Fractons-Elasticity Duality

Center for Theory of Quantum Matter

CTQM

with Michael Pretko

M. Pretko and L. R., arXiv 2017, to appear PRL

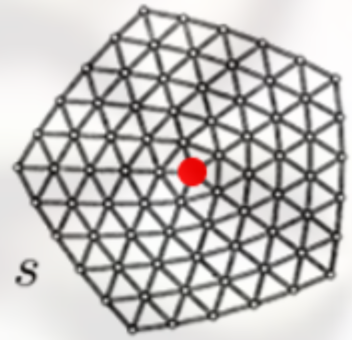
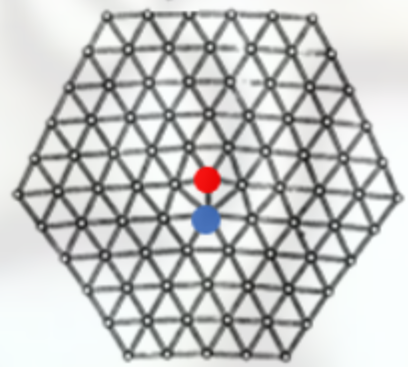
$: NSF-MRSEC, Simons Foundation

Duality, Aspen, March 19, 2018

M. Pretko, L. R., 2017

Fracton-elasticity duality

2D

Fracton $\partial_i \partial_j E^{ij} = \rho$	Disclination $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s$ 
Dipole	$\vec{b}$ Dislocation 
Gauge Modes	Phonons
Electric Field E_{ij}	Strain Tensor u_{ij}
Magnetic Field B_i	Lattice Momentum π_i

Outline

M. Pretko and L. R., arXiv 2017

- New type of quantum liquids: fractons
- Higher rank gauge theories
- Elasticity
- Duality
- Further directions

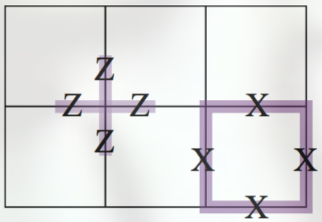
States of quantum matter

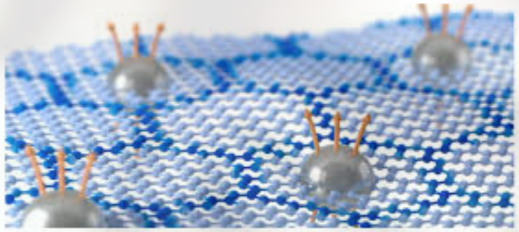
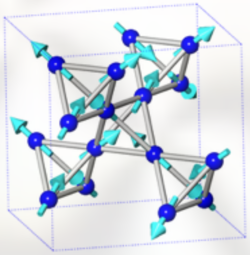
- "conventional" *ordered* states, e.g., AFM, SF, …
  - Local order parameter
  - Classified by broken symmetry
  - Short-range entangled
  - Landau theory description

P. W. Anderson
X. G. Wen
R. Laughlin
A. Kitaev
S. Sachdev
Fisher, Balents, Hermele
...

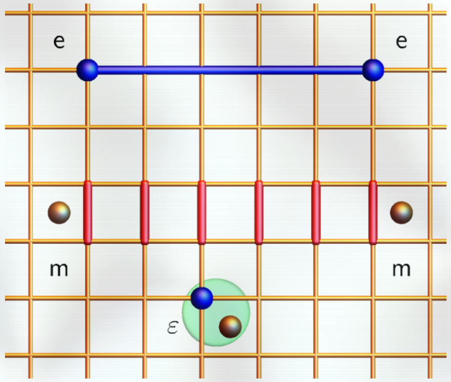
States of quantum matter

- "conventional" quantum *'liquid'* states, e.g., FQHE, spin ice, toric code, ...



  - Non-local, fractionalized bulk excitations as ends of strings: anyons *– free to move but with statistical "interaction"*
  - Topological order with O(1) ground state degeneracy
  - Long-range entangled
  - Gauge theory (Z_2, U(1), ...) description



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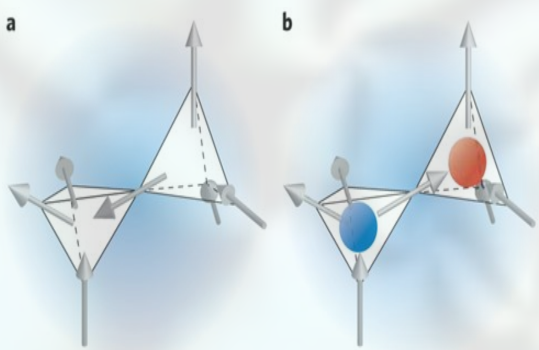
States of quantum matter

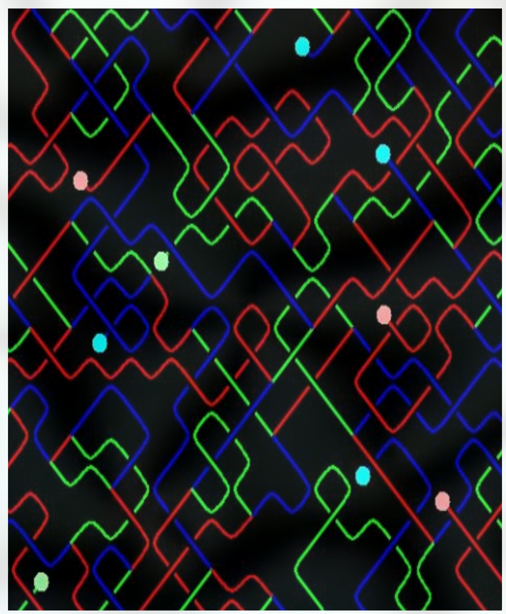
- "conventional" quantum *'liquid'* states, e.g., FQHE, spin ice, toric code, …
  - Gauge theory (Z_2, U(1), …) description

Hermele, Fisher, Balents

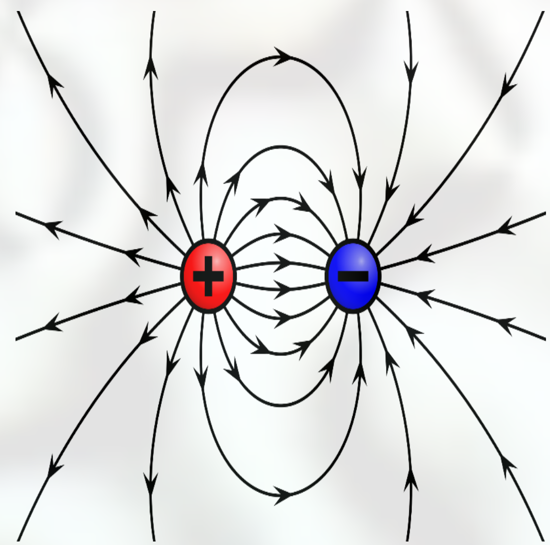
Example: U(1) Spin Liquid



Geometric frustration
(e.g. "spin-ice rules")



String condensate (*Wen*)
vanishing line tension



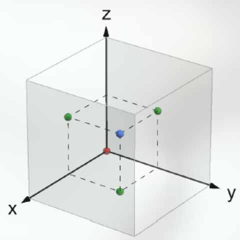
$$\mathcal{L}_{\mathcal{EM}} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu}$$

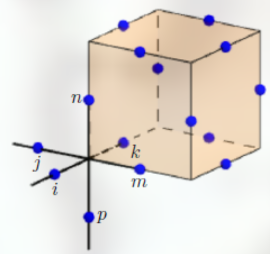
Emergent
electromagnetism!

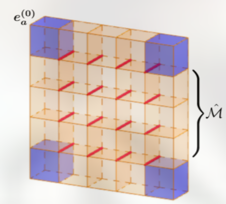
C. Chamon, 2005
A. Rasmuss, et al., 2016
J. Haah, 2011, '13
S. Bravyi, et al., 2011
B. Yoshida, 2013
S. Vijay, L. Fu, 2015, '16

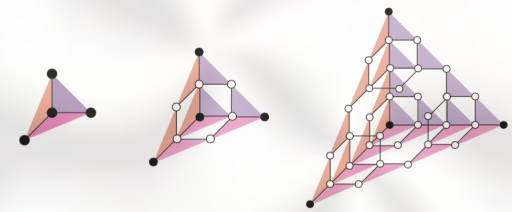
States of quantum matter

- *new class of* quantum *'liquids': Z_2 fractons*, e.g., Haah's code, X-cube, lattice rotors,…









Fractal operator

  – Non-local, fractionalized excitation
    - -> at corners of extended objects: fractons – *immobile* in isolation

    - -> at ends of undeformable string: dipoles – *subdimensional*

C. Chamon, 2005
A. Rasmussen, et al., 2016
J. Haah, 2011, '13
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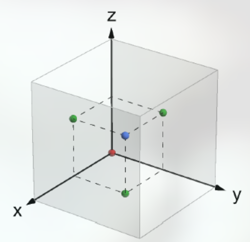
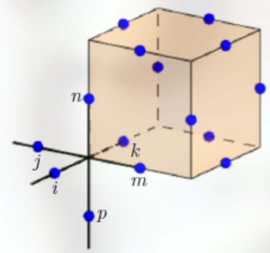
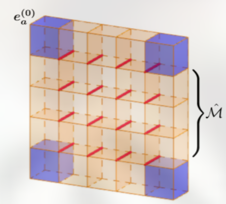
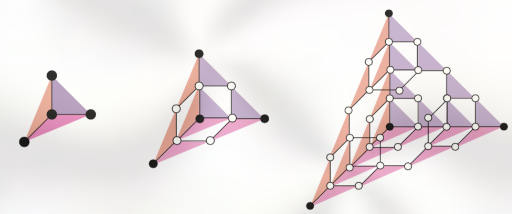








Fractal operator

  – Non-local, fractionalized excitations
    -> at corners of extended objects: fractons – *immobile*
    -> at ends of undeformable string: dipoles - *subdimensional*

  – Topological order with O(L) 3d ground state degeneracy

  – Long-range entangled

  – No (?) field theory description

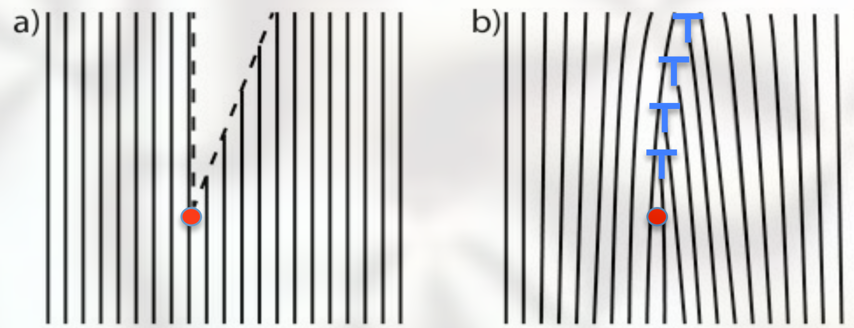
L. R., 2016
classical duality

Topological crystal defects

$$\mathcal{H}_{el} = \frac{1}{2} B (u_{ij})^2 \longrightarrow \tilde{\mathcal{H}}_{el} = \frac{1}{2} B^{-1} (\nabla^2 \phi)^2 - i\phi(s + \hat{z} \cdot \nabla \times \mathbf{b})$$

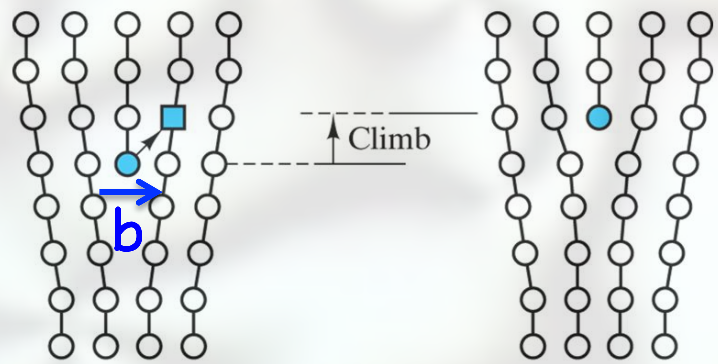
- ***Disclination:***

  immobile



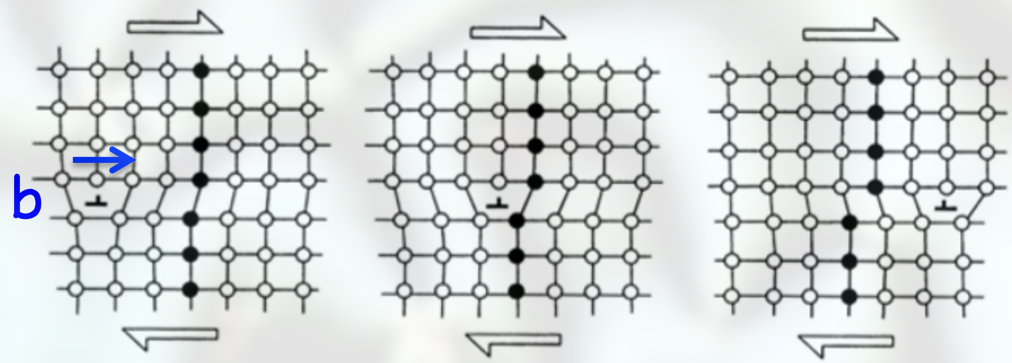
- ***Dislocation*** ***climb:***

  constrained by v/i diffusion



- ***Dislocation*** ***glide:***

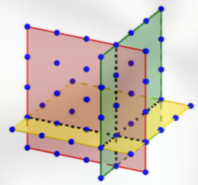
  subdimension (d-1) motion



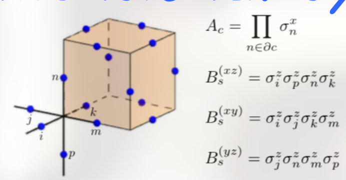
S. Vijay, et al., 2015, '16
M. Pretko, 2016, '17
T. Hsieh, et al., 2017
K. Slagle, Y. B. Kim, 2017
H. Ma, et al., 2017

Fracton developments

- *"gauging" global Z_2 subdimensional symmetry spin model*



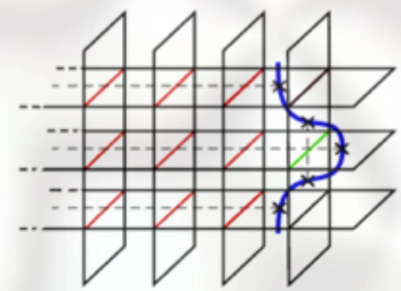
Planar



X-Cube Model

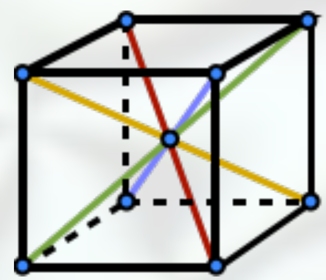
S. Vijay, J. Haah, L. Fu, 2016

- *Coupled-layers construction*



Ma, Lake, Chen, Hermele, 2017

- *Coupled-chains construction*



Halasz, Hsieh, et al., 2017

- *Parton construction*

- *Higher rank tensor gauge theory* $\partial_i \partial_j E_{ij} = \rho_f$

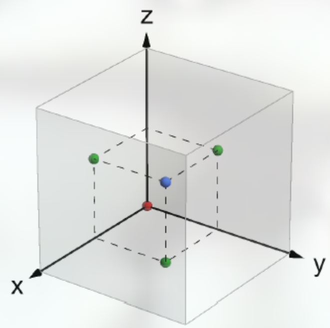
M. Pretko, 2016

Rasmussen, You, C. Xu, 2016

Pretko, 2016

Tensor gauge theory

- U(1) symmetric tensor gauge lattice rotor models

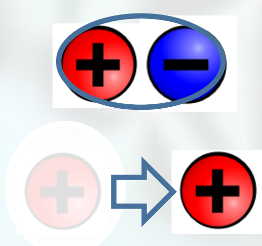


Rasmussen, You, C. Xu, 2016

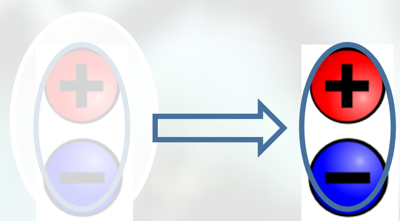
M. Pretko, 2016

- Conservation of charges and of *dipoles* ---> *fracton* phenomenology!

-> moving charge changes dipole moment -> forbidden by dipole conservation



-> dipole motion constrained



Rasmussen, You, C. Xu, 2016

Pretko, 2016

Tensor gauge theory

2D

- U(1) symmetric tensor gauge and electric fields:

-> $\mathcal{H} = \frac{1}{2} E_{ij} E_{ij} + \frac{1}{2} B_i B_i \quad [E^{ij}, A_{ij}] = i\delta^{(2)}(\mathbf{x}) \quad B^i = \epsilon_{jk} \partial^j A^{ki}$

- Gauss's law: $\partial_i \partial_j E^{ij} = \rho$

- Gauge freedom: $A_{ij} \rightarrow A_{ij} + \partial_i \partial_j \alpha$

- Two gapless gauge modes, with: $\omega \sim k$

- Other generalizations *(e.g., vector charges,... see M. Pretko)*

Pretko, 2016

Tensor gauge theory

$$\partial_i \partial_j E^{ij} = \rho$$

A “hidden” extra conservation law:

Charge Conservation

$$Q = \int d^2x \, \rho = \oint \partial_i E^{ij} da_j$$

(boundary term)

Total ***charge*** only changes by particles entering/leaving the system

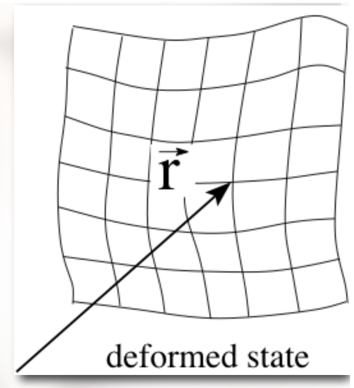
Dipole Conservation

$$P^i = \int d^2x \, \rho x^i = \oint (x^i \partial_j E^{jk} - E^{ik}) da_k$$

(boundary term)

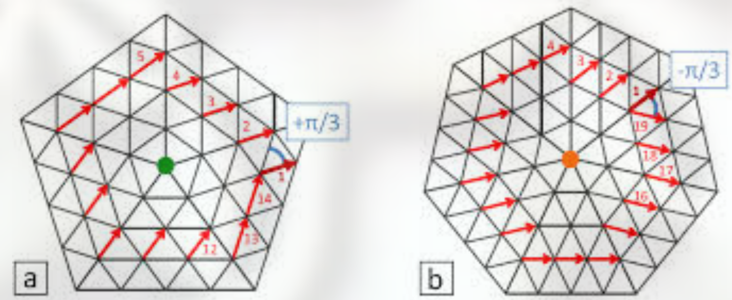
Total ***dipole*** moment only changes by particles entering/leaving the system

Elasticity theory and defects

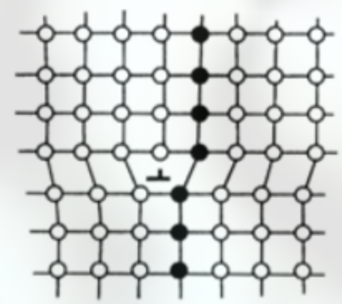


- Eulerian phonons: $\vec{r} = \vec{R} + \vec{u}(\vec{r})$
- Strain: $u_{ij} = \frac{1}{2}(\partial_i \vec{R} \cdot \partial_j \vec{R} - \delta_{ij}) \approx \frac{1}{2}(\partial_i u_j + \partial_j u_i)$
- Hamiltonian: $\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}C_{ij,kl}u_{ij}u_{kl} = \frac{1}{2}\pi^2 + \mu(u_{ij})^2 + \frac{1}{2}\lambda(u_{ii})^2$
- Topological defects
  - Disclinations: $\nabla \times \nabla\theta = s\,\delta^2(\vec{r}) \equiv s(\vec{r})$

    (bond angle: $\theta = \epsilon_{ij}u_{ij}$)

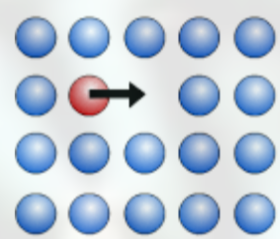


  - Dislocations: $\nabla \times \nabla u_i = b_i\delta^2(\vec{r}) \equiv b_i(\vec{r})$.



  - Vacancies/interstitials:

$$n_d = n_v - n_i$$

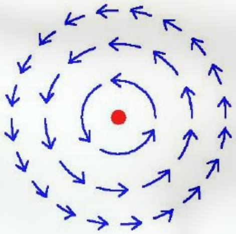


Dasgupta, Halperin
Fisher, Lee

Boson-vortex duality

Superfluid

vortices



topological winding

$$\epsilon^{ij}\partial_i\partial_j\phi = \rho$$

Goldstone mode

$$H = \int d^2x \frac{1}{2}\Big((\partial_i\phi)^2 + n^2\Big)$$

Maxwell Gauge Theory (with matter)

particles

Gauss's law:

$$\partial_i E^i = \rho$$

photon

$$H = \int d^2x \frac{1}{2}\Big(E^i E_i + B^2\Big)$$

M. Pretko, L. R.,2017

Fracton-elasticity duality

- Elastic Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial_t u^i)^2 - \frac{1}{2}C^{ijk\ell}u_{ij}u_{k\ell}$

---> $\mathcal{L} = \frac{1}{2}C^{-1}_{ijk\ell}\sigma^{ij}\sigma^{k\ell} - \frac{1}{2}\pi^i\pi_i - \sigma^{ij}(\partial_i\tilde{u}_j + u^{(s)}_{ij}) + \pi^i\partial_t(\tilde{u}_i + u^{(s)}_i)$

- Disclinations: $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s(\mathbf{x})$
- Momentum conservation (Newton) constraint: $\partial_t\pi^i - \partial_j\sigma^{ij} = 0$
- Electric, magnetic fields: $B^i = \epsilon^{ij}\pi_j$ $\quad E^{ij}_\sigma = \epsilon^{ik}\epsilon^{j\ell}\sigma_{k\ell}$

-> Faraday law: $\partial_t B^i + \epsilon_{jk}\partial^j E^{ki}_\sigma = 0$

-> Gauge fields: $B^i = \epsilon_{jk}\partial^j A^{ki}$ $\quad E^{ij}_\sigma = -\partial_t A^{ij} - \partial_i\partial_j\phi$

-> Gauge freedom: $A_{ij} \to A_{ij} + \partial_i\partial_j\alpha$ $\quad \phi \to \phi + \partial_t\alpha$

M. Pretko, L. R.,2017

Fracton-elasticity duality

- Elastic Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial_t u^i)^2 - \frac{1}{2}C^{ijk\ell}u_{ij}u_{k\ell}$
- Disclinations: $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s(\mathbf{x})$
- Electric, magnetic fields: $B^i = \epsilon^{ij}\pi_j$ $\quad E^{ij}_\sigma = \epsilon^{ik}\epsilon^{j\ell}\sigma_{k\ell}$

$B^i = \epsilon_{jk}\partial^j A^{ki}$ $\quad E^{ij}_\sigma = -\partial_t A^{ij} - \partial_i\partial_j\phi$

-> Fracton Hamiltonian: $[E^{ij}, A_{ij}] = i\delta^{(2)}(\mathbf{x})$

$$\mathcal{H} = \frac{1}{2}\tilde{C}^{ijk\ell}E_{ij}E_{k\ell} + \frac{1}{2}B^iB_i + \rho\phi + J^{ij}A_{ij}$$

-> Fracton charges, dipole current currents:

$$\rho = s \qquad J^{ij} = \epsilon^{ik}\epsilon^{j\ell}(\partial_t\partial_k - \partial_k\partial_t)u_\ell = \epsilon^{(i\ell}v^{j)}b_\ell$$

-> Gauss's law, continuity: $\partial_i\partial_j E^{ij} = \rho$ $\quad \partial_t\rho + \partial_i\partial_j J^{ij} = 0$

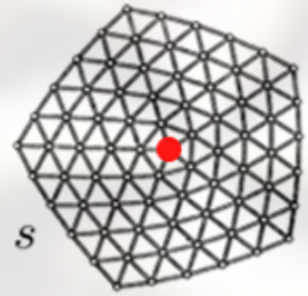
-> Ampere's law: $\partial_t E^{ij} + \frac{1}{2}(\epsilon^{ik}\partial_k B^j + \epsilon^{jk}\partial_k B^i) = -J^{ij}$ $\quad \partial_t n_d + \partial_i J^i_d = -J^i{}_i$

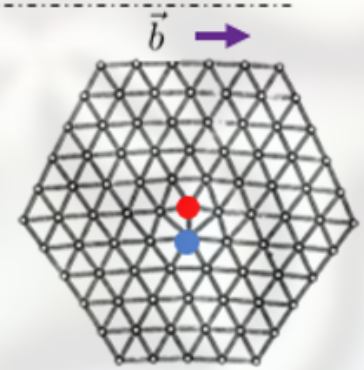
Marchetti, L.R. 1998

M. Pretko,
L. R., 2017

Fracton-elasticity dictionary

Fracton $\partial_i\partial_j E^{ij} = \rho$	Disclination $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s$
Dipole	Dislocation
Gauge Modes	Phonons
Electric Field E_{ij}	Strain Tensor u_{ij}
Magnetic Field B_i	Lattice Momentum π_i





$$\partial_t B^i + \epsilon_{jk}\partial^j E^{ki}_\sigma = 0. \quad \Longleftrightarrow \quad \partial_t \pi^i - \partial_j \sigma^{ij} = 0$$

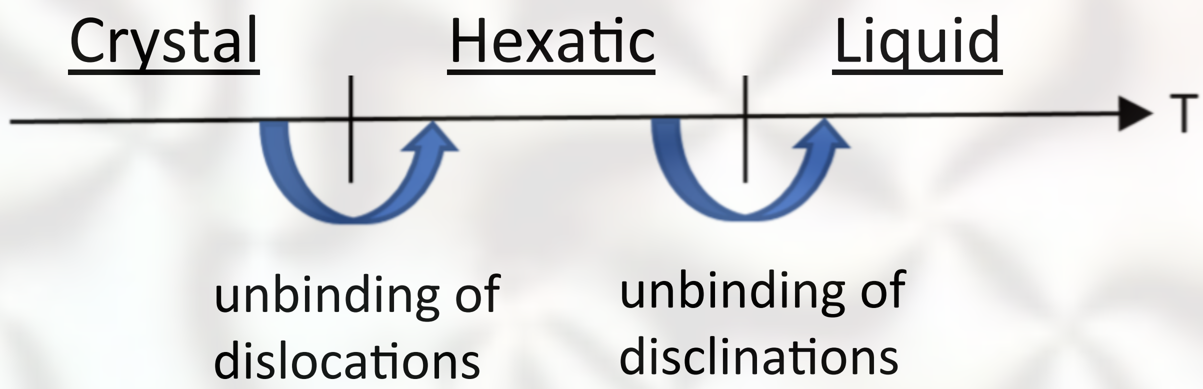
Faraday <-> Newton

Fracton condensaton transition

2D

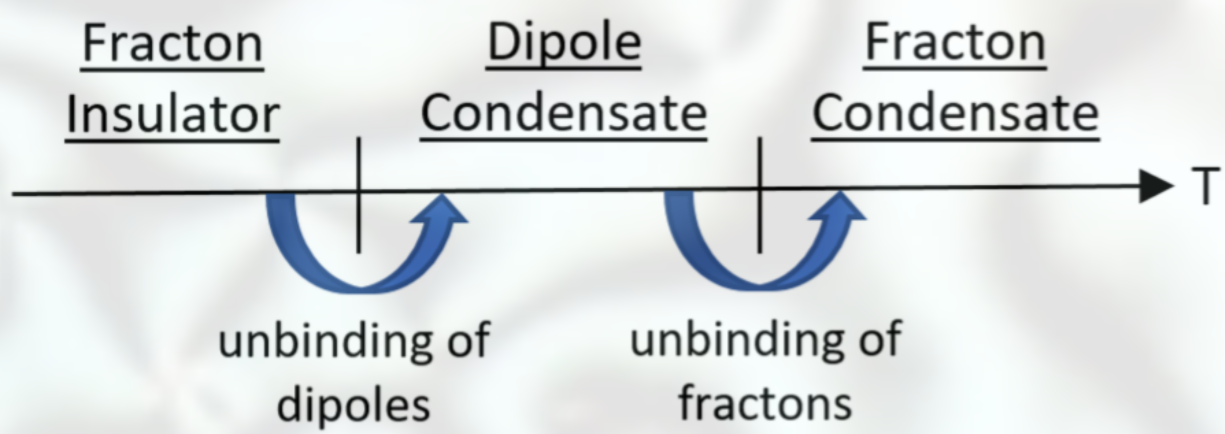
Kosterlitz, Thouless
Halperin, Nelson
Young

2D crystal:



M. Pretko, L. R.,2017

2D scalar fracton model:



$$\tilde{\mathcal{H}} = \frac{1}{2} B^{-1} (\nabla^2 \phi)^2 - g_s \cos\left(\frac{2\pi}{6}\phi\right) - g_b \sum_{n=1,2,3} \cos\left(\mathbf{b}_n \cdot \hat{z} \times \nabla \phi\right)$$

M. Pretko, L. R., 2017

Open questions and conclusions

- *Fractons are realized as topological defects in quantum crystals*

- New fracton phases and transitions, e.g., fracton 'superconductor' ?
- Supersolid – interplay of vortices and lattice defects ?
- Quantum melting ?
- Anisotropic crystal melting *(Halperin, Ostlund)* ?
- Relation to Z_2 lattice models via Higgs'ing *(Ma, Chen, Hermele '18; Bulmash, Barkeshli '18)* ?
- Chiral fracton models ? *(Pretko, et al. '17; Fractional topological elasticity: A. Gromov, arXiv: 1712.06600)*
- 3D generalization ?
- Classification of topological *crystalline* insulators via gauging *(Else, Thorngren)* ?
- …

Thank you