Lecture 9

Quantum physics of light

Announcements:

- midterm 1 solutions are posted
- lecture 8 is posted
- homework 3 solutions are posted
- homework 4 today, Feb 12, *in class*
- reading for this week is:
  - Ch 3, 4 in TZD

Last Time

recall lecture 8:

Atomic structure of matter:

- atoms, molecules
- electrons, protons, neutrons
- solids, liquids, gases,...and much, much more
- atomic number (Z), mass number (A), atomic unit (u), Avagadro number (N_A),...

Today

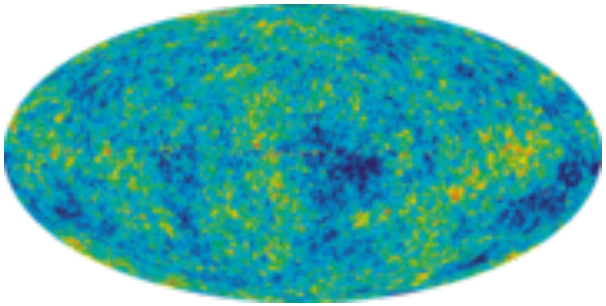
Problems with classical physics: *quantization of light*

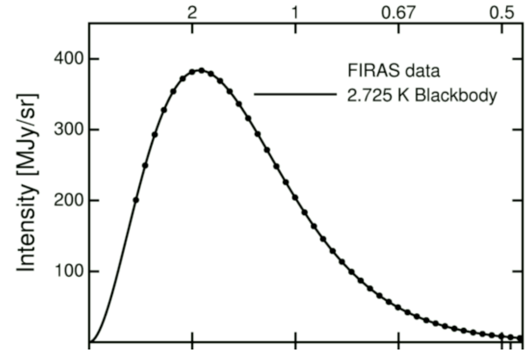
- Planck's blackbody radiation
- Photoelectric effect
- X-ray diffraction
- Compton effect

⟹ particle nature of light: the photon

Blackbody radiation

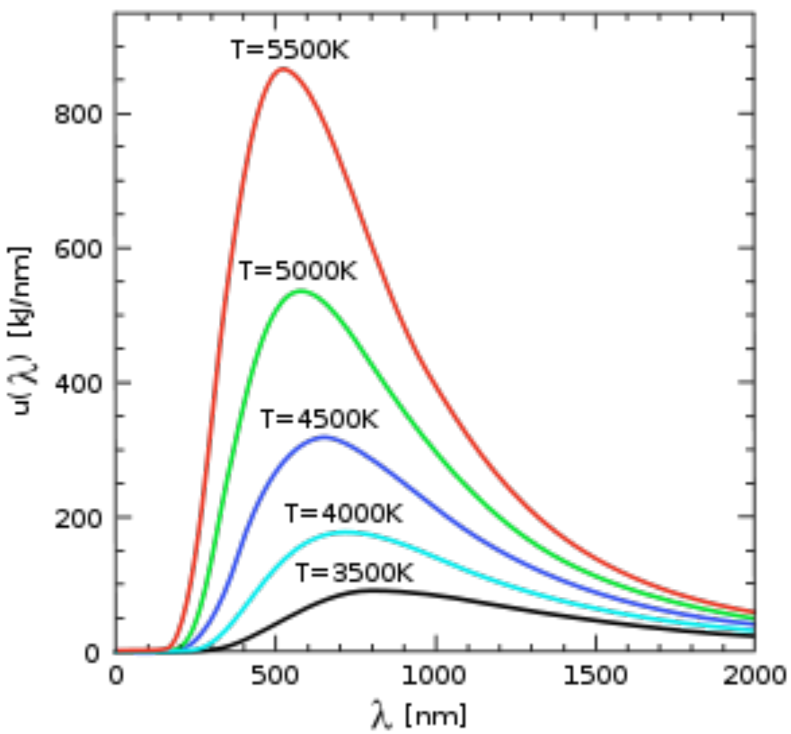
- "black" body radiates: *hot oven, poker, Sun, glowing coal, CMB,…*

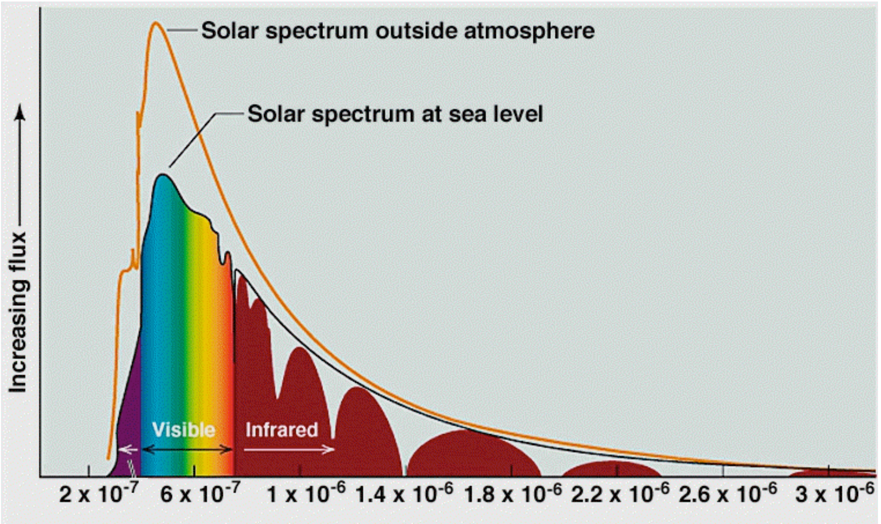




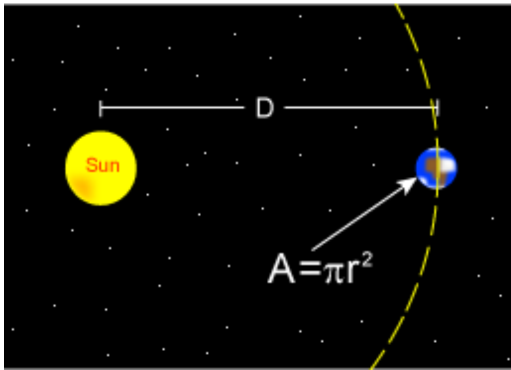
T_{CMB} = 3K, Big Bang, 13.3 billion years ago, cooled from 3000K

- independent of material, just T: *shorter λ higher T*

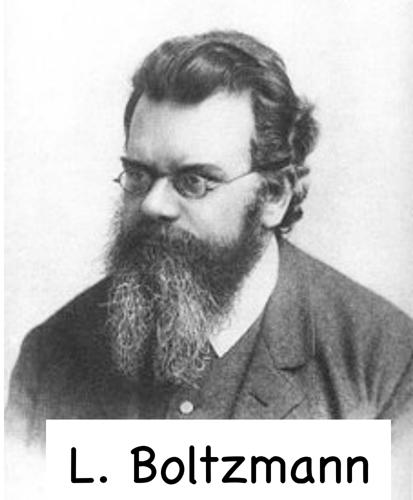




- determines Earth's average temperature:

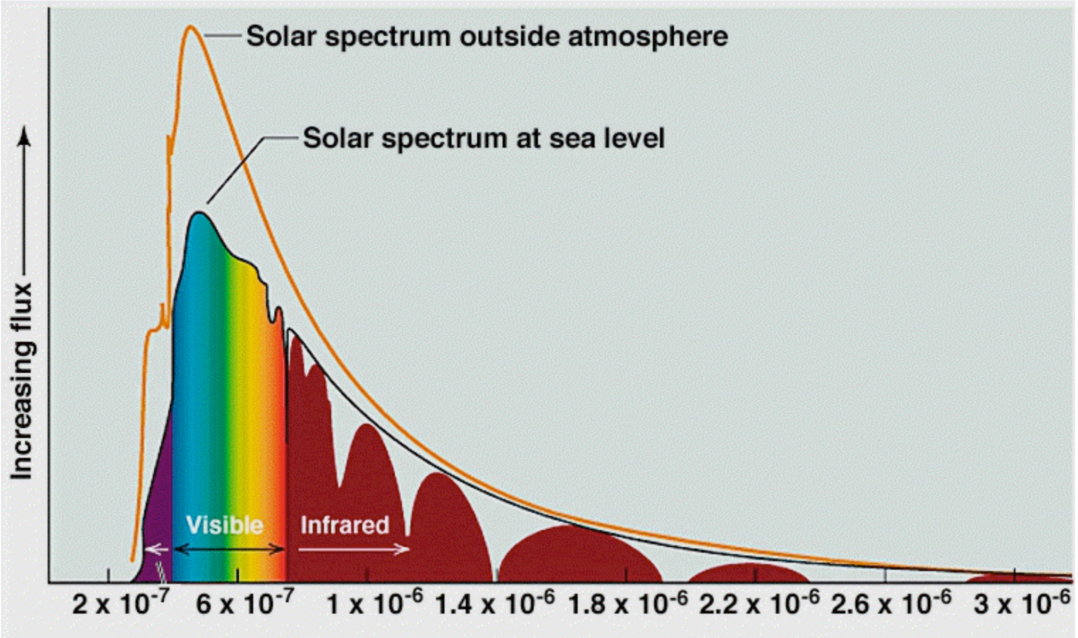


Temperature - energy



L. Boltzmann
1844-1906

- Thermal energy = $k_B T$
- Boltzmann constant $k_B = 1.38\times10^{-23}$ J/K $= 8.6\times10^{-5}$ eV/K
- 1 eV = 12000 Kelvin



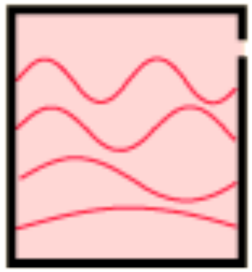
$$k_B T \approx E_{peak}/3$$

Planck's black body radiation: quantization of light

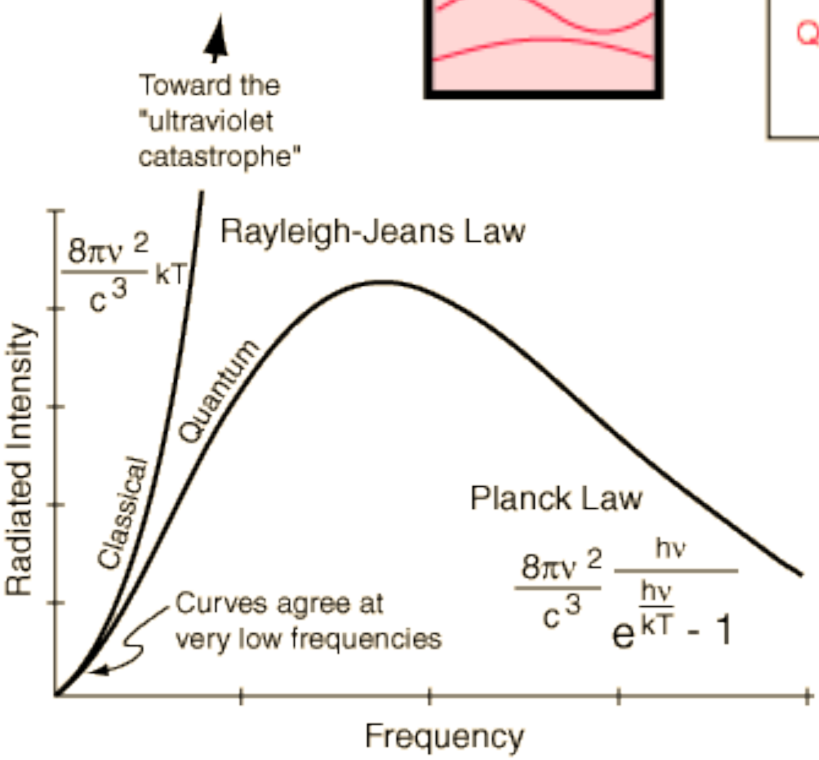
- Theory:

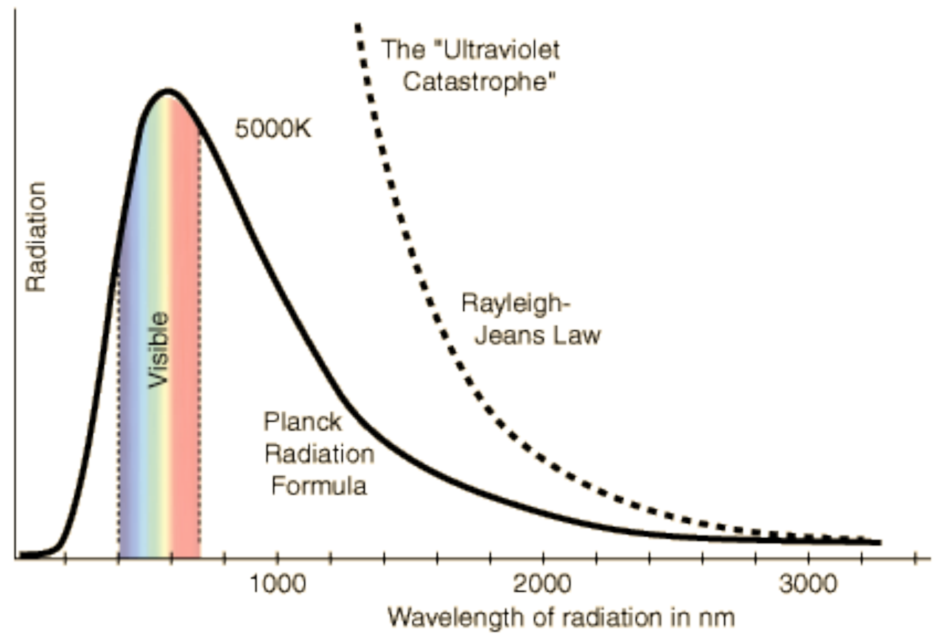
(Planck, Einstein)
1900 1905

Radiation modes in a hot cavity provide a test of quantum theory



	#Modes per unit frequency per unit volume	Probability of occupying modes	Average energy per mode
CLASSICAL	$\frac{8\pi\nu^2}{c^3}$	Equal for all modes	kT
QUANTUM	$\frac{8\pi\nu^2}{c^3}$	Quantized modes: require $h\nu$ energy to excite upper modes, less probable	$\frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$



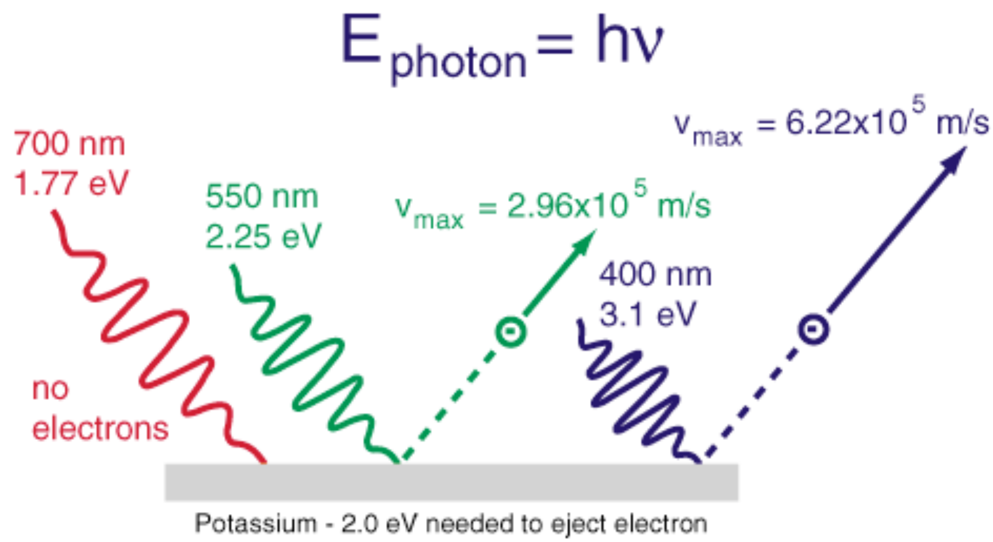


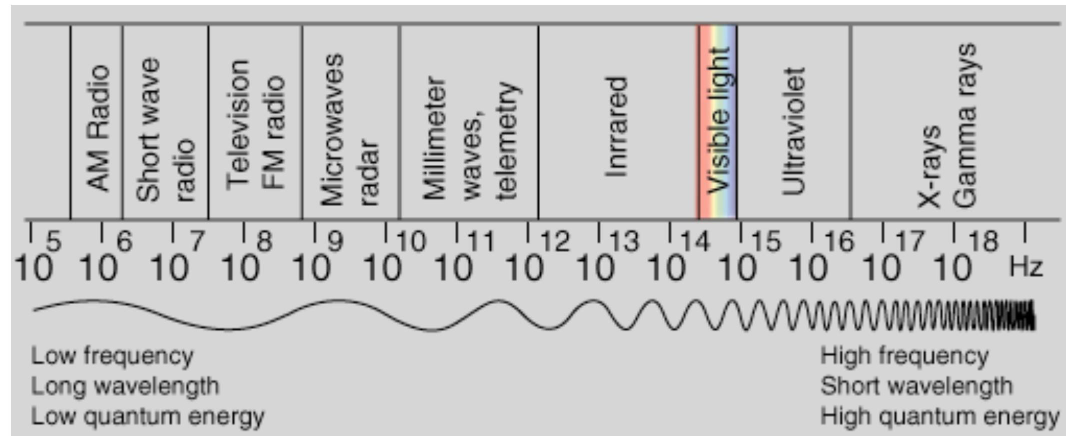
- Planck's constant: $h = 6.63\times10^{-34}$ J-sec $= 4.1\times10^{-15}$ eV-sec
- $E_\nu = 0, h\nu, 2h\nu, 3h\nu, \ldots$ discrete quanta → *photons* of frequency ν
- Stephan-Boltzmann law: $P = \sigma T^4$ – total power radiated *(e.g., Earth's T)*

Photoelectric effect

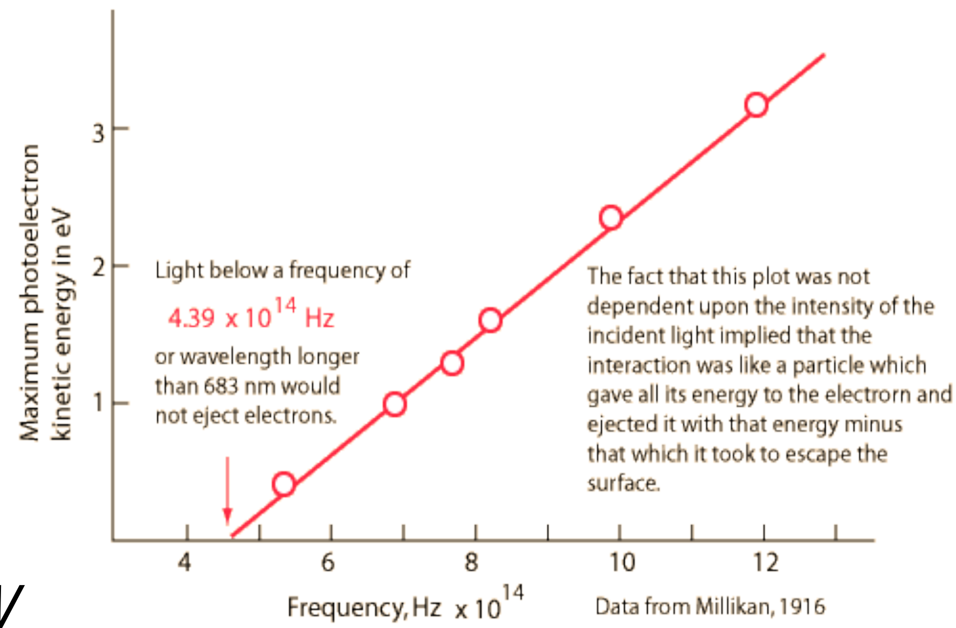
Hertz (1887), Einstein (1905)

- electron emission from metal surface irradiated by light





- threshold for emission to overcome work function W:
  - $\nu < \nu_c \rightarrow$ *zero current*
  - $\nu > \nu_c \rightarrow$ *nonzero current*
  - *independent of light intensity* *(increases N_{ph}, but not photon energy)*



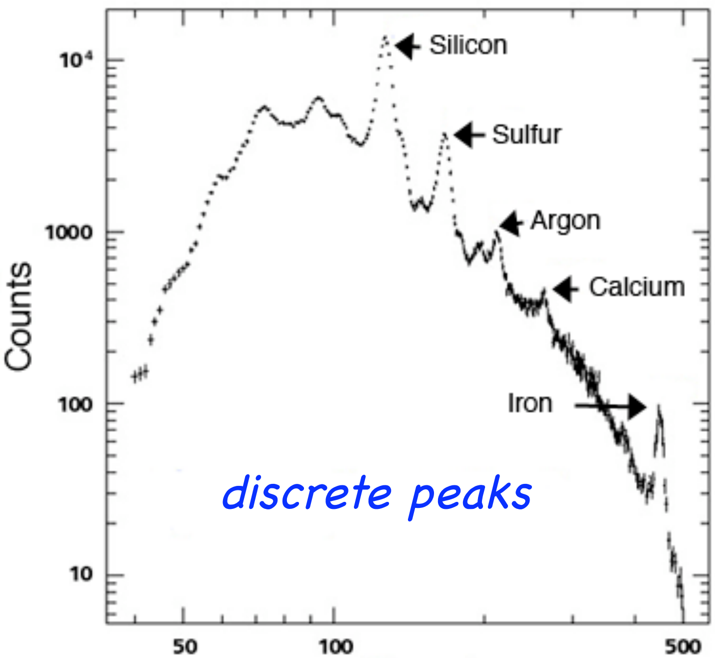
- Einstein: *photons of $E = h\nu$ give electron kinetic energy $K_e = h\nu - W$*

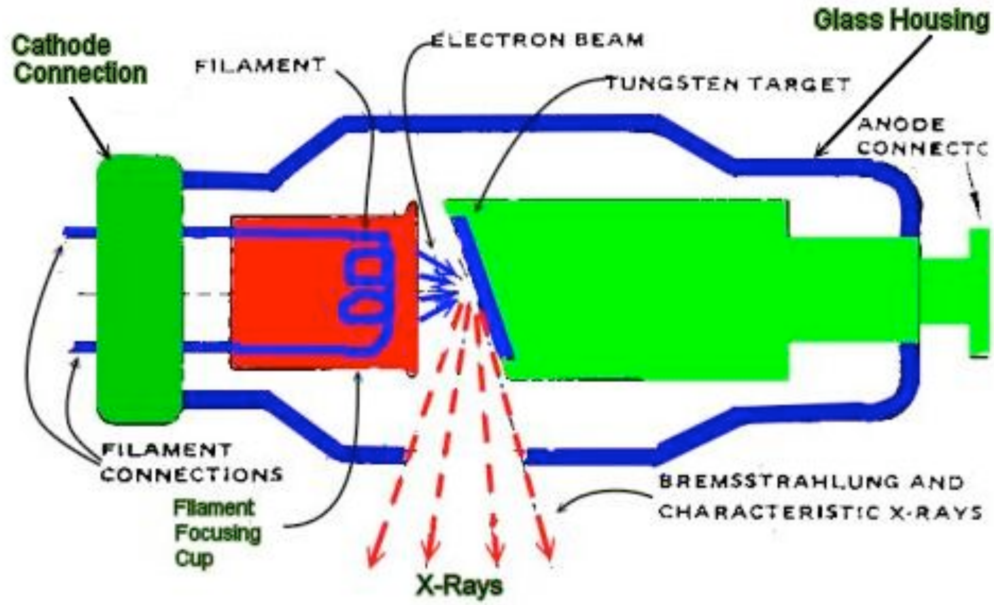
Roentgen (1895)

X-rays and Bragg diffraction

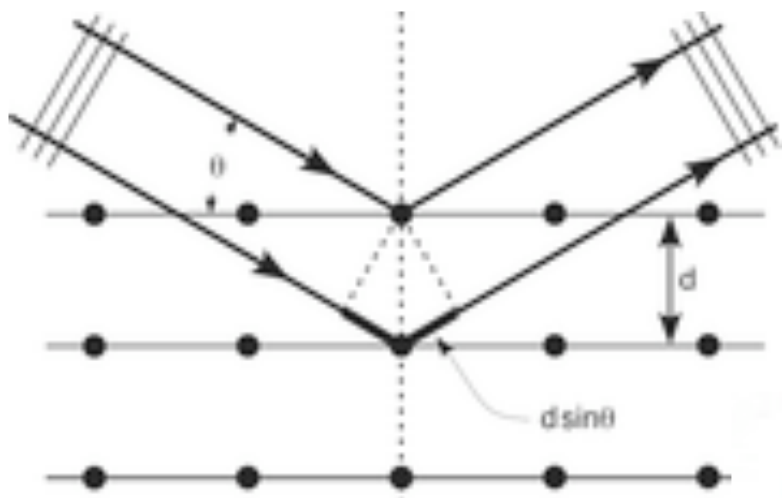
Bragg (1912)

- X-rays (light with 1/100Å < λ < 10Å)



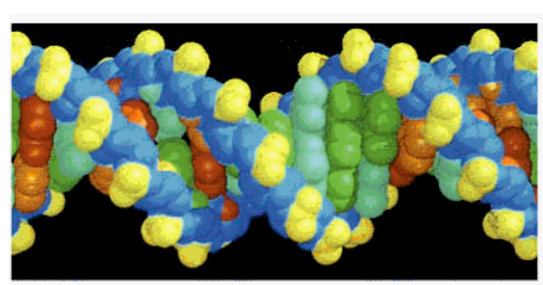


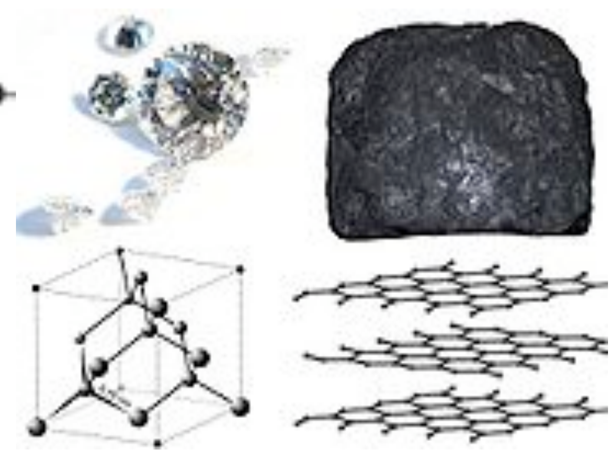
- Bragg diffraction:
  - *spectroscopy*
    (rainbow, prism)
  - *crystallography:*
    - *crystals*
    - *DNA*
    - *proteins*



Bragg condition:

$2d\sin\vartheta = n\lambda$



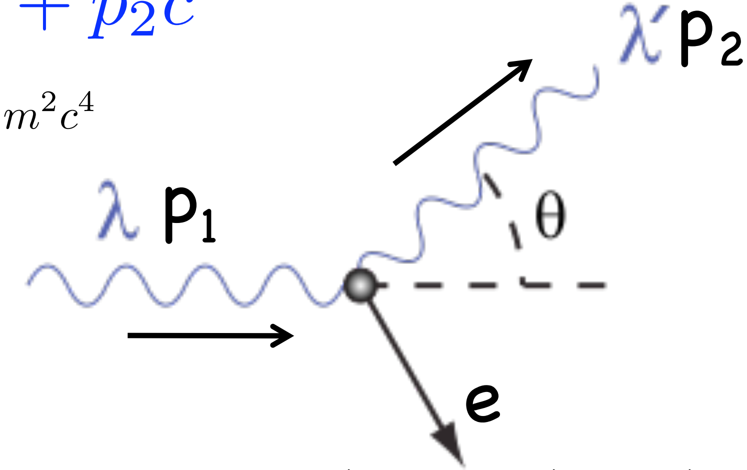


Compton effect

Compton (1923)

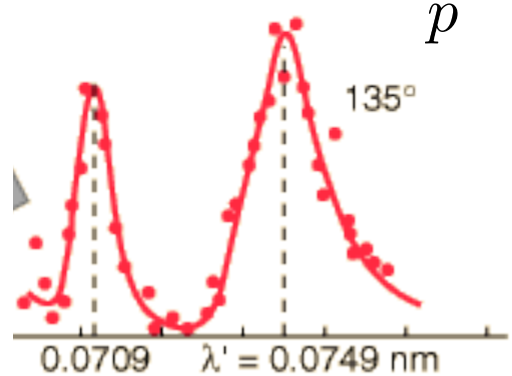
- *particle-like* scattering of light (x-ray) by electron
- $p_1 c + mc^2 \quad = \quad \sqrt{(\vec{p}_1 - \vec{p}_2)^2 c^2 + m^2 c^4} + p_2 c$

$$
\begin{aligned}
(p_1 - p_2)^2 c^2 + m^2 c^4 + 2(p_1 - p_2) mc^3 &= p_1^2 c^2 + p_2^2 c^2 - 2 p_1 p_2 c^2 \cos\theta + m^2 c^4 \\
\frac{1}{p_2} - \frac{1}{p_1} &= \frac{1}{mc}(1 - \cos\theta)
\end{aligned}
$$



$$\vec{p} \quad = \quad \vec{p}_1 - \vec{p}_2$$

$$\Longrightarrow \quad \lambda_2 - \lambda_1 \quad = \quad \frac{h}{mc}(1 - \cos\theta)$$



- Compton wavelength $\lambda_C = h/mc$ *= 2.4x10⁻¹² meters (electron)*
- discrete shift in λ for fixed angle ϑ *(contradicts classical EM)*

$\Longrightarrow$ particle nature of light: the photon

clicker question

Photoelectric effect

Q: *Blue light (whose wavelength λ corresponds to $h\nu$=3eV) illuminates surface of potassium whose workfunction is 2 eV.*
Are any electrons ejected from the surface, and if so what is their maximum speed?

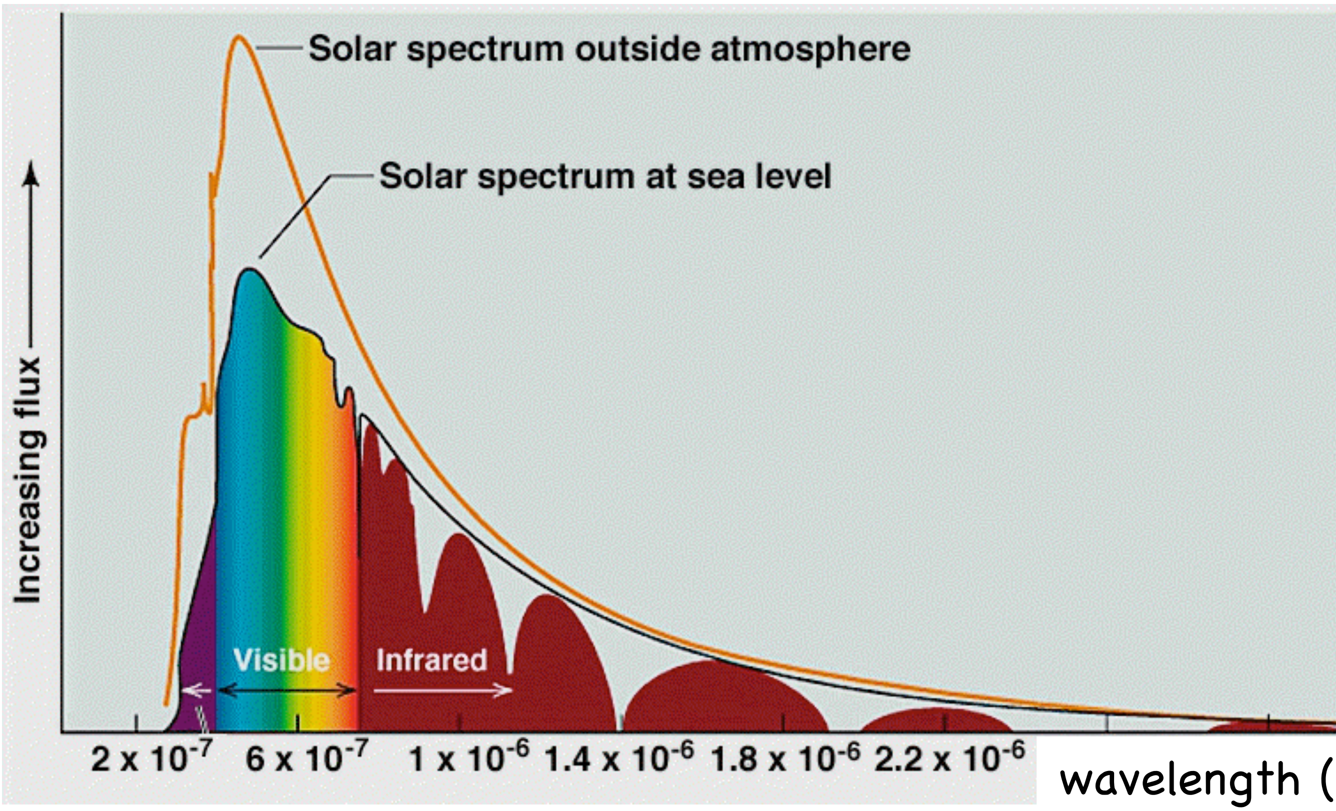
a) no, b) yes, 1 m/s, c) yes, 600 m/s, d) yes, 6×10^5 m/s

A: $E_K = h\nu - W = 1\ eV = \frac{1}{2} m v^2 \rightarrow v/c = 2\times10^{-3}$

clicker question

Blackbody radiation of the Sun

Q: *The Sun is approximately a blackbody radiator as can be seen from its emitted spectrum below. Estimate Sun's T from its spectrum below, given k_B=8.6x10^{-5}eV/K, h = 4.1x10^{-15}eV-sec*



wavelength (meters)

$$k_B T \approx E_{peak}/3$$

a) 100 K, b) 10^{10} C, c) 6000 K, d) 2 trillion degrees

A: *The peak is roughly at around 6x10^{-7} m ≈ 2 eV = 24000 K*
→ T_{sun} ≈ 6000 Kelvin