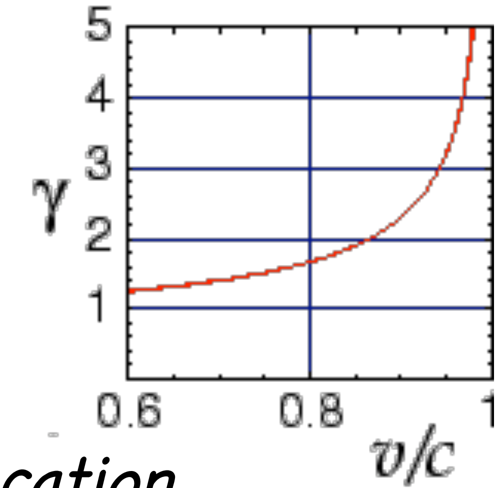
Relativity of length

Announcements:

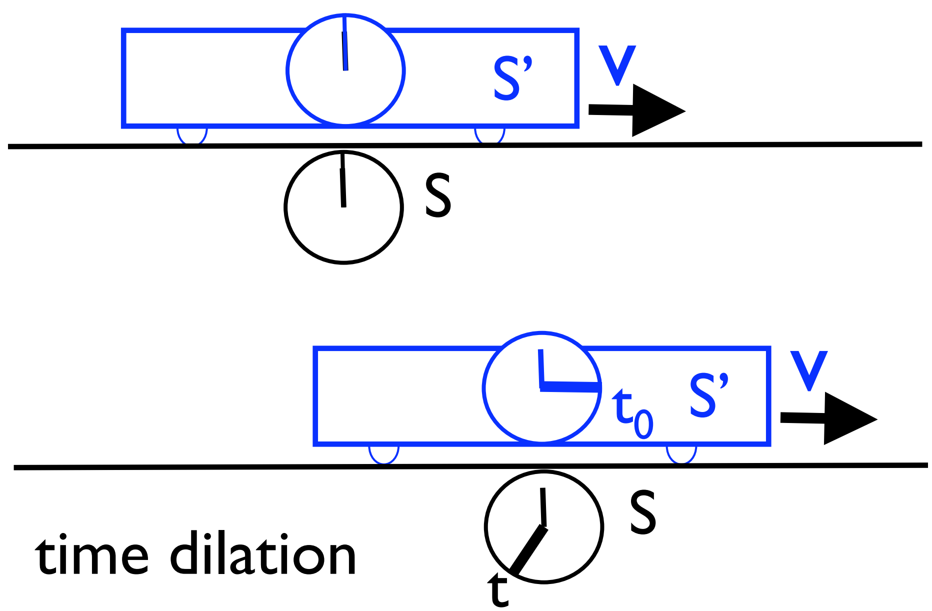
- midterm exam scheduled: Wed, Feb 16, in class
- lecture 5 is posted
- homework 3 *(due Friday, Feb 4 in class)* is posted on CULearn
- homework 1, 2 solutions are posted on CULearn
- reading for this week is:
  - Ch 2 in TZD

Last time

recall lecture 5:

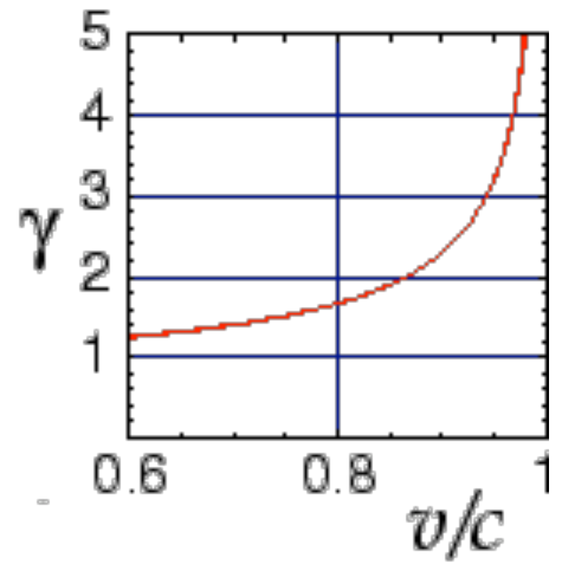


- Time dilation
  - proper time t_0 between two events in *same location* is always shortest $t = \gamma\, t_0 > t_0$
  - $c^2 t_0^2 = c^2 t^2 - x^2 = c^2 t'^2 - x'^2$ frame independent
  - a clock keeps proper time between its ticks
  - clocks moving relative to it measure it to be running slower



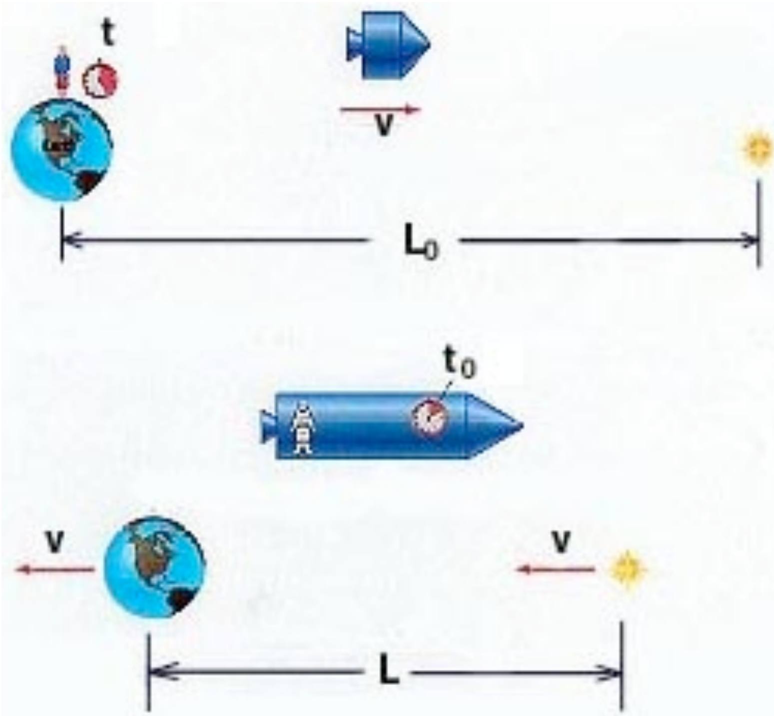
time dilation

Today



Time to talk about length:

- physics behind length contraction
- Lorentz transformation
- Doppler effect

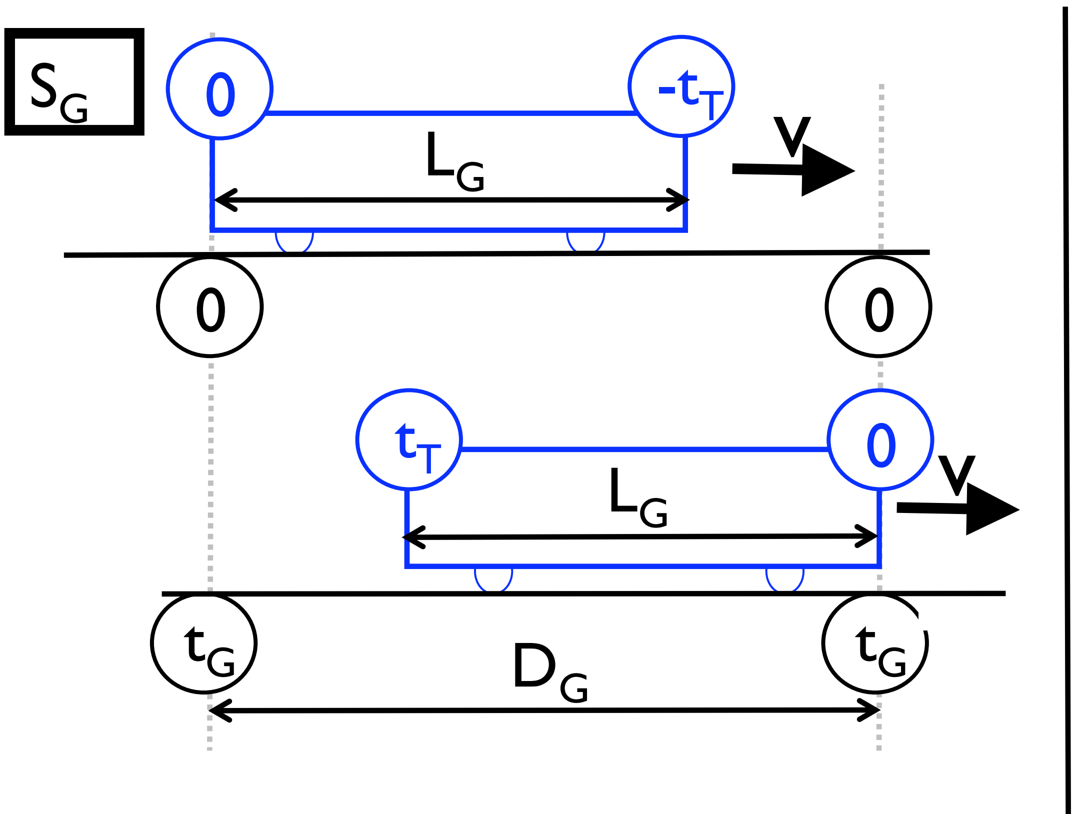


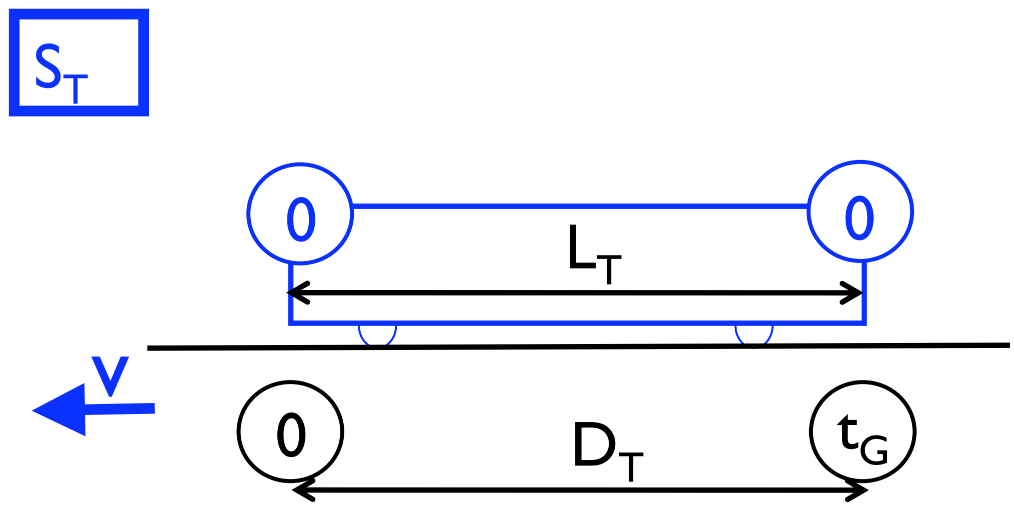
Relativity of length: length "contraction"

- from l = v t and l′ = v t′ → <u>length contraction</u>

$\longrightarrow$ $L_G = L_T\sqrt{1 - v^2/c^2} < L_T$ (*simultaneous* ends)

- get it quickly from pion lifetime: *lives shorter in its rest frame → must see shorter lengths, so that v is same*





$$\left(\frac{L_G}{L_T}\right)\left(\frac{D_T}{D_G}\right) = \frac{1}{\gamma^2} = 1 - v^2/c^2$$

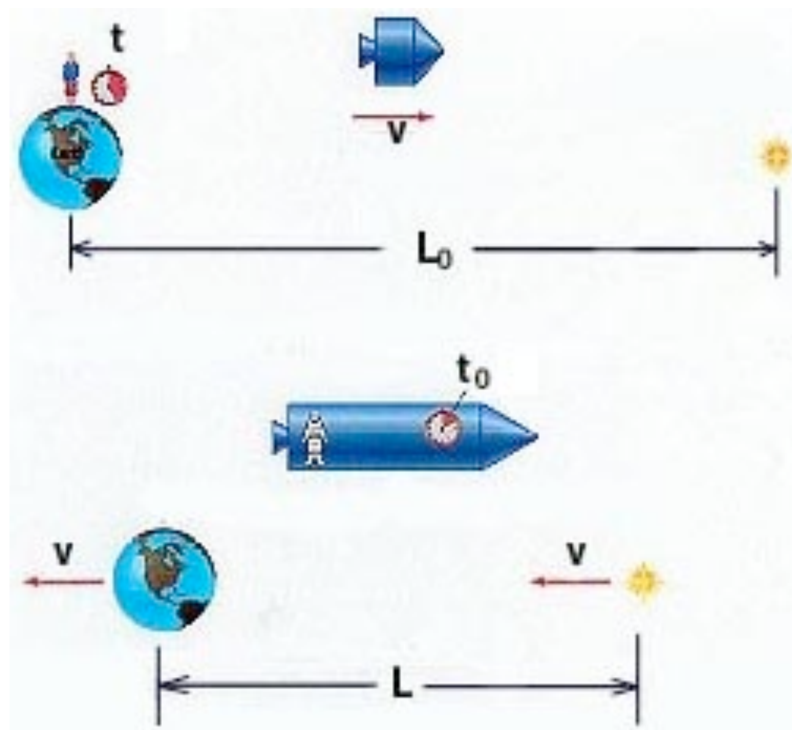
$$\rightarrow \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

<u>*study the "snake" example in TZD*</u>

Proper length

- *proper length l_0 :*
  - l_0 is length of an object in its own reference frame
  - $l_0^2 = x^2 - c^2t^2 = x'^2 - c^2t'^2$ frame independent
  - length contraction is a consequence of time dilation
  - length of an object, L measured from a frame moving relative to the object is *shorter than the proper length*:

$$L = L_0 / \gamma < L_0$$



clicker question

Length contraction

Q: *Alice takes a ship moving at speed 0.8c (γ=5/3) to a planet that is 8 light-years away (as judged by Bob, who stays back home on earth). What is the distance that Alice needs to travel to get to the planet as measured from her point of view ?*

a) 8 light-years, $\boxed{\text{b) 4.8 light-years,}}$ c) 1.2 light-year

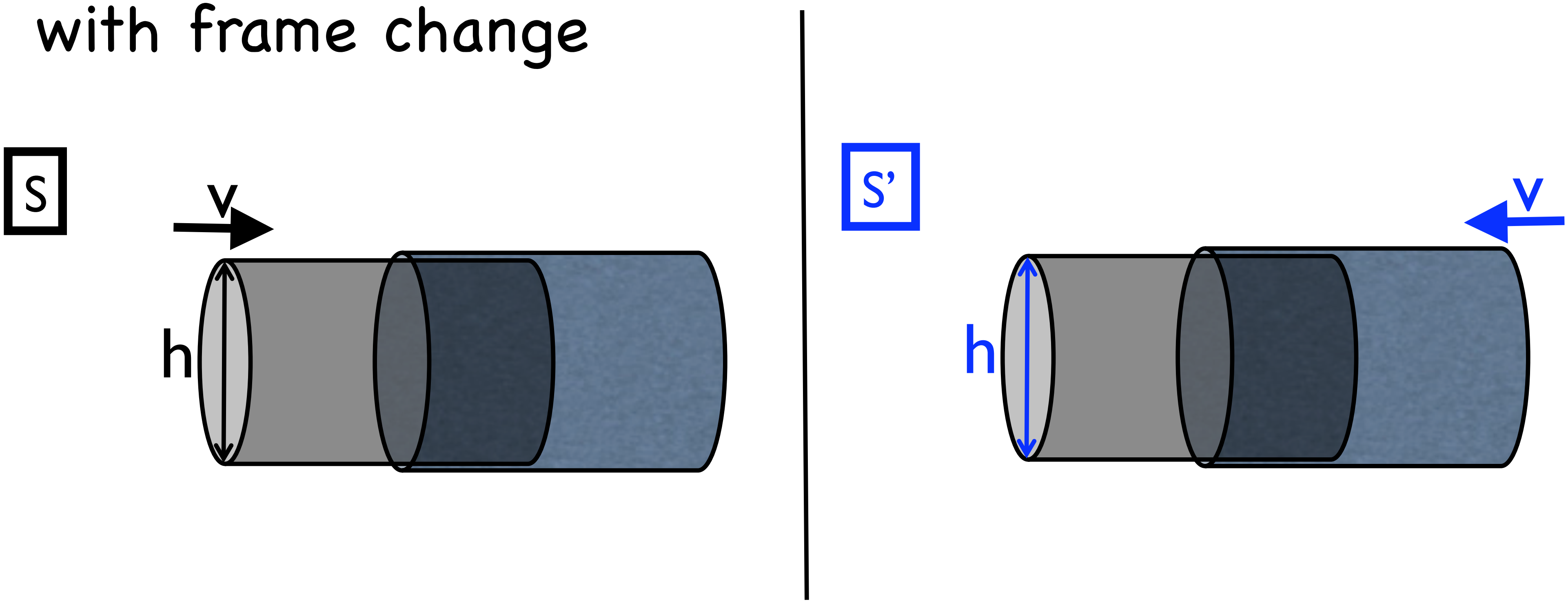
A: *By the principle of relativity Alice and Bob must agree on the velocity v of the other's reference frame. From Alice's frame, it is the planet that is moving toward her spaceship with velocity –v. Since Alice's clock runs slower then for her to agree with Bob about velocity v, her distance measurement must also be shorter. Thus,*

$L_A = L_B/\gamma < L_B = 3/5 \times 8$ light-years $= 4.8$ light-years.

Transverse length are invariant

- Note:

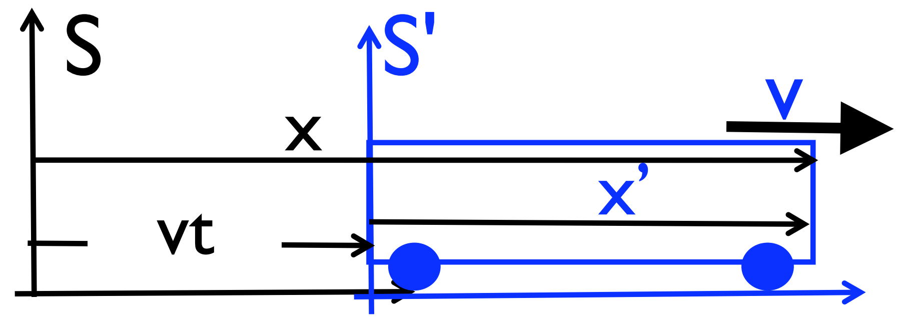
by symmetry of S and S′ conclusions, lengths *perpendicular* to direction of motion *do not change* with frame change



that's why from time to travel *vertical* distance is easier to get γ for time dilation

Lorentz (-Fitzgerald) transformation

- Galilean transformation (GT):
$$\begin{aligned} x' &= x - vt \\ t' &= t \end{aligned}$$
- Rotation:
$$\begin{aligned} x'_1 &= \cos\theta x_1 + \sin\theta x_2 \\ x'_2 &= -\sin\theta x_1 + \cos\theta x_2 \end{aligned}$$
- Lorentz transformation (LT):



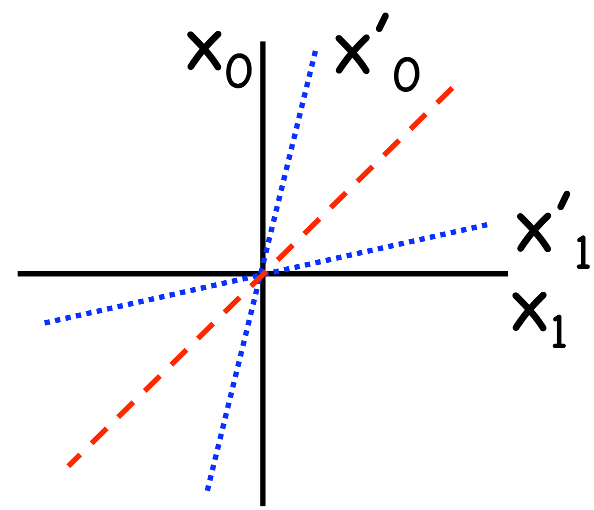
  - from length contraction, inversion
  - reduces to GT for v << c

$$\boxed{\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma\left(t - \frac{v}{c^2}x\right) \end{aligned}}$$

  - preserves proper length/time:

    $l_0^2 = x^2 - c^2t^2$, $t_0^2 = t^2 - x^2/c^2$ (hyperbolas)

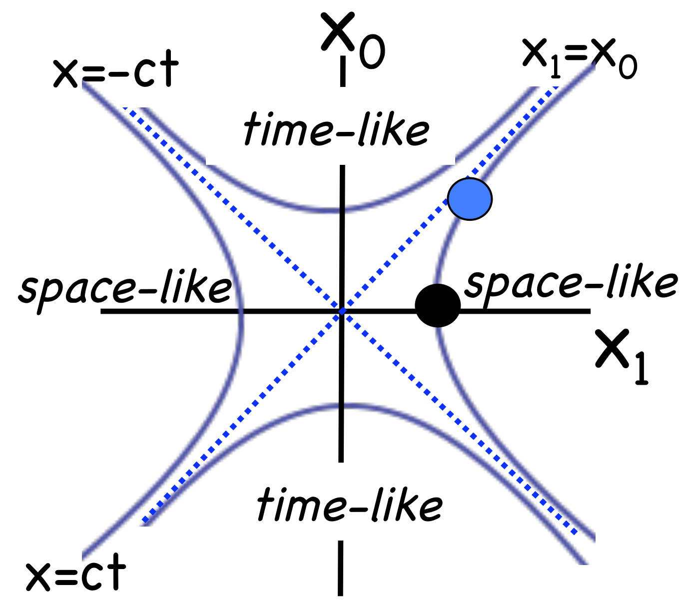
  - space-time symmetry (x_0 = ct, β=v/c):
  - inversion:
$$\begin{aligned} x_1 &= \gamma(x'_1 + \beta x'_0) \\ x_0 &= \gamma(x'_0 + \beta x'_1) \end{aligned}$$



Lorentz transformation of space-time

- time-like and space-like events
- analogy with rotations
- LT in matrix form:

$$\begin{pmatrix} x_1' \\ x_0' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x_1 \\ x_0 \end{pmatrix}$$

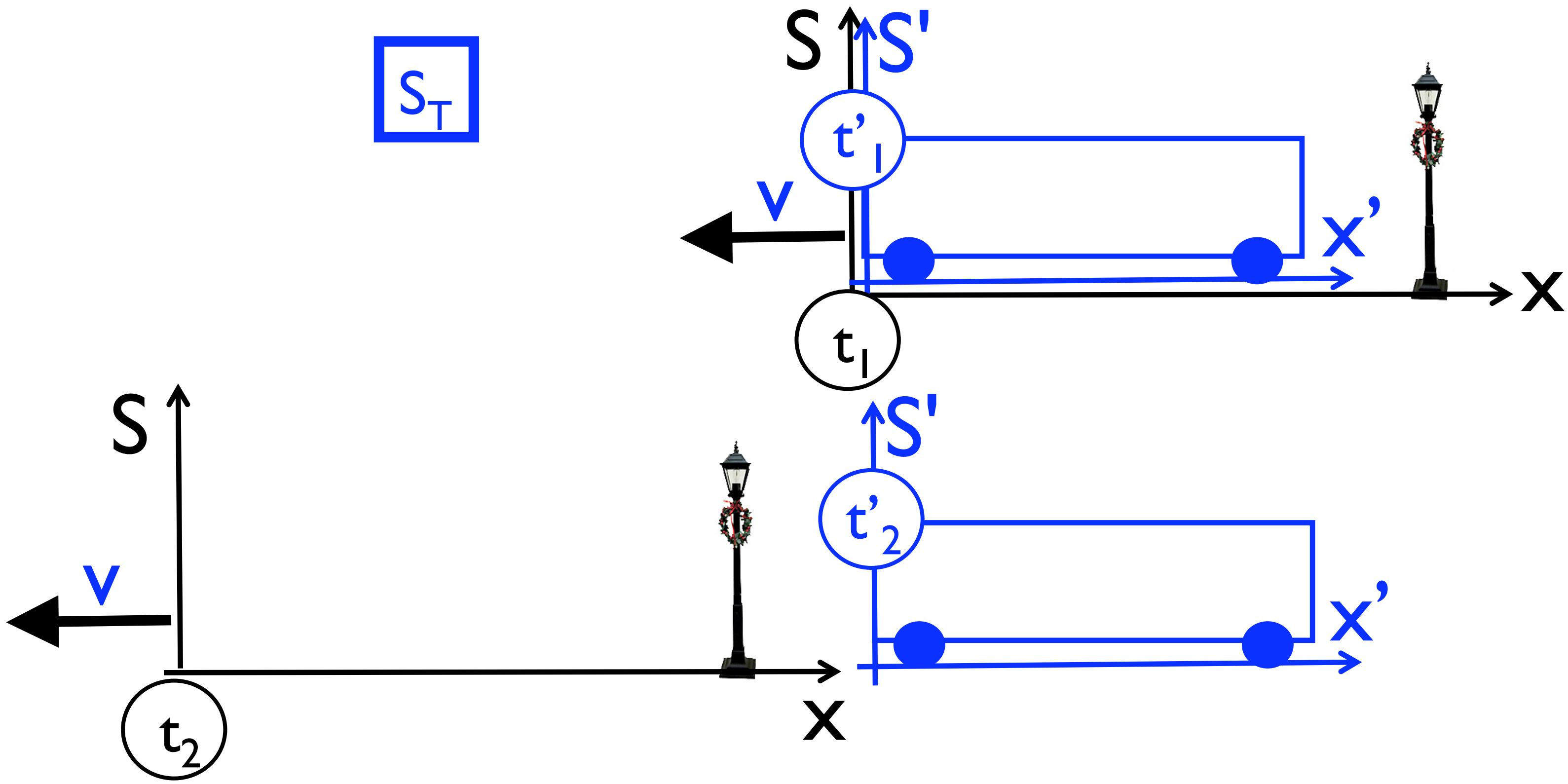


- combining LT's → velocity transformation:

$$\begin{pmatrix} \gamma_3 & -\gamma_3\beta_3 \\ -\gamma_3\beta_3 & \gamma_3 \end{pmatrix} = \begin{pmatrix} \gamma_1 & -\gamma_1\beta_1 \\ -\gamma_1\beta_1 & \gamma_1 \end{pmatrix} \begin{pmatrix} \gamma_2 & -\gamma_2\beta_2 \\ -\gamma_2\beta_2 & \gamma_2 \end{pmatrix}$$

- parallel: $$u_{||} = \frac{u_{||} + v}{1 + u_{||}v/c^2}$$
- perpendicular: $$u'_{\perp} = \frac{u_{\perp}}{\gamma(1 + u_{||}v/c^2)}$$

Time dilation via Lorentz transformation



- Time:

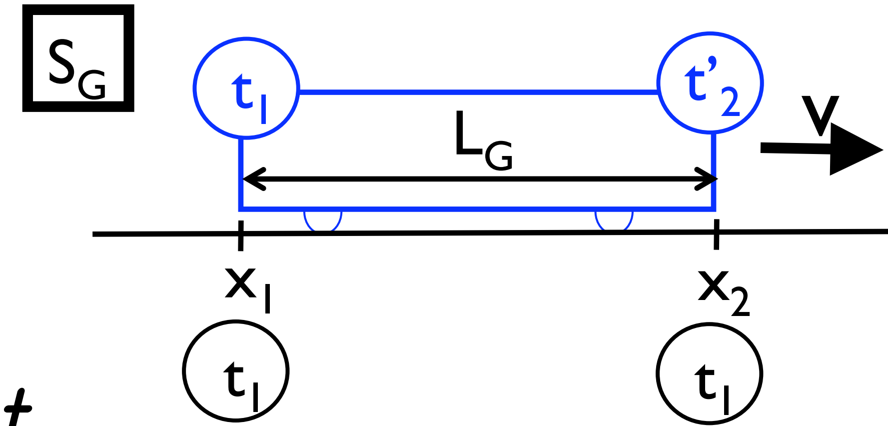
duration between two events located at same position in the clock's frame

$$t_1 = \gamma\left(t'_1 + \frac{v}{c^2}x'_1\right) \Longrightarrow T_G = t_2 - t_1 = \gamma(t'_2 - t'_1) + \gamma\frac{v}{c^2}(\cancel{x'_2} - \cancel{x'_1})$$

$$t_2 = \gamma\left(t'_2 + \frac{v}{c^2}x'_2\right) \Longrightarrow \boxed{T_G = \gamma T_T > T_T}$$

Note: be careful about relativity of simultaneity

Length contraction via Lorentz transformation



- Length:

distance between left and right ends at equal times in measured frame

$$\begin{aligned} x'_1 &= \gamma(x_1 - vt_1) \\ x'_2 &= \gamma(x_2 - vt_1) \end{aligned} \Longrightarrow L_T = x'_2 - x'_1 = \gamma(x_2 - x_1) - \gamma v(\cancel{t}_2 - \cancel{t}_1)$$

$$\Longrightarrow \boxed{L_G = L_T/\gamma < L_T}$$

Note: be careful about relativity of simultaneity clicker question

Transverse length transformations

Q: *A driver of a large truck (height 10 feet) stops in front of an underpass with a clearance of only 8 feet. The driver decides to clear the underpass by driving fast at speed v and thereby utilizing the length contraction about which he learned long ago in college.*

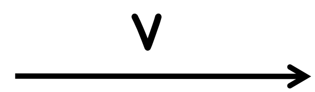
How fast does he need to be moving in order to just clear underneath the underpass?

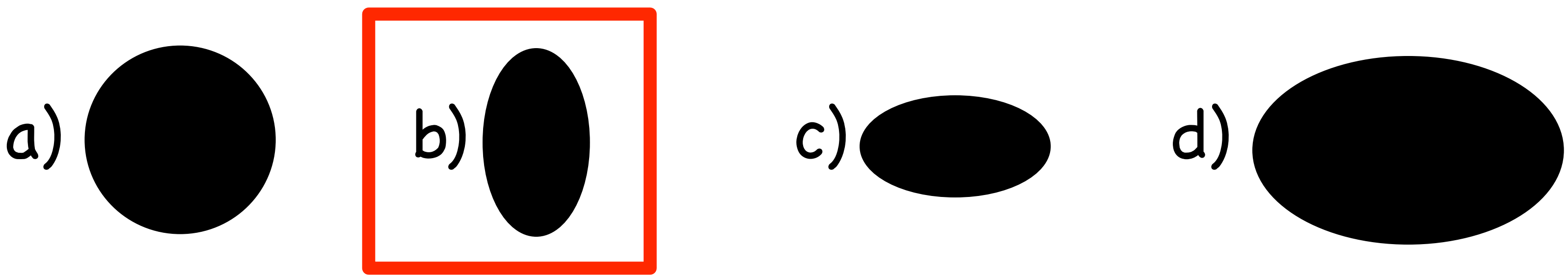
a) c, b) c/3, c) 0.1c, d) 2c $\boxed{\text{e) none of the above}}$

A: *There is no change in length transverse to the direction of motion*

Longitudinal length transformations

Q: A rigid ball is observed from a moving frame.
What is its shape as described from the moving frame ?





A: From the moving frame it is the ball that is moving. Hence its length along the direction of motion is contracted by the factor $1/\gamma$, but it dimensions are unchanged along the transverse axes.
Hence the answer is b)

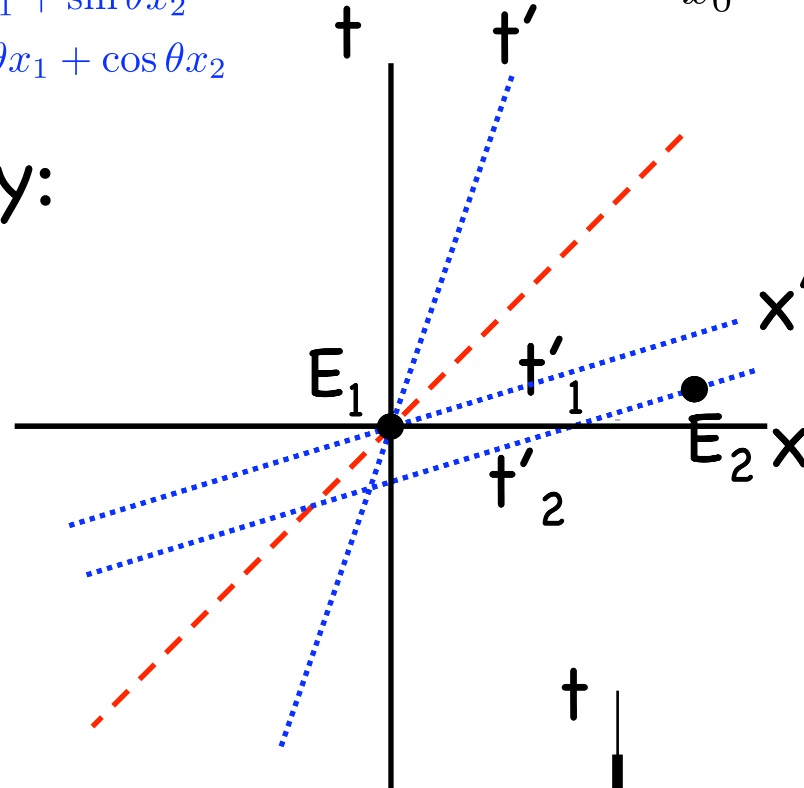
$$\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma(t - \frac{v}{c^2}x) \end{aligned}$$

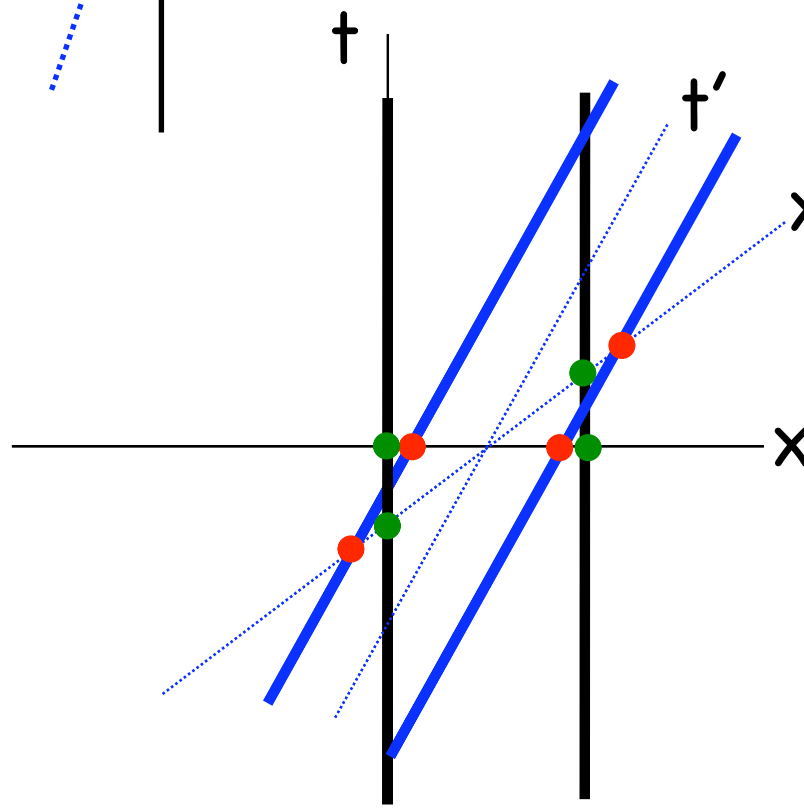
Minkowski space-time (1)

$$\begin{aligned} x_1 &= \gamma(x'_1 + \beta x'_0) \\ x_0 &= \gamma(x'_0 + \beta x'_1) \end{aligned}$$

$$\begin{aligned} x'_1 &= \cos\theta x_1 + \sin\theta x_2 \\ x'_2 &= -\sin\theta x_1 + \cos\theta x_2 \end{aligned}$$

- relativity of simultaneity:
  (reorder space-like events)





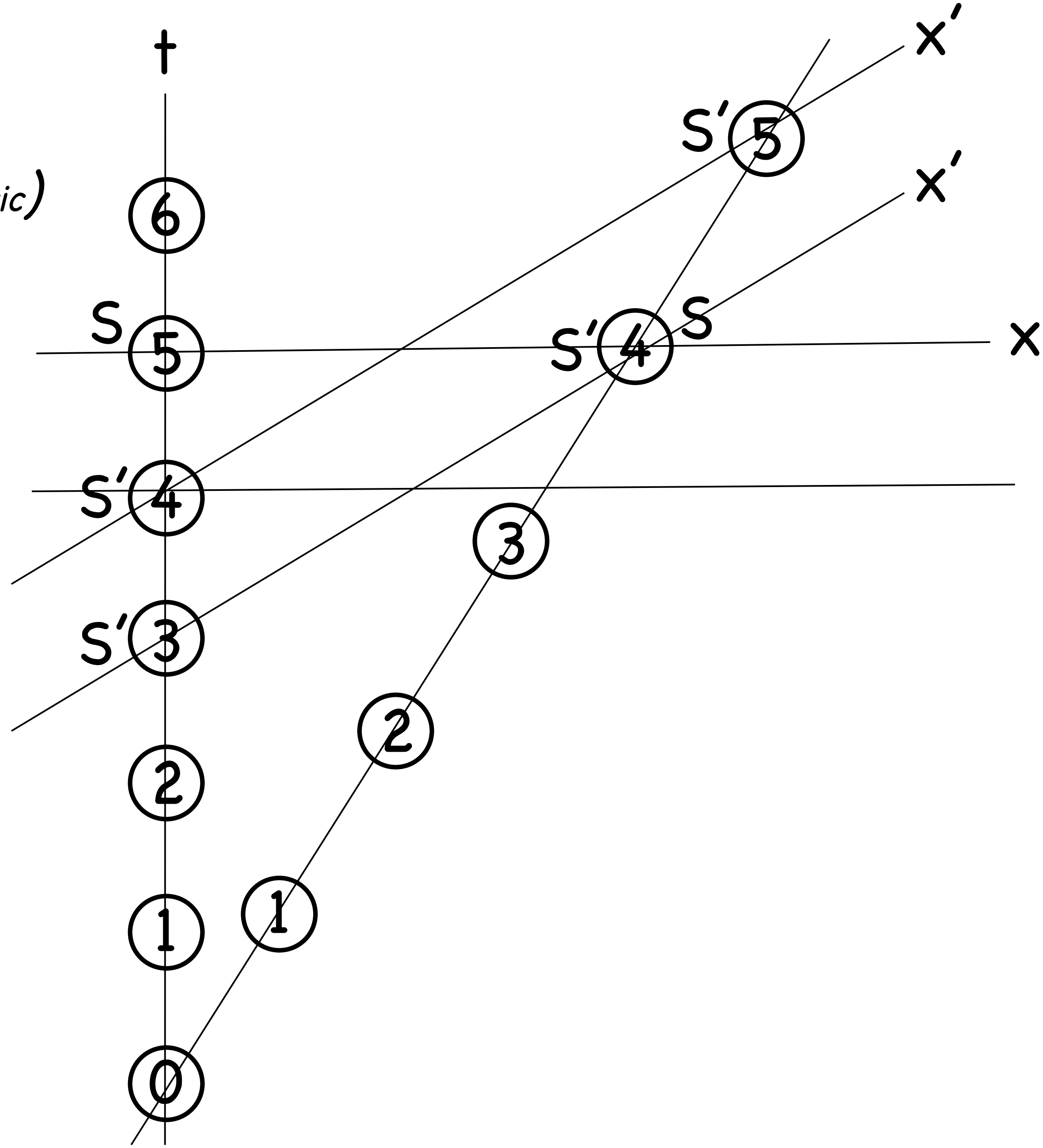
- relativity of lengths:
  (length contraction and dilation)

$$x' = \gamma(x - vt)$$
$$t' = \gamma(t - \frac{v}{c^2}x)$$

Minkowski space-time (2)

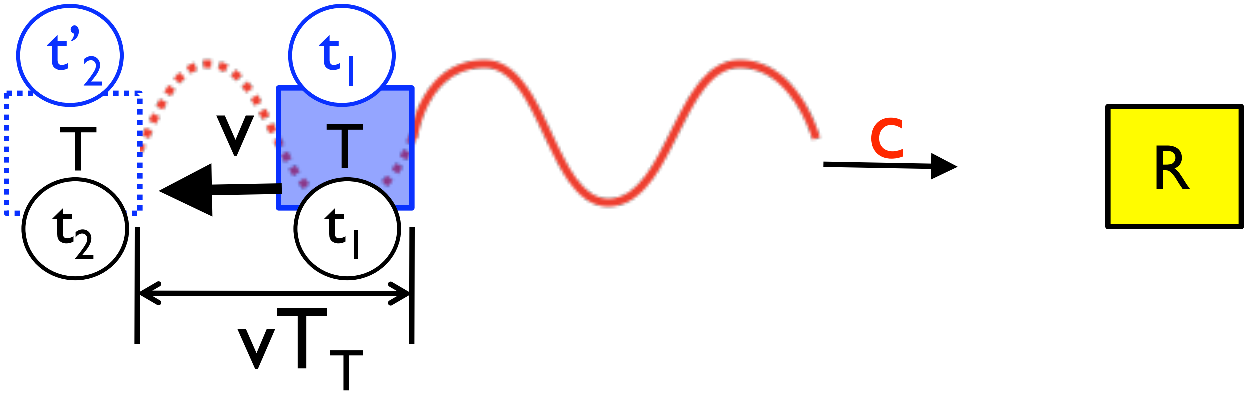
$$x_1 = \gamma(x_1' + \beta x_0')$$
$$x_0 = \gamma(x_0' + \beta x_1')$$

- relativity of time:
  (time dilation, not symmetric)



- twin paradox:

Relativistic Doppler effect



- $T'_T = t'_2 - t'_1$, transmitting frequency $f_T = 1/T'_T$

$$
\begin{aligned}
T_R^{away} &= \gamma T'_T + \frac{v}{c}\gamma T'_T \\
&= T'_T\sqrt{\frac{1+v/c}{1-v/c}}
\end{aligned}
$$

- another way to derive γ
- same whether T or R move
- compare to nonrelativistic limit
- applications:
  - *police radar*
  - *wind speed*
  - *laser cooling in JILA*
  - *red/blue shift of star light -> Hubble, expanding universe*

$$
\begin{aligned}
f_R^{away} &= f'_T\sqrt{\frac{1-v/c}{1+v/c}} < f'_T \\
f_R^{toward} &= f'_T\sqrt{\frac{1+v/c}{1-v/c}} > f'_T
\end{aligned}
$$

Hubble's expanding universe

$$f_R^{away} = f_T' \sqrt{\frac{1 - v/c}{1 + v/c}} < f_T'$$

$$f_R^{toward} = f_T' \sqrt{\frac{1 + v/c}{1 - v/c}} > f_T'$$

- In 1929 Edwin Hubble showed that the velocity of galaxies (measured using redshift) was proportional to their distance, giving first evidence for the Big Bang theory