Smectic vortex glass

(a trip down a memory lane)

Boulder

Center for Theory of Quantum Matter

CTQM

L.R. arXiv:2105.05247

$: Simons Investigator, NSF-MRSEC

SMRC

Tutorial & Seminar, Caltech Physics, 3 June 2021

Outline

- ***Background tutorial***
  - vortices in superconductors and related problems
- ***"Smectic" vortex glass***
  - *Transverse Bose-glass geometry*
  - *Predictions*
  - *Harmonic Larkin analysis*
  - *Nonlinear pinning and functional RG -> transverse Meissner effect*

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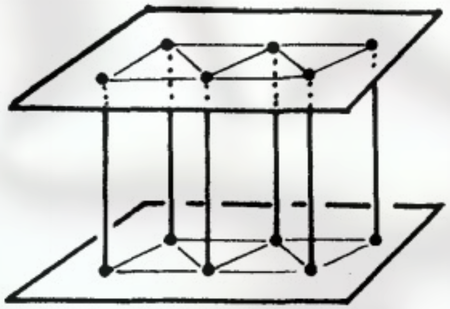
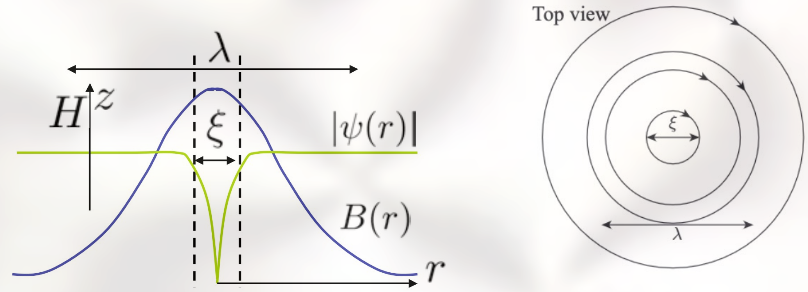
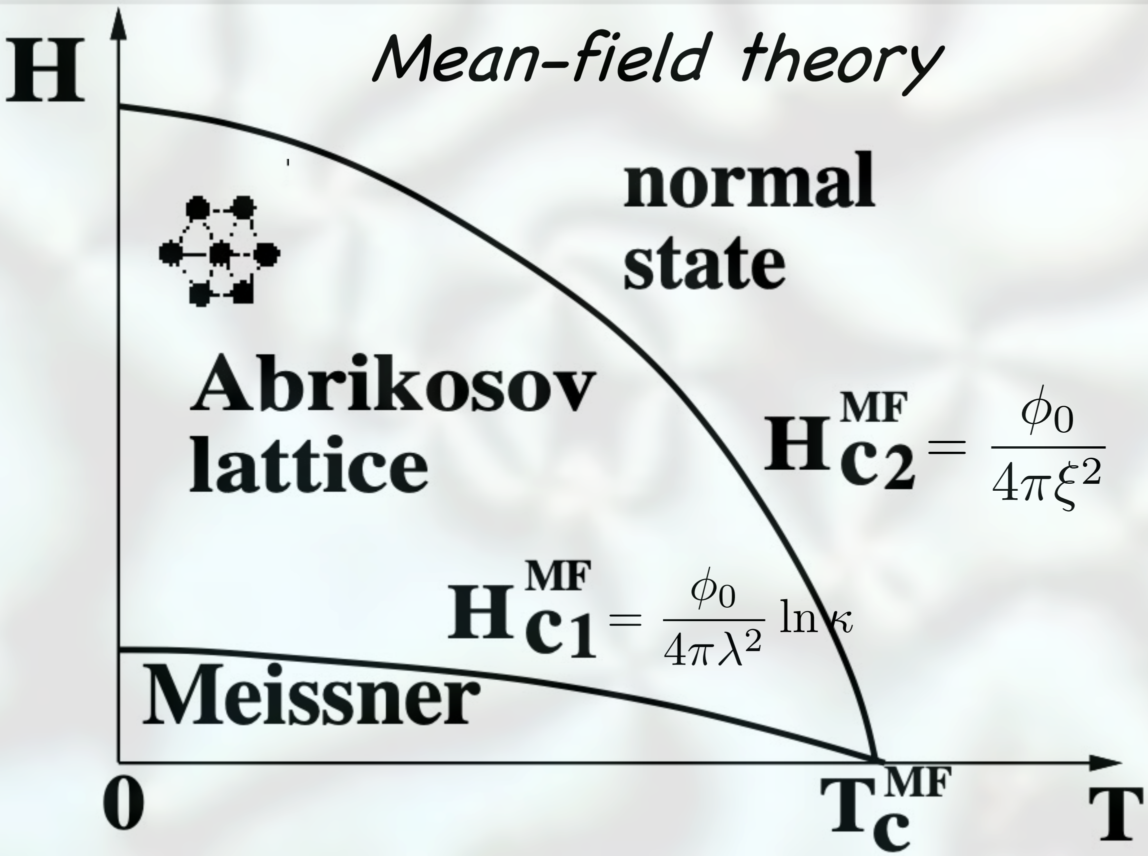
Type-II superconductors H-T phase diagram

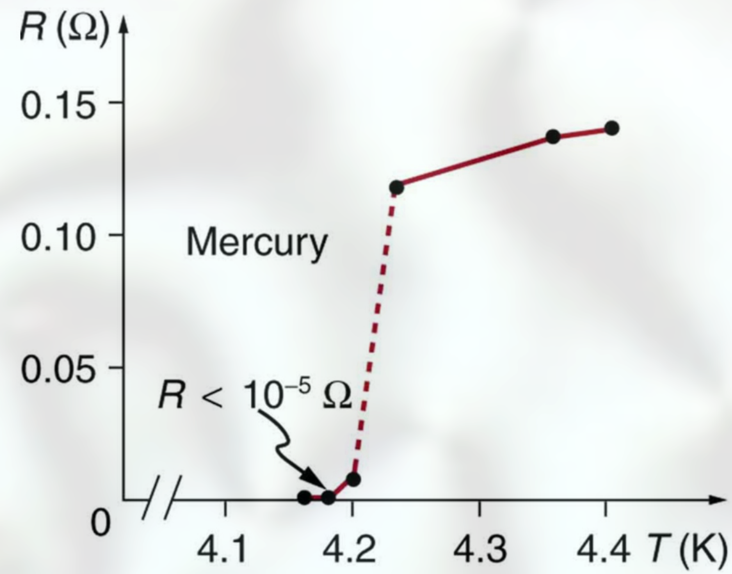
Abrikosov, 1957

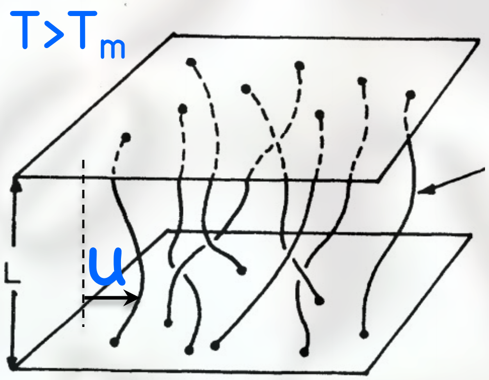




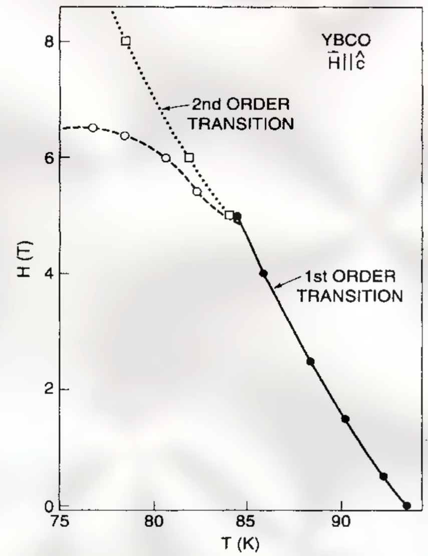
$$\kappa \equiv \lambda/\xi > 1/\sqrt{2}$$







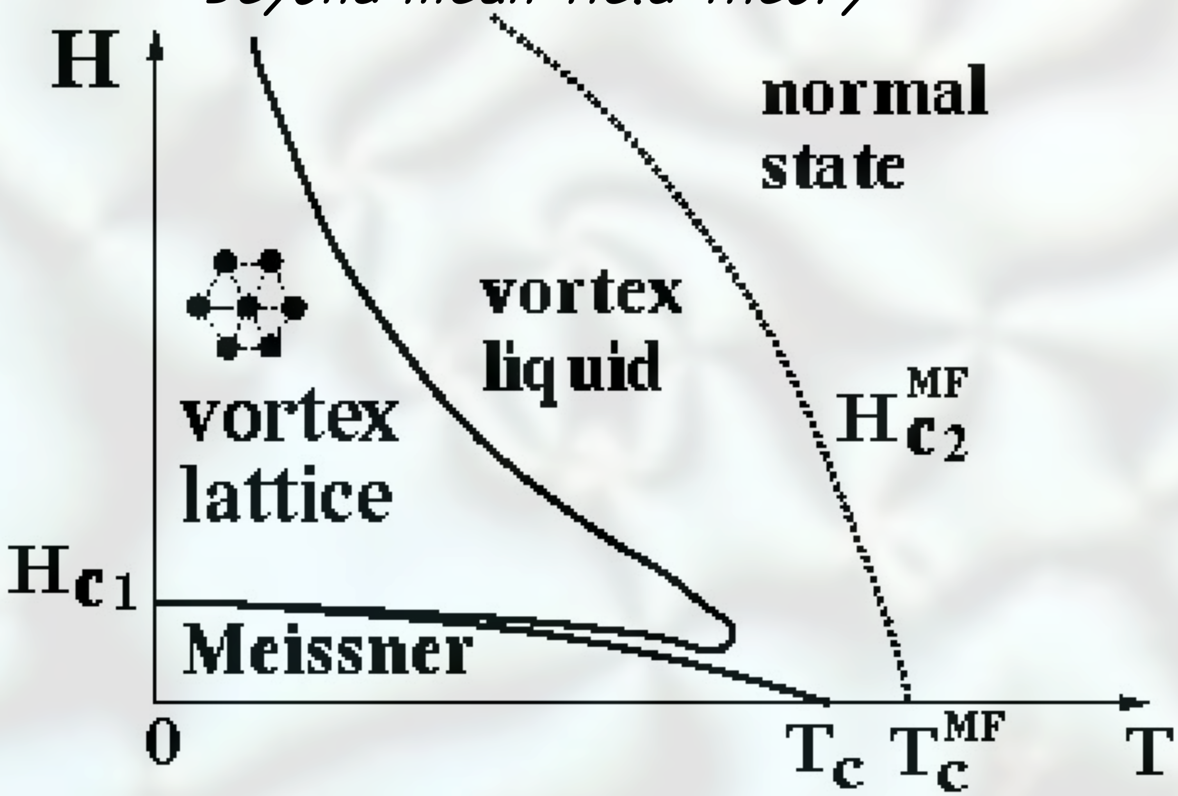
Vortex lattice melting

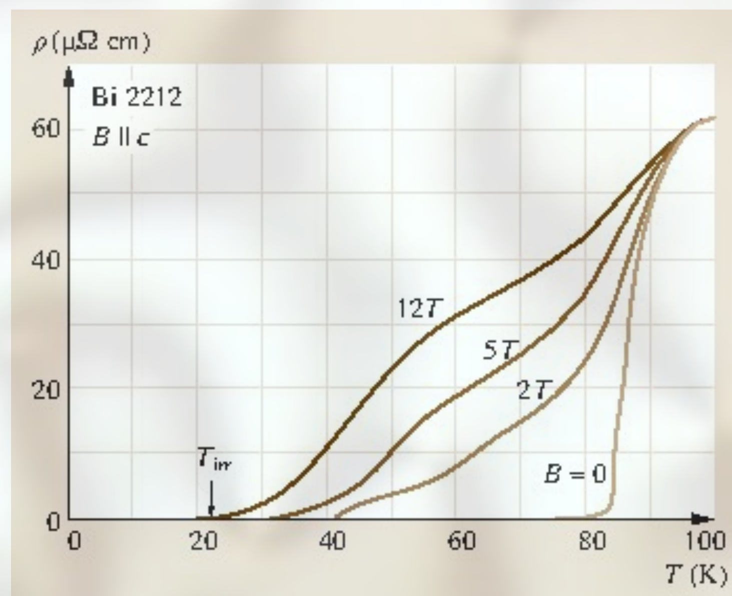


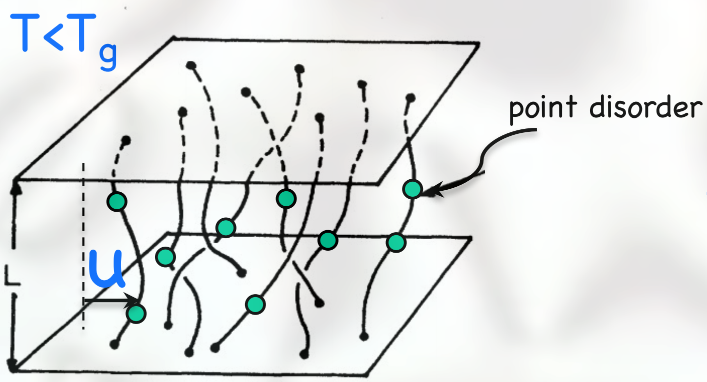
Eilenberger, 1967
D.S. Fisher, 1980
D. R. Nelson, et al, 1988
H. Safar, et al, 1992
W. Kwok, et al., 1992
E. Zeldov, 2000

Lindemann criterion: $\langle u^2 \rangle_{T_m} = c_L a^2$

Beyond mean-field theory







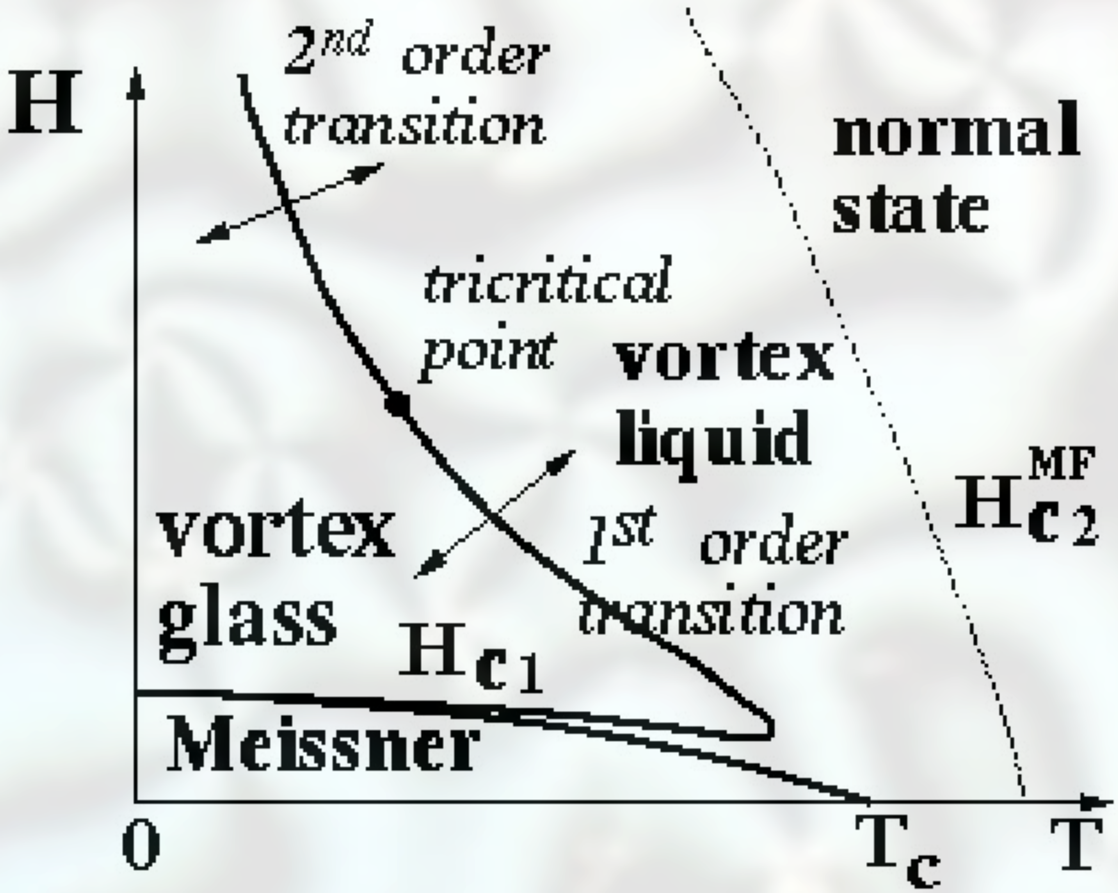
Vortex glass

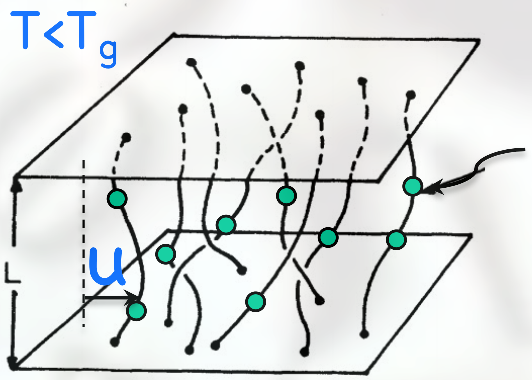
(lattice point pinning)

Larkin, Ovchinnikov, '70, '74
Imry-Ma, '75
Fisher, Fisher, Huse '91

$$E_{\text{exc}} \sim Y L^{\theta}$$

Larkin length: $\langle u^2 \rangle_{\xi_L} \sim a^2 \quad \Longrightarrow \quad \xi_L \sim \left(\frac{K}{V_p}\right)^{2/(4-d)}$



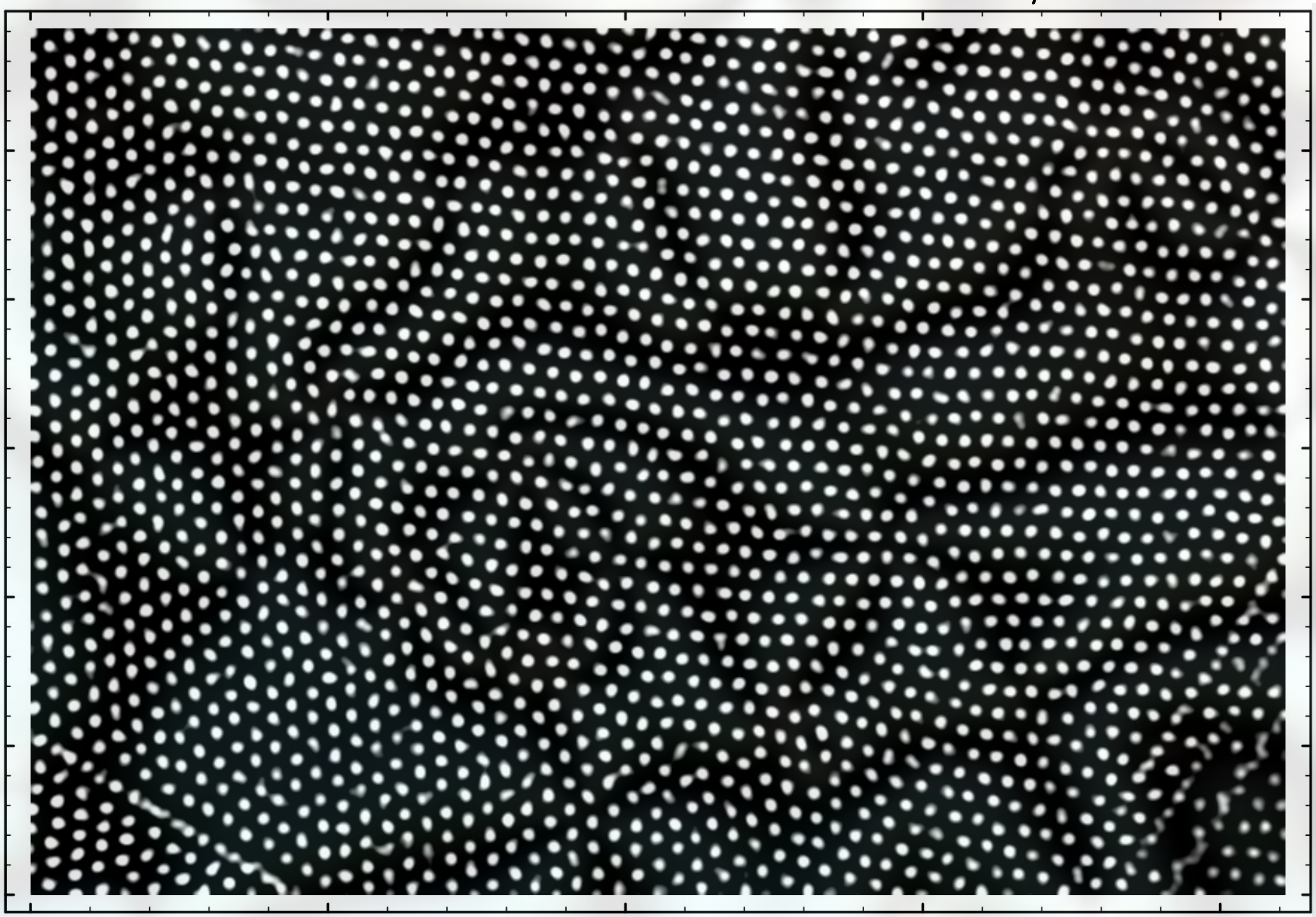


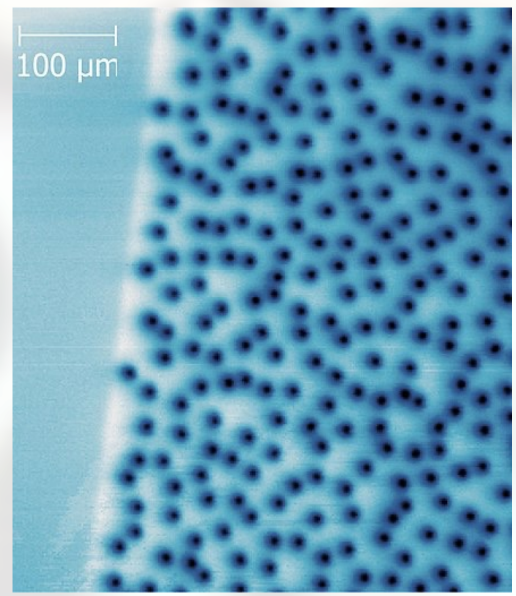
Pinned elastic media

('baby' glass)

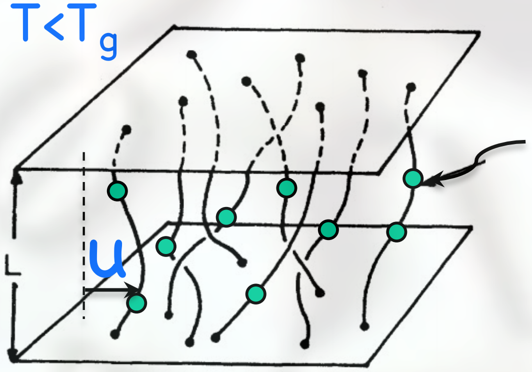
Larkin, Ovchinnikov, '70, '74
Imry-Ma, '75
Fisher, Fisher, Huse '91

Bitter decoration of pinned vortices





YBCO SQUID microscopy
Wells, et al. 2015



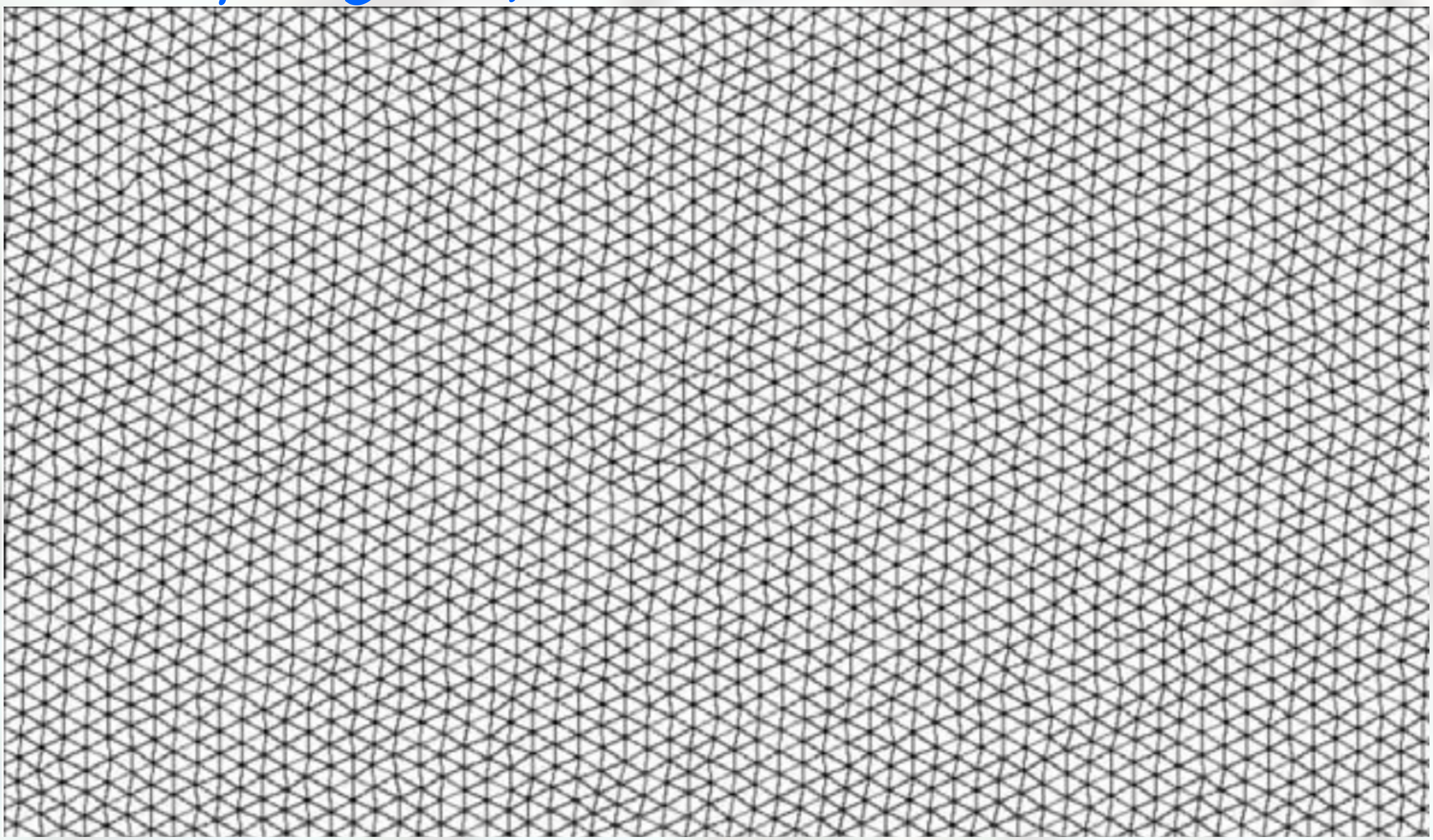
'Bragg' glass

Giamarchi, Le Doussal '95
D. S. Fisher '85, '97
Kierfeld, Nattermann, Hwa '97
T. Klein, et al., '01

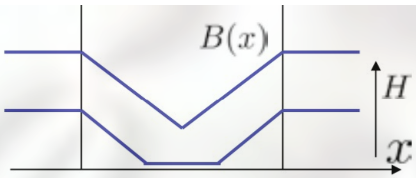
$$\overline{\langle (u(x) - u(0))^2 \rangle} \approx a^2 A_d \ln(x/a)$$

universal

topologically ordered: no dislocations



BSCCO, P. Kim, et al., '99



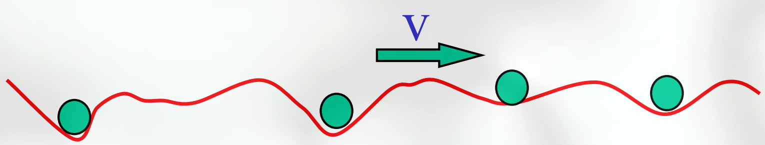
Bean profile -> m relaxation: $m(t) \sim \frac{m_0}{(\ln t)^{1/\mu}}$

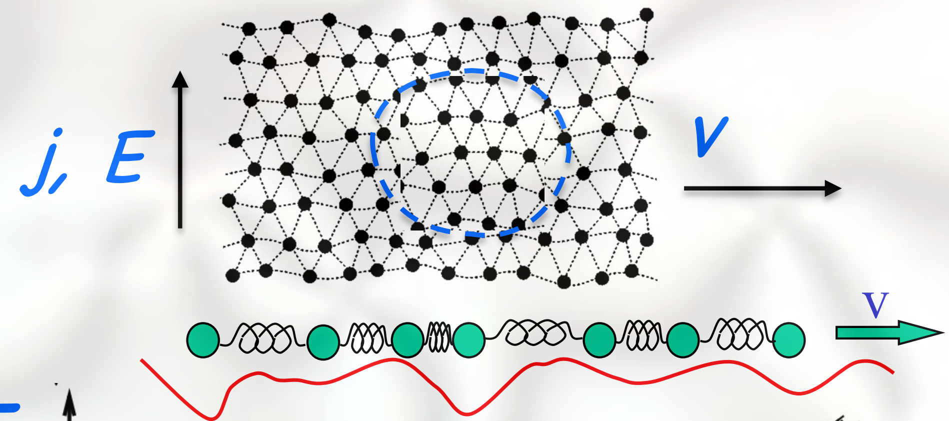
Depinning elastic media

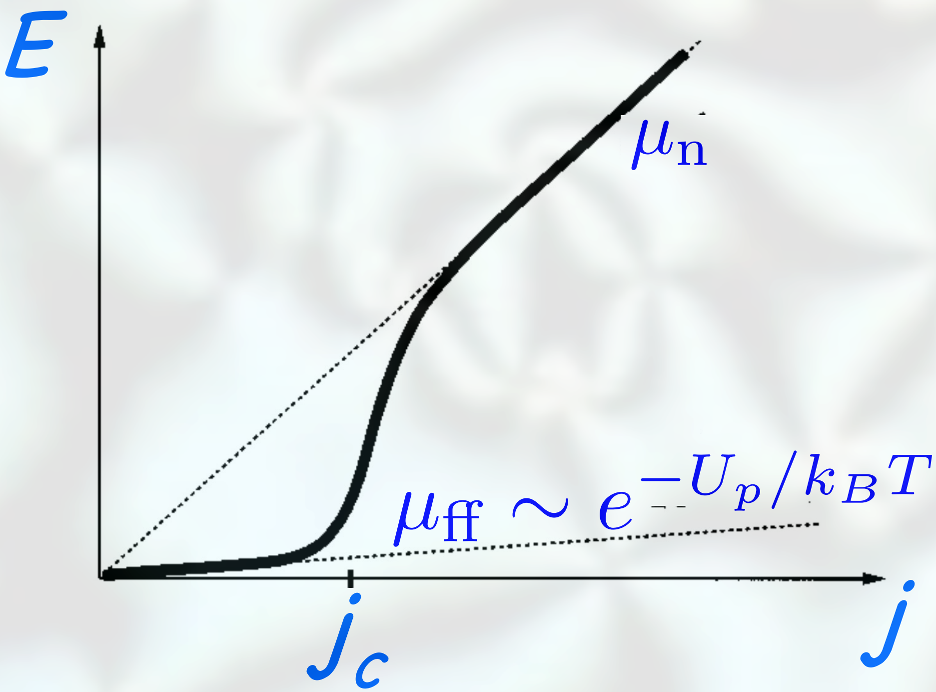
transport

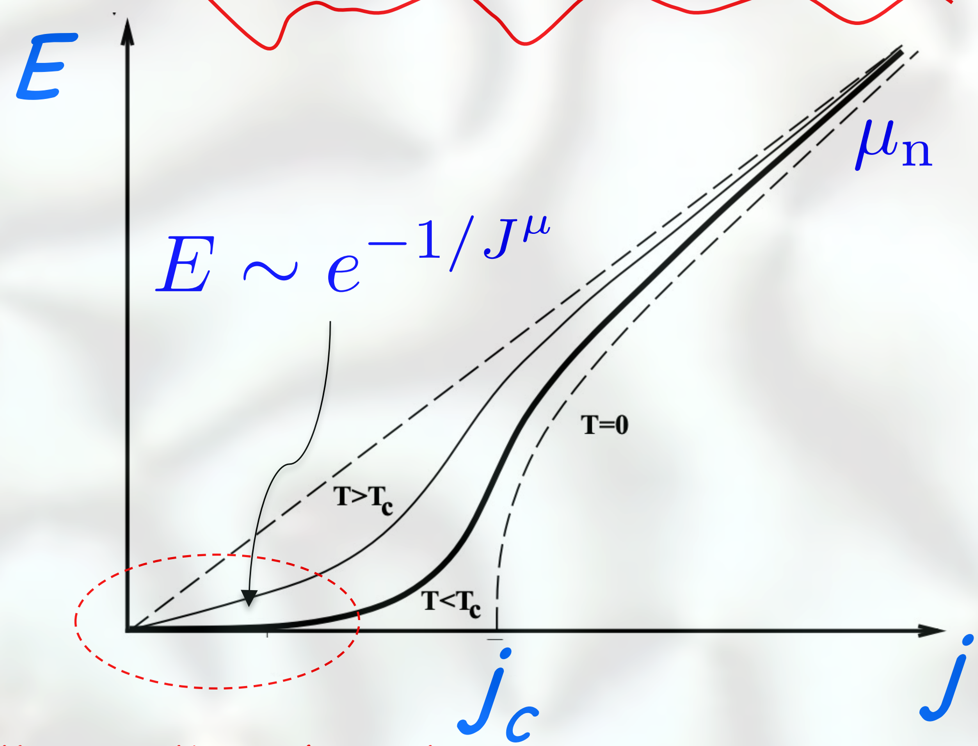
Larkin, Ovchinnikov,
Imry-Ma,
Fisher, Fisher, Huse
O. Narayan, Fisher
Giamarchi, Le Doussal

Anderson-Kim (single particle) vs collective pinning (interacting)







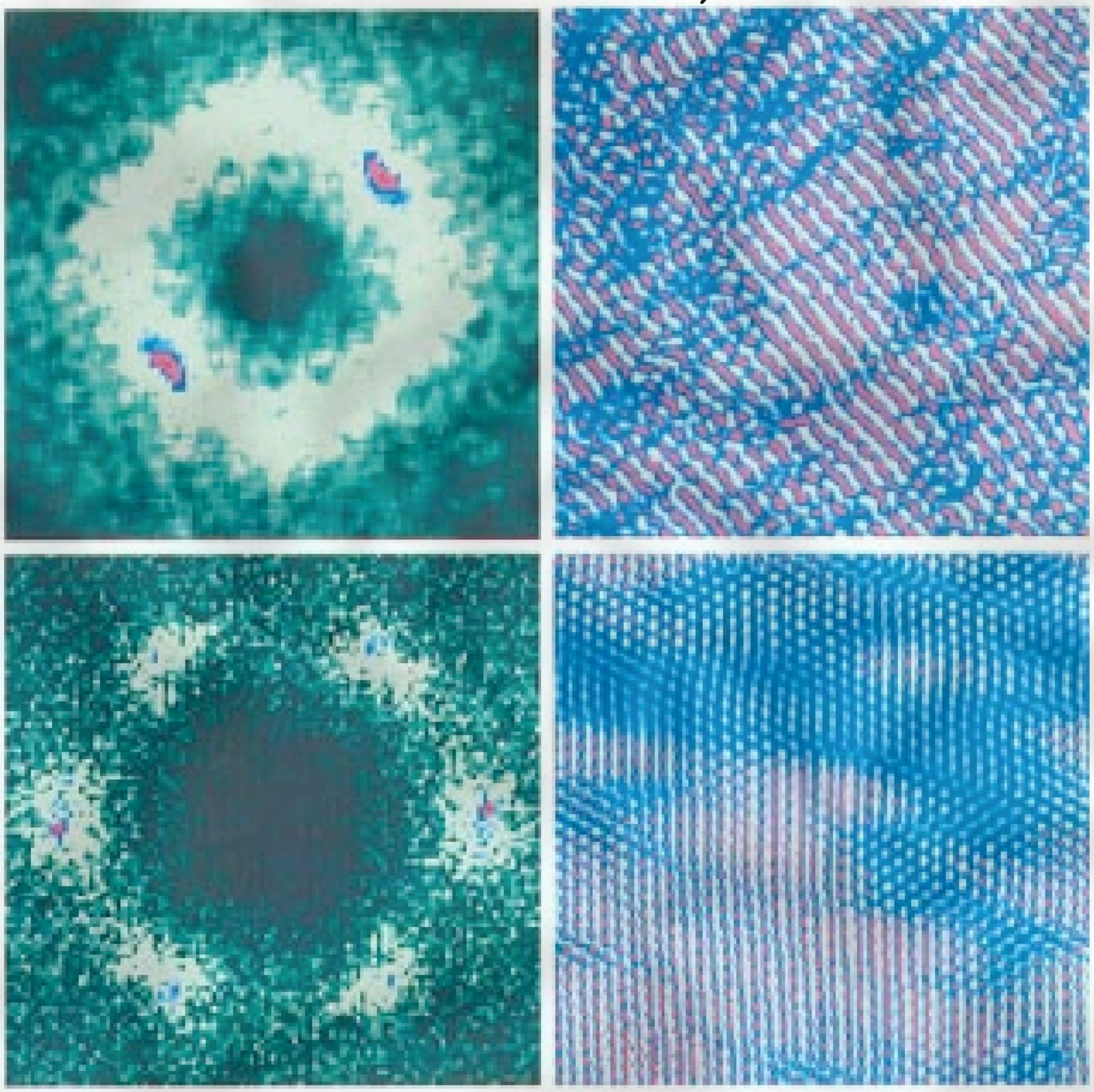


more general: friction, earth quakes, etc

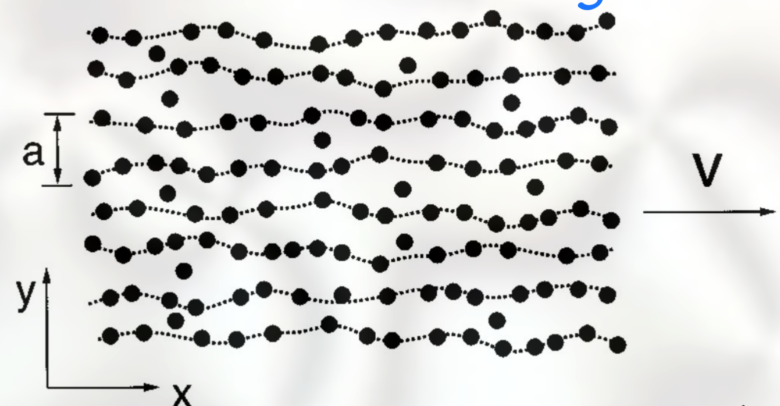
Moving elastic media
transport

Fukuyama, Lee '78
Nattermann '90
Narayan, Fisher '92
Koshelev, Vinokur '94
Giamarchi, Le Doussal '96
Balents, Marchetti, LR '97
Vinokur, Scheidl '98

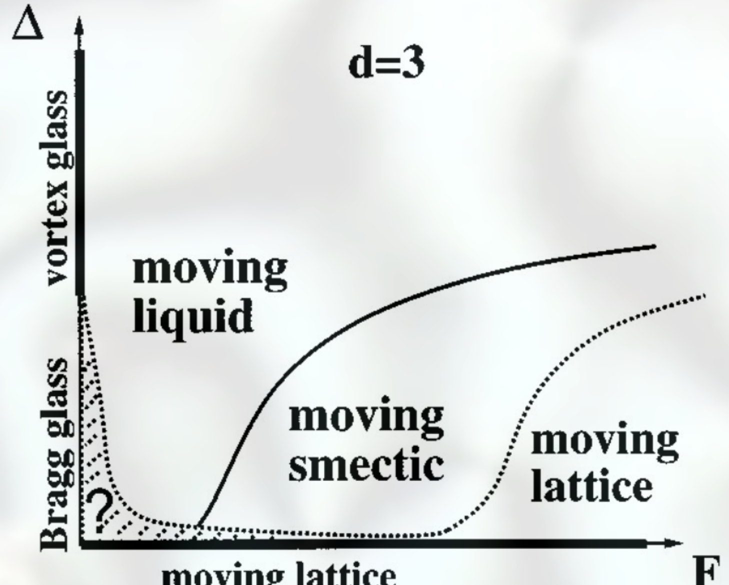
Moon, Scalettar, Zimanyi '96



"transverse smectic glass"



Balents, Marchetti, LR '97

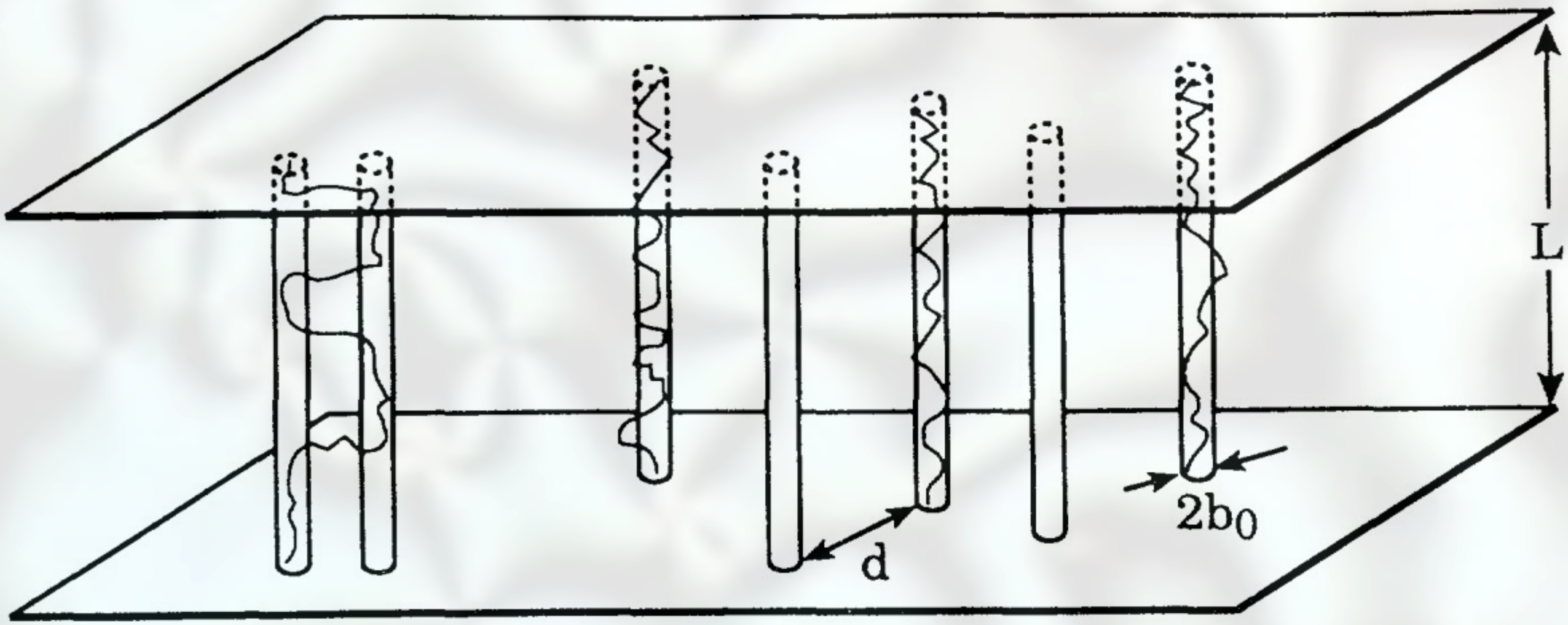


(Anisotropic) 'Bose' glass

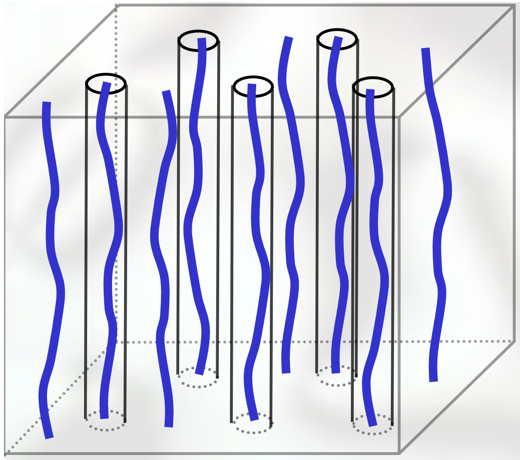
correlated columnar disorder

Fisher, et al., 1989
Nelson, Vinokur, 1992
Hwa, et al. '93
Balents, '93
L.R. '95
Nelson, L.R. '96

Heavy ion irradiation tracks, screw dislocations



Nelson, Vinokur, 1992



‘Bose’ glass

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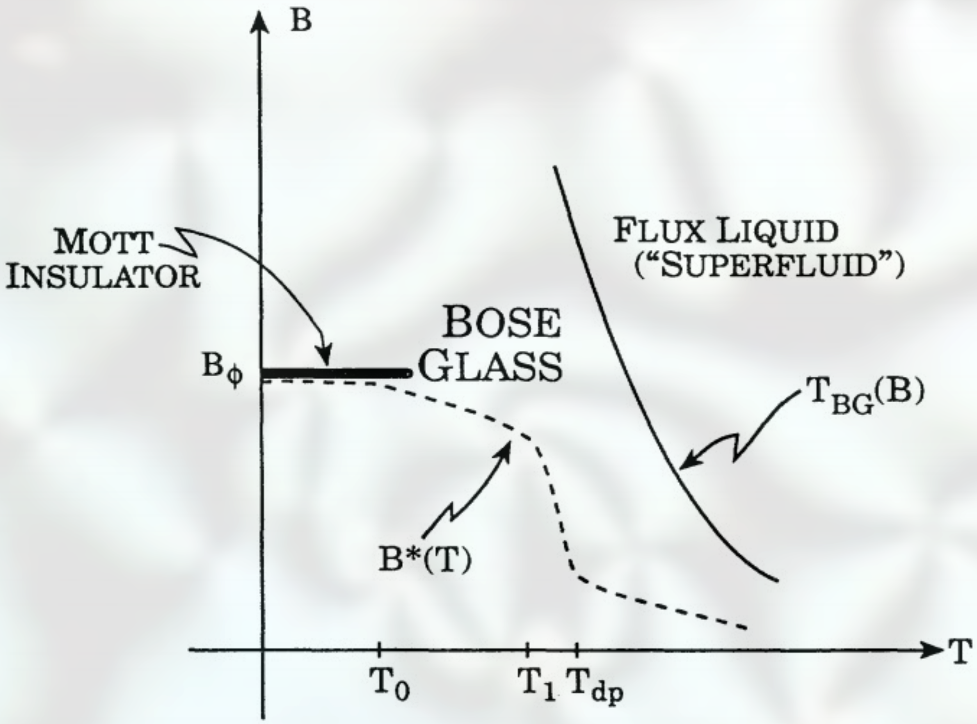
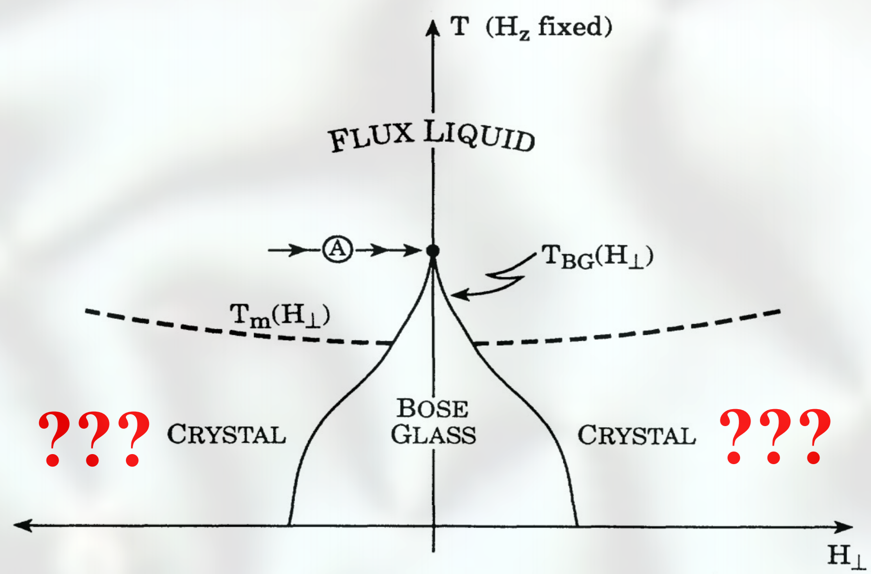


Nelson, Vinokur, 1992

Transverse Meissner effect



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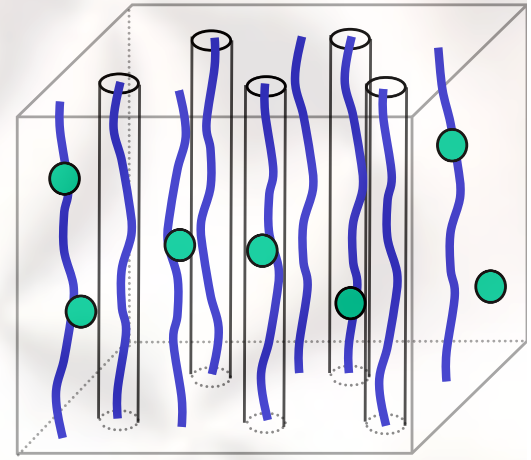
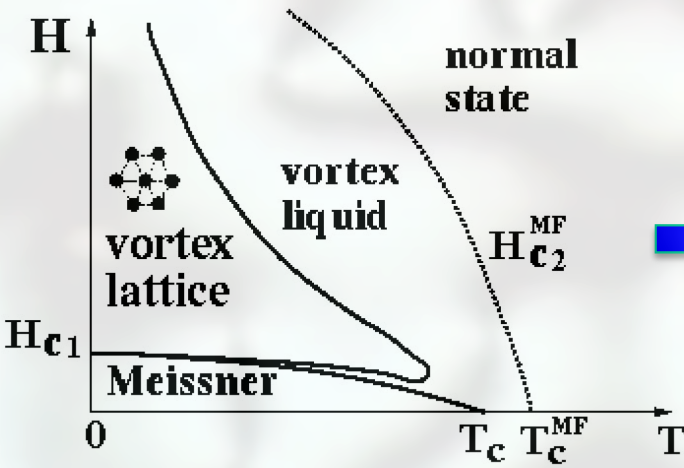
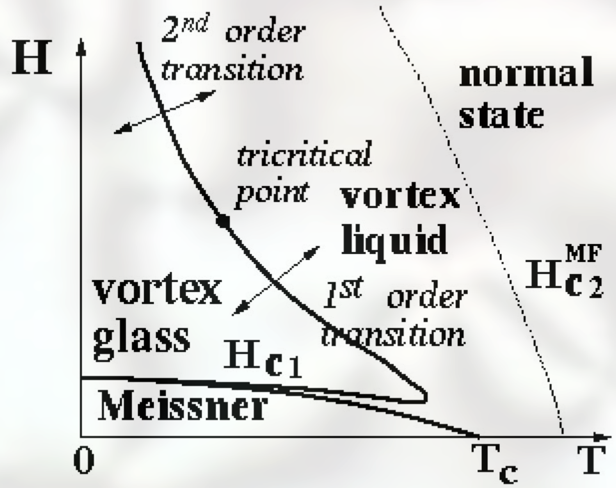
Fisher, Fisher, Huse '89
Nelson, Vinokur '92

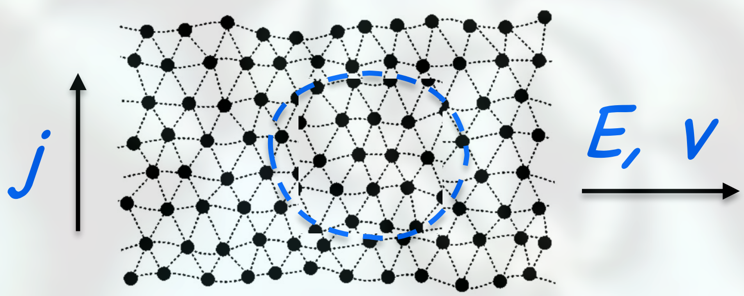
Motivation

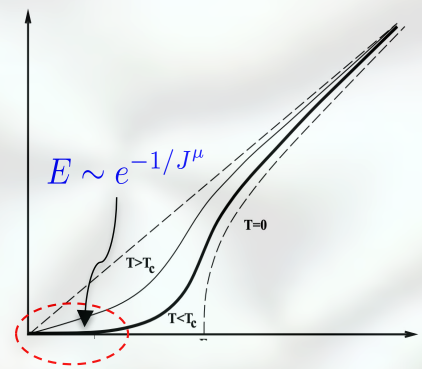


- vortex lattice pinning by point and columnar defects

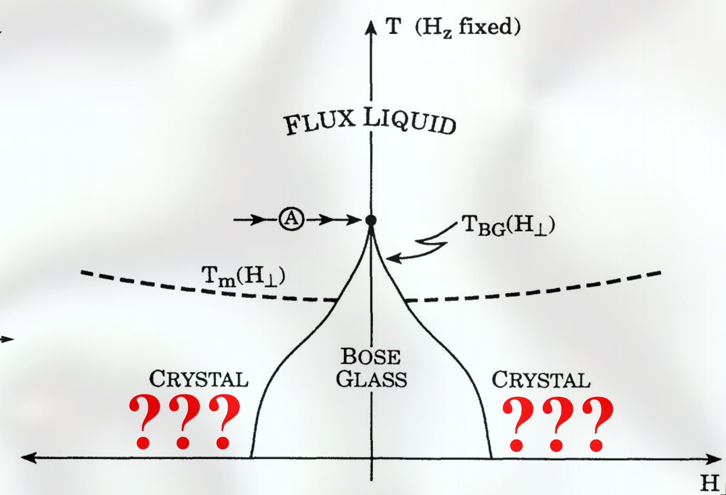




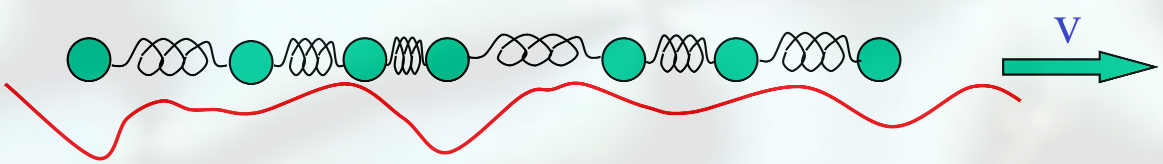


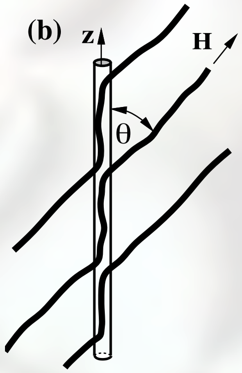


Hwa-Nelson-Vinokur



- dynamics over a random substrate

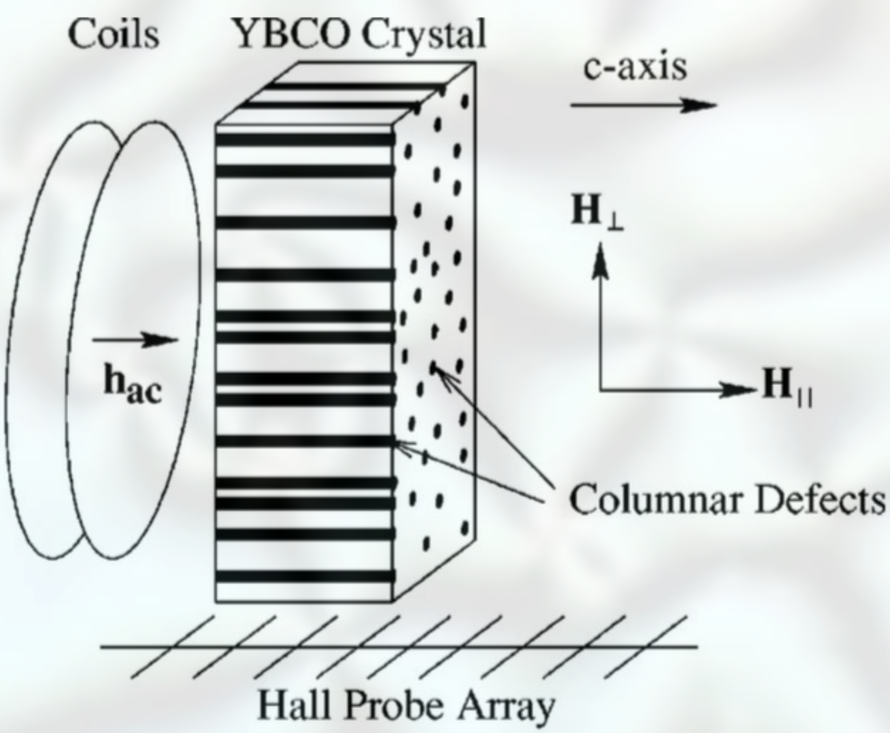


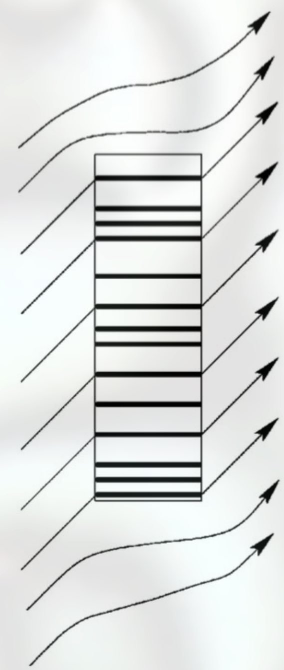


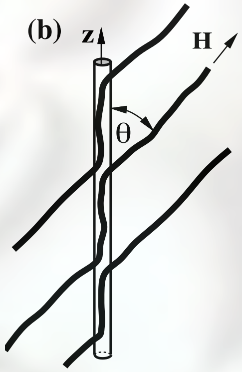
Experimental puzzles

Smith, Jaeger, Rosenbaum, et al., 01

- measure transverse Meissner effect $T_s(H_\perp)$, melting $T_m(H_\perp)$







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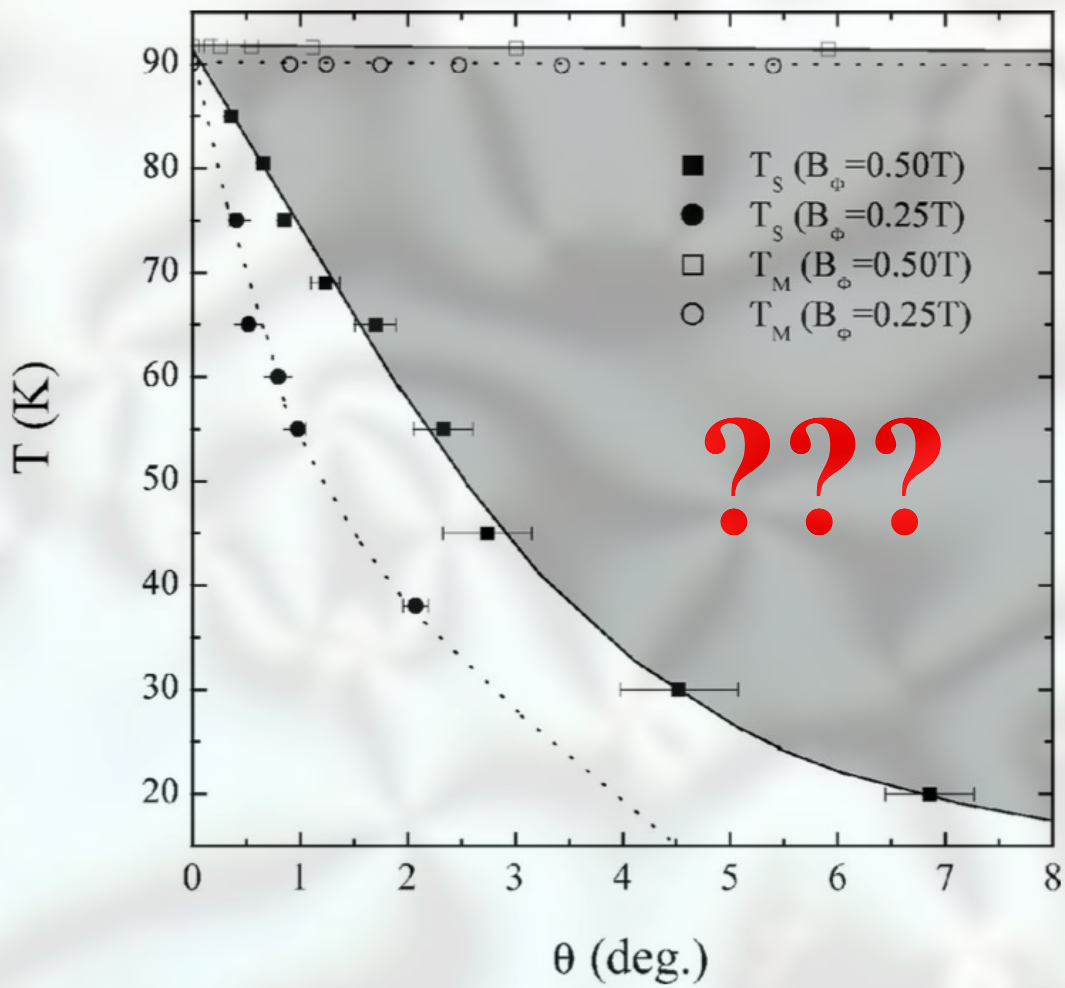
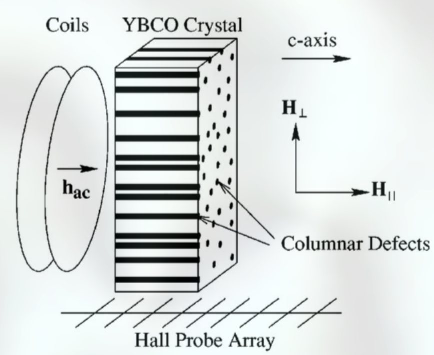
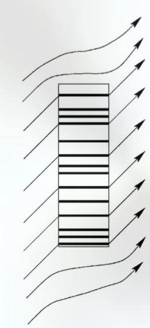






Tilting a vortex array pinned by a columnar defect

L.R., PRB 2006

Model

L.R., PRB 2006

- Bulk (1+1)d model: $H = \int dx dz [\underbrace{K(\partial_z u - h)^2 + B(\partial_x u)^2}_{\text{elasticity with tilt } h \propto H_\perp} - \underbrace{V_{pin} n_v(x,z)}_{\text{pinning}}]$

$$H \approx \int dx dz [K(\partial_z u - h)^2 + B(\partial_x u)^2] - v \int dz \cos[G u(0,z)]$$

- Reduction to (0+1) d model: $(K\partial_z^2 + B\partial_x^2) u(x,z) = B u_0(z) \partial_x \delta(x).$

-> $\tilde{u}(x, q_z) = \tilde{u}_0(q_z) e^{-(K/B)^{1/2} |q_z| |x|}$

Sine-Hilbert model $H_0 = \overline{K} \int_z \int_{z'} \underbrace{\left(\frac{u_0(z) - u_0(z') - h(z-z')}{z - z'} \right)^2}_{} - v \int_z \cos[G u_0(z)]$

long-range elasticity induced by deformation of the bulk

$$\frac{1}{\pi} \int dz' \frac{\phi(z) - \phi(z')}{(z-z')^2} + \sin\phi(z) = 0 \qquad \text{with} \qquad \partial_z \phi(z)|_{z=0,L} = h$$

soliton: $u_s(z) = -a/\pi \tan^{-1}(1/z)$

Peierls '40, Nabarro '47
Ablowitz '87

Soliton tilt transition

L.R., PRB 2006

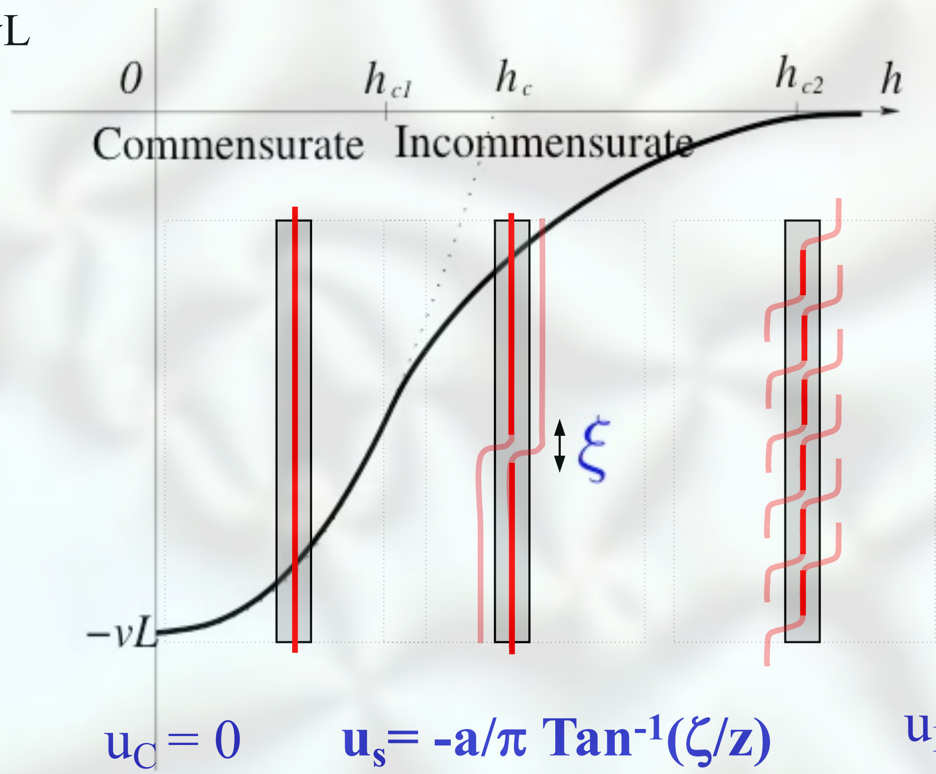
$$H_0 = \overline{K} \int_z \int_{z'} \left(\frac{u_0(z) - u_0(z') - h(z-z')}{z-z'} \right)^2 - v \int_z \cos[G u_0(z)]$$

- Soliton (commensurate-incommensurate) tilt transition at $h_{c1} = \frac{a}{2L} \ln \frac{L}{\xi}$:

$$E_{N_s} \approx E_C + L^2 \left[2(h_{c1} - h) n_s + \frac{1}{2} V_s(L/2) n_s^2 \right]$$

$E_C = K\, h^2 L^2 - vL$

$E_I = 0$



$\langle \partial_z u \rangle \propto n_s \sim |H^\perp - H^\perp_{c1}|$

$$\xi = \left(\frac{a}{2\pi}\right)^2 \frac{\overline{K}}{v}$$

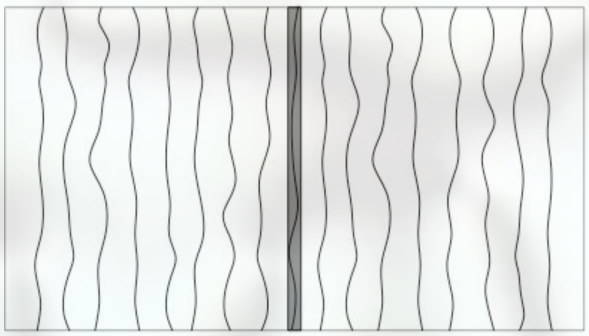
$u_C = 0$ **$u_s = -a/\pi \, \mathrm{Tan}^{-1}(\zeta/z)$** $u_I \approx h\, z$

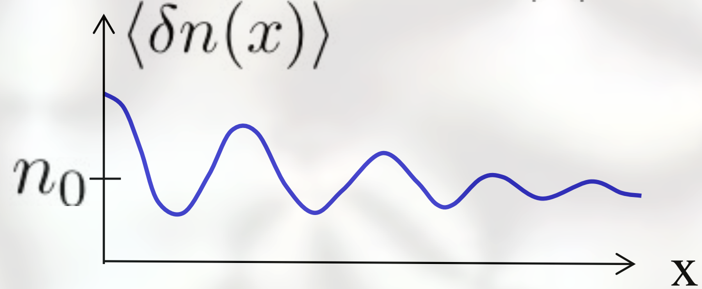
Results

L.R., PRB 2006

- Thermal depinning (roughenning) transition at $T_p = \frac{\sqrt{KB}}{\pi a_0^2}$

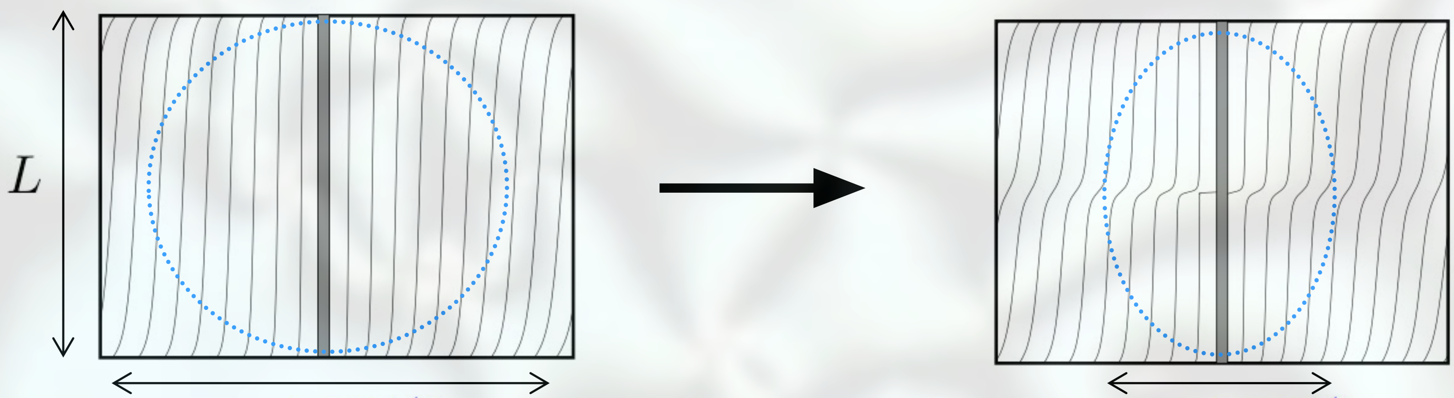
Friedel oscillations in the vortex density: $\langle n(x,z)\rangle_0 - n_0 \approx \frac{c}{|x|^{\eta/2}} \cos(2\pi n_0 x)$





Hofstetter, et al, 2004

- Tilting transition at $H_{c1}^{\perp} \approx \frac{\phi_0}{w}\frac{1}{L}\ln\frac{L}{\xi}$, crossover at $H_{c2}^{\perp}(0) \approx H_{c1}^{\perp}(0) + \frac{\phi_0}{2\pi w \xi}$



L.R., PRB 2006

$\lambda_h^0 = \left(\frac{B}{K}\right)^{1/2} L$

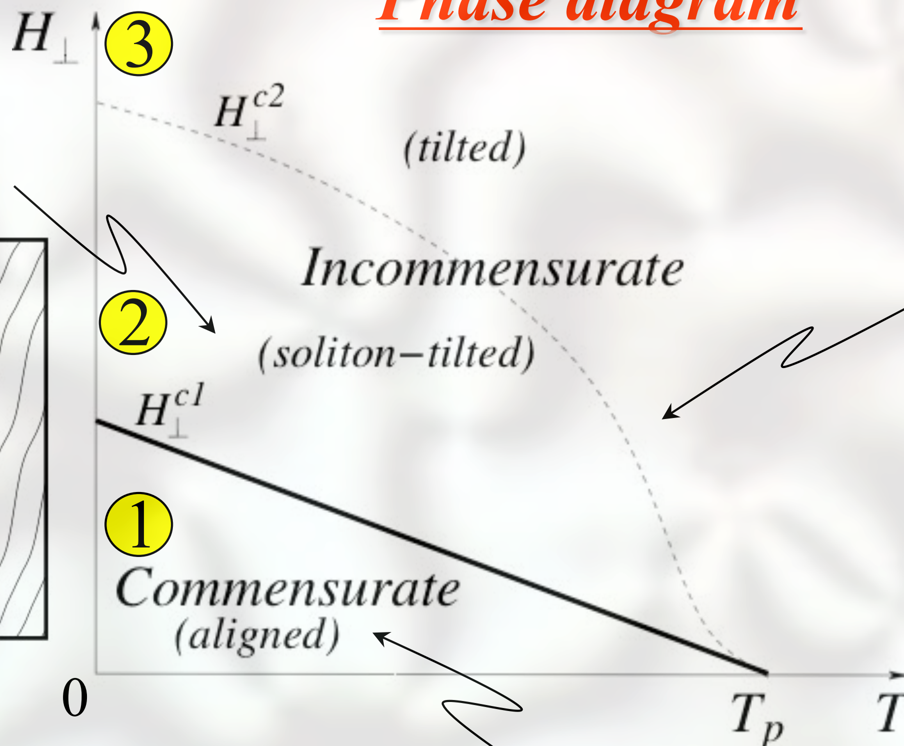
$\lambda_h = \left(\frac{B}{K}\right)^{1/2} \frac{1}{2\pi n_s(H_\perp)} \sim \frac{1}{|H^{\perp} - H_{c1}^{\perp}|}$

$H_{c1}^{\perp}(T) \approx H_{c1}^{\perp}(0)(1 - T/T_p)$

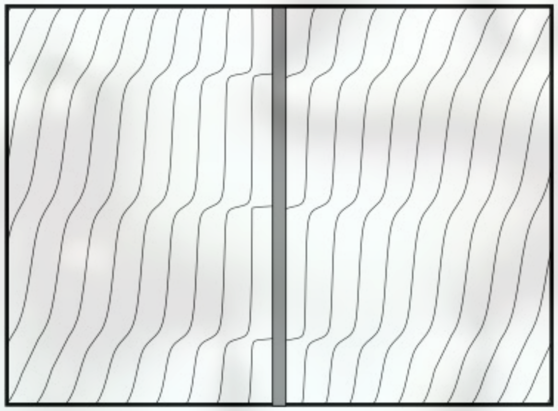
$H_{c2}^{\perp}(T) \approx H_{c1}^{\perp}(T) + H_{c2}^{\perp}(0)e^{-T/|T_p - T|}$

Phase diagram

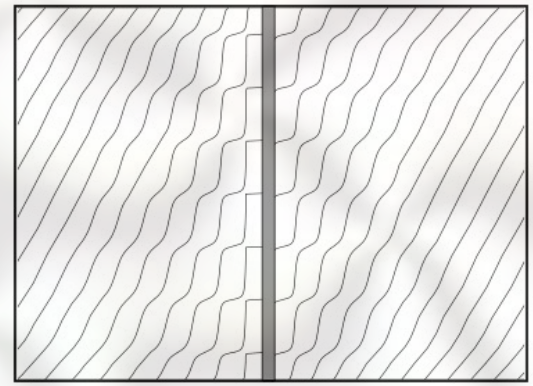
L.R., PRB 2006



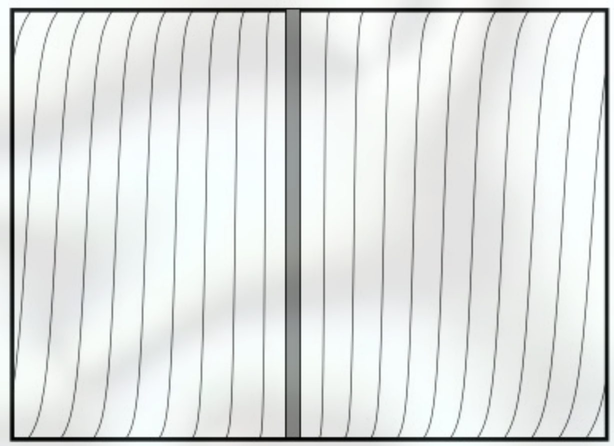
nonlinear tilt response
via solitons proliferation

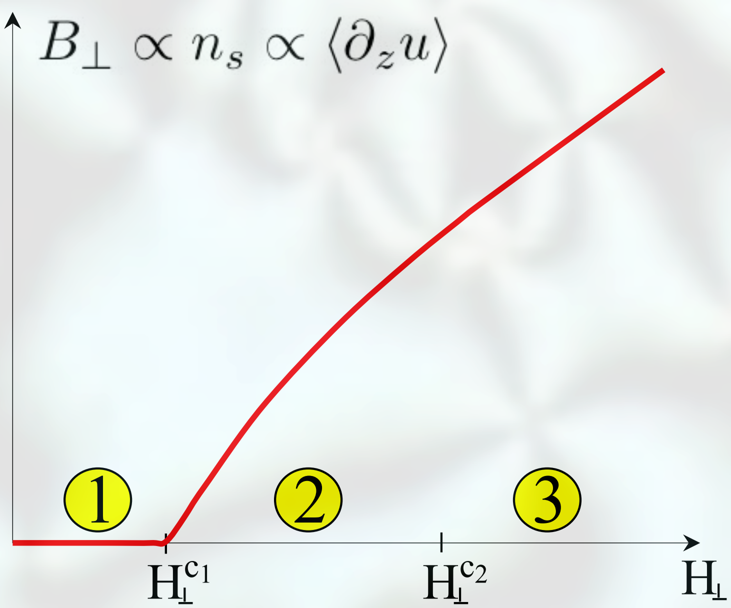


linear tilt response
overlapping solitons

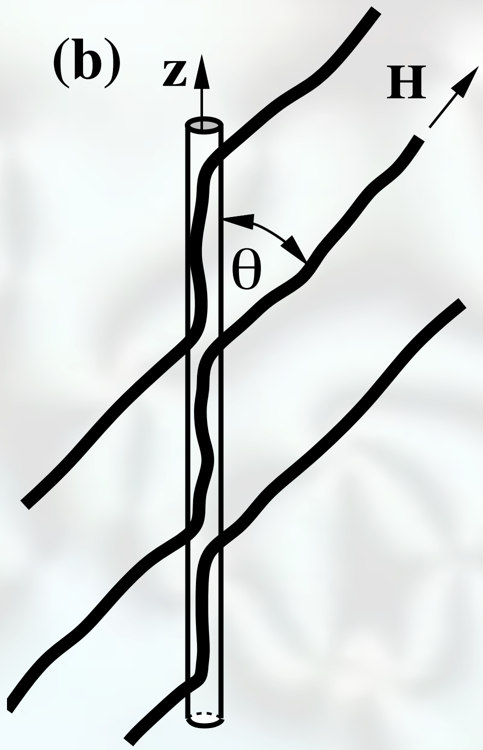


vanishing tilt response (transverse Meissner effect)

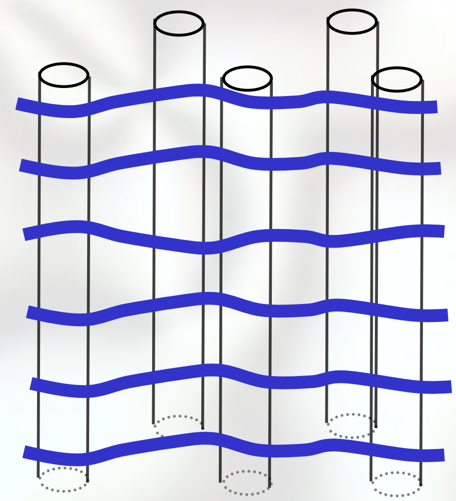




Large-angle tilt



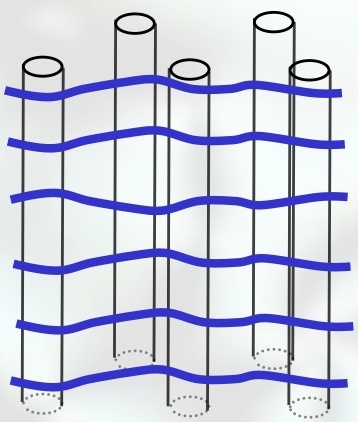
θ = π/2 tilt

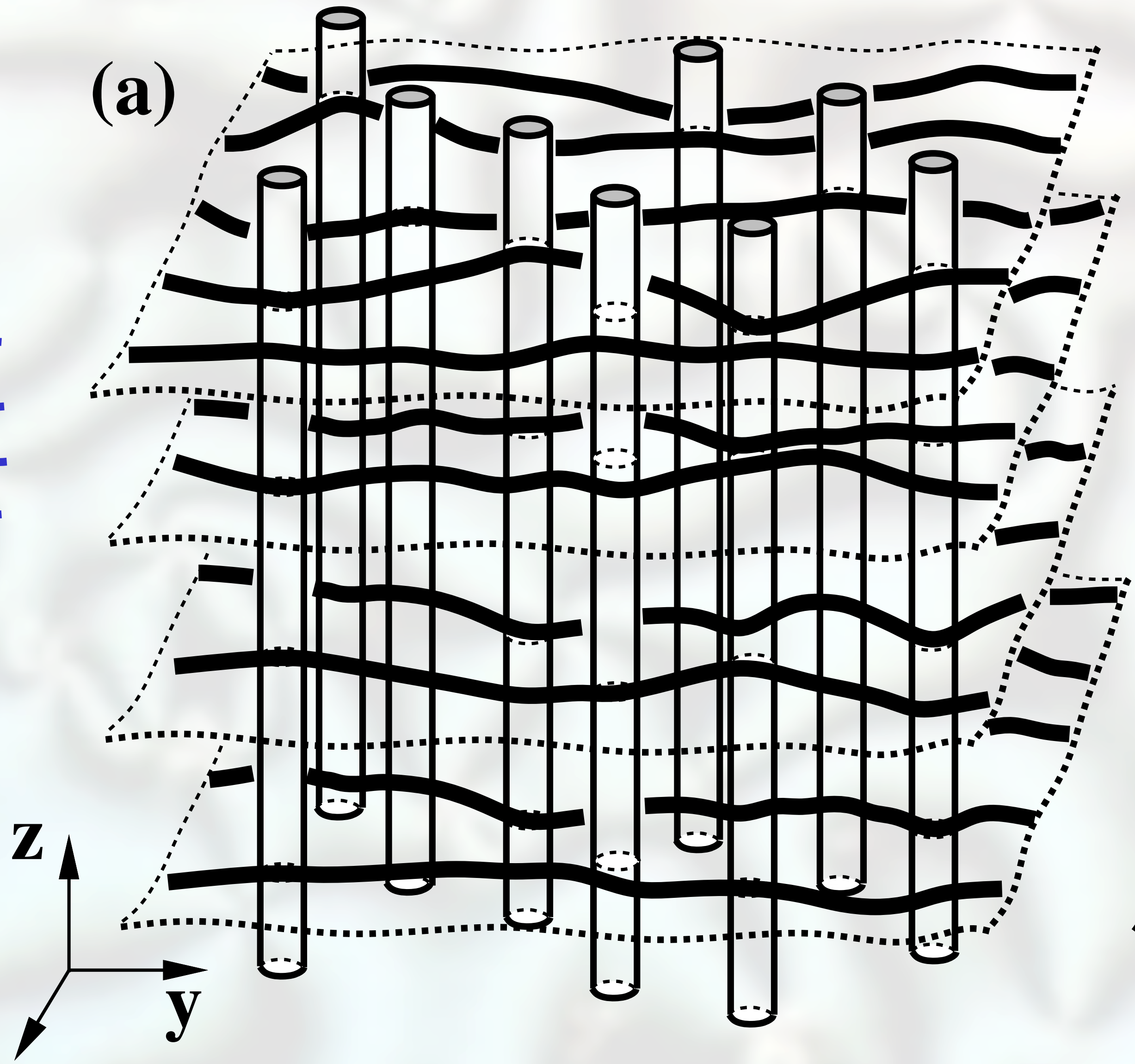


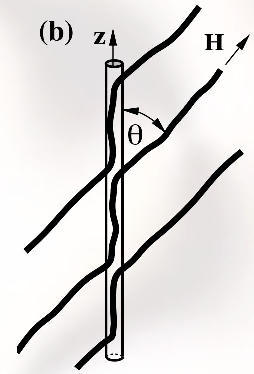
L.R., arXiv 2021

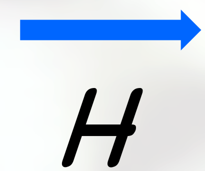
Smectic vortex glass

- θ = π/2 tilt





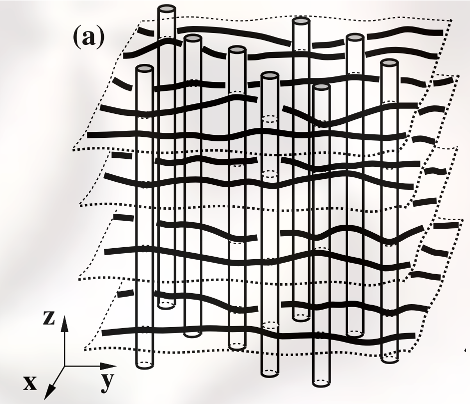




L.R., arXiv 2021

Smectic vortex glass

structure



- Periodic along columns (z), $u_z \approx 0$

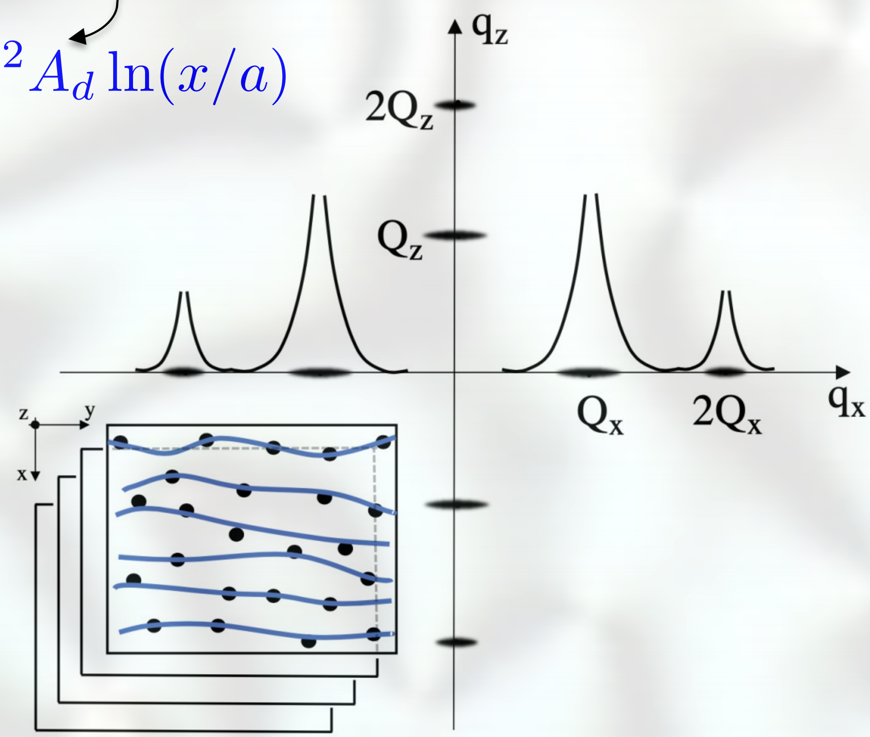
-> *Bragg peaks:* $S(0,q_z) \sim \sum_n I_{nQ_z}\delta(q_z - nQ_z)$

- Power-law rough transverse to columns (x), $\overline{\langle(u(x)-u(0))^2\rangle} \approx a^2 A_d \ln(x/a)$

universal

-> *power-law peaks:*

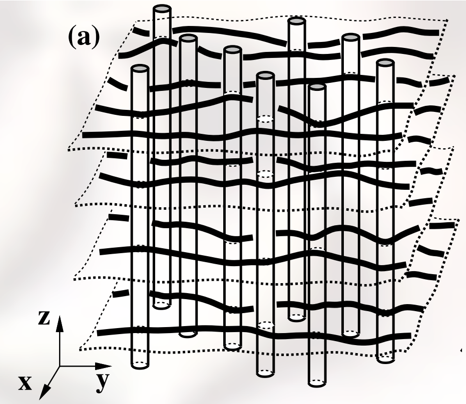
$$S(q_x,0) \sim \sum_n \frac{1}{|q_x - nQ_x|^{1-n^2\eta}}$$



L.R., arXiv 2021

Smectic vortex glass

elasticity



- *divergent shear modulus:* $\mu_{zx} \to \infty$

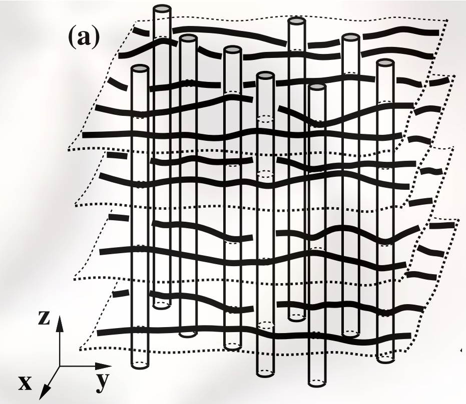
- *anomalous elasticity:* $\delta H = \sigma_c \int_{\mathbf{r}} |\partial_z u_x|$

-> *"shear Meissner" effect*

L.R., arXiv 2021

Smectic vortex glass

transport

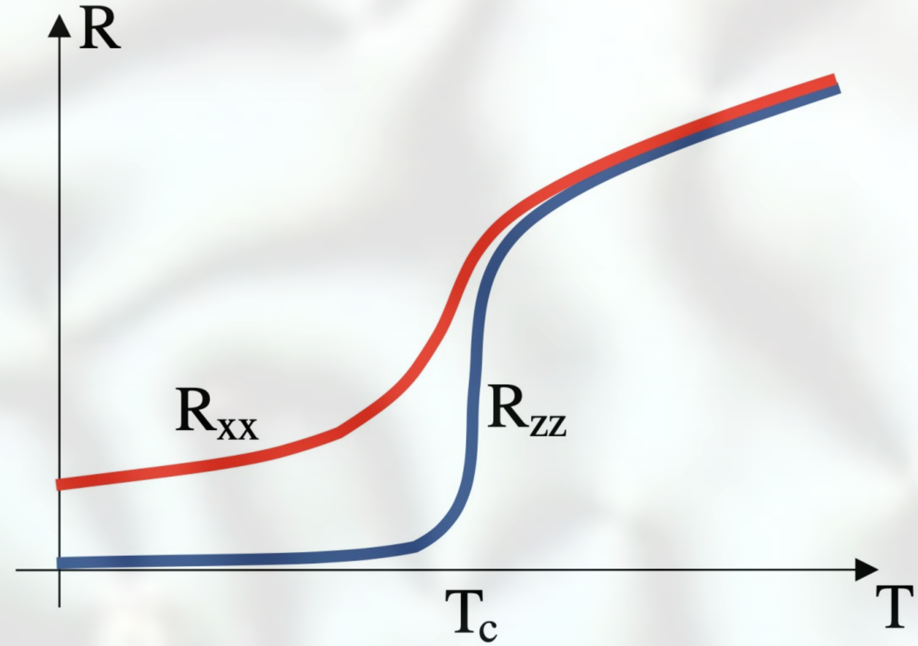


- *vanishing resistivity* ρ_{zz} *along columns*

- *flux-flow resistivity* ρ_{xx} *transverse to columns*

- *divergent anisotropy:*

  $\rho_{xx} / \rho_{zz} \to \infty$



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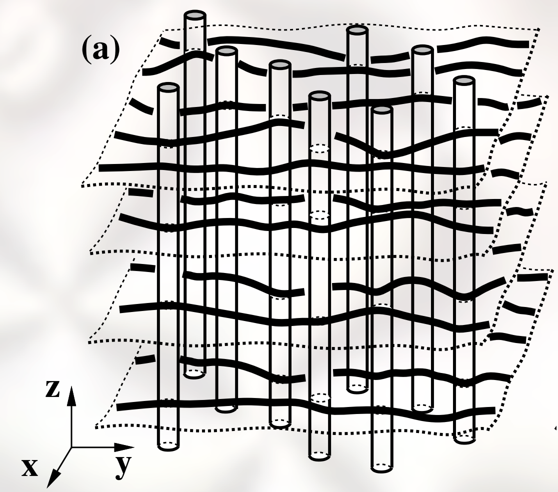
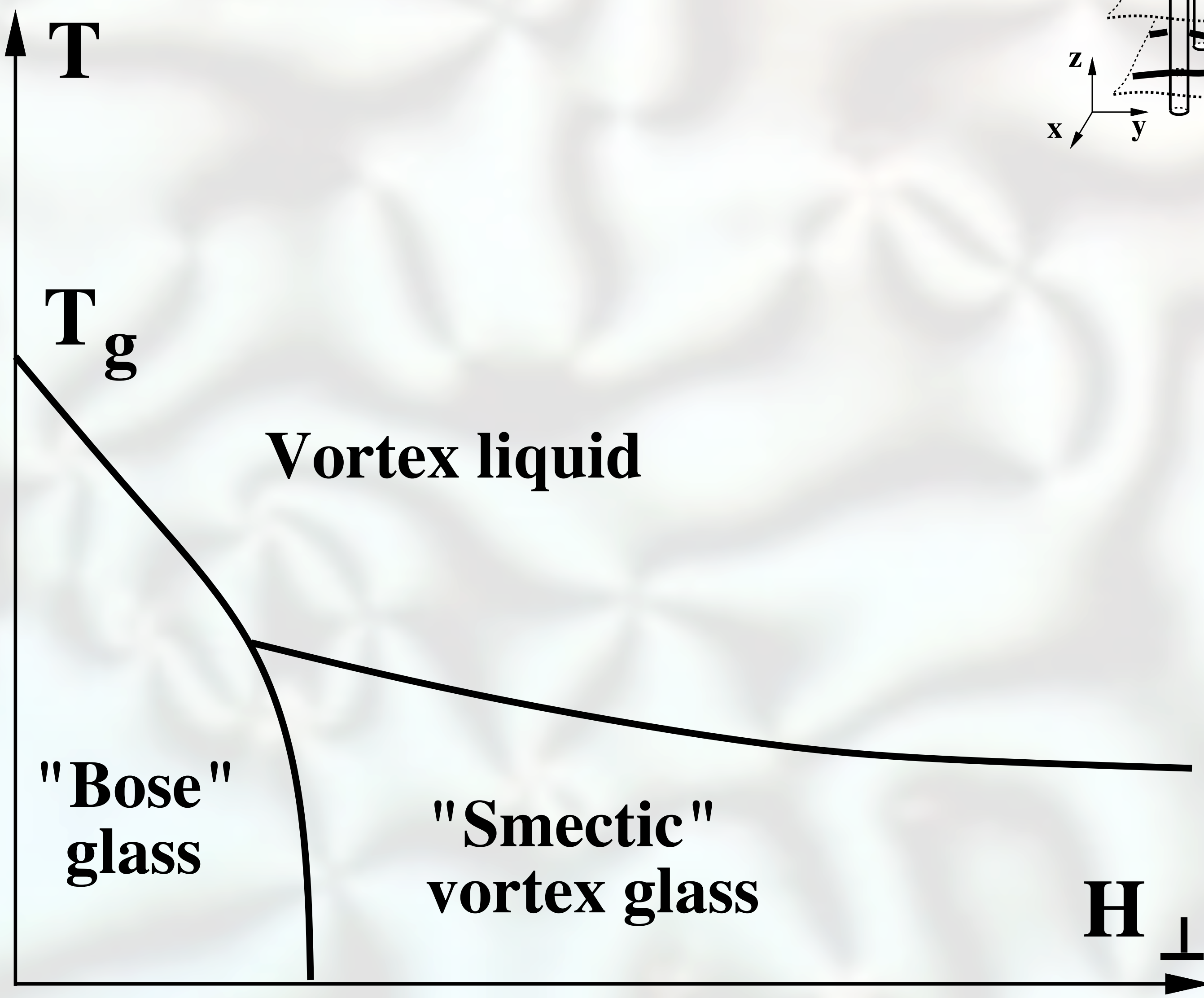
Phase diagram

- θ = π/2 tilt





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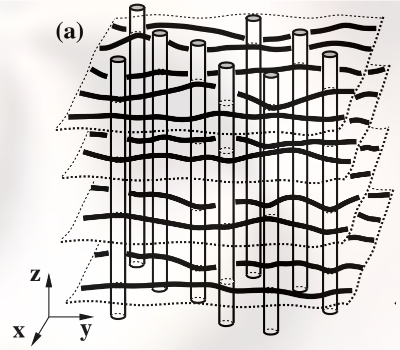
Elasticity & correlated pinning



- *model:* $H = \int_{\mathbf{r}} \left[\frac{1}{2}K_i(\partial_y u_i)^2 + \mu_{ij}(\partial_i^{\perp} u_j)^2 - V(x,y)n(x,y,z)\right]$

disorder only couples to u_x

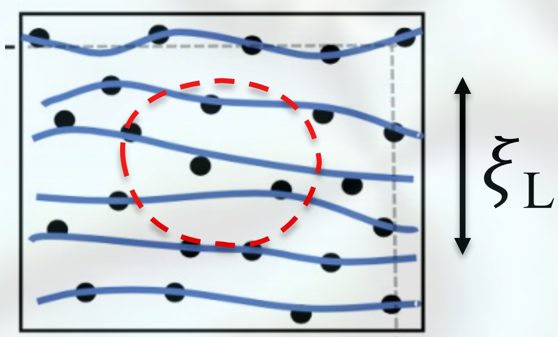
$$= \int_{\mathbf{r}} \left[\frac{1}{2}u_i\hat{\Gamma}_{ij}u_j + U_0(x,y)\partial_x u_x + U(x,y,u_x(\mathbf{r}))\right]$$

$$\delta n(\mathbf{r}) \approx -n_0\nabla_{\perp}\cdot\mathbf{u} + \sum_{\mathbf{Q}} n_{\mathbf{Q}}\, e^{i\mathbf{Q}\cdot(\mathbf{r}_{\perp}+\mathbf{u}(\mathbf{r}))} \qquad \overline{U(x,y,u_x)U(x',y',u'_x)} = R(u_x-u'_x)\delta(x-x')\delta(y-y')$$

- *melting via Lindemann criterion:* $\langle u_z^2\rangle_{T_m} = c_L a^2$

$$\Rightarrow \quad T_{\text{melt-Sm}} \approx c_L a^3 \overline{K}$$

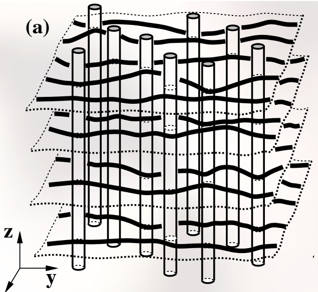
- *columnar pinning and Larkin (Imry-Ma) analysis:* $\overline{\langle u_x^2\rangle}_{\xi_L} \approx \frac{\Delta}{K^2}\xi_L^{5-d} = a^2$



$$\Rightarrow \quad \xi_{\text{L}} = \left(a^2K^2/\Delta\right)^{1/2}$$

L.R., arXiv 2021

Beyond Larkin scale: functional RG



- *replicated model for u_x :* $\overline{F} = -T\overline{\ln Z} = -T \lim_{n\to 0} \frac{\overline{Z^n}-1}{n}$ $\quad \overline{Z^n} = \int [du_\alpha] e^{-H^{(r)}[u_\alpha(\mathbf{r})]/T}$.

$$H^{(r)} = \frac{1}{2}\sum_{\alpha}^{n} \int_{\mathbf{x},z} \left[u_\alpha \hat{\hat{\Gamma}} u_\alpha + \mu_{zx}(\partial_z u_\alpha)^2 \right] - \frac{1}{2T}\sum_{\alpha,\beta}^{n} \int_{\mathbf{x},z,z'} R[u_\alpha(\mathbf{x},z) - u_\beta(\mathbf{x},z')]$$

- *ϵ=5-d, RG for pinning U(r, u) :* $\overline{U(x,y,u_x)U(x',y',u'_x)} = R(u_x - u'_x)\delta(x-x')\delta(y-y')$

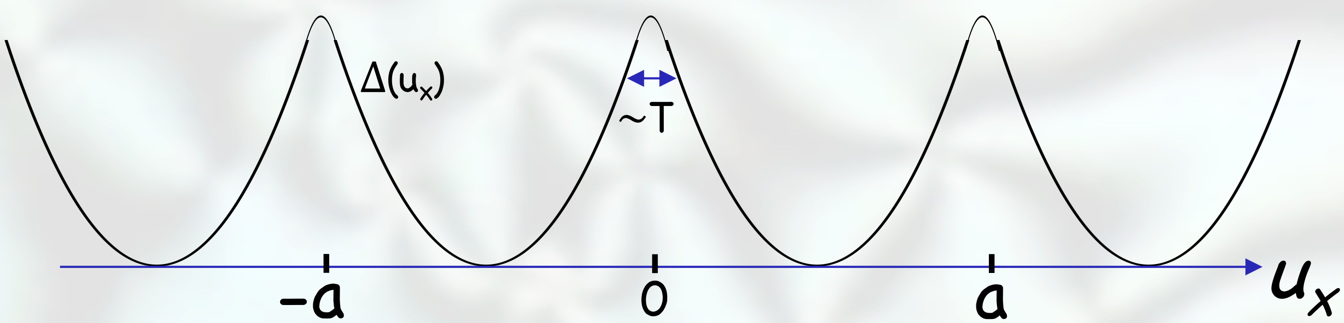
$$\partial_\ell \hat{R}(u) = \epsilon \hat{R}(u) + \frac{1}{2}\hat{R}''(u)\hat{R}''(u) - \hat{R}''(u)\hat{R}''(0) \qquad T(\ell) = e^{-(d-3+\omega)\ell}T \equiv e^{-\Theta\ell}T$$

cf. D. S. Fisher '85

(dangerously irrelevant)

- *T=0 fixed point :* $\hat{\Delta}_*(u) = -\hat{R}''_*(u) = \frac{\epsilon}{6Q^2}\left[(Qu-\pi)^2 - \frac{\pi^2}{3}\right]$, $\quad \hat{\Delta}''(0,\ell) \sim -\frac{\epsilon}{T(\ell)} \to -\infty$

Giamarchi, Le Doussal '96



$$\overline{\langle u(\mathbf{x}) - u(0)\rangle^2} \approx \frac{a^2}{9}\ln(x/a)$$

for 2<d<5

- *divergent shear modulus:* $\delta\mu_{zx} \approx -g_2\Delta''(0)\mu_{zx}\delta\ell$

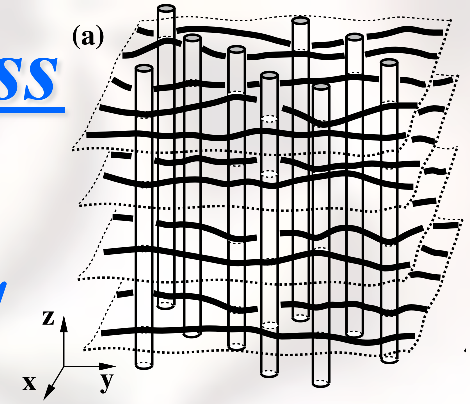
μ_{zx} -> ∞ ➡ $\delta H = \sigma_c \int_{\mathbf{r}} |\partial_z u_x|$

for Bose glass, Balents '93

"Shear Meissner" effect: *vanishing response to shear stress* $\quad -\sigma_{zx}\partial_z u_x$

L.R., arXiv 2021

Absence of dislocations: Bragg glass



- *dislocations proliferate for weak disorder? No!*
  - *(i) $u_z = 0$, at $T = 0$*
  - *(ii) convergent thermal fluctuations in 3d*
  - *(iii) all observables are z-independent at $T = 0$*

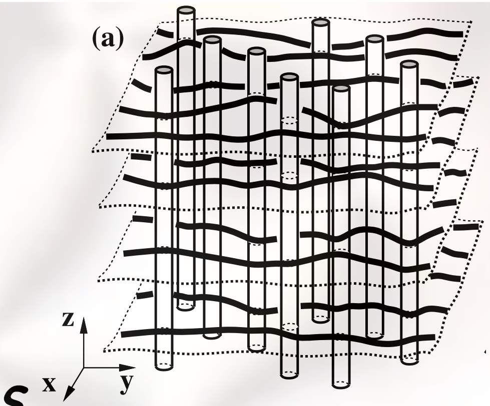
-> *no dislocations in xz plane:* $\hat{\mathbf{y}} \cdot \nabla \times \nabla u_i = b_i(\mathbf{r}) = 0$

-> *no dislocations in xy plane:* $\mathbf{B} = B_0(\partial_y u_x, -\nabla \cdot \mathbf{u}, \partial_y u_z)$

$u_z = 0$ *gives* $B_x = B_0 \partial_y u_x \;, \quad B_y = -B_0 \partial_x u_x$

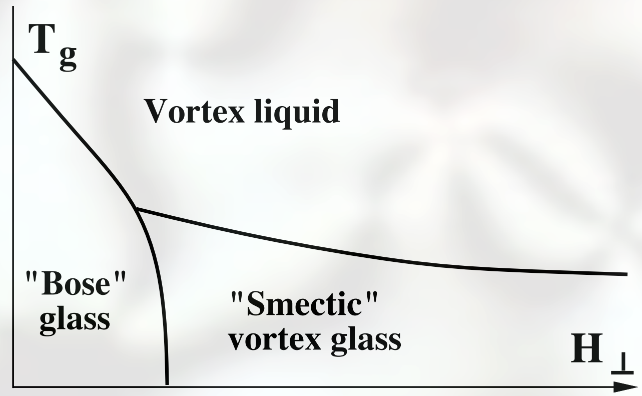
$\nabla \cdot \mathbf{B} = 0$ -> $\partial_x \partial_y u_x - \partial_y \partial_x u_x = 0$

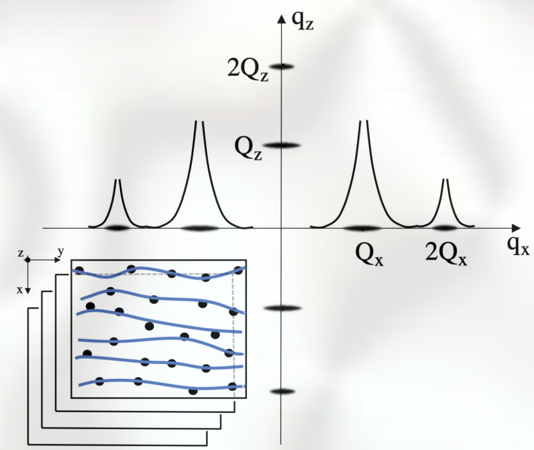
Summary



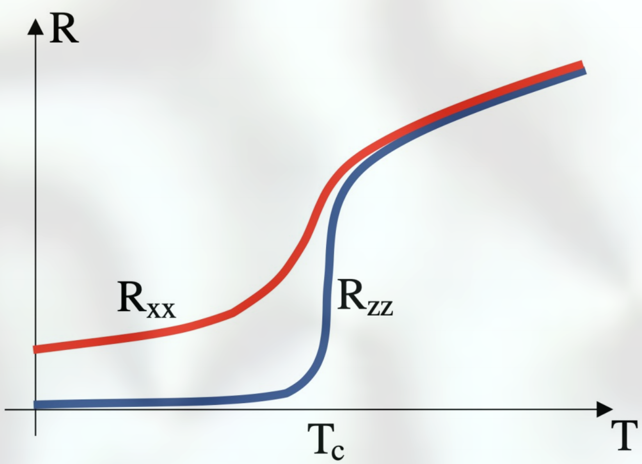
- Overview of 30+ years of vortex physics in type-II superconductors with strong fluctuations

- 'Smectic vortex glass'





  - ➢ Bragg glass
  - ➢ novel structure function
  - ➢ divergent resistive anisotropy: $\rho_{xx} / \rho_{zz} \to \infty$
  - ➢ divergent shear modulus, $\mu_{zx} \to \infty$
  - ➢ 'shear Meissner' effect



Thank you