Relativity of time

Announcements:

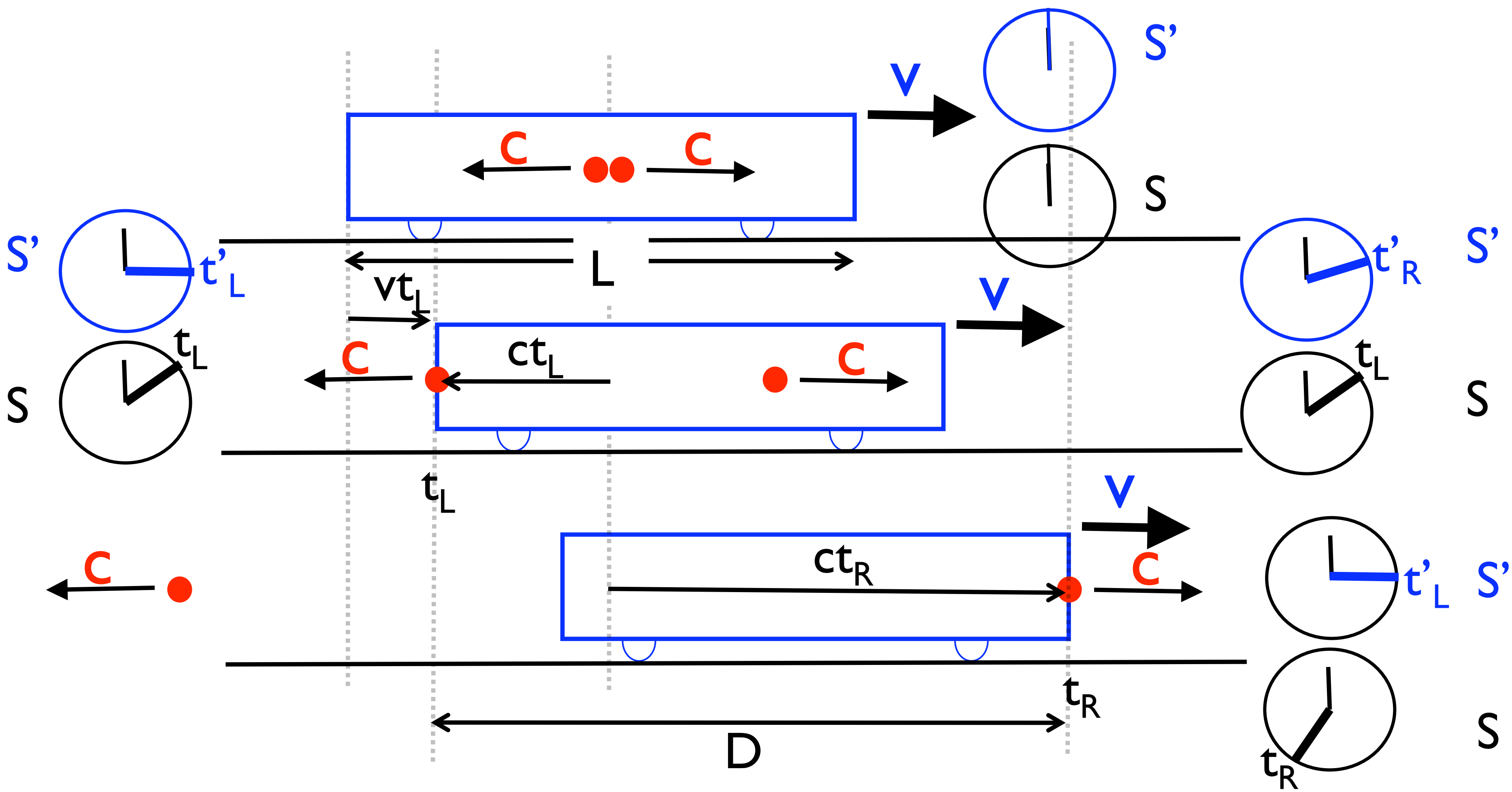
- midterm exam scheduled: Wed, Feb 16, in class
- lecture 4 is posted
- homework 3 *(due Friday, Feb 4 in class)* is posted on CULearn
- homework 1, 2 solutions are posted on CULearn
- reading for this week is:
  - Ch 2 in TZD

Last Time

recall lecture 4:

- relativity of simultaneity
  - events simultaneous in one frame are not simultaneous in a moving frame
  - clocks synchronized in one frame are not in the moving frame; front clock in the moving frame falls behind the back clock by $\Delta T = - D\, v/c^2$ (D is distance between events in measured frame)

Recall in more detail: **Simultaneity is a relative concept**



- S′: photons arrive *simultaneously* (at left/right ends) at $t'_L = t'_R = L'/2c$
- S: photons arrive at right *after* left ends: $t_L = \frac{L/2}{c+v} < t_R = \frac{L/2}{c-v}$

not simultaneous in S !

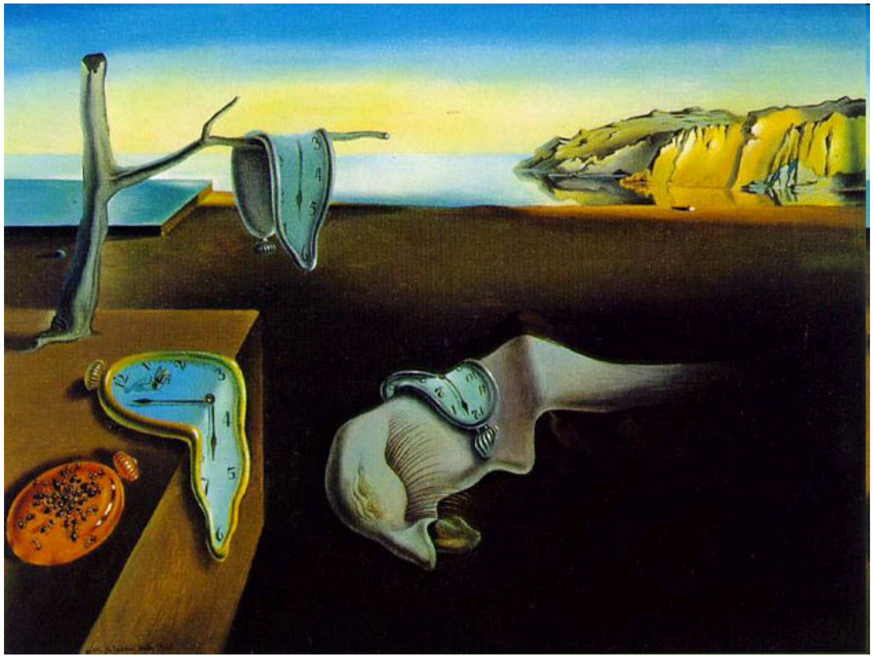
$$t_R - t_L = \frac{Lv}{c^2 - v^2} = D\frac{v}{c^2} > 0$$

front (right) clock on moving train lags behind back (left) one

Today

Time to talk about time:

- physics behind time dilation
- proper time



clicker question

Relative speed vs velocity transformation

Q: *Suppose two rockets are observed (from e.g., earth) to be moving in opposite directions with equal speeds v. What is their relative speed, as observed from the same reference frame (e.g., earth)?*

a) v, b) 2v, c) c, d) none of the above

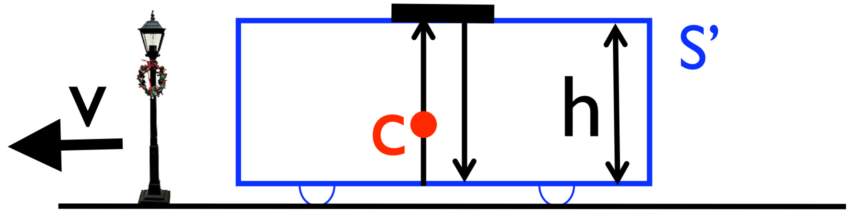
A: *Note that the relative speeds (as viewed from another reference frame) indeed simply add even for relativistic systems. This is to be contrasted to transformation of speeds as one changes points of views between different reference frames. Note that former can easily give relative speed to be larger than c, and in fact as large as 2c.*

Relativity of time: time "dilation"

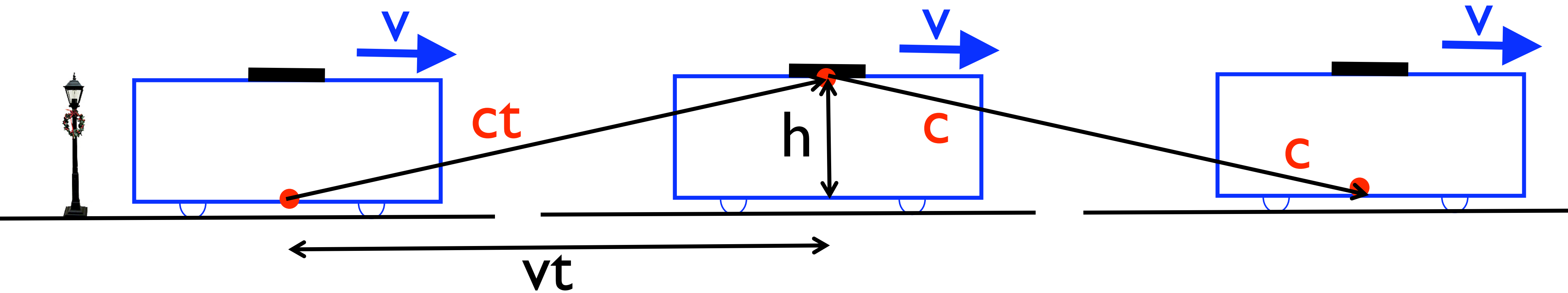
- *"Absolute, true, and mathematical time, of itself and from its own nature, flows equably without relation to anything external."* – Isaac Newton

- *"It came to me that time was suspect!"* – Albert Einstein

- time dilation:



$t' = h/c$



- $(ct)^2 = (vt)^2 + h^2 \Longrightarrow t = \dfrac{h}{\sqrt{c^2 - v^2}} = \dfrac{t'}{\sqrt{1 - v^2/c^2}} \equiv \gamma t' > t'$

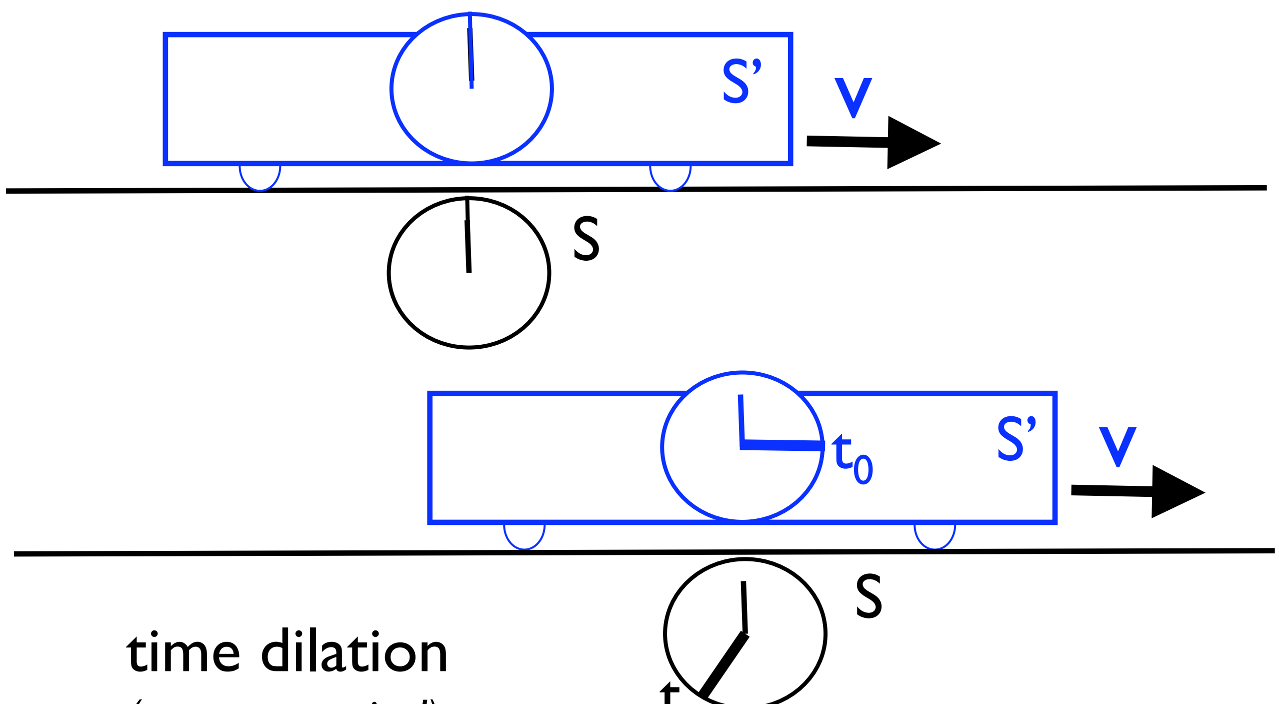
- moving clocks run slower by a factor $\gamma^{-1} = \sqrt{1 - v^2/c^2}$ ("twin paradox")

(first hint of inaccessibility of speed of light limit)

- realization in pion lifetime

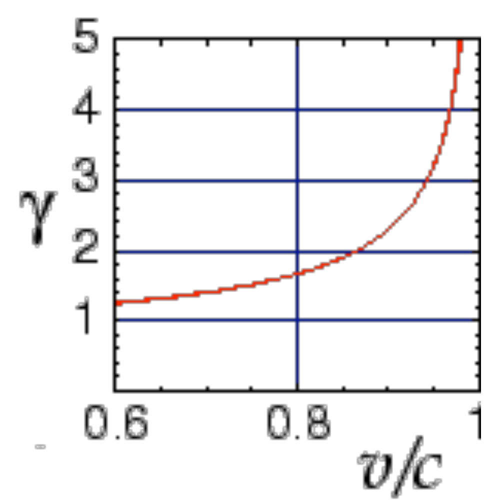
Proper time

- *proper time t_0:*
  - time duration t_0 between two events on a clock at same x
  - t_0 is always shortest, with $t = \gamma\, t_0 > t_0$
  - $c^2 t_0^2 = c^2 t^2 - x^2 = c^2 t'^2 - x'^2$ frame independent
  - a clock keeps proper time between its ticks
  - clocks moving relative to it, measure it to be running slower



time dilation

(not symmetrical)



clicker question

Time dilation 1

Q: *In a clock's reference frame a duration between its two ticks is 5 second. What can the time duration between these two ticks be when viewed from a moving truck?*

a) 2 sec,   b) 5 sec,   c) 7 sec,   d) 1.5 sec

A: *Proper time, (i.e., a time measured in the frame of reference stationary with the clock) is always the shortest. Thus c) is the answer, as the only one larger than the proper time of 5 seconds.*

Time dilation in practice

- *estimate time dilation in an everyday situation:*
  - How much time does your watch falls behind the ground clock when you fly from Los Angeles to Sydney at 1000 ft/s (700mi/h)?
  - $t = \gamma\ t_0$
  - $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \left(\frac{3\times10^2 m/s}{3\times10^8 m/s}\right)^2}} = \frac{1}{\sqrt{1 - 10^{-12}}} \approx 1.0000000000005$
  - 11 hours + 20 billions of a second
  - verified in 1971 via atomic clocks carried on such a trip

clicker question

Time dilation 2

Q: *Alice takes a ship moving at speed 0.8c to a planet that is 8 light-years away (as judged by Bob, who stays back home on earth). How many years will Bob age during Alice's roundtrip?*

a) 1 year, b) 2 years, **c) 20 years,** d) 0.5 year

A: *From Bob's point of view Alice's trip takes*
$t_B = 2 \times 8$ light-years / 0.8c = 20 years.

clicker question

Time dilation 3

Q: *Alice takes a ship moving at speed 0.8c (γ=5/3) to a planet that is 8 light-years away (as judged by Bob, who stays back home on earth). How many years will Alice age during her roundtrip?*

a) 1 year, **b) 12 years,** c) 20 years, d) 0.5 year

A: *Alice's time is the proper time since it is at the same location (her spaceship). Hence her time is shorter (running slower) than that in Bob's frame that is moving relative to her. Consequently Alice will age by $t_A = t_B / \gamma$ = 3/5 x 2 x 8 light-years / 0.8c = 12 years.*

Equivalently, in Alice's reference frame, 8 light-years are length contracted by a factor of 3/5, leading to a 3/5 briefer trip -> 12 years