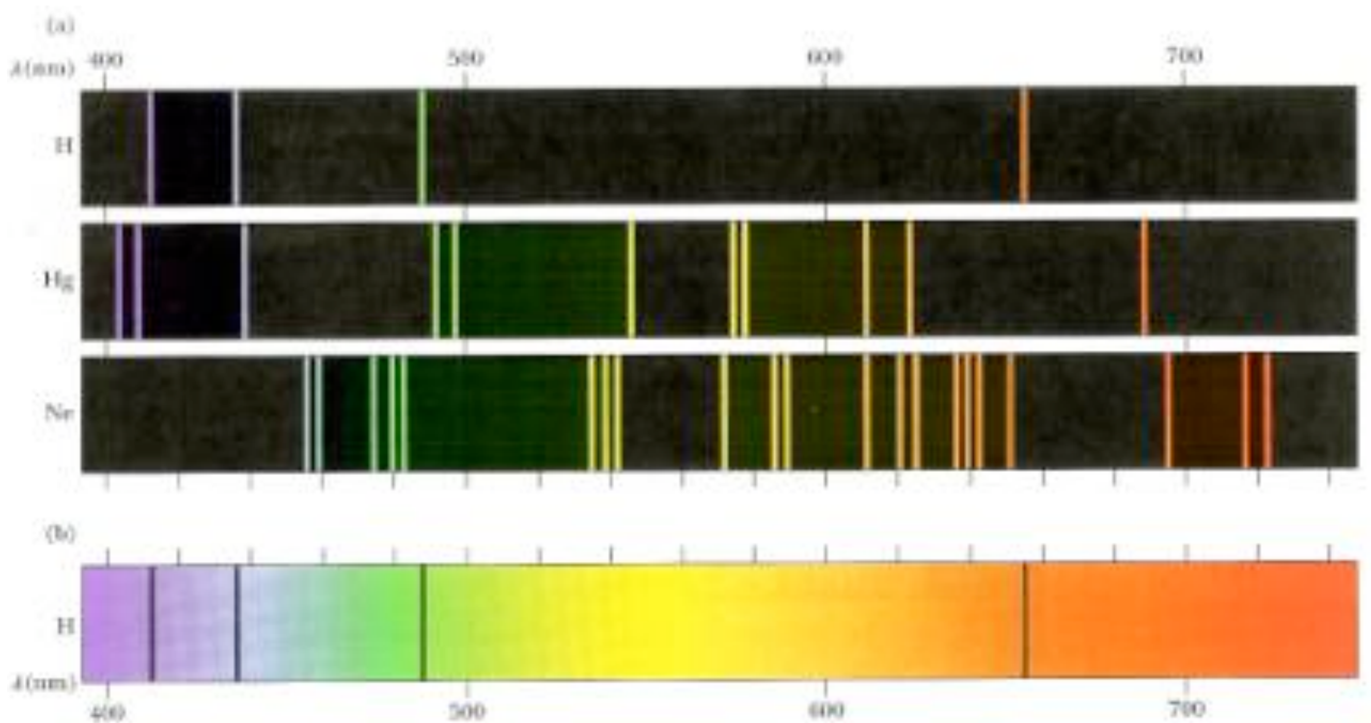
Quantum physics (particle nature) of light:

- Planck's black body radiation
- Photoelectric effect
- X-ray diffraction
- Compton effect

Today

Problems with classical physics: *atomic spectra, Franck-Hertz experiment*

- Atomic spectra
- Atomic instability in classical theory
- Bohr's theory of atomic spectra



Atomic spectra

- observed emission/absorption spectra for Hydrogen:

- Balmer-Rydberg formula:

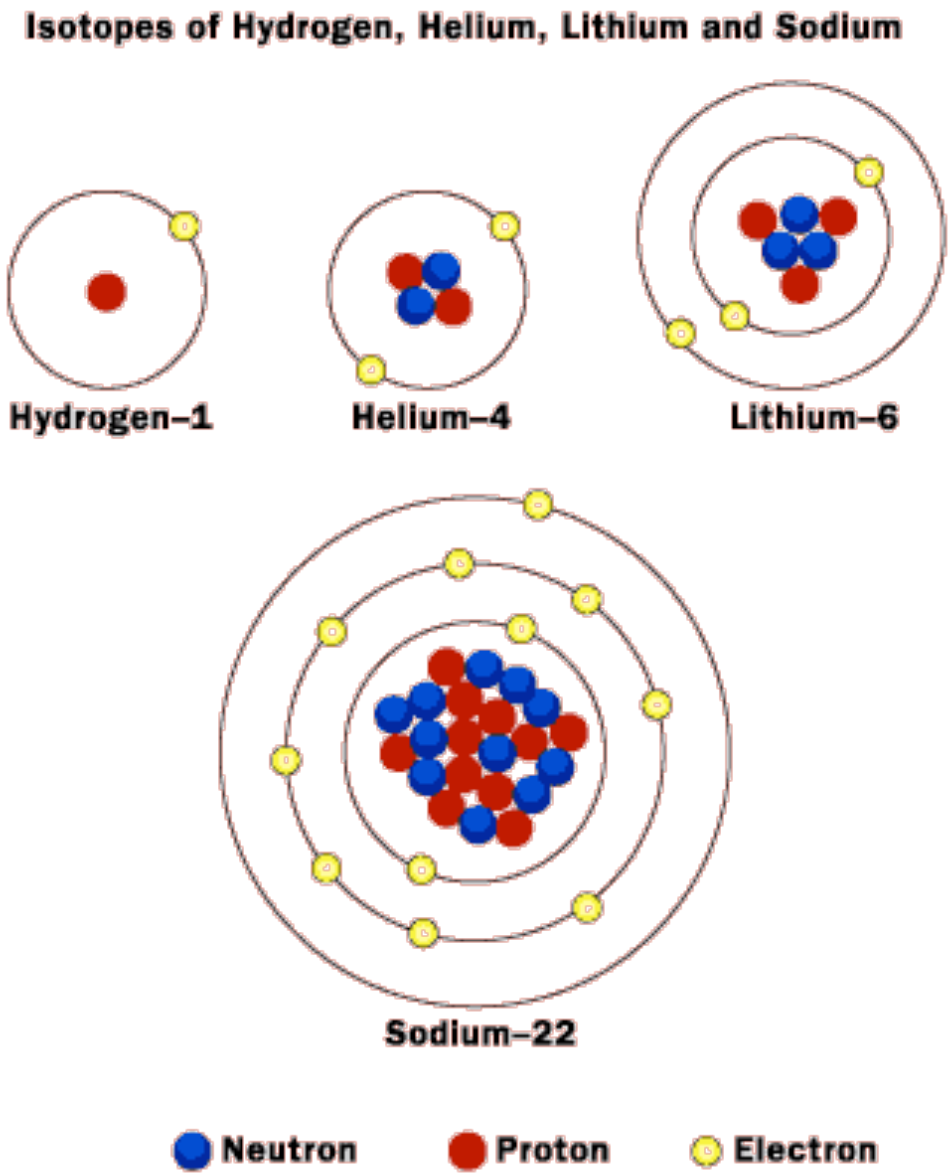
$$\frac{1}{\lambda} = R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right), \quad n > n', \text{both integers}$$

Rydberg constant R = 0.001 Å^{-1}

$$\Delta E_{n,n'} = hcR\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$$

Bohr-Rutherford's picture of atoms

- planetary semi-classical model (1913) inspired by Rutherford's scattering



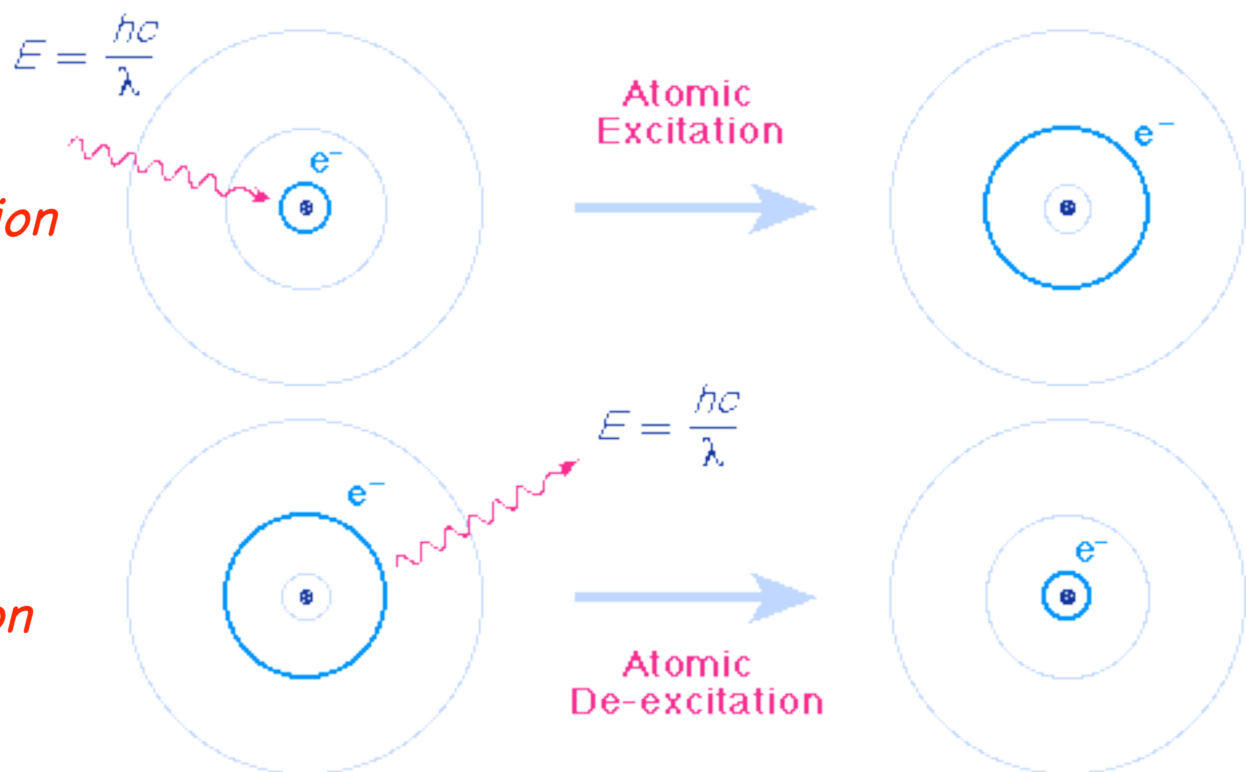
1885-1962

Bohr's picture of atomic emission/absorption (1)

- electron's discrete transition between a set of allowed "orbits":

photon absorption

photon emission

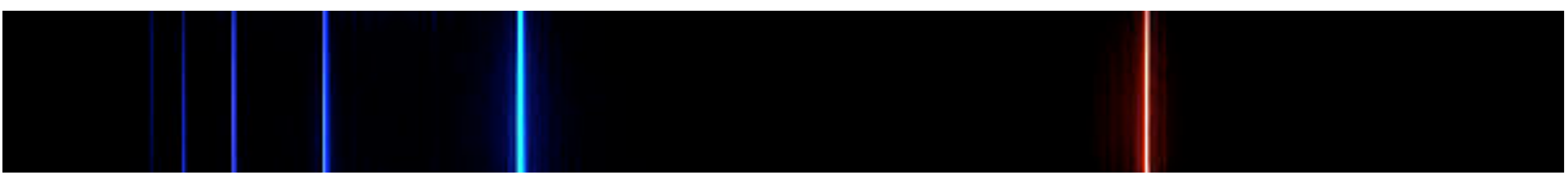


- energies of emitted/absorbed photon: $$\Delta E_{n,n'} = hcR\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right), \quad n > n', \text{both integers}$$

Rydberg constant R = 0.001 Å^{-1}

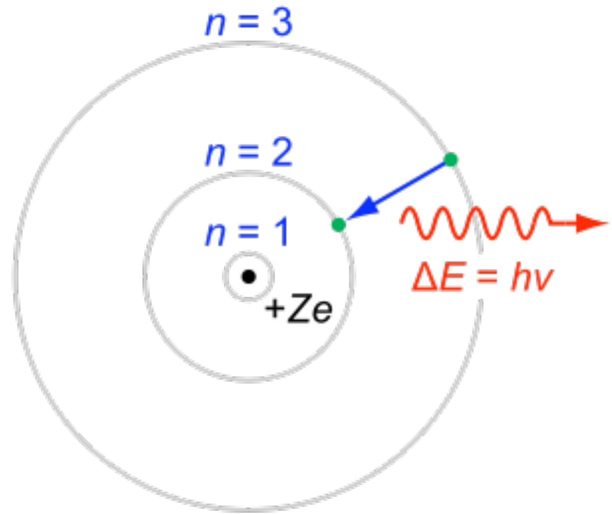
Balmer series of Hydrogen:

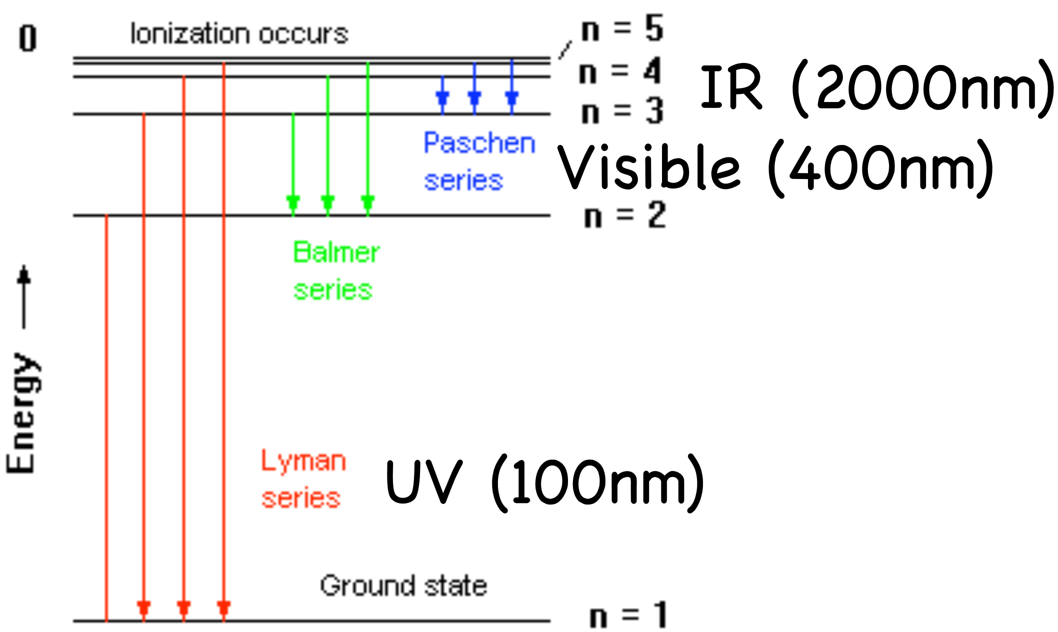


Bohr's picture of atomic emission/absorption (2)

- electron's discrete transition between a set of allowed "orbits":

$$\Delta E_{n,n'} = hcR\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$$

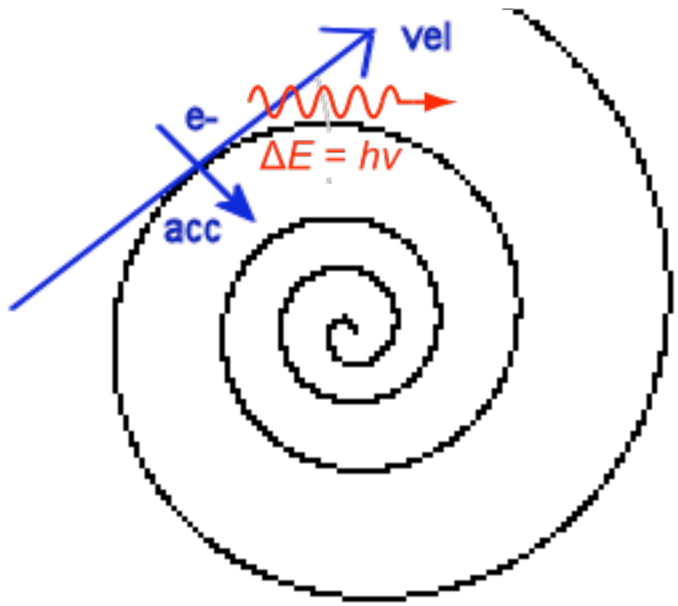




Balmer series of Hydrogen (n->2 transitions):

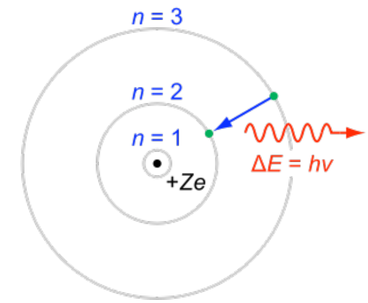
Atomic instability in classical picture

- classical radiation from accelerating electron



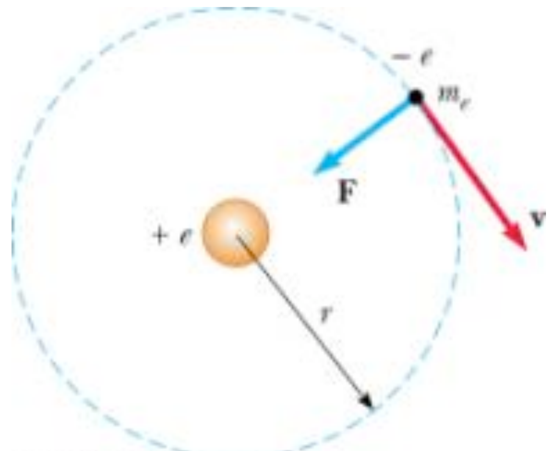
- → atom (electron orbiting proton) is *classically unstable*

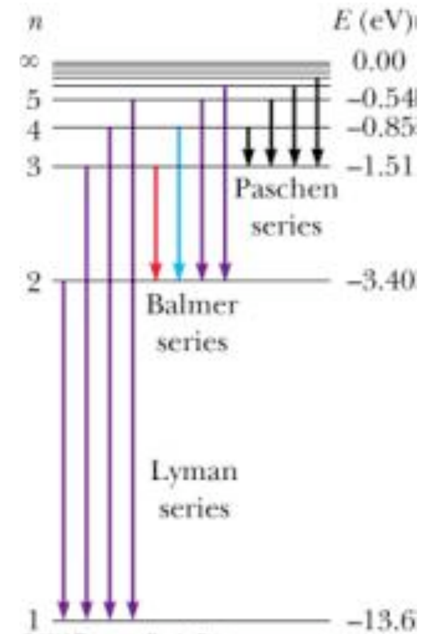
Bohr's theory of Hydrogen spectrum



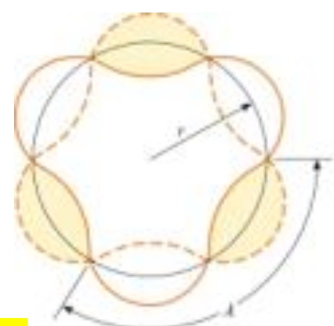
- electrons' "orbits":
  - *ma=F* → $\frac{mv^2}{r} = \frac{Ze^2}{r^2}$
  - $E_r = \frac{1}{2}mv^2 - \frac{Ze^2}{r}$

    $= -\frac{Ze^2}{2r}$





- discretization of electrons' orbits:
  - *angular momentum quantization:* $L_n = n\hbar, \quad (\hbar \equiv h/2\pi)$
  - *deBroglie waves:* $p = h/\lambda$, $2\pi r_n = n\lambda$ → $pr_n = n\,h/2\pi$
  - $r_n = \frac{n^2\hbar^2}{Zme^2} = (0.53\mathring{A})n^2$



(a)

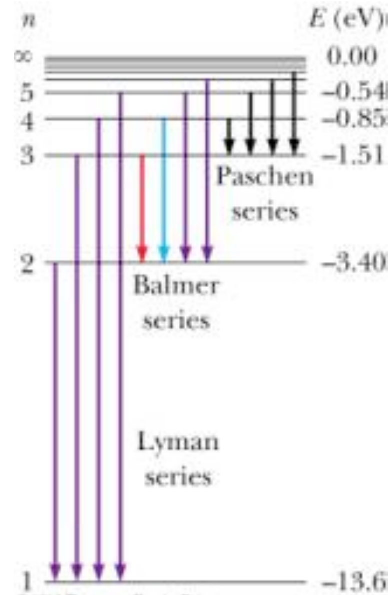
"waves" on guitar

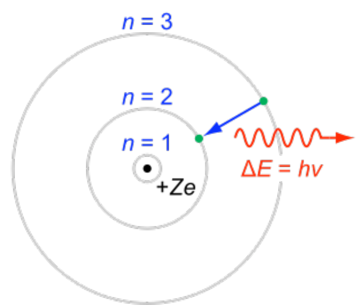
(b)

- → $E_n = -\frac{Z^2e^4m}{2\hbar^2n^2} = -\frac{13.6eV}{n^2}Z^2$

Fine structure constant

- electrons' Bohr "orbits" and fine structure constant
  - $$\frac{mv^2}{r} = \frac{Ze^2}{r^2} \qquad E_r = \frac{1}{2}mv^2 - \frac{Ze^2}{r} = -\frac{Ze^2}{2r}$$
  - $$E_n = -\frac{Z^2e^4m}{2\hbar^2n^2} = -13.6eV\frac{Z^2}{n^2}$$
  - $$r_n = \frac{n^2\hbar^2}{Zme^2} = (0.53\mathring{A})\frac{n^2}{Z}$$

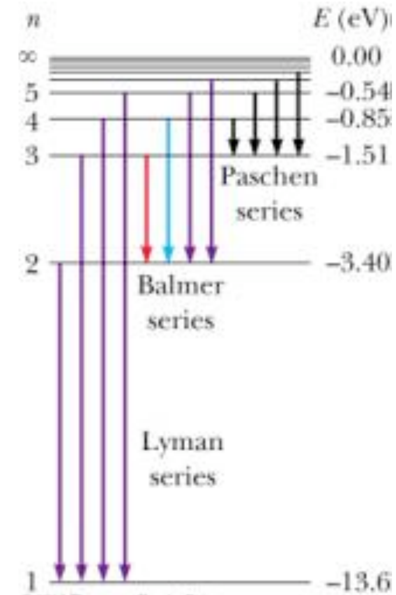




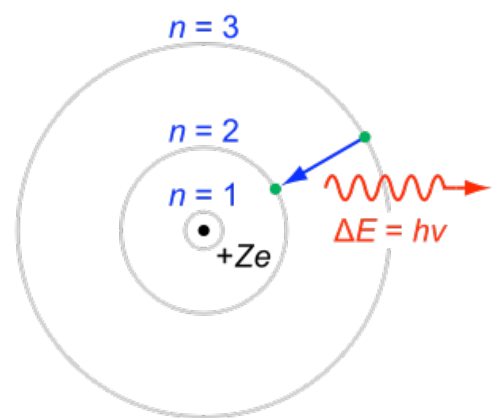
- nonrelativistic electron:

$$\left(\frac{v}{c}\right)^2 = \frac{e^2/r}{mc^2} = \left(\frac{e^2}{\hbar c}\right)^2 \equiv \alpha^2 = \left(\frac{1}{137}\right)^2 \ll 1$$

Questions for Bohr's model



- Why is angular momentum quantized?
- Why don't electrons radiate, leading to instability of atoms?
- What determines the relative rates of transitions between different states? (predict intensities of spectral lines?)



- How to generalize to more complex, multi-electron atoms?
- Shapes of molecular orbits that determine chemical bonds?

clicker question

Key new idea

Q: *Which of these is the key new idea/ingredient of Bohr's theory?*

(a) Energy is conserved

(b) Energy is quantized according to E = n hν

(c) Angular momentum is quantized, $L_n = n\hbar$

(d) Angular momentum is conserved

A: *(c) Atom is a cavity with electrons waves, with only special wavelengths fitting into it;* $2\pi r_n = n\lambda \rightarrow p r_n = n\, h/2\pi$