lecture 7

# Mass, momentum and energy in relativity

## Announcements:

- lecture 6 is posted
- homework 3 *due date is postponed to Monday, Feb 7, in class*
- homework 2 solutions are posted
- reading for this week is:
    - Ch 2 in TZD

# Last Time

recall lecture 6:

- physics behind time dilation

  $\longrightarrow$ $$t = \frac{t'}{\sqrt{1 - v^2/c^2}} > t'$$

- physics behind length contraction

  $\longrightarrow$ $$\ell = \ell' \sqrt{1 - v^2/c^2} < \ell'$$

*relativity of simultaneity*

- Lorentz transformation

$$x' = \gamma(x - vt)$$
$$t' = \gamma(t - \frac{v}{c^2}x)$$

- Doppler effect

$$f_R^{away} = f_T' \sqrt{\frac{1 - v/c}{1 + v/c}} < f_T'$$
$$f_R^{toward} = f_T' \sqrt{\frac{1 + v/c}{1 - v/c}} > f_T'$$

clicker question

# Doppler shift

Q: *An alien sends a laser beam toward Earth using a souped up green laser pointer. The people on Earth observe this light to be red.*

*Is the alien spaceship moving toward or away from Earth?*

a) away from Earth

b) toward Earth

c) impossible to tell

A:

$$f_R^{away} = f'_T\sqrt{\frac{1-v/c}{1+v/c}} < f'_T$$

$$f_R^{toward} = f'_T\sqrt{\frac{1+v/c}{1-v/c}} > f'_T$$

clicker question

# Longitudinal velocity transformations

Q: *A spaceship traveling with speed $v = c/2$ relative to Earth shoots forward a bullet with speed $u' = c/2$ relative to the spaceship.*
*What is the speed of the bullet relative to Earth?*

a) $u=4c/5$, b) $u=c$, c) $u=0$, d) $5c/4$, e) $c/4$

A: *Using relativistic velocity transformation, $u=(c/2+c/2)/(1+1/4)$ gives the answer in a), $u=4c/5$.*

*Note: be careful with signs*

# Today

## Relativistic dynamics:

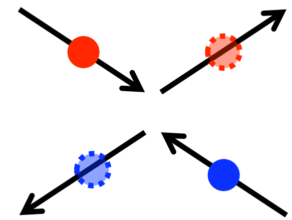
- momentum
- mass
- energy
- relativistic mechanics

# New (relativistic) mechanics is needed

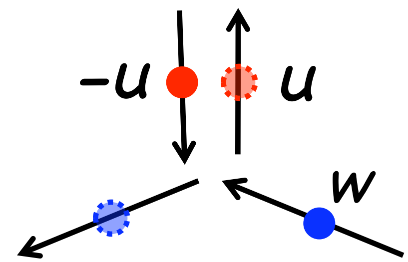
- Problem:
  - *E&M is invariant under Lorentz transformations*
  - *Newton's equation ma = F is not*
- Solution:
  - *reformulate mechanics*
  - *use momentum, energy conservation principles*

# Relativistic momentum

- p – *conserved in translationally invariant system, but* mv *is not*
- Newton's law: ma = F ⟷ m dv/dt = F ⟷ dp/dt = F
- need a formulation of a relativistic momentum, so that p is conserved in collisions:



  - *postulate velocity-dependent mass:* $p = m(v)v$
  - $u_x^A = 0$, $u_y^A = -u$; $u_x^B = -v$, $u_y^B = u/\gamma$ → $w^2 = (v^2 + u^2/\gamma^2)$
  - *x-component conservation*
  - *y-component conservation*




$$-m(u)u + m(w)u/\gamma = m(u)u - m(w)u/\gamma$$

$$\Longrightarrow \quad p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \qquad m_0 = m(u=0)$$

$$= m_0 dx/dt_0$$

⟹ **Newton's law:** dp/dt = F

# Relativistic velocity-dependent "mass"

- $m_0$ - rest mass: mass in rest frame $\longrightarrow$ Newtonian mass
- $m(u) = \dfrac{m_0}{\sqrt{1 - u^2/c^2}}$ $\longrightarrow$ ∞ for u → c
  - *second hint of inaccessibility of speed of light limit:*

    *the larger the speed u, the larger the m(u),*

    *the more force it takes to accelerate object further...*

# Relativistic energy

- Work

$$
\begin{aligned}
&= \int_a^b \vec{F}\cdot d\vec{r}, \\
&= \int_a^b \frac{d\vec{p}}{dt}\cdot d\vec{r}, \\
&= \int_a^b \frac{d}{dt}\left[\frac{m\vec{v}}{\sqrt{1-v^2/c^2}}\right]\cdot\frac{d\vec{r}}{dt}dt \\
&= E_b - E_a
\end{aligned}
$$

$$\Longrightarrow \quad E = \frac{m_0c^2}{\sqrt{1-v^2/c^2}}$$

- Einstein's energy expression:
  - $E(0) = m_0c^2$ , *mass* $\longleftrightarrow$ *energy*
  - $E(v) = \dfrac{m_0c^2}{\sqrt{1-v^2/c^2}} \approx_{v\to 0} m_0c^2 + \dfrac{1}{2}mv^2 + \ldots$
  - $E(p) = \sqrt{(pc)^2 + (m_0c^2)^2} \overset{m=0}{\Longrightarrow} E_{photon} = pc$

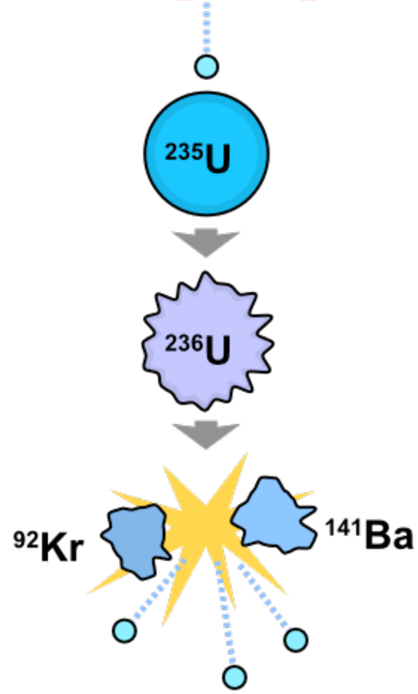
# Energy-momentum

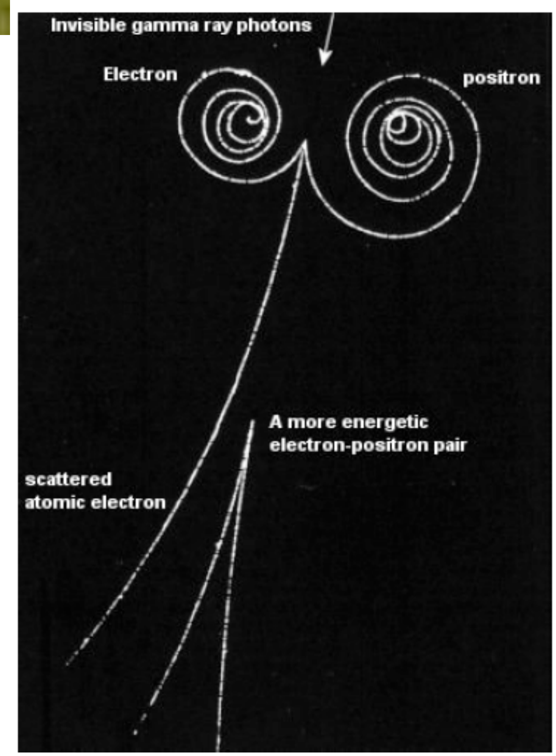
- conservation in collisions: $(p, E)$
- transforms under LT: $(p, E) \rightarrow (p', E')$, like $(x,t)$
- frame invariant: $m^2c^4 = E^2 - p^2c^2$, like $x^2-c^2t^2$
- mass not conserved <-> kinetic energy K:

  $\Delta K = \Delta mc^2$, usually $\Delta m$ too small to notice

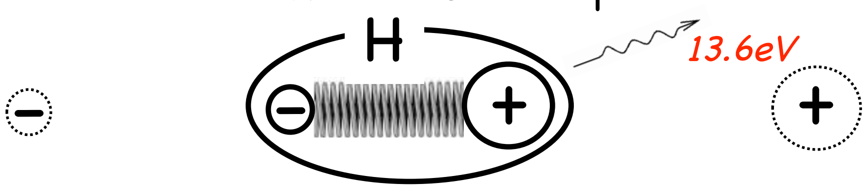
# Mass-energy

- Energy out of mass: $M - (m_1 + m_2) = K/c^2$
  - Nuclear and chemical reactions (fission and fusion):
    - *Chemical reactions* $2H_2 + O_2 \rightarrow 2H_2O$ + heat *(only $10^{-10}$g, tiny)*
    - *Power plants*
    - *Atomic bomb*




- Mass out of energy
  - particles and antiparticles created out of (e.g., photon) energy
  - H atom: $M_H < m_e + m_p$ out of attractive $V_{coulomb}$




lighter by 13.6eV

clicker question

# Energy in rest mass 1

Q: *If we could release the energy of a mass entirely into energy, what mass would be needed to supply the entire world with energy each year? (Globally $4 \times 10^{20}$ joules is used each year)*


a) 1kg,  b) 100kg,  c) 5000kg,  d) 1,000,000kg

A: $m = E/c^2 = 4 \times 10^{20}$ Joules$/(3 \times 10^8 \text{ m/s})^2 \approx 5000$ kg

# Energy in rest mass 2

Q: *Suppose that the speed of light was one half as big. What would be the change in amount of firewood that would need to be burned in order to generate the same amount of heat?*

a) half,  b) twice,  c) four times,  d) same

as much firewood would need to be burned

A: *$E=mc^2$ , hence half c, need 4 times large m to get same E.*

# Caution

- Be careful about relativity of simultaneity
- Time slows as speed increases:
  - *only when viewed from another reference frame*
- Objects shorten as speed increases:
  - *only when viewed from another reference frame*
- Nothing can travel faster than speed of light:
  - *only physical objects and signals cannot propagate faster than light. Relative speed between two objects as observed from another observer can be faster than c. Crossing point of scissors or laser pointer projected on the Moon can move faster than c*